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A two-scale damage model for dynamic rock behavior

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ABSTRACT: Modeling the dynamic damage response of rocks requires taking into account the mechanics of micro-cracks and their overall response to applied loading. This paper presents a new micro-mechanical damage model accounting for inertial effects. The two-scale damage model is fully deduced from small-scale descriptions of dynamic micro-crack propagation, without macroscopic assumptions. The passage from the micro-scale to the macro-scale is done through an asymptotic homogenization approach. An appropriate micromechanical energy analysis is proposed leading to a dynamic damage evolution law. Numerical simulations are presented in order to illustrate the ability of the model to describe known behaviors upon high strain rate, like size dependency of the structural response or strain-rate sensitivity.

1 INTRODUCTION

The failure behavior of quasi-brittle materials such as rocks is known to be sensitive to internal fractures, which are commonly characterized as damage. When rocks are subjected to high and rapid changes of stresses (due to dynamic loading as earthquakes or underground explosions), the damage process is governed by dynamic behaviour of fractures. One of the ways to address this issue is the analysis of micro-structural damage process. The macroscopic damage evolution is due to micro-crack propagation. At that scale, the damage mechanism occurs in mode I (tensile), even if the macroscopic loading is in compression (Brace and Bombalakis 1963).

In the present paper, we propose a new approach for dynamic damage propagation. Considering the micro-crack dynamic propagation, the dynamic model of damage is deduced through the mathematical homogenization method based on asymptotic developments and locally periodic distribution of micro-cracks (Sanchez-Palencia 1980) including the effect of the size of the microstructure and the loading rate on the behavior of the materials.

A two-scale approach for damage was deduced in Dascalu et al. (2008) for brittle damage. More general formulations of this model, including non-brittle behaviors or more complex crack evolutions, were given in Dascalu (2009), François and Dascalu (2010) and Dascalu et al. (2010). The present development extends these results to the case of dynamic evolution of damage and inertial effects.

2 TWO-SCALE PROBLEM

In this paper, we consider the dynamic evolution of the elastic solid containing a large number of micro-cracks. We suppose that the micro-crack distribution is locally periodic (see Fig. 1a), with ε the size of a periodicity cell or, equivalently, the distance between the centers of neighbouring micro-cracks and by l the micro-crack length. The length l is assumed to have small spatial variations such that, locally, the distribution of micro-cracks may be considered as

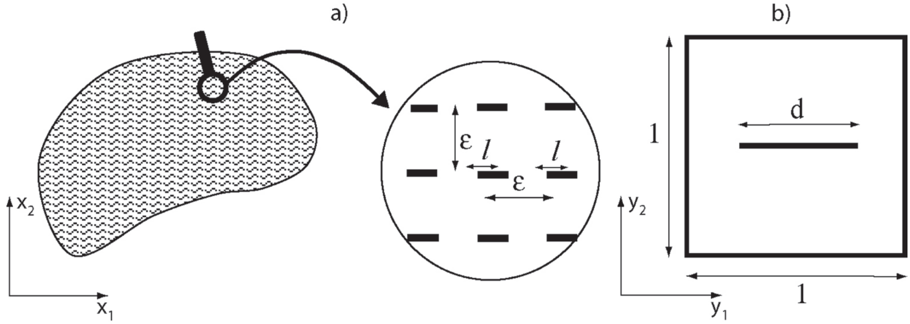

Figure 1. (a) Micro-fissured medium with locally periodic microstructure, ε is the size of a period and l is the local micro-crack length. (b) Unit cell with rescaled crack of length d.

periodic. Each crack is assumed to be horizontal (parallel to the x_1-axis) and straight. We define the damage variable, included between 0 (for undamaged material) and 1 (for complete damaged material), as the ratio between the micro-crack length l and the period size ε:

$$d = \frac{l}{\varepsilon} \tag{1}$$

2.1 *Elastodynamics equations*

We consider the elastodynamics equations for the initial heterogeneous medium that we assume to be a two-dimensional isotropic elastic medium containing a locally periodic array of micro-cracks. Let $B_S = B/C$ be the solid part of B, with C the union of all micro-cracks inside B. The momentum balance equation is:

