MEGNO: fast method to study dynamics of planetary systems

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Solar System in an Exoplanetary Context

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The Mean Exponential Growth factor of Nearby Orbits (MEGNO)

The Mean Exponential Growth factor of Nearby Orbits (MEGNO) method is introduced by Cincotta et al. (e.g., Cincotta & Simó 2000; Cincotta et al. 2003) and it belongs to the group of fast chaos indicators such as the well known Lyapunov Charascteristic Number (LCN), but it is faster in computation. This chaos indicator evaluates the stability of the body trajectories after a small perturbation of the initial conditions and has demonstrated its power for studying the long-term evolution of Hamiltonian systems. The MEGNO provides a clear picture of resonant structures and establishes the locations of stable and unstable orbits. To calculate MEGNO, Y(t), each body's six-dimensional displacement vector, δ_i, it is considered as a dynamical variable in both position and velocity from its shadow particle, i.e., a particle with slightly perturbed initial conditions. Then, for each δ_i differential equations applying a variational principle to the trajectories of the original bodies are obtained. Then, the MEGNO is straightforwardly computed from the variations as:

$$Y(t) = \frac{2}{t}\int_0^t \frac{\|(\dot{\delta})\|}{\|\delta\|}\, s\, ds$$

Along with its time-average mean value:

$$\langle Y(t)\rangle = \frac{1}{t}\int_0^t Y(t)\, ds$$

Thanks to the time weighting factor we get an amplification of stochastic behavior allowing the detection of hyperbolic regions in the interval (t_o, t). To distinguish between chaotic and quasi-periodic trajectories in the phase space we evaluated the <Y(t)>. Then:

If the system is chaotic: $\langle Y(t\to\infty)\rangle \to \infty$

If the system is quasi-periodic: $\langle Y(t\to\infty)\rangle \to 2$

As is the case for every numerical indicator of chaos, the quasi-periodic orbits are only valid up to the integration time. The dynamical character of the system for longer times is not known, and could eventually turn out to be chaotic. The time-averages MEGNO converges faster to its limit value compared to the standard calculation of the LCN. This property allows a more rapid exploration of the dynamical phase space topology of a given system. MEGNO has been used in widespread applications within dynamical astronomy ranging from galactic dynamics to stability analysis of extrasolar planetary systems and Solar System small body dynamics. In this work, we focus on the last two, and we present some examples of its application.

Dynamics of the Jovian irregulars satellites

There are a large number of satellites in orbit around giant planets of the Solar System. These satellites can be divided into two groups: regular and irregulars. Regular satellites are characterized by circular orbits placed close to the equatorial plane of the planet, e.g., Galilean satellites of Jupiter. On the other hand, irregular satellites are located close to the Hill sphere boundary of the parent plane and show large eccentricities and highly inclined orbits in either prograde or retrograde motion. The most abundant irregular satellite population exists on Jupiter with more than 60 members. Regular satellites were formed during the accretion processes within a circumjovian planetary disk. However, the formation of irregular satellites is still under discussion, the most favored scenario being the capture from an initial heliocentric orbit. Hinse et al. (2010) studied the dynamical nature of irregular satellites through MEGNO parameter for first time. The authors computed the MEGNO indicator over a large grid in (a_0, i_0) space, where both prograde and retrograde irregular satellites are located. From this study they found that prograde satellites are more chaotic than retrograde. Moreover, they found several dynamical structures. After a large number of tests they concluded that these structures are chaotic mean motion resonances.

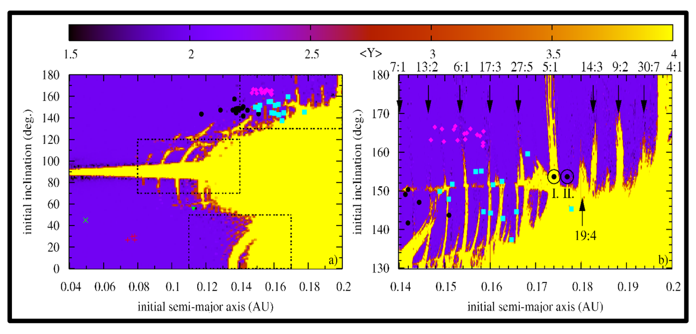


Fig 1: MEGNO maps in (a_0,i_0) space of test satellites with e_o=0.20. Different symbols refer to different irregular satellites families as follows: Themisto x symbol, Himalia + symbol, Carpo triangle Symbol, Ananke dot symbol, Carme rhomb symbol, and Pasiphae Square symbol. Left panel is a zoom of the upper right box in the Right panel. Mean motion resonances are identify. Extracted from Hinse et al. (2010).

