A TWO-SCALE DAMAGE LAW FOR CREEPING ROCKS

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Summary. The theoretical developments and the numerical applications of a time-dependent damage law are presented. This law is deduced from considerations at the micro-scale where non-planar growth of micro-cracks, following a subcritical propagation criterion, is assumed. The passage from micro-scale to macro-scale is done through an asymptotic homogenization approach. Results of numerical simulations of time-dependent damage behavior are presented.

Keywords: crack rotation, homogenization, subcritical propagation, time-dependent damage.

1 INTRODUCTION

The evolution of damage in rocks is often time-dependent. At the micro-scale, such materials contain micro-cracks that evolve on non-planar paths in a rate-dependent manner, and this complex propagation strongly affects the macroscopic mechanical behavior. We assume that the micro-crack distribution may be locally approximated by a periodic one that is characterized by a micro-structural length — the distance between two adjacent micro-cracks. The source of time-dependency is the subcritical propagation of micro-cracks. This paper combines concepts of asymptotic homogenization [1,2] and subcritical and mixed crack propagation to deduce a time-dependent damage model.

2 MATHEMATICAL FORMULATION

Consider a two-dimensional isotropic elastic medium containing a locally periodic distribution of micro-cracks. Each crack is straight with a length $2a$ and an orientation of angle θ with respect to the x_1 direction (abscissa of the referential system considered at the macro-scale). The damage variables are d - the ratio between the crack length $2a$ and the distance between two micro-cracks ε that also represents the size of the periodicity cell (Fig. 1) :

$$d = \frac{2a}{\varepsilon} \tag{1}$$

and the angle θ. On the crack faces, traction free opening or frictionless contact conditions are assumed.

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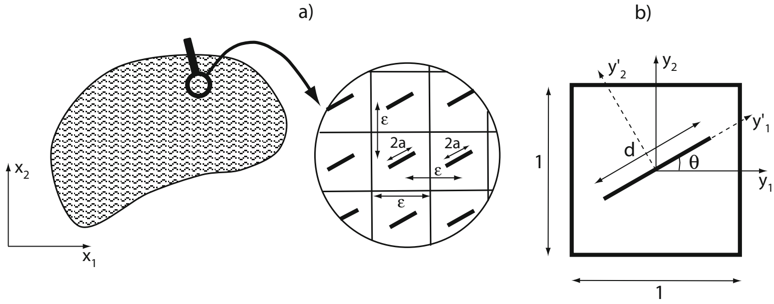

Fig. 1. (a) Fissured medium with locally periodic microstructure. (b) Unit cell with rescaled crack of length d.

2.1 Asymptotic Homogenization

The locally periodic microstructure is constructed from a unit cell $Y = [-0.5, 0.5] \times [-0.5, 0.5]$ expressed in a (y_1, y_2) orthogonal axis system centered in the middle of the crack, corresponding to the center of the cell. Then this unit cell is rescaled by the parameter ε so that the period of the material is εY. The two distinct scales are represented by the variable $\mathbf{x}$ (the macroscopic variable) and $\mathbf{y} = \mathbf{x}/\varepsilon$ (the microscopic variable). After asymptotic homogenization, we obtain the following relationships for the overall response of the material (e.g. [2,3]):

$$\frac{\partial}{\partial x_j}\Sigma_{ij}^{(0)} = 0 \;\; ; \;\; \Sigma_{ij}^{(0)} = C_{ijkl}e_{xkl}(\mathbf{u}^{(0)}) \;\; ; \;\; C_{ijkl}^{\pm}(d,\theta) = \frac{1}{|Y|}\int_{Y_s}(a_{ijkl} + a_{ijmn}e_{ymn}(\xi_{\pm}^{kl}))\,dy \tag{2}$$

where $\Sigma_{ij}^{(0)}$ is the macroscopic stress and C_{ijkl} the homogenized coefficients, respectively. Eq. (2) is the homogenized equation of equilibrium. $\xi_{\pm}^{pq}$ are elementary solutions of the microscopic correction $\mathbf{u}^{(1)}$ for particular $e_{xpq}(\mathbf{u}^{(0)}) = \delta_{pq}$. The distinction $\pm$ corresponds to opening (+) or contact (-) conditions of the crack lips [2]. The homogenized coefficients C_{ijkl} depend on the state of damage of the material (d and θ) and on the mechanical properties of the solid matrix (E and ν). From the integral (2), the coefficients can be initially computed for a large number of d and θ for both opening and closure states. After interpolation, polynomial expressions of $C_{ijkl}(d,\theta)$ are obtained [2,3].