$$\frac{\partial \sigma_{ij}^{\varepsilon}}{\partial x_j} = \rho \frac{\partial^2 u_i^{\varepsilon}}{\partial t^2} \quad \text{in } B_S \tag{2}$$

including the linear elasticity constitutive relation:

$$\sigma_{ij}^{\varepsilon} = a_{ijkl} e_{xkl}\left(u^{\varepsilon}\right) \tag{3}$$

where a_{ijkl} is the elasticity tensor, $\sigma_{ij}^{\varepsilon}$ is the stress field and u^{ε} the displacement field from which the strain tensor is calculated in the small deformations hypothesis:

$$e_{xij}\left(u^{\varepsilon}\right) = \frac{1}{2}\left(\frac{\partial u_i^{\varepsilon}}{x_j} + \frac{\partial u_j^{\varepsilon}}{x_i}\right) \tag{4}$$

Traction free conditions are assumed on the crack faces:

$$\sigma^{\varepsilon} N = 0 \tag{5}$$

where N is a unit normal vector on the crack faces.

2.2 *Asymptotic developments*

The locally periodic microstructure is constructed from a reference unit cell Y (Fig. 1b) referred to microscopic coordinates (y_1, y_2). Rescaled with the small parameter ε, the unit cell

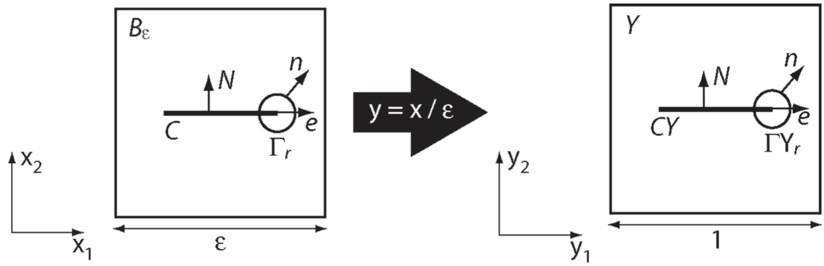

Figure 2. Rescaling of the unit cell to the microstructural period of the material.

becomes the physical period of the material εY as in Figure 2. We assume the microstructural period ε to be small enough with respect to the characteristic dimensions of the whole body and the wavelength of the elasto-dynamic field. We consider distinct variables at different scales: the macroscopic variable x and the microscopic variable $y = x/\varepsilon$. For a variable depending on both x and y the total spatial derivative takes the form

$$\frac{d}{dx_i} = \frac{\partial}{\partial x_i} + \frac{1}{\varepsilon}\frac{\partial}{\partial y_i} \tag{6}$$

The unit cell Y contains the scaled crack CY and its solid part $Y_S = Y/CY$. Following the method of asymptotic homogenization (e.g. Sanchez-Palencia 1980), we look for two-scale expansions of u^ε and σ^ε in the form:

$$u^{\varepsilon}(x,t) = u^{(0)}(x,y,t) + \varepsilon u^{(1)}(x,y,t) + \varepsilon^2 u^{(2)}(x,y,t) + \cdots \tag{7}$$

$$\sigma^{\varepsilon}(x,t) = \frac{1}{\varepsilon}\sigma^{(-1)}(x,y,t) + \sigma^{(0)}(x,y,t) + \varepsilon\sigma^{(1)}(x,y,t) + \cdots \tag{8}$$

where = $u^{(i)}(x,y,t)$ and $\sigma^{(i)}(x,y,t)$, $x \in B$, $y \in Y$ are smooth functions an Y—periodic in y.

2.3 *Homogenization analysis*

Substituting the asymptotic development of u^ε and σ^ε (Eq. 7 and 8) in the elasto-dynamic equations (2, 3, 4) and taking into account the relation (6), one can obtain the followings expressions

$$\frac{\partial}{\partial x_j}\sigma_{ij}^{(I)} + \frac{\partial}{\partial y_j}\sigma_{ij}^{(I+1)} = \rho\frac{\partial^2 u_i^{(I)}}{\partial t^2} \tag{9}$$

$$\sigma_{ij}^{(I)} = a_{ijkl}\left(e_{xkl}\left(u^{(I)}\right) + e_{ykl}\left(u^{(I+1} \right.\right. \tag{10}$$

that have to be solved for each I.