Modelling the inner debris disc of HR 8799

The HR 8799 is a young system composed by at least by four giant planets and two debris belts. The planets are most likely trapped in a double Laplace resonance, with orbital period ratios of 1:2:4:8 after rapid migration that followed their formation. The two debris belts were detected by infrared observation. The inner one, the "hot" disk, is located between the inner planet and the star, while the outer one, the "cool" disk is beyond the outer planet, from 100 to 300 au. However, the inner is unresolved and poorly understood. This lead Contro et al. (2016) to study the dynamical nature of the inner belt to better understand its structure and collisional environment. The authors performed a grid of 400x600 initial conditions in the parameters space (a_0, e_0) in the region of interest and they computed the MEGNO parameter for 100 kyr. They found that the inner belt is highly structured, with gaps between regions of dynamical stability. They concluded that the belt is likely constrained between sharp inner and outer edges located at 6 and 8 au. The inner edge being cleared by the 4:1 mean motion resonance with the inner planet.

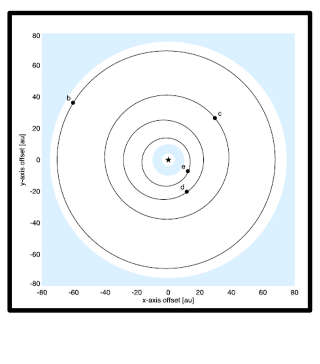


Fig 2: Schematic vision of the HR 8799 planetary system with the 4 giant planets and the two debris disks in blue.

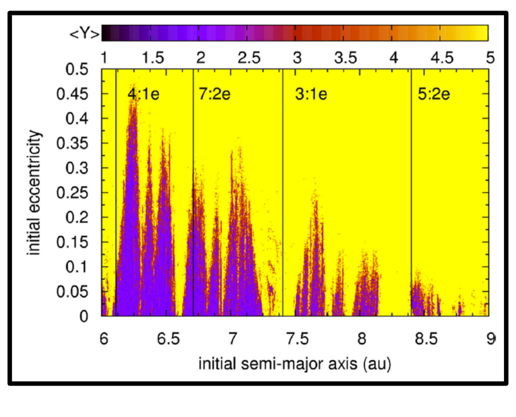


Fig 3: MEGNO mapping of the region likely occupied by the inner HR 8799 debris belt. The resonances which are sculpting the edges of the belt are also shown. Extracted from Contro et al. (2016).

Predicting a third planet in the Kepler-47 circumbinary system

The analysis of the light curve of Kepler-47 binary system reveled the transits of two planets. At the same time, a single transit event that could not be explained by them was detected. It was suggested that if this transit event was due to a third planet, given its depth of 0.2%, the planet must have about 4.5 Earth-radii. In Hinse et al. (2015) dynamical considerations to examine that possibility were used. The authors used the MEGNO technique to identify regions in the phase-space where the hypothetical third planet could follow quasi-periodic orbits. Their results identify ranges of semi-major axis and eccentricity that would render the third planet stable over a period of 10 Myrs. To examine the extent of the dependence of the results on the mass of this planet, the authors carried out integrations for different values of its mass, and showed that a third planet with a mass as high as 50 Earth masses can still maintain stable orbits either between the known planets or beyond the orbit of the outer one.

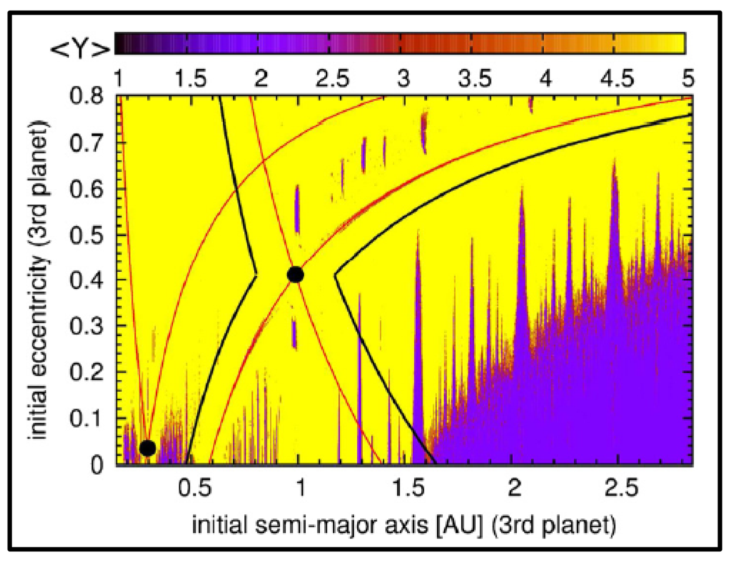


Fig 4: Dynamical MEGNO map in the (a_0, e_0) space for the third planet on the Kepler-47 system. The two known planets are shown as black circles. The general quasi-periodic region for a third planet starts from around 1.6 au. Extracted from Hinse et al. (2015).

Luyten planetary system: Does it harbor similar structures to Solar System?

