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From Basic Particle Gradation Parameters to Water Retention Curves of Unsaturated Sandy Soils

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Abstract

It has long been realised that the water retention curve (WRC) of an unsaturated soil is related to its particle size distribution (PSD) as the pore structures are determined by PSD. After a series of studies on the relationship between the WRC and PSD, pedotransfer functions (PTFs) have been proposed in different forms in literature to predict the WRC in drying path. However, most of these PTFs use a number of parameters estimated from the full PSD curve, which makes it somehow complicated for a straightforward application in geotechnical engineering. In this study, a pedotransfer function is raised to formulate the parameters in the van Genuchten's water retention model for sandy soils from a semi-statistical approach with the key features of a monodispersed granular material and an extremely polydisperse material being considered theoretically. Only basic soil gradation parameters of d_{60} (particle size at 60% passing) and the coefficient of uniformity ($C_u = d_{60} / d_{10}$) are employed in this model after a dimensional analysis. Water retention tests on 4 types of glass beads and 4 kinds of uniform sands are carried out in this study to obtain the WRCs of monodispersed granular materials. Data of another 70 sandy soils from the UNSODA database are adopted for the analysis of particle size uniformity effect. Finally, in this model, the air entry value is expressed by d_{60} and the parameter n in van Genuchten's model (which is related to slope of WRC) is formulated as a function of C_u. Validation of this model shows that, although it has fewer and easily obtained parameters comparing to previous PTFs, it has a better fitness on sandy soils.

Keywords: unsaturated soil, pedotransfer function, particle size distribution, water retention curve.

1. Introduction

The water retention curve (WRC) or the soil-water characteristic curve (SWCC) is the relationship between the degree of saturation and suction. It is an important hydraulic property for unsaturated soils which may also affect the soil mechanical behaviours. It has been recognised that there is a connection between the WRC and basic soil properties such as particle size distribution (PSD), soil texture and relative density [1–3]. This relationship is usually referred as pedotransfer functions (PTFs) [4]. The microstructure or fabric is complicated for a clayey soil or soil with a large fraction of organic matters. For a sandy soil, its void ratio variation is relatively small, and its pore size distribution usually has a single peak, which associated to its PSD [5]. Thus, the WRC may be predicted from the PSD which will save the experiment time and cost.

The first approach to predict WRC from PSD is to develop a physico-empirical model, which requires a simplified grain shape and pore space, such as the classic Arya-Paris model [6]. In this model, it is assumed that the PSD is composed of a number of single particle size fractions. The pore size distribution is estimated from the fractions, and then the WRC can be deduced with a scaling factor. Another method to estimate WRC from PSD is a statistical way based on the parameters of closed-form WRC functions. The relationship between the fitting parameters in the closed-form equations and the PSD is investigated based on regression analysis. Early attempts can be seen in [2,3], formulations are proposed by using soil texture fractions (fractions of sand, silt and clay). Later, a measure of mean grain size and a measure of grain size uniformity are found to be the key factors affecting the WRC parameters [7–9].

The basic physical properties are considered in the physico-empirical approach, but it has a set of simplifications on the studied system and cannot avoid using some empirical coefficients. It usually requires a large number of parameters to be measured, which makes it less applicable in engineering practice. In the statistical approach, on the other hand, the physical background is not involved. The conclusion may not be valid beyond the studied database.

In this study, the relationship between the PSD and WRC is explored in a semi-physical and semi-statistical way. The classic van Genuchten's model [10] is adopted to describe the WRC. Dimensional analysis [11] is applied to clarify the key parameters which may determine the WRC. A monosized granular packing and an extremely polydisperse packing are considered to enhance the physical basis. Regression analysis is implemented on the first drying WRCs of 70 soils from the UNSODA database [12] and 8 newly tested granular soils by the authors. A couple of new PTFs are proposed by using basic parameters of d_{60} and C_u. A validation of the PTF is also carried out by comparing the prediction performance with a model in literature.

