A capacitated Vehicle Routing Problem with pickups, time windows and 3D packing constraints: a mathematical formulation

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1 Introduction

E-commerce becomes widespread and more and more retailers offer an online service. In addition, an unsatisfied customer can usually return the product from large-scale e-commerce platforms, but this is not always easy in local areas as the box collection is scarce. Retailers could combine forces to visit customers and collect their boxes. We investigate how the collection of boxes from these different retailers or customers is organised. While literature about last mile delivery is extended, first mile pickup, which corresponds to the first movement of the goods within the supply chain, has not been well studied yet. However, parcels collection tends to be dynamic and the pickup process, in which the vehicle is initially empty, is much more able to react to disruptions occurring during the day.

In some real-world applications, the dimensions of the boxes and of the vehicles should be taken into account since the packing plan is crucial. Hence, we face a combination of two $\mathcal{NP}$-hard problems, namely, the Capacitated Vehicle Routing Problem and the 3D Loading Problem (3L-CVRP) and even its extension with time windows (3L-CVRPTW). The work presented in [1] is one of the closest works related to delivery aspects. The authors describe the problem with a linear formulation and then solve it using two heuristics while taking into account the Last In First Out policy. Moreover, a formulation for the 3L-CVRPTW from [2] considers heterogeneous vehicles, but does not tackle the stability of the cargo, the potential fragility of the boxes and the multi-drop constraints.

In order to deal with real-life problems, we called on a consulting group to conduct a survey among Belgian transport service providers (SP) using vehicles up to 3.5 tons. Eight respondants' interviews helped to identify the main constraints and issues faced by the SPs and the drivers during pickup trips. For instance, most of the SPs do not know the dimensions of the boxes in advance and so, the packing is left to the driver. These may lead to reloading efforts or sending a new vehicle to meet the request or not collecting the boxes. Moreover, a large majority of the boxes are rectangular, time windows correspond to regular opening hours and the SPs often allow split pickup.

In this work, we develop a mathematical formulation for the 3L-CVRPTW with pickup operations, split pickups and possible outsourcing of some customer's requests to another SP. In order to check the validity of the model and see its computational limitations, our mathematical model was tested on a set of small instances.

2 Mathematical formulation

A SP has to satisfy the requests of N customers by collecting, for each customer i, a set of $\mathcal{I}_i$ rectangular boxes of various dimensions and weight. He has a fleet of F identical vehicles with specific dimensions and maximum weight capacity, based at a single depot (denoted by node 0), in order to pick up the boxes in a single or several visits to the customers (split pickup). He can also call on a subcontractor for transporting some or all of the requests with a penalty cost per outsourced request.

The linear model holds $O(N^2(F + \max_i |\mathcal{I}_i|^2))$ variables and $O(N^2 F \max_i |\mathcal{I}_i|^2)$ constraints and thus cannot be thoroughly detailed here. Therefore, only the main elements are now presented.

The main decisions of the SP are

1. which vehicles will leave the depot and for those he needs to determine a route:
   $\Psi_{ijf} = 1$ if vehicle f travels over arc (i, j), 0 otherwise.
   and the boxes to be loaded at every customer visited:
   $\gamma_{k_i f} = 1$ if box k_i of customer i is transported by vehicle f, 0 otherwise.
2. which (whole) requests to outsource:
   $\rho_i = 1$ if customer i is outsourced, 0 otherwise.

His objective is to minimise his transportation and outsourcing costs, while satisfying several constraints.

Customer satisfaction. Every customer i should have his boxes k_i transported either by a vehicle f of the SP or by a subcontractor.

Routing constraints. A vehicle leaving the depot ends up at this depot. Each vehicle may leave the depot at most once. A customer i is visited by a vehicle f if at least one box of the customer is loaded in the vehicle.

Time constraints. The duration to complete a route does not exceed the maximum driver working duration. Pickup operations must occur within the customer's time-windows. These sets of constraints requires, among other things, a set of variables a_{if} representing the service starting time of the customer i by the vehicle f.