(i) Eq (9) for I = −1 and Eq (10) for I = 0 give:

$$\frac{\partial}{\partial x_j}\sigma_{ij}^{(0)} = 0 \tag{11}$$

$$\sigma_{ij}^{(0)} = a_{ijkl}\left(e_{xkl}\left(u^{(0)}\right) + e_{ykl}\left(u^{(1)}\right)\right) \tag{12}$$

For given $u^{(0)}$, taking into account Eqs (11–12) one obtains the boundary value problem for the corrector $u^{(1)}$:

$$\frac{\partial}{\partial y_j}\left(a_{ijkl}e_{ykl}\left(u^{(1)}\right)\right)=0, \quad \text{in} \quad Y_S \tag{13}$$

$$\left(a_{ijkl}e_{ykl}\left(u^{(1)}\right)\right)N_j=-\left(a_{ijkl}e_{xkl}\left(u^{(0)}\right)\right)N_j, \quad \text{on} \quad CY \tag{14}$$

with periodicity conditions on the external boundary of the cell. The microscopic corrector $u^{(1)}$ depends linearly on the macroscopic deformations

$$u^{(1)}(x,y,t)=\xi^{pq}(y)e_{xpq}\left(u^{(0)}\right)(x,t) \tag{15}$$

Here the characteristic functions $\xi^{pq}(y)$ are elementary solutions of Eqs (13–14) for the particular macroscopic deformation $e_{xkl}(u^{(0)})$. The equilibrium equation (11) shows that inertial effects are not directly present at the microscopic level.

(ii) Eq (9) for $I=0$ and Eq (10) for $I=1$ give

$$\frac{\partial}{\partial x_j}\sigma_{ij}^{(0)}+\frac{\partial}{\partial y_j}\sigma_{ij}^{(1)}=\rho\frac{\partial^2 u_i^{(0)}}{\partial t^2} \tag{16}$$

$$\sigma_{ij}^{(1)}=a_{ijkl}\left(e_{xkl}\left(u^{(1)}\right)+e_{ykl}\left(u^{(2)}\right)\right) \tag{17}$$

By applying the mean value operator $\langle\cdot\rangle=1/|Y|\int_{Y_S}\cdot\,dy$ (where $|Y|$ is the area of Y) to Eq (16) and remembering that $u^{(0)}$ is y-independent, one can obtain:

$$\frac{\partial}{\partial x_j}\left\langle\sigma_{ij}^{(0)}\right\rangle=\langle\rho\rangle\frac{\partial^2 u_i^{(0)}}{\partial t^2} \tag{18}$$

as describing the overall dynamic behavior of an elastic body with a given distribution of (non-evolving) micro-cracks. We define the macroscopic stress

$$\left\langle\sigma_{ij}^{(0)}\right\rangle=C_{ijkl}e_{xkl}\left(u^{(0)}\right) \tag{19}$$

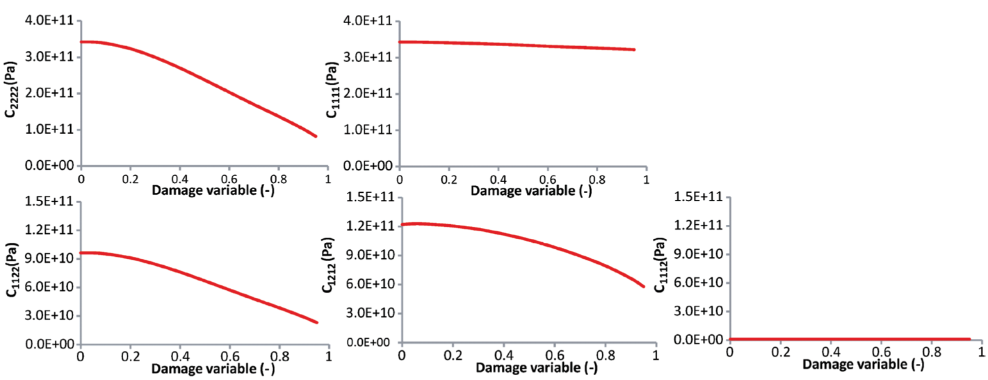

Figure 3. Homogenized coefficients for horizontal crack orientation for elastic parameters E = 2 GPa and ν = 0.3.

where

$$C_{ijkl}(d) = \frac{1}{|Y|}\int_{Ys} \left(a_{ijkl} + a_{ijmn}e_{ymn}\left(\xi^{kl}\right)\right)dy \tag{20}$$

are the homogenized coefficients.

