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Efficient Estimation of the Skewness of a Linear Oscillator Subjected to a Non-Normal and Non-Polynomial Wind Loading

Margaux Geuzaine[1)*], Michele Esposito Marzino[2)], Vincent Denoël[3)]

1) PhD Candidate, University of Liège & F.R.S.-FNRS, Belgium, mgeuzaine@uliege.be
2) PhD Candidate, University of Liège, Belgium, michele.espositomarzino@uliege.be
3) Associate Professor, University of Liège, Belgium, v.denoel@uliege.be

ABSTRACT: A fast spectral analysis of a linear oscillator subjected to a non-normal and non-polynomial wind loading is presented in this paper. It is based on the Multiple Timescale Spectral Analysis, which generalizes the Background/Resonant decomposition and offers therefore a rapid yet accurate estimation of the statistics of the structural responses at any order. In particular, simple expressions are derived in this paper for the third central moments of the structural responses under such a loading whose bispectrum is actually complex. These statistics are eventually obtained 100 times faster than through the numerical integration of the response bispectrum.

Keywords: multiple timescale spectral analysis, skewness, bispectrum, complex-valued

1. INTRODUCTION

In a spectral context, the second and third cumulants of a given real-valued process are typically obtained by integrating the real parts of its power spectral density and its bispectrum over a one- and a two-dimensional frequency space, respectively. Meanwhile, due to symmetry, the imaginary parts do not contribute to the cumulants, which are hence real, as expected. When dealing with the response of a slightly damped oscillator whose natural frequency is much higher than the characteristic frequency of the loading, these spectra are however expected to feature sharp peaks and their numerical integration thus requires using a lot of points to provide accurate results.

From a perturbation perspective, the sharpness and the distinctness of the peaks in the functions to integrate can fortunately be turned into an advantage (Hinch 1995). The contributions of such separated peaks to the integral can be considered separately and be expressed by easily interpretable semi-analytical formulas. Regarding the variance, it yielded the famous background/resonant decomposition, which is widely used by the wind engineering community (Davenport 1961). Then, it allowed to formalize the general framework of the multiple timescale spectral analysis, which helps to find similar expressions for higher order or crossed statistics (Denoël 2015). They are eventually computed much faster than before thanks to the resulting reduction in the dimensions of the integrals which drastically decreases the number of integration points required to compute them with a sufficient accuracy.

The third cumulant has already been decomposed into two parts in previous works (Denoël and Carassale 2015; Denoël 2011). To do so, the imaginary part of the loading bispectrum was discarded. This assumption is licit provided that the loading is a time-reversible process, e.g. a polynomial transformation of a Gaussian input, in which case the imaginary part of the loading bispectrum is exactly equal to zero (Williams 1992). In a more general context, however, the bispectrum of a process can be complex-valued. Experimental evidence shows that the imaginary part might even be of the same order of magnitude as the real part (Esposito 2019), see for instance

*) *Corresponding author*

the bispectrum of the wind pressure on a high-rise building which is represented in Figure 1 and is based on the data given in (Kikuchi et al. 1997).

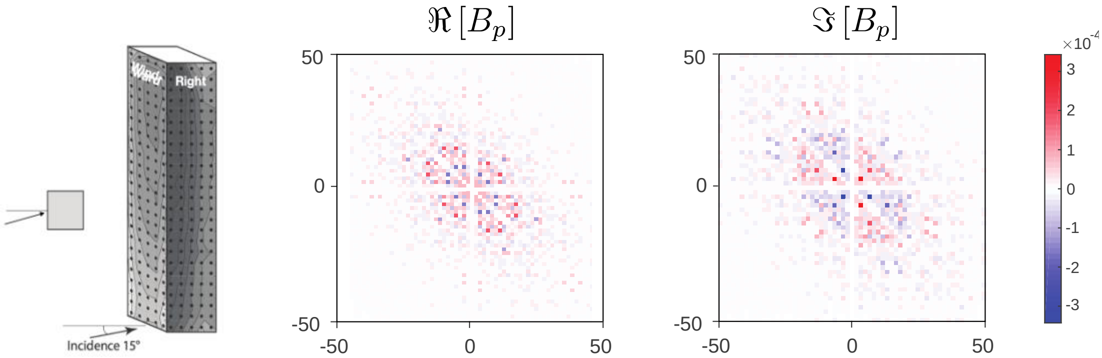

Figure 1. Bispectrum of the wind pressure at a given point on the building (Kikuchi et al. 1997).

The real part of the response bispectrum is therefore expressed as a function of both the real and imaginary parts of the loading bispectrum, see Figure 2-(a) to Figure 2-(g). As a consequence, the third moment of the response includes one background component and two biresonant components: one related to the real part of the loading bispectrum and one to the imaginary part. Their expressions are presented and discussed in this paper in the light of a parametric analysis.