Packing constraints can be split into geometric constraints and specific constraints. In the former, the packing plan has to respect the vehicle maximum weight capacity, boxes should lie entirely in the vehicle, two boxes may not overlap. In the latter, to fit with real-life settings, we (1) ensure that the boxes are vertically stable (box either on the floor or the four basis corners supported by other boxes), (2) respect the forbidden rotations for boxes, and (3) prevent fragile boxes from supporting other boxes. All those constraints require, among other things, a set of variables (x_k, y_k, z_k) representing the position of the front left bottom corner of the box k.
In addition to those constraints, the multi-load constraints are crucial for the integrated problem considered here and state that if a customer is visited, his boxes cannot be packed below those of previous customers in the route while ensuring that each successive packing is feasible, i.e. at every pickup point.

Finally, some valid inequalities are added to the model to speed up the search in the Branch-and-Bound tree. For instance, to remove some symmetric feasible solutions such as the label of the vehicle used: $\sum_{j=1}^{N} \Psi_{0jf} \leq \sum_{j=1}^{N} \Psi_{0j(f-1)}$ for each vehicle f.

3 Results and discussion

To the best of our knowledge, there are no instances available in the literature for this particular problem with customer locations, large time windows and few boxes per customer. Thus, we combined the benchmark instances from Solomon [3] and Bortfeldt and Yi [4] available in literature for the VRP and the packing respectively. We generated 10 instances for 5, 10, 15, 20 customers respectively for small and large time windows.

The linear formulation is implemented in Java using IBM ILOG CPLEX 12.10 library as Brand-and-Bound (B&B) solver. Tests were performed on a personal laptop with a computation time limit of one hour for every instance run.

In Table 1, one can see the proportion of instances solved to optimality on the first row, the mean time for instances solved to optimality and the mean gap for instances for which the optimisation stopped due to the time limit. We note that, with three vehicles ($F = 3$), when the number of customers is 15 or more, each with around two boxes on average to be collected, the time limit is reached with an optimality gap close to 80%. This highlights that instances beyond 15 customers are difficult to solve optimally when considering the proposed model.

Based on these results, we are able to identify the maximum size of the instances that can be solved with the B&B in order to apply a decomposition algorithm. In the literature, many works decompose the problem into the routing and packing separately as in [5]. Our next step is to develop a constructive matheuristic that will decompose the problem, such as the Insert-and-Fix ([6]), but keeping at every step the integration of

			Number of customers (N)			
			5	10	15	20
Small TW	Instances solved at optimality [%]		100.00	90.00	30.00	0.00
	Time [sec.]	Mean (sd.)	0.19 (0.07)	392.05 (1140.90)	533.72 (328.34)	/
	GAP [%]	Mean (sd.)	/	84.94 (0.00)	84.06 (15.75)	96.75 (2.19)
Large TW	Instances solved at optimality [%]		100.00	90.00	10.00	0.00
	Time [sec.]	Mean (sd.)	1.41 (2.51)	685.48 (1099.04)	3461.55 (0.00)	/
	GAP [%]	Mean (sd.)	/	6.47 (0.00)	94.26 (4.55)	97.81 (0.84)

Table 1: Evolution of the computational time and percentage of outsourcing ($F = 3$)

routing and packing aspects, and reducing the size of the problem by reducing the number of customers considered, for instance, only focusing on the closest customers to the depot.

4 Conclusion

As a contribution, we developed a complete mathematical formulation for the vehicle routing problem with pickups, time windows and various practical loading constraints encountered by some service providers. We tested this formulation on small instances and the results enabled us to identify the maximum instance size that the B&B is able to solve. Our next step is to develop constructive matheuristic such as the Insert-and-Fix.

References

[1] A. Moura and J.F. Oliveira, "An integrated approach to the vehicle routing and container loading problems", *OR spectrum* 31(4), 775-800, 2009.

[2] A. Moura, "A model-based heuristic to the vehicle routing and loading problem", *International Transactions in Operational Research*, 26(3), 888-907, 2019.

[3] M. M. Solomon, "Algorithms for the vehicle routing and scheduling problems with time window constraints", *Operations research*, 35(2), 254–265, 1987.

[4] A. Bortfeldt and J. Yi, "The split delivery vehicle routing problem with three-dimensional loading constraints", *European Journal of Operational Research*, 282(2), 545–558, 2020.

[5] M. Iori, J.J. Salazar Gonzalez and D. Vigo, "An exact approach for the vehicle routing problem with two-dimensional loading constraints", *Transportation Science*, 41, 253–264, 2007.

[6] Y. Pochet and L. A. Wolsey, *Production Planning by Mixed Integer Programming.* Springer, 2006.