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Background/resonant decomposition of modal response correlations of coupled aeroelastic models submitted to buffeting loads

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ABSTRACT: Design against flutter is an important task when designing a flexible structure. Spectral flutter analysis necessitates the integrations of displacement power spectral densities for evaluating the covariance matrix of responses. This paper presents a light semi-analytical alternative to classical quadrature methods to integrate PSDs efficiently, decreasing the number of integration point by one or two orders of magnitude. It is based on a multiple timescale spectral analysis (MTSA), and constitutes the extension of the background/resonant decomposition of Davenport to aeroelastic systems. This approach is restricted to modal analysis, and assumes small damping ratios as well as low modal coupling but possibly highly correlated modes.

Keywords: flutter, MTSA, turbulence, aeroelasticity, long span bridges, BR decomposition.

1. INTRODUCTION

Last few decades have seen consequent progress in long span structures, driving civil engineers to design more and more slender structures with innovative shapes. Design against flutter is known as one of the most concerning issues for such flexible structures.

The first insights in aeroelasticity are due to [Theodorsen (1935)] who studied the behaviour of aircraft airfoils with a flat plate model. Few decades later, research migrated to civil engineering, involving study of complex profile bridge girders. Among others, [Scanlan (1993)] proposed a canonical formulation relating the self-excited forces with the flutter derivatives. These derivatives are determined experimentally by testing in wind laboratory or numerically using computational fluid dynamics. The structural response is determined by superimposing self-exciting forces to the buffeting forces. In the frame of a spectral analysis, the structural response is characterized by the variances and the co-variances of the response, obtained by integration of the displacement power spectral densities.

The presented semi analytical method offers a lightweight and time effective alternative to classical numerical integration methods to evaluate these integrals. Numerical models of bridges used to perform a flutter analysis have been so far dramatically limited by the computational resources available, and therefore consist most of the time in a 2-DOF pitch-plunge model. The proposed formulation allows significant acceleration of an essential step of the flutter design process.

2. PROBLEM FORMULATION

The dynamics of a M-DOF model subjected to buffeting and aeroelastic loads is described in frequency domain by

$$[-\omega^2 \mathbf{M}_s + i\omega \mathbf{C}_s + \mathbf{K}_s]\mathbf{X}(\omega) = \mathbf{F}_{ae}(\omega) + \mathbf{F}_{bu}(\omega) \qquad (1)$$

where $\mathbf{M}_s$, $\mathbf{C}_s$ and $\mathbf{K}_s$ refer respectively to the mass, damping and stiffness matrices. The right-hand side contains the self-excited $\mathbf{F}_{ae}$ and buffeting forces $\mathbf{F}_{bu}$. The buffeting forces are most often

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described by their power spectral density matrix (PSD matrix) that depends on the geometrical properties of the structure and the buffeting spectrum. The self-excited forces are modelled by $\mathbf{F_{ae}}(\boldsymbol{\omega}) = i\omega\mathbf{C_{ae}}(\boldsymbol{\omega}) + \mathbf{K_{ae}}(\boldsymbol{\omega})$ [Scanlan (1993)] so that they can be incorporated as aeroelastic stiffness and damping, functions of the flutter derivatives. As a result, the transfer function reads

$$\mathbf{H}(\omega) = [-\,\omega^2\mathbf{M}(\omega) + i\omega\mathbf{C}(\omega) + \mathbf{K}(\omega)]^{-1} \tag{2}$$

with $\mathbf{M}(\omega) = \mathbf{M}_s(\omega) + \mathbf{M}_{ae}(\omega)$, $\mathbf{C}(\omega) = \mathbf{C}_s(\omega) + \mathbf{C}_{ae}(\omega)$ and $\mathbf{K}(\omega) = \mathbf{K}_s(\omega) + \mathbf{K}_{ae}(\omega)$. The spectral analysis consists in the determination of the displacement PSDs, expressed as

$$\boldsymbol{S_x}(\omega) = \mathbf{H}(\omega)\mathbf{S}_{\mathbf{F}_{\mathrm{bu}}}(\omega)\mathbf{H}^{\mathrm{T}}(\omega). \tag{3}$$

The integration of this matrix provides the covariance matrix that is used to recombine modal responses into physical displacements and, along with the extreme value theory, establish extreme displacements. The latter displacement, added to the average response serves as basis to perform the design of the structure. Because these power spectral densities are the result of a quadratic product in $\mathbf{H}(\omega)$, they experience very sharp peaks. Classical integration methods such as the trapezoidal rule are typically used to integrate (3). However, they turn out to be highly resource consuming since a very high frequency resolution must be used to represent correctly the acute resonant peaks affecting the PSDs. Furthermore, the span of the frequency interval must be large enough to capture all the energy of the process. As a result, conventional quadrature methods struggle to integrate accurately the displacement spectrum because they require a large number of integration points [Heremans & Al. (2021)]. The method presented here offers a lightweight semi-analytical alternative for the evaluation of this integral.