2. ANALYTICAL AND NUMERICAL RESULTS

2.1. Proposed Decomposition

In the context described above, the bispectrum of the response is given by

$$B_q(\omega_1,\omega_2) = K_b(\omega_1,\omega_2)\, B_p(\omega_1,\omega_2) \tag{1}$$

where $B_p(\omega_1,\omega_2)$ is the bispectrum of the loading. It is supposed to be known, either experimentally, either analytically from a loading model. The structural kernel reads

$$K_b(\omega_1,\omega_2) = H_s(\omega_1)\, H_s(\omega_2)\, H_s^*(\omega_1+\omega_2) \tag{2}$$

at third order. It depends on the frequency response function of the oscillator, which is defined as

$$H_s(\omega) = \left[k - m\omega^2 + 2i\omega\xi\sqrt{km}\right]^{-1} \tag{3}$$

with k its stiffness, m its mass, ξ its damping ratio, and hence $\omega_0 = \sqrt{k/m}$ its natural frequency.

The third central moment of the response can therefore be obtained as

$$\kappa_{3,q} = \iint \Re[B_q(\omega_1,\omega_2)]\mathrm{d}\omega_1\mathrm{d}\omega_2 \tag{4}$$

or in an alternative way, with the multiple timescale spectral analysis, as

$$\kappa_{3,q} = \kappa_{3,b} + \kappa_{3,r} + \kappa_{3,i} \qquad \text{with} \qquad \kappa_{3,b} = \frac{\kappa_{3,p}}{k_s^3}\ ; \tag{5}$$

$$\text{and} \qquad \kappa_{3,r} = 6\pi\frac{\omega_0^3}{k_s^3}\xi\int\frac{\Re\left[B_p(\omega_0,\omega_2)\right]}{(2\xi\omega_0)^2+\omega_2^2}\,\mathrm{d}\omega_2\ , \tag{6}$$

$$\text{and} \qquad \kappa_{3,i} = 3\pi\frac{\omega_0^2}{k_s^3}\int\frac{\Im\left[B_p(\omega_0,\omega_2)\right]}{(2\xi\omega_0)^2+\omega_2^2}\omega_2\mathrm{d}\omega_2\ . \tag{7}$$

2.2. Parametric Analysis

Globally, the results presented in Figure 3-(h) and Figure 3-(i) can be explained quite easily thanks to the simple expressions that have been obtained for the main components of the third central moment of the response. First, the approximations provided by their sum are verified as its ratio with the reference values is close to one and is getting even closer when the frequency ratio and the damping ratio decrease. This is indeed to be expected given that the multiple timescale spectral analysis is based on perturbation methods and its accuracy is consequently conditioned upon the smallness of these two parameters. Second, the background to bi-resonant ratio decreases with the damping ratio. By contrast, the bi-resonant component increases with the frequency ratio. Small damping ratios and large frequency ratios result in relatively more resonant structural response, which is less skewed and thus more Gaussian, as shown in Figure 3-(i). This corroborates the central limit theorem. At last, the bi-resonant component related to the imaginary part of the loading bispectrum is clearly not negligible in the example at stake where it has been estimated by a parametric approach based on an AR model of the loading (Raghuveer and Nikias 1985):

$$B_p(\omega_1, \omega_2) = H_p(\omega_1) H_p(\omega_2) H_p^*(\omega_1 + \omega_2) \quad \text{with} \quad H_p(\omega) = \frac{1}{1 + \alpha i \omega} \tag{8}$$

where $\alpha = \omega_p/\omega_0$ stands for the frequency ratio.

