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Asymptotic approximation of flutter and buffeting response of torsional aeroelastic oscillator

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ABSTRACT: Flutter design consists in the repetitive computation of the structural response for increasing wind velocities until reaching the instability of the aeroelastic system. A linear model of aerodynamic forces such as that based on Scanlan derivatives can be studied by means of equivalent frequency-dependent stiffness and damping. Therefore, the analysis in the frequency domain is suitable. In a spectral analysis, the variance of the response is the integral of the corresponding power spectral density characterized by its sharp peaks in the vicinity of the natural frequencies of the system. In this paper, we present an alternative solution to standard integration methods which extends the Background/Resonant decomposition to a single degree-of-freedom frequency dependent oscillator. The proposed method avoids the need for a dense frequency discretization, which significantly cuts the CPU load. While providing a decent accuracy, it also consists in simple closed form equations which give physical understanding. The investigation is limited to a single degree-of-freedom system but provides a significant insight into more complex models where such an approximation can become more valuable. A companion paper deals with the multi-degree-of-freedom case.

Keywords: Flutter, Multiple Timescale Spectral Analysis, Background, Resonant.

1. INTRODUCTION

Flutter is among the aeroelastic phenomenon of outmost concern in the design of long span bridges exposed to wind effects. The deplorable collapse of the Tacoma Bridge in 1940 has warned designers and researchers about this type of instability severely detrimental to structural integrity.

Building on the advances in aeronautics, extensive investigations have been conducted in civil engineering in order to predict flutter of bridge decks by means of wind tunnel testing, numerical simulations using sophisticated models and analytical approximations. Civil engineers use the Scanlan formulation (Scanlan, 1993) to express the self-excited forces by means of coefficients coming from wind tunnel tests called *flutter derivatives*. These coefficients model the interaction between the fluid and the structure and are the equivalent of the Küssner coefficients (Küssner, 1936) used in aeronautics. Methods of flutter analysis are already available in the literature for bridge decks subjected to buffeting effects (Abbas et al., 2017). Pioneers like (Bleich, 1948) have demonstrated that bridge flutter is most of the time governed by the interaction between the fundamental vertical bending and its frequency-nearby torsional mode. This 2-mode model is considered to be sufficiently precise to predict the critical flutter speed especially for a predesign of deck sections. Based on the nature of aerodynamic forces, represented as frequency dependent stiffness and damping, the analysis can be performed in the frequency domain. Considering the

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turbulent wind as a stationary random process defined by its power spectral density (PSD), the spectral response is obtained through the multiplication of the PSD of the buffeting loads by the FRF of the aeroelastic system. The variance of the response is then computed by integrating the resulting spectra. This operation is crucial as it is repeated for several wind velocities ranging from zero to the critical flutter velocity, in order to investigate the progressive increase of the structural response while approaching flutter. In this paper, an alternative method to full numerical integration of the PSD is developed in order to address the need of computational efficiency. This method considers the coexistence of multiple timescales in the PSD of the response (Denoël, 2015), and provides analytical approximations of the variance in relevant regions of the frequency domain, particularly in the background region (very low frequencies) emerging from the turbulence and the resonant region inherent to the faster vibrations of the aeroelastic system. This represents a generalization of the quasi-steady formulation presented in Davenport's works (Davenport, 1962). In this paper, the structural model is limited to the torsional degree-of-freedom, which does not cover the interaction between adjacent modes. However, it provides a general framework that can be followed in higher dimensional systems (Heremans et al. 2022).

2. PROBLEM STATEMENT

In time domain, the motion of a single dof structure subjected to aerodynamic and buffeting loads is governed by

$$m_{\mathrm{s}}\ddot{q}(t) + c_{\mathrm{s}}\dot{q}(t) + k_{\mathrm{s}}q(t) = f_{\mathrm{ae}}(t) + f_{\mathrm{bu}}(t) \tag{1}$$

where m_{s}, c_{s} and k_{s} are the structural mass, viscosity and stiffness. The loading is split into two components: the aerodynamic forces $f_{\mathrm{ae}}(t)$ and the buffeting forces $f_{\mathrm{bu}}(t)$. The buffeting loading is characterized as a linear combination of the turbulent components of the wind, while the aerodynamic loading is a linear combination of the velocity and the displacement of the structure.