2.2 Subcritical Growth of Micro-crack

The evolution of the micro-crack length is described through a subcritical criterion adapted from the Charles' law:

$$\frac{dl}{dt} = v_0\left(\frac{K_I^{\star}}{K_0}\right)^n \tag{3}$$

where K_0, v_0 and n are material parameters. $K_I^{\star}$ is the stress intensity factor for the tensile mode of rupture (Mode I) of the kinked crack (Fig. 2a). The determination of the stress intensity factors at the crack tips is made through the computation of path-independent J-, L- and M- integrals [4] for straight trajectory of micro-cracks and from

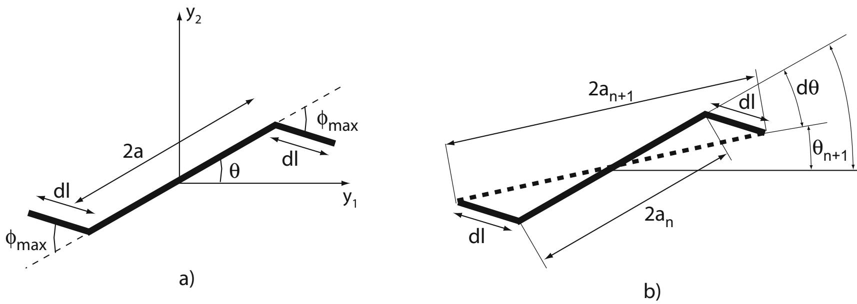

Fig. 2. Kinked crack. (a) The out-of-plane crack growth propagates in the direction that maximize the energy release rate. (b) The kinked crack (solid line) and its equivalent replacement crack (dashed line).

the polynoms given by Leblond [5] for the kinked cracks. The crack extension is assumed to propagate in the direction that maximizes the energy release rate, with a kinking angle (Fig. 2a) [6]:

$$\phi_{max} = \text{sgn}(K_{II})[0.70966\lambda^3 - 0.097725\text{sin}^2(3.9174\lambda) - 13.1588\text{tanh}(0.15199\lambda)] \; ; \; \lambda = \frac{|K_{II}|}{K_I + |K_{II}|} \tag{4}$$

At each time increment, the kinked crack is replaced by an equivalent straight crack deduced by joining the tips of the real branched crack (Fig. 2b). We obtain in this way the evolution of damage parameters d and θ in the form of differential equations [4]:

$$\frac{dd}{dt} = \frac{2}{\varepsilon}\cos(\phi_{max})\frac{dl}{dt} \quad ; \quad \frac{d\theta}{dt} = \frac{2}{\varepsilon d}\sin(\phi_{max})\frac{dl}{dt} \tag{5}$$

3 NUMERICAL EXAMPLES

All the simulations presented in the following have been made considering biaxial loading. Plane-strain condition is considered in the third direction. Figure 3 illustrates the response of a material submitted to an uniaxial tension loading at constant vertical strain rate. Under a low strain rate, the effect of time becomes predominant and the