2. Theoretical analysis of the relationship between WRC and PSD

2.1 The water retention curve in van Genuchten's model

With the increase of soil suction, the degree of saturation starts to decrease when it reaches the air entry value (AEV) and air bubbles are entered the water phase. Then the degree of saturation reduces significantly with suction increase and both water and air phases become continuous (see Fig 1 where large pores start to drain while smaller pores are still filled with water). When the suction value is very high, water phase exists as isolated water bridges and adsorption layers. Change of suction has little further influence on the degree of saturation. This is named as the residual state. The definition of the effective degree of saturation is usually adopted to characterise the water retention behaviour as:

$$S_e = \frac{\theta - \theta_r}{\theta_s - \theta_r} = \frac{S_r - S_r^r}{1 - S_r^r} \tag{1}$$

where θ, θ_s and θ_r are water content, saturated water content and residual water content respectively and S_r and S_r^r are the degree of saturation and the residual degree of saturation. In the classic van Genuchten's model [10], the effective degree of saturation is expressed as:

$$S_e = \left(1 + \left(\frac{s}{\alpha}\right)^n\right)^{\frac{1}{n}-1} \tag{2}$$

where s is soil suction, α is a parameter related to air entry value and n is a parameter related to WRC slope.

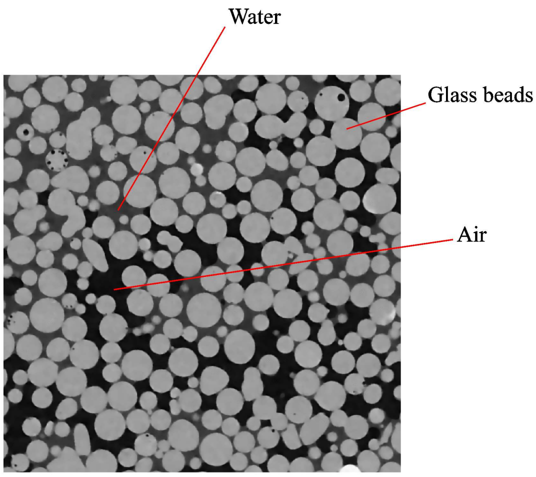


Fig. 1. A cross section of an unsaturated granular material by X-ray tomography (0.2-0.4mm glass beads, S_r=46.7%).

2.2 Dimensional analysis of the relationship

In geotechnical engineering, d_{10}, d_{30} and d_{60} (particle sizes at 10%, 30% and 60% passing by weight) are three important particle sizes in describing soil gradation [13]. With the coefficient of uniformity ($C_u = d_{60}/d_{10}$), the coefficient of curvature ($C_c = d_{30}^2/(d_{60}d_{10})$) and a measure of mean particle size, the shape of PSD can be determined. In this study, for model simplicity d_{60} is used to quantify the mean grain size.

Dimensional analysis, namely Buckingham's Pi-theorem [11], is widely used in analyzing science and engineering problems which can clarify the key factors and simplify the physical relationship. It rewrites a physical relationship as independent and dimensionless groups of variables and reduces a certain degree of freedom in the system. In WRC, the effective degree of saturation (S_e) relies on the PSD, suction and also the water surface tension value (noted as γ). The variation of void ratio for sandy soil is assumed to be small. S_e[-] can then be expressed as a function of variables of C_u[-], C_c[-], d_{60}[L], s[ML^{-1}T^{-2}] and γ[MT^{-2}], where - means dimensionless and L, M and T represent length, mass and time units respectively. By applying Buckingham's Pi theorem, S_e can be simplified as a f' function:

$$S_e = f\left(C_u, C_c, d_{60}, s, \gamma\right) = f'\left(C_u, C_c, \frac{sd_{60}}{\gamma}\right) \tag{3}$$

By analysing the particle size data in the UNSODA database, it is found that C_c is a dependent parameter on C_u, then the relationship can be simplified as:

$$S_e \approx f''\left(C_u, \frac{sd_{60}}{\gamma}\right) \tag{4}$$

The van Genuchten's equation can also be rewritten by normalized suction ($s^* = sd_{60}/\gamma$) and normalized α ($\alpha^* = \alpha d_{60}/\gamma$) as:

$$S_e = \left(1 + \left(\frac{s^*}{\alpha^*}\right)^n\right)^{\frac{1}{n}-1} \tag{5}$$

Combining the above two equations, one can deduce that the parameter n can be a function of the coefficient of uniformity (C_u) and the normalized parameter $\alpha^* = \alpha d_{60}/\gamma$ is either related to C_u or a constant.