3. PROPOSED APPROXIMATION

The proposed approximation extends Davenport's background/resonant decomposition to aeroelastic systems. The PSDs are split into two contributions: the content located in the low frequency range defines the background component, while the peak(s) in the neighbourhood of the resonant frequencies defines the resonant component. The derivation of the analytical expressions of these components is based on the Multiple Timescale Spectral Analysis (MTSA) [Denoël (2015)].

3.1. Modal Analysis

The cross power spectral densities expressed in nodal basis as proposed in (3) depicts up to 2N distinct peaks, if N refers to the number of nodes of the structure, and therefore the number of peaks in an FRF. The use of a modal analysis decreases this number to two distinct peaks at most in the resonance regime, which is highly appreciated since the MTSA then turns out to be competitive by providing approximation of the resonant component peak by peak. Expressed in a modal basis, equation (3) reads

$$\boldsymbol{S_q}(\omega) = \mathbf{H}^*(\omega)\mathbf{S}_{\mathbf{F}^*_{\mathrm{bu}}}(\omega)\mathbf{H}^{*\mathrm{T}}(\omega) \tag{4}$$

where the overhead asterisk indicates modal quantities.

3.2. Small Coupling Assumption

Classical modal analysis in dynamics generally assumes Rayleigh damping as it allows to form a diagonal flexibility matrix in the modal basis $\mathbf{J}^*(\omega) = -\omega^2\mathbf{M}^* + i\omega\mathbf{C}^* + \mathbf{K}^*$, whose inverse is easily calculated. Unfortunately, the frequency dependency of stiffness and damping matrices does not allow an exact diagonalization of the flexibility matrix. Instead, a modal basis is chosen such that the dynamic flexibility matrix is nearly diagonal for all frequencies. This small coupling assumption, along with the first order approximation of slightly coupled matrices leads to the following expression for the FRF matrix [Denoël, Degée (2009)]

$$\mathbf{H} \approx \mathbf{J}_d^{-1} + \mathbf{J}_d^{-1}\mathbf{J}_o\mathbf{J}_d^{-1}, \tag{5}$$

for which no full matrix inversion is required. In the latter equation, $\mathbf{J}_{\boldsymbol{d}}$ refers to the diagonal of the modal dynamic flexibility, and $\mathbf{J}_o = \mathbf{J}^* - \mathbf{J}_d$ contains only the coupling terms of $\mathbf{J}^*$.

3.3. MTSA Approximation

The spirit of the multiple timescales analysis is founded on processing separately two phenomena characterized by different timescales. The buffeting action brings most of its energy in the low frequency range, and constitutes therefore the background component. The dynamics of the structure is generally characterized by a higher frequency response, materialized by the resonant peaks. The sum of the two components gives an approximation of the modal displacement PSD

$$\mathbf{S}_q(\omega) = \mathbf{S}_q^B(\omega) + \mathbf{S}_q^R(\omega) \tag{6}$$

The background component is obtained by expanding the expressions of the modal transfer functions of modes i and j, and substituting them in (5). If ω_i and ω_j refer respectively to the circular natural frequencies of the modes i and j, the background component reads

$$S_{q,ij}^B(\omega) = H_{d,ii}(\omega) S_{p,ij}(\omega) \overline{H}_{d,jj} \approx \frac{S_{p,ij}(\omega)}{K_{d,i}(\omega) K_{d,j}(\omega)} \tag{7}$$

in which the dynamics response is approximated by a quasi-static response in each mode. This PSD is then integrated to give the variance $\sigma_{q,ij}^{B^2}$. Because $S_{p,ij}(\omega)$ is a smooth function of ω, the evaluation of this integral does not require much integration points. The resonant component is obtained introducing a stretched coordinate $\omega = f(\eta, \varepsilon, \delta, \rho)$ with η a variable of order $\mathcal{O}(1)$, $\varepsilon = (\omega_j - \omega_i)/(\omega_i + \omega_j)$ a small parameter, and ρ and δ two parameters functions of ω_i and ω_j. This stretching is introduced in $H_{d,i}(\omega)$ and $J_{o,ij}(\omega)$, and an asymptotic expansion of (5) in the sense of the perturbation method reviewed by [Denoël (2015)] is obtained. This series development is truncated at leading order, to get the following expression

$$S_{q,ij}^R(\omega) = \frac{1}{\varepsilon^2} \frac{S_{p,ij}(\omega_{ij})}{\omega_i \omega_j} \frac{1}{\mathcal{L}_1(\eta)\mathcal{L}_2(\eta)} \tag{8}$$