In frequency domain, the governing equation takes the following form via a Fourier transform

$$[-m_{\mathrm{s}}\omega^2 + \mathrm{i}\omega c(\omega) + k(\omega)]Q(\omega) = F_{\mathrm{bu}}(\omega) \tag{2}$$

where $c(\omega) = c_{\mathrm{s}} - c_{\mathrm{ae}}(\omega)$ and $k(\omega) = k_{\mathrm{s}} - k_{\mathrm{ae}}(\omega)$ gather both the structural and aerodynamic viscosity and stiffness, $c_{\mathrm{ae}}(\omega)$ and $k_{\mathrm{ae}}(\omega)$ are frequency dependent functions. They can be expressed as a function of Scanlan derivatives for bridge decks. The buffeting loading is assumed to be a zero-mean Gaussian stochastic process; it is characterized by its power spectral density $S_{f,\mathrm{bu}}(\omega)$. The spectra of the response is obtained by

$$S_q(\omega) = |H(\omega)|^2 S_{f,\mathrm{bu}}(\omega) \tag{3}$$

where

$$H(\omega) = \left[-\frac{\omega^2}{\omega_{\mathrm{s}}^2} + 2\mathrm{i}\xi_s \frac{\omega}{\omega_{\mathrm{s}}}\mathcal{C}(\omega) + \mathcal{K}(\omega)\right]^{-1} \tag{4}$$

represents the frequency response function of the aeroelastic system. $\mathcal{K}(\omega)$ and $\mathcal{C}(\omega)$ are dimensionless aeroelastic stiffness and damping defined as

$$\mathcal{K}(\omega) \coloneqq \frac{k(\omega)}{k_{\mathrm{s}}} = 1 - \frac{k_{\mathrm{ae}}(\omega)}{k_{\mathrm{s}}}\,, \mathcal{C}(\omega) \coloneqq \frac{c(\omega)}{c_{\mathrm{s}}} = 1 - \frac{c_{\mathrm{ae}}(\omega)}{c_{\mathrm{s}}} \tag{5}$$

The integration of (3) provides the variance of the displacement of the structural response

$$\sigma_q^2(U) = \int_{-\infty}^{+\infty} S_q(\omega; U)\, d\omega \tag{6}$$

where U is the mean velocity of the incident wind.

3. MULTIPLE TIMESCALE SPECTRAL ANALYSIS

3.1. Assumptions

The approximation of the response is built on the following assumptions:

- (i) The structural damping ratio ξ_s is supposed as small as 5%-10%, beyond these values the quality of the approximation is deteriorated;
- (ii) The damping and stiffness vary slowly around the resonance frequencies;
- (iii) The buffeting loading varies slowly around the resonance frequencies;
- (iv) The aeroelastic system and the buffeting loading have separate timescales: the natural vibrations of the main system are considered as fast dynamics and can be treated separately from the slow dynamics emerging from the turbulence of the wind.

From assumption (iv), the response can be seen as the sum of a Background component (low frequencies) and a Resonant component (in the close neighbourhood of the natural frequencies)

$$S_q(\omega) = S_{q,B}(\omega) + S_{q,R}(\omega) \Rightarrow \sigma_q^2 = \sigma_{q,B}^2 + \sigma_{q,R}^2 \tag{7}$$

3.2. Background component

In the range $\omega \ll \omega_s$, the frequency response function in Equation (4) can be approximated by $\hat{H}(\omega) = \left(k_s \mathcal{K}(\omega)\right)^{-1}$. Therefore, the background contribution is given by

$$S_{q,B}(\omega) = \frac{S_{f,\mathrm{bu}}(\omega)}{\left(k_\mathrm{s}\mathcal{K}(\omega)\right)^2} \Rightarrow \sigma_{q,B}^2 = \frac{1}{k_\mathrm{s}^2}\int_{-\infty}^{+\infty} \frac{S_{f,\mathrm{bu}}(\omega)}{\left(k_\mathrm{s}\mathcal{K}(\omega)\right)^2} d\omega. \tag{8}$$

3.3. Resonant component

From Equation (7) and (8), the resonant contribution is obtained by approximating the residual

$$S_{q,R}(\omega) = \left(|H(\omega)|^2 - \frac{1}{\left(k_\mathrm{s}\mathcal{K}(\omega)\right)^2}\right) S_{f,\mathrm{bu}}(\omega). \tag{9}$$

Following the general methodology of the Multiple Timescale Spectral Analysis (Denoël, 2015), we introduce the stretched coordinate η to zoom on the resonance frequency $\bar{\omega}$ and its close neighbourhood such that $\omega(\eta) = \bar{\omega}(1 + \xi_\mathrm{s}\eta)$, where $\bar{\omega}$ is the resonant frequency, i.e. the solution of the nonlinear eigenvalue problem $-\frac{\bar{\omega}^2}{\omega_\mathrm{s}^2} + \mathcal{K}(\bar{\omega}) = 0$. Invoking assumptions (i) and (iv) which are common to the classical (B/R) decomposition, we can write

$$\mathcal{K}\left(\omega(\eta)\right) = \mathcal{K}(\bar{\omega}) + \xi_\mathrm{s}\eta\bar{\omega}\partial_\omega\mathcal{K}(\bar{\omega}) + \mathcal{O}(\xi_\mathrm{s}^2),\ \mathcal{C}\left(\omega(\eta)\right) = \mathcal{C}(\bar{\omega}) + \xi_\mathrm{s}\eta\bar{\omega}\partial_\omega\,\mathcal{C}(\bar{\omega}) + \mathcal{O}(\xi_\mathrm{s}^2) \tag{10}$$