2.3 Two extreme conditions

To make the analysis more applicable on data beyond the database, two extreme conditions are considered. One is a monosized granular material, thus the coefficient of uniformity $C_u = 1$. If the particles are assumed to be spheres, pore size in the granular medium will be uniform. This means when suction reaches the air entry value all pores start to drain. The slope of the WRC is nearly infinite ($n \rightarrow \infty$). Another scenario is that C_u is very high. This means that large pores are filled with finer particles and the soil is very difficult to be desaturated. Therefore, the slope of the WRC will be a rather flat shape. The minimum value of parameter n will be reduced to approach 1.

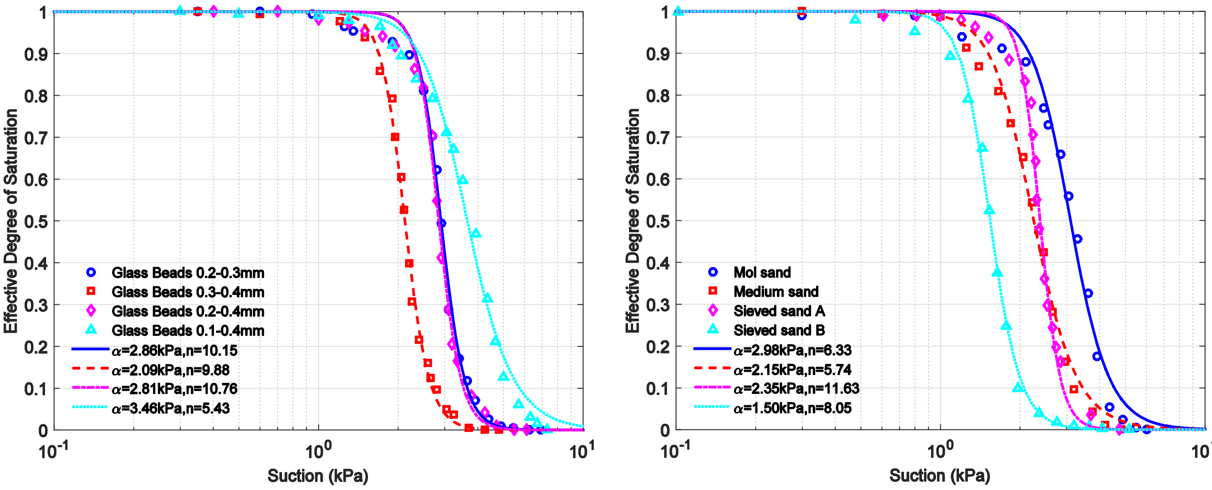
3. Estimating WRC from C_u and d_{60}

3.1 The supplementary water retention tests

Most of the soils in the UNSODA database have relatively high C_u values ($C_u > 2$). The two extreme conditions are not considered. Before the analysis of water retention behaviors of different sandy soils from the database, a set of supplementary water retention tests are carried out on nearly monodisperse granular materials. Four types of rounded glass beads and four kinds of sands with irregular shapes are tested. The PSDs of the four glass beads range from 0.2mm to 0.3mm, 0.3mm to 0.4mm, 0.2mm to 0.4mm and 0.1mm to 0.4mm respectively. C_u values are 1.27, 1.23, 1.43 and 1.74 accordingly. d_{60} values are 0.27mm, 0.36mm, 0.32mm and 0.28mm. Four sands are one silica 'Mol sand', one medium sand from a construction site and another two types of sands (sands A and B) sieved from the medium sand. Particle size of Mol sand is ranging from 0.1mm to 0.5mm with

$C_u = 1.43$ and $d_{60} = 0.28$mm. The medium sand size ranges from 0.1mm to 1.68mm with $C_u = 2.26$ and $d_{60} = 0.52$mm. The particle sizes of sand A and B are sieved from 0.208mm to 0.417mm and from 0.417mm to 0.833mm respectively. The corresponding C_u values are approximated as 1.43 and 1.47 and d_{60} values are about 0.32mm and 0.66mm.

Fig. 2 shows the measured water retention curves of the glass beads and sand in which the points are measured data and the lines are the best-fitted curves by the van Genuchten's model (with least squares errors). It can be seen that the van Genuchten's model gives a good description of the water retention behaviors of granular materials. It can also be observed that the slope of the WRC is closely associated with the slope of the PSD. Higher polydispersity generally makes it harder to desaturate.



(a) (b)

Fig. 2. Water retention curve of relatively uniform sands and glass beads. (a) WRC of glass beads; (b) WRC of sands.