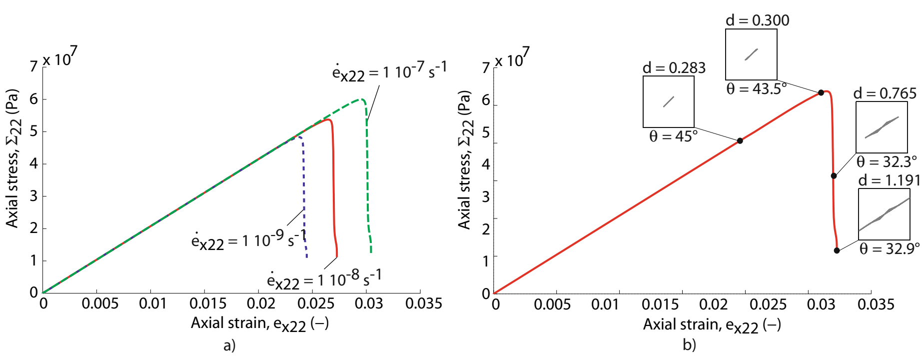

Fig. 3. Axial tension tests at various constant strain rate $\dot{e}_{x22}$. (a) The strength increases when the strain rate increases. (b) Evolution of the micro-crack in the periodic cell for $\dot{e}_{x22} = 1.10^{-8}\text{s}^{-1}$.

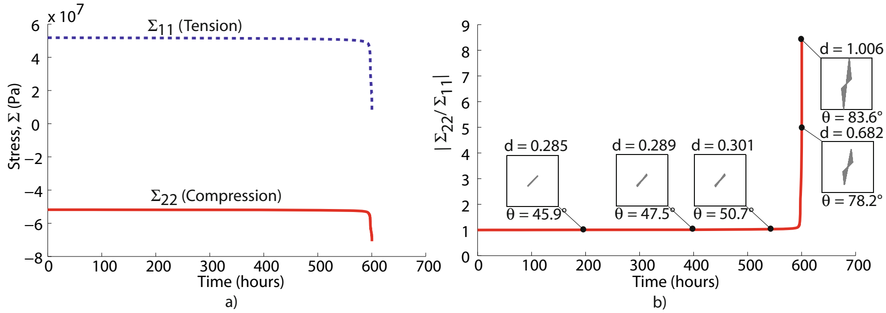

Fig. 4. Relaxation test under biaxial conditions. $e_{x22} = -0.035$ (compression) and $e_{x11} = 0.035$ (tension). Evolution with time of (a) the horizontal and vertical stresses and of (b) the ratio of anisotropy.

failure appears for a lower strain level than in the case of faster loading (Figure 3a). As the damage increases, a kinked angle forms and the equivalent crack rotates, tending to be perpendicular to the principal tensile strain (Figure 3b). Figure 4a shows the evolution of horizontal and vertical stresses with time. During relaxation tests, under a biaxial combined tensile/compressive constant strain field, the subcritical micro-crack growth produces a gradual stress relaxation upon failure. As long as the crack propagates, the direction of the equivalent crack tends toward a vertical direction inducing an anisotropic response of the material (Figure 4b).

4 CONCLUSIONS

The subcritical growth of micro-cracks is responsible for the time-dependent behavior of many creeping rocks. A subcritical criterion together with a crack rotation model has been considered at the micro-scale. Upscaled equations have been obtained by an adapted asymptotic homogenization procedure. A time-dependent macroscopic damage model has been deduced. Numerical simulations in a macroscopic point of loading at constant strain rate and relaxation tests have shown the ability of the developed model to reproduce the time-dependent damage response.

REFERENCES

[1] Dascalu, C.: A two-scale damage model with material length. C.R. Mécanique 337, 645–652 (2009)
[2] Dascalu, C., Bilbie, G., Agiasofitou, E.: Damage and size effect in elastic solids: A homogenization approach. Int. J. Solid Struct. 45, 409–430 (2008)
[3] Dascalu, C., François, B., Keita, O.: A two-scale model for subcritical damage propagation. Int. J. Solid Struct. 47, 493–502 (2010)
[4] François, B., Dascalu, C.: A two-scale time-dependent damage model based on non-planar growth of micro-cracks. J. Mech. Phys. Solids 58, 1928–1946 (2010)
[5] Leblond, J.B.: Crack paths in three-dimensional elastic solids. I: two-term expansion of the stress intensity factors - application to crack path stability in hydraulic fracturing. Int. J. Solids Struct. 36, 79–103 (1999)
[6] Schütte, H., Bruhns, O.T.: On a geometrically nonlinear damage model based on a multiplicative decomposition of the deformation gradient and the propagation of microcracks. J. Mech. Phys. Solids 50, 827–853 (2002)