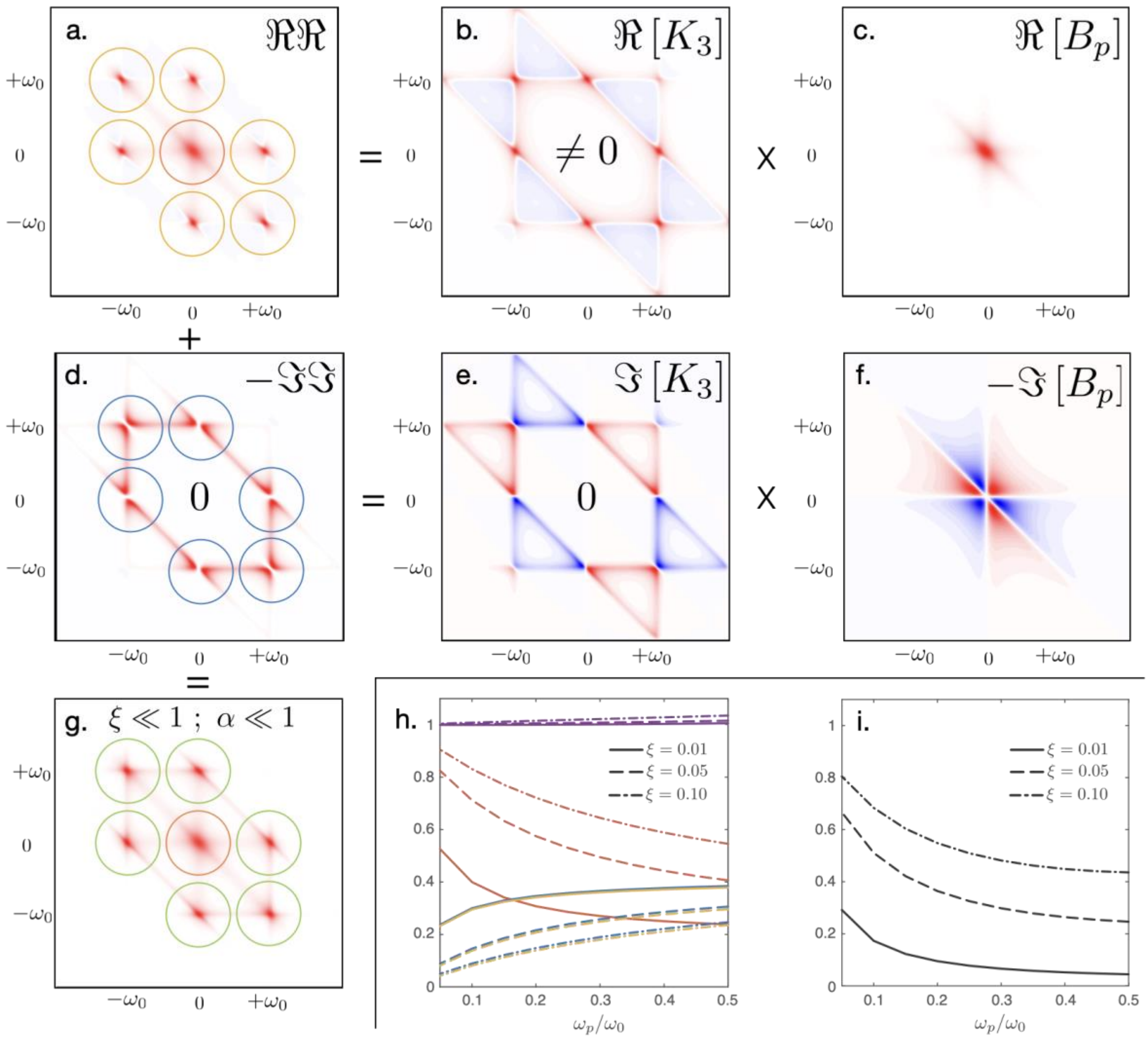

Figure 2. First line of graphs: (a) product of (b) and (c) with (b) the real part of the structural kernel and (c) the real part of the loading bispectrum; Second line of graphs: (d) minus the product of (e) and (f) with (e) the imaginary part of the structural kernel and (f) the imaginary part of the loading bispectrum. Third line of graphs: (g) sum of (a) and (d) which gives the real part of the response bispectrum; (h) in purple, the ratio between the third central moments of the response obtained through the proposed decomposition and through the numerical integration of the bispectrum, then in orange, yellow and blue, the repartition of the estimated results between the background and the bi-resonant components associated to either the real part, either the imaginary part of the loading bispectrum, respectively; (i) dynamic amplification of the skewness of the response with respect to the skewness of the loading.

3. CONCLUSIONS AND PERSPECTIVES

All in all, the Multiple Timescale Spectral Analysis provides meaningful analytical approximations for the main components of the third central moments. It helps to understand how the response is influenced by the loading and by the structural parameters, respectively. They thus allow to determine how the structure can be modified to exhibit a safer dynamical behavior, based on simple and sound mathematics. Thanks to the perturbation theory, the discrepancy is also known to remain limited provided that the damping ratio and the frequency ratio are sufficiently smaller than unity. Last but not least, the computational time is divided by about one hundred at least when using the expressions derived in this paper for the third central moments of the responses. This is explained by the fact that integrating their bispectrum over a two-dimensional frequency space is avoided.

The Multiple Timescale Spectral Analysis has thus proved its worth and is currently being developed to provide such expressions for the statistics of structures with many degrees-of-freedom, subjected to self-excited forces (Heremans, Mayou, and Denoël 2021), or wave forces which trigger the inertial regime as well (Geuzaine and Denoël 2020), on top of the background and the resonant ones.

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