3.2 Analysis of the experimental results and the database and the estimation equation

The UNSODA database comprises 780 soils. Soils used for the analysis are selected from the database. For the purpose of analysing the data from a more efficient and reliable way, two criteria are considered in the soil selection. One is that it has a clear particle distribution curve which covers the range of d_{10} to d_{60} (this gives the d_{60} and C_u values). Another aspect is that the soil should have a water retention curve of the first drying with the residual state reached at high suction in the data. It is considered to be within the residual state if the slope of the three data points at the highest suction in the WRC is less than 0.15 in the semi-logarithm plot and the final degree of saturation is also less than 20%. Finally, 70 soils are obtained from the database. The water retention curves of these soils are also fitted by the van Genuchten's equation with the least squares method and the parameters of α and n are obtained.

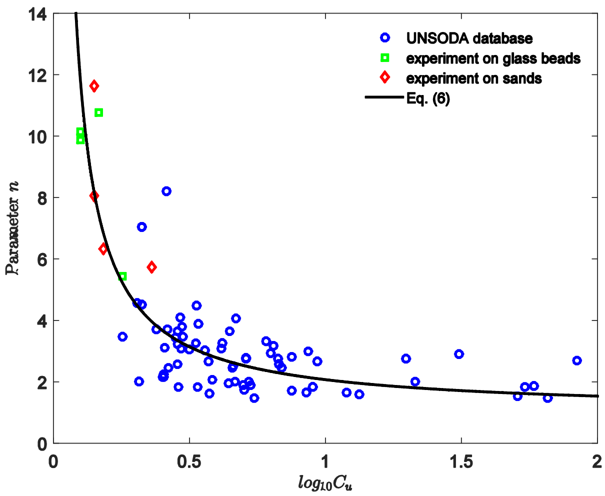


Fig. 3. The relationship between grain size uniformity and parameter n.

In Fig. 3, the best fitted n parameter of the 8 tested granular materials and the 70 selected sandy soils from the UNSODA database are plotted against $\log_{10}(C_u)$. It can be seen that when the material is nearly monosized, C_u approaches to 1 and the parameter n tends to be infinite. When the grain size is highly dispersed (C_u is a very high value), the parameter n decreases to about 1. The equation can be adopted to represent the n and $\log_{10}(C_u)$ relationship:

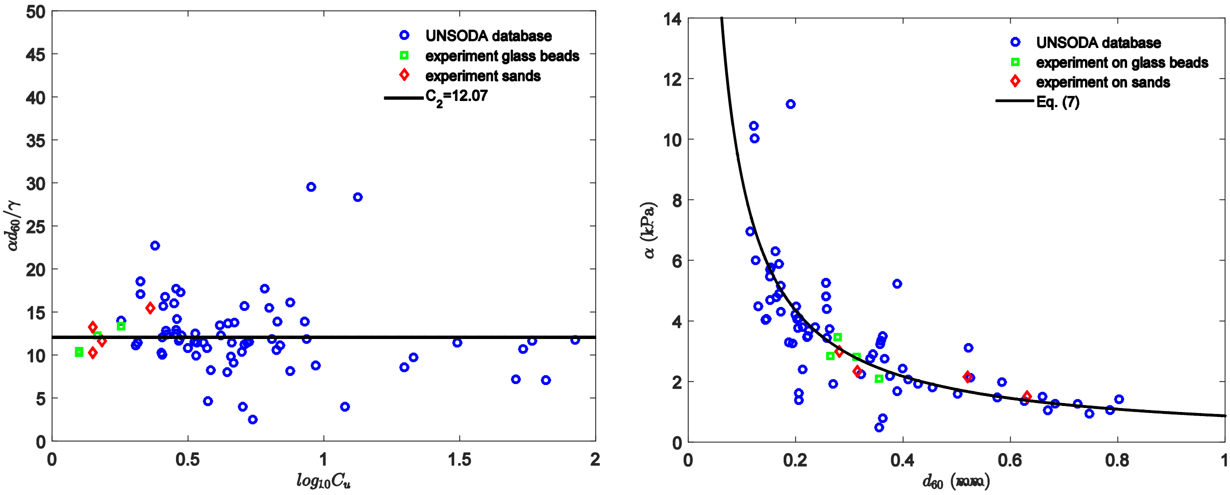
$$n = \frac{C_1}{\log_{10}(C_u)} + 1 \tag{6}$$

where C_1 is a fitting parameter by the least squares method that $C_1 \approx 1.07$.