These coefficients C_{ijkl} can be computed by solving the unit cell problems (13) and (14) for several d and interpolating between the obtained values (Fig. 3). Those unit cell problems have been computed by the FEAP finite element code.

3 ENERGY RELEASE RATES AND DYNAMIC DAMAGE EVOLUTION

We consider in this section the problem of evolving micro-cracks and we extend our homogenization method in order to obtain macroscopic evolution laws for damage.

The dynamic energy release rate can be expressed at the tip of a micro-crack as follows

$$G^{d\varepsilon} = \lim_{r=0}\int_{\Gamma Yr} \left((U+T)n_1 - \sigma_{ij}^{\varepsilon}\frac{\partial u_i^{\varepsilon}}{dx_1}\right)ds \tag{21}$$

where

$$U = \frac{1}{2}a_{mnkl}e_{xkl}\left(u^{\varepsilon}\right)a_{mnkl}e_{xmn}\left(u^{\varepsilon}\right) \tag{22}$$

$$T = \frac{1}{2}\rho\frac{\partial u^{\varepsilon}}{\partial t}\frac{\partial u^{\varepsilon}}{\partial t} \tag{23}$$

are respectively the energy of deformation and the density of kinetic energy (Freund 1998).

Here Γ_r is a circle of an infinitesimal radius r surrounding the crack tip and n is the unit normal vector on Γ_r (see Fig. 2).

The physical energy release rate in dynamics is expressed as function of the stress intensity factors as follows (Freund 1998):

$$G^{d\varepsilon} = \frac{1-v^2}{E}A(\dot{l})k(\dot{l})^2\left(K_I^{\varepsilon}\right)^2 \tag{24}$$

For crack propagation in mode I, Freund (1998) established the relation

$$A(\dot{l})k(\dot{l})^2 \approx \left(1 - \frac{\dot{l}}{C_R}\right) \quad \text{with} \quad C_R = \frac{0.862+1.14v}{1+v}\sqrt{\frac{E}{2(1+v)\rho}} \tag{25}$$

where C_R is the Rayleigh wave speed.

Including the asymptotic development (7, 8) into relation (21), and comparing the obtained results with the physical energy release rate in dynamics (24), we establish the dynamic damage evolution law (see Keita et al. (2013) for more details):

$$\frac{dd}{dt} = \frac{2C_R}{\varepsilon}\left(\frac{G^c}{\varepsilon^*\frac{dC_{ijkl}}{dd}e_{xkl}\left(u^{(0)}\right)e_{xij}\left(u^{(0)}\right)} + \frac{1}{2}\right) \tag{26}$$

4 LOCAL MACROSCOPIC RESPONSE

4.1 *Parametric study*

For each step of calculation, the problem is strain controlled and the corresponding stress is obtained by combination of Equations (26), (20) and (19). The response of the model was simulated under a horizontal tension rate of 1.10^{-3} s^{-1} while the vertical deformation is constrained. The parameters used in the simulations are reported in Table 1. Figure 4 illustrates the influence of the microstructure size on the material behaviour. As long as the microstructural size ε decreases, the material resistance increases. Figure 5 exhibits the ductility induced when the rate of straining $\dot{e}$ increases. In dynamic conditions, under high load rate, the inertia effect becomes more important and crack propagation is slower than the loading rate which makes the behavior more ductile.

4.2 *Model validation*

To validate our model efficiency we compare the model strain-rate sensitivity of tensile strength curve of AL23 alumina to the strain-rate sensitivity of tensile strength curve form GEPI test (Erzar and Buzaud 2012). For the microstructural size of $\varepsilon = 3.10^{-3}$ m and the initial damage value $d_0 = 0.3$ (the other parameters being similar to Table 3), our dynamic damage model gives a good agreement with the experimental results (Fig. 6) showing an increase of strength along with a faster loading.

Table 1. Material parameters and loading conditions used in the simulations.