where $\mathcal{L}_1(\eta)$ and $\mathcal{L}_2(\eta)$ are two functions linear in η but still quite cumbersome whose expressions are omitted here. The two poles corresponding to the roots of $\mathcal{L}_1(\eta)$ and $\mathcal{L}_2(\eta)$ suggests the use of Cauchy's residue theorem to obtain the resonant contribution

$$\sigma_{q,ij}^{R\;2} = \int_{-\infty}^{+\infty} S_{q,ij}^R(\eta)\, d\omega = \frac{4\pi i\, \omega_{ij}}{\varepsilon \omega_i \omega_j} \frac{1}{\overline{\mathcal{L}_3}(i)} \left[i c_j(\omega_j) + i c_i(\omega_i) + \delta\varepsilon(1+\rho)\mathcal{L}_3(i) \right]^{-1} \tag{9}$$

with $\mathcal{L}_3(a) = \partial_\omega k_a(\omega_a) - \omega_a(2m_a(\omega_a) + \partial_\omega m_a(\omega_a))$. This approximation is conditioned by 4 important hypotheses: the separation of the timescales of the phenomena, the assumption of small modal damping and small ε and that Scanlan's derivatives with respect to ω are smooth and not varying too fast across resonance peaks. The last hypothesis is probably the most restrictive, as it restrains, in principle, the application of the method to modes with close natural frequencies. The proposed method is therefore less accurate, relatively speaking, for pairs of modes with distinct frequencies, i.e. little correlation. This does not appear as a pragmatic limitation since errors on small correlations will not expose the quality of the recombination of modal responses.

4. ILLUSTRATION

The efficiency of the method is demonstrated on a pitch/plunge model of the Storebelt bridge, an example borrowed from the benchmark described in [Diana & al. (2019)] where a flat plate model is considered. In this application, the fundamental torsional and bending eigen frequencies have been modified to 0.25Hz and 0.2375Hz to illustrate the method with significantly correlated modal responses.

The results are presented in Figure 1 in terms of PSDs for two different wind speeds. Despite a light and local underestimation of the PSD in the fundamental resonant peak, the MTSA method provides a trustful approximation of the modal PSD. The Figure 2 displays the correlation coefficient and the absolute errors for different wind speeds. The method provides an excellent approximation of the correlation coefficient, with errors of the order of 1%.

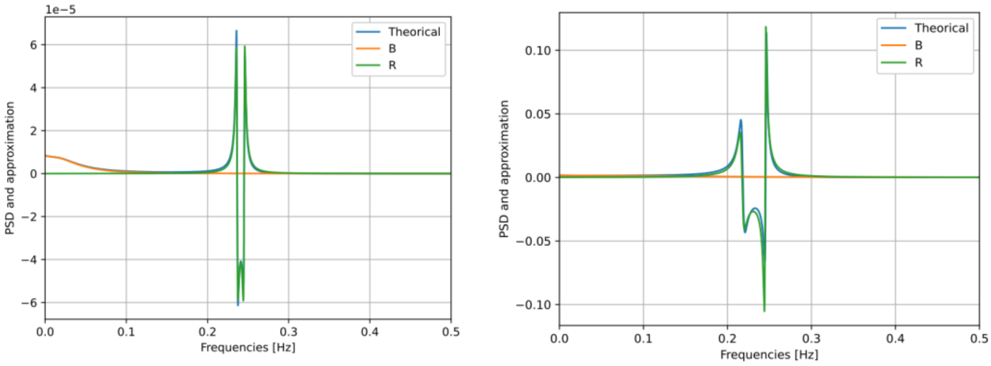

Figure 1. Cross-PSD of the modal displacements, and its background and resonant decomposition for U=10m/s (left) and U=40m/s (right).

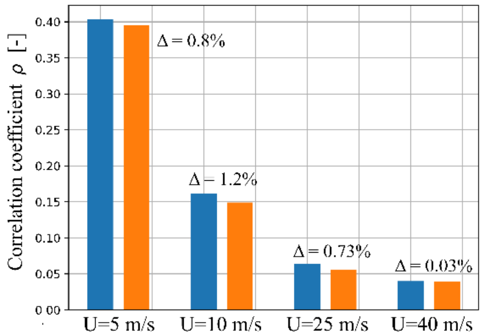

Figure 2. Theorical modal covariances (orange), proposed approximation (blue) and absolute errors for different subcritical wind speeds.

5. CONCLUSIONS

The presented method offers a very simple alternative to classical integration methods for evaluating the integral of the power spectral densities during a spectral analysis. Its efficiency holds from the use of the analytical expression of the modal transfer function in the neighbourhood of resonance peaks, similarly to what is done in the classical background/resonant decomposition. The proposed method allows a significant reduction of the numerical burden associated with numerical integration of cross-PSDs. As such, it opens interesting perspectives on the analysis of large structures. The method was shown to work efficiently on a chosen application, providing a fine approximation of the correlation coefficient with an error limited to about 1%.

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