Luyten's star is one the closest stars to the Solar System and the closest multi-planetary system known so far, located only at 3.8 pc. Its age is estimated to be 2.75 Gyr, so it is younger than our Solar System. Luyten is a M dwarf with a mass of 0.29 Sun masses. In a recent paper, Astudillo-Defru et al. (2017) reported the detection of two planets of Earth-mass size by radial velocities. However, the uncertainties of the masses are subjected to the orbital inclination angle, so the real masses remain unknown with a serious degree of uncertainty (see table 1). In this context, in order to explore the (a_o, e_o) space parameter we constructed MEGNO maps for a test massless particles (see Fig. 5) considering different combinations of the planetary parameters and orbital inclination ranging from 90º to 10º.

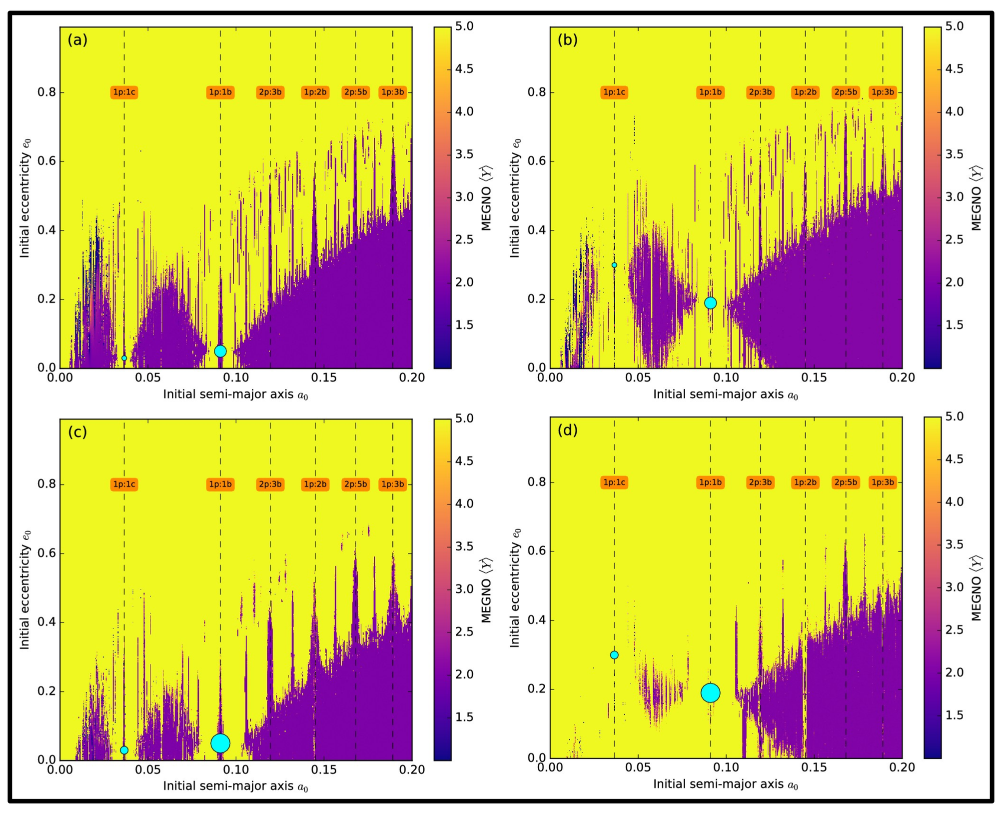
	GJ 273 b	GJ 273 c
Period [days]	$18.6498^{+0.0059}_{-0.0052}$	$4.7234^{+0.0004}_{-0.0004}$
e	$0.10^{+0.09}_{-0.07}$	$0.17^{+0.13}_{-0.12}$
a [au]	$0.09110^{+1.9e-5}_{-1.7e-5}$	$0.03646^{+2.0e-6}_{-2.0e-6}$
Msin(i) [M_{Earth}]	$2.89^{+0.27}_{-0.26}$	$1.18^{+0.16}_{-0.16}$

Table 1: Planet parameters for the Keplerian fitted by Astudillo-Defru et al. (2017).

We found that the dynamical environment changes dramatically. If we consider the most relax scenario, i.e., lower values of planets' eccentricities and higher values of the orbital inclination (which implies lower values of planetary masses) we found many stable structures and mean motion resonances which could harbor minor bodies such as comets and asteroids. These structures would be similar to the asteroid belt and Kuiper belt in the Solar System. However, when we change to the scenario with the largest values of eccentricities and lowest values of orbital inclination most of the stable structures disappear completely. This work is still in progress and future results will be published.

Fig 5: Dynamical MEGNO maps in the (a_0, e_0) space for a massless particle. Different panels refer to different planetary configurations in the range of their uncertainties as follows:

a) M_c=1.18 M_{Earth}, e_c=0.03, M_b=2.89 M_{Earth}, e_b=0.05
b) M_c=1.18 M_{Earth}, e_c=0.30, M_b=2.89 M_{Earth}, e_b=0.19
c) M_c=6.79 M_{Earth}, e_c=0.03, M_b=16.64 M_{Earth}, e_b=0.05
d) M_c=6.79 M_{Earth}, e_c=0.30, M_b=16.64 M_{Earth}, e_b=0.19



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