Materials parameters							Loading conditions	
E [Pa]	ν [–]	ε [m]	G^c [jm^{-2}]	C_R [m/s]	ρ [kg/m³]	d_0 [–]	$\dot{e}_{11}$ [s^{-1}]	$\dot{e}_{22}$ [s^{-1}]
$2.\ 10^{11}$	0.3	1.10^{-3}	164	486	3800	0.2	0	1.10^{3}

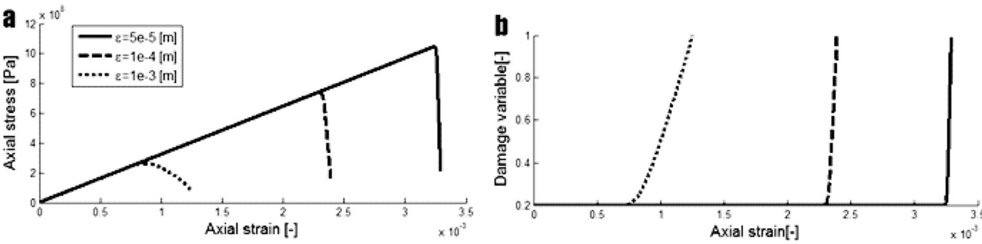

Figure 4. Effect of the internal length ε on numerical results of a horizontal tension test at constant strain rate. Evolution of (a) Stress (b) Damage variable with strain.

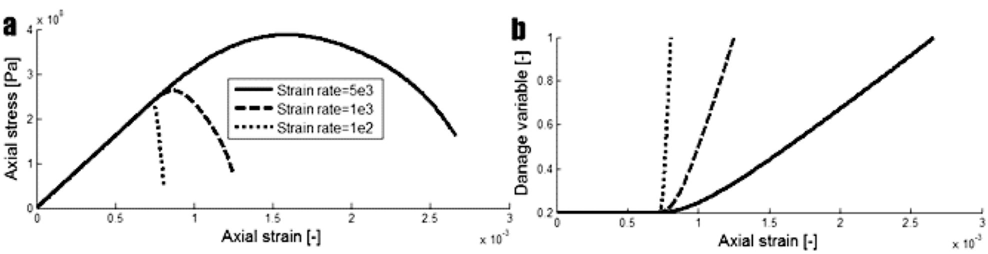

Figure 5. Effect of the strain rate of loading $\dot{e}$ on the behavior of materials subject to a horizontal tension test at constant strain rate. Evolution of (a) Stress (b) Damage variable with strain.

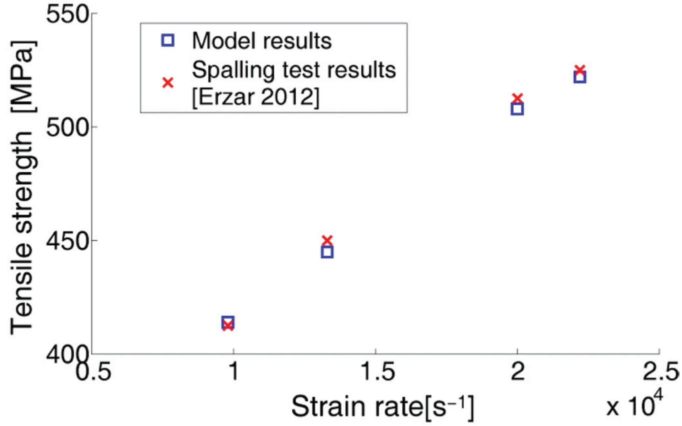

Figure 6. Strain-rate sensitivity of tensile strength: comparison of model results with spalling test results.

5 CONCLUSION

The dynamic micro-crack propagation criterion has been applied at the micro-scale and up-scaled by the asymptotic homogenization procedure. Dynamic macroscopic damage model has been deduced and the stress-strain response of materials, depending on time, has been obtained in a macroscopic point. The dynamic damage law contains a microstructural length allowing for the prediction of size effects, and Rayleigh wave speed parameter that measures the speed of dynamic crack growth. Efficiency of the model is demonstrated by comparing the strain-rate sensitivity of tensile strength, obtained experimentally from spalling test with the prediction of the model.

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