The replacement in Equation (4) and the truncation at leading order yields

$$H\left(\omega(\eta)\right) = \frac{1}{2\xi_\mathrm{s}k_\mathrm{s}}\left[\left(-\frac{\bar{\omega}^2}{\omega_\mathrm{s}^2} + \frac{1}{2}\bar{\omega}\partial_\omega\mathcal{K}(\bar{\omega})\right)\eta + \mathrm{i}\frac{\bar{\omega}}{\omega_\mathrm{s}}\mathcal{C}(\bar{\omega})\right]^{-1} \tag{11}$$

We can also transcribe assumption (iii) as $S_{f,\mathrm{bu}}\left(\omega(\eta)\right) = S_{f,\mathrm{bu}}(\bar{\omega}) + \mathcal{O}(\xi_\mathrm{s})$. From the last two expressions and via some standard calculus, an asymptotic approximation is found for the variance

$$\sigma_{q,R}^2 = \frac{S_{f,\mathrm{bu}}(\bar{\omega})}{\left(k_\mathrm{s} - k_\mathrm{ae}(\bar{\omega})\right)^2}\frac{\pi\bar{\omega}}{2\bar{\xi}}\frac{1}{1 + \frac{1}{2}\frac{\bar{\omega}\partial_\omega k_\mathrm{ae}(\bar{\omega})}{k_\mathrm{s} - k_\mathrm{ae}(\bar{\omega})}} \tag{12}$$

where $\bar{\xi} \coloneqq \frac{c_\mathrm{s} - c_\mathrm{ae}(\bar{\omega})}{2\sqrt{m_\mathrm{s}\left(k_\mathrm{s} - k_\mathrm{ae}(\bar{\omega})\right)}}$ is an equivalent damping ratio including the aeroelastic contribution.

4. ILLUSTRATION OF THE METHOD

The B/R decomposition is validated through the case study of the Golden Gate bridge (Heremans, 2021). The governing failure mode is limited to the pitching dof. The PSD of the response is depicted in Figure 1-(a), the resonant peak slightly shifts to the left as U increases, and becomes more acute on approaching the flutter instability. Figure 1-(b) compares the standard deviation of the response obtained with the proposed approach (purple), which is virtually superimposed with the reference results obtained with a dense numerical integration (blue). The relative error is less than 1%. It also provides a consistent measure of the critical flutter speed ($U_{\mathrm{cr}} \approx 24\ m/s$).

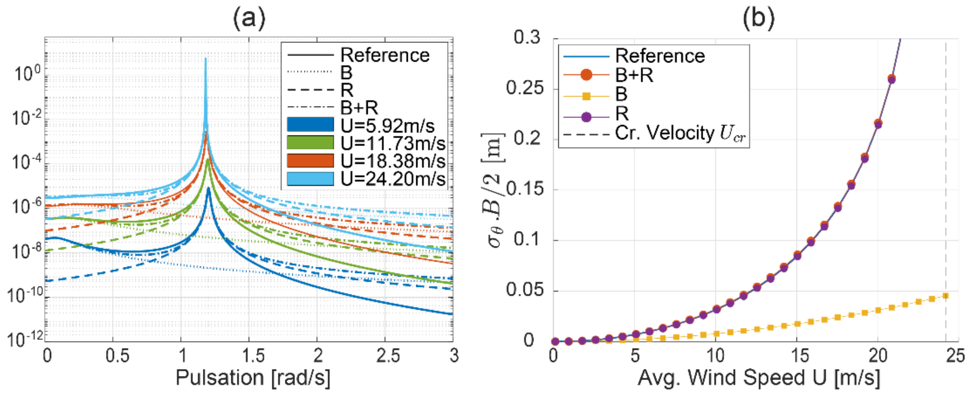

Figure 1 (a) PSD of the response numerically computed (Reference) and its B/R approximation (B+R). (b) Scaled standard deviations obtained with trapezoidal integration (Reference) and using the asymptotic solution (B+R). (Structural characteristics : Natural frequency $f_s = 0.19\ Hz$, Damping ratio $\xi_s = 0.5\%$, Deck width $B = 27.43\ m$, Moment of inertia $I_s = 4.4 \cdot 10^6\ kgm^2/m$)

5. CONCLUSIONS

The generalization of Davenport's theory to aeroelastic systems is accomplished through the proposed method, where the timescale separation plays a major role. The analytical formulation remains simple and allows an immediate interpretation of the results. It is a generalization since the proposed expressions for the background (8) and resonant (12) contributions to the response degenerate into the well-known formulation for constant damping and stiffness. Within the assumptions of the study, it offers a serious alternative to intensive integration methods. While the formulation developed in this paper is limited to 1-dof aeroelastic systems, extension to multiple-dof structures is also possible, following the same general derivation.

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