In the dimensional analysis, the normalized parameter $\alpha^* = \alpha d_{60}/\gamma$ is believed to be either related to C_u or to be a constant. Fig. 4a shows that the normalized α is not obviously influenced by C_u. Therefore, it is assumed that $\alpha d_{60}/\gamma$ is approximately a constant C_2. This means α is inversely proportional to the mean particle size as other authors already introduced [9]. The parameter α can be approximated from d_{60} as:

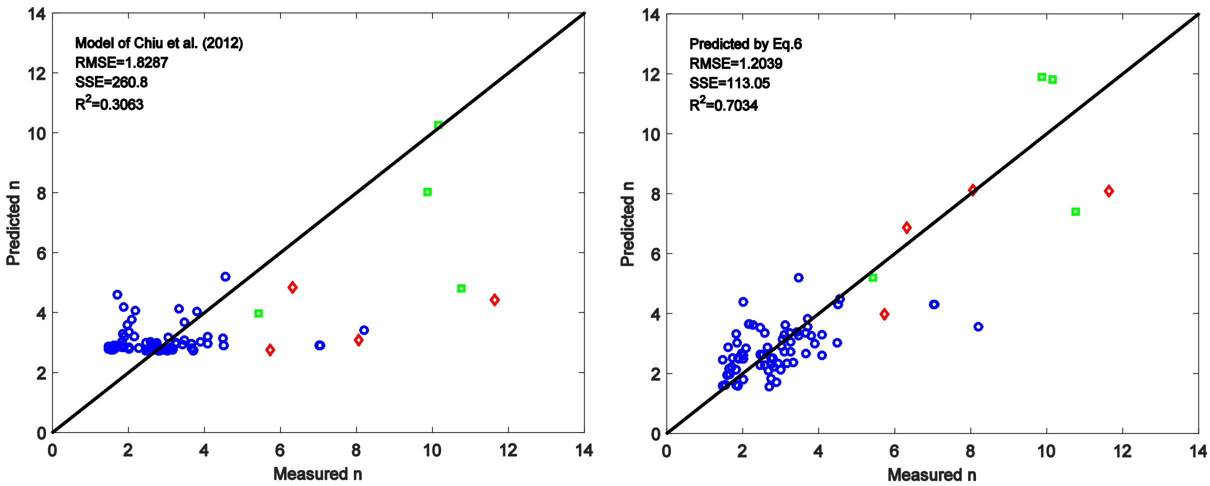
$$\alpha = \frac{C_2 \gamma}{d_{60}} \tag{7}$$

Fig. 4b demonstrates the correlation between α and d_{60}. C_2 is fitted to be 12.07 by the 78 materials.



(a) (b)

Fig. 4. (a) Relationship between normalized α and C_u; (b) Relationship between α and d_{60}.



(a) (b)

Fig. 5. The performance of prediction in parameter n. (a) Prediction by the model in [9]; (b) Prediction by Eq. (6).

3.3 Validation of the equations

Validation of the model can be carried out by comparing the estimated n and α values with the measured ones on the 78 materials. Our predictions are compared with the predictions of the PTFs in Chiu's model [9]. The fitting parameters are both from the 78 materials by least-squares method. Statistical parameters of the Root-Mean-Square Error (RMSE), the Sum of Square Errors (SSE) and the coefficient of determination (R^2) of the different predictions are calculated for comparison. We find that the prediction performance of α is almost the same (in Chiu's model α is estimated from d_{50}). For the prediction of parameter n, it can be observed from Fig. 5 that the expression of Eq. (6) is better, as it has lower errors and a higher coefficient of determination. The prediction of n is especially improved for the materials we tested with relatively narrow grain size distribution.

4. Conclusions

Basic soil gradation parameters of d_{60} and C_u are employed to estimate the WRCs of unsaturated granular soils. Two extreme scenarios are considered to quantify the monosized material and the highly polydisperse sand, which enhances physical features to the statistical approach. Water retention tests are carried out on 8 nearly monodispersed materials. New PTFs are proposed to estimate the WRC by applying dimensional analysis and regression analysis on the 8 tested granular materials and 70 soils from a database. The parameter α, related to inversed air entry value, is inversely proportional to d_{60}. The parameter n, proportional to the WRC slope, is inversely proportional to logarithmic C_u. Validation shows that the new PTFs improve the estimation especially in prediction of parameter n.

Acknowledgements

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