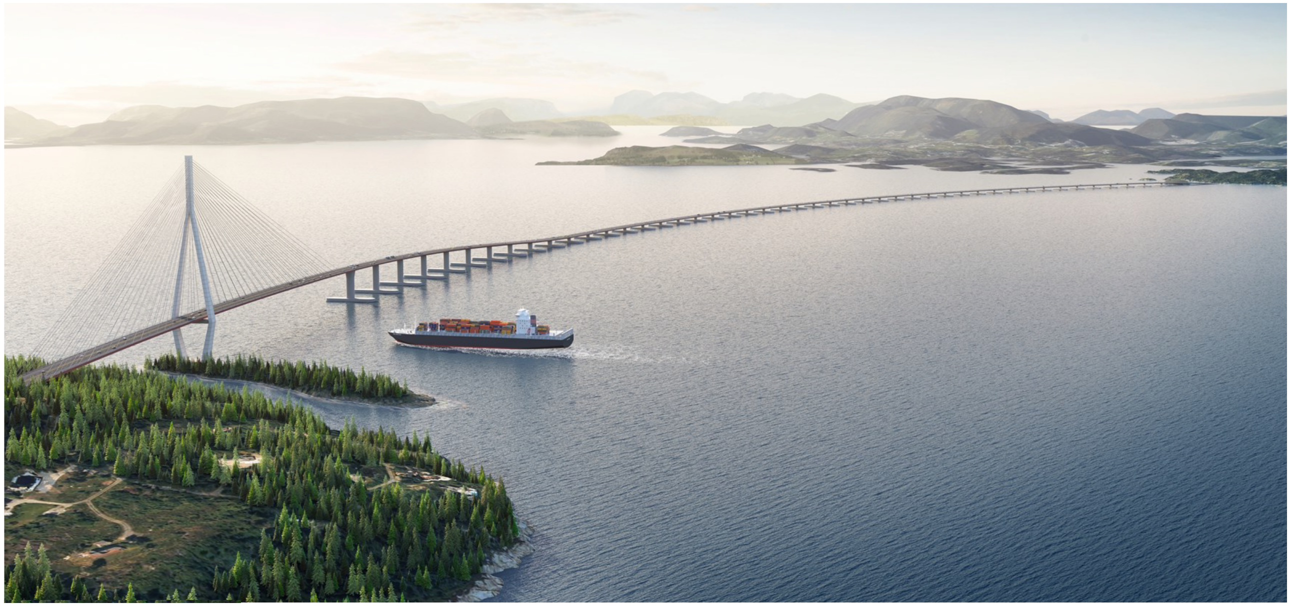


Multiple Timescale Spectral Analysis of Wave-Loaded Floating Structures

-- Public Defence of Margaux Geuzaine --

The need for floating structures increases throughout the world.

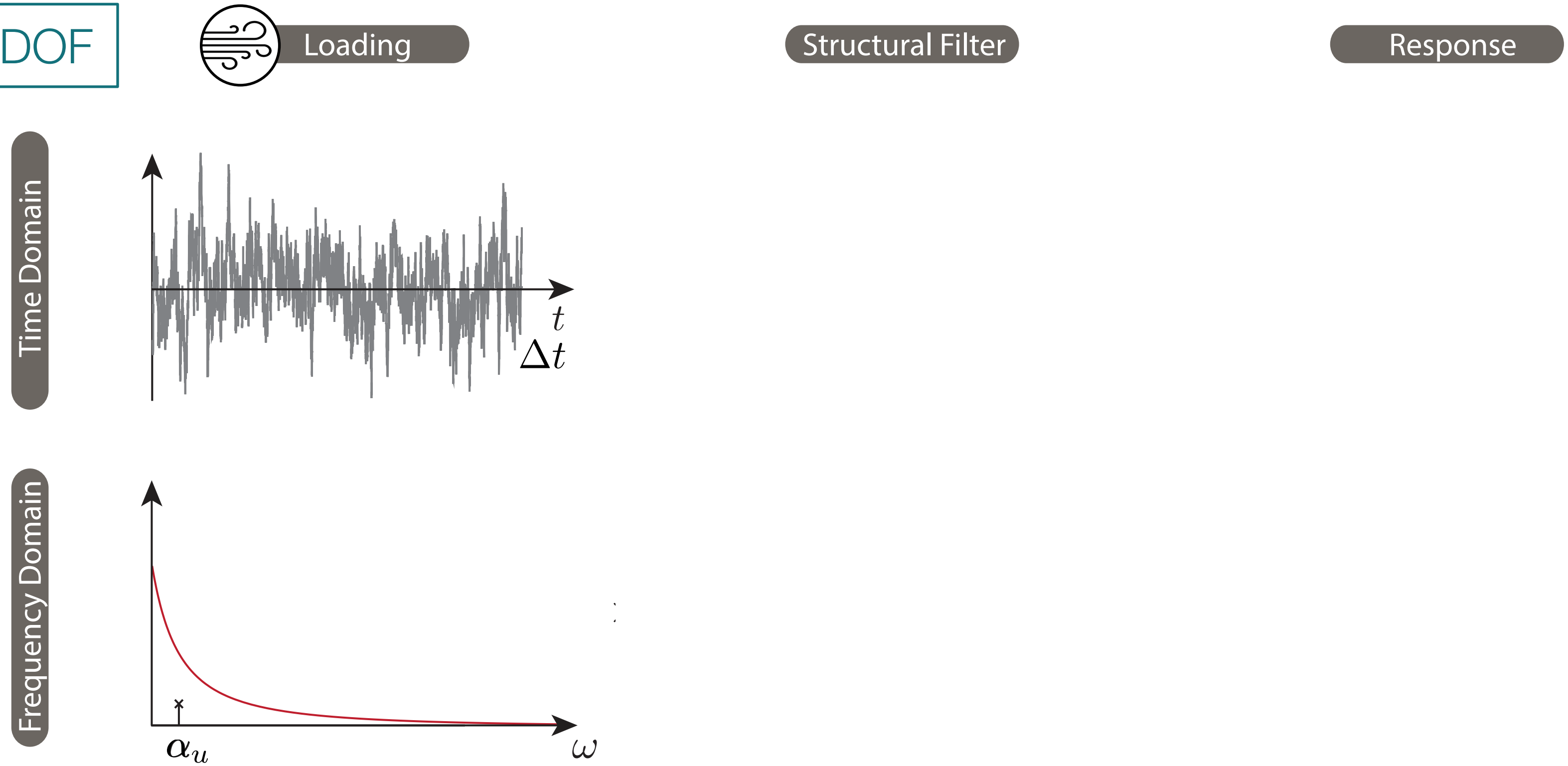
Opportunities for Cleaner Energy Production

Alternatives to Land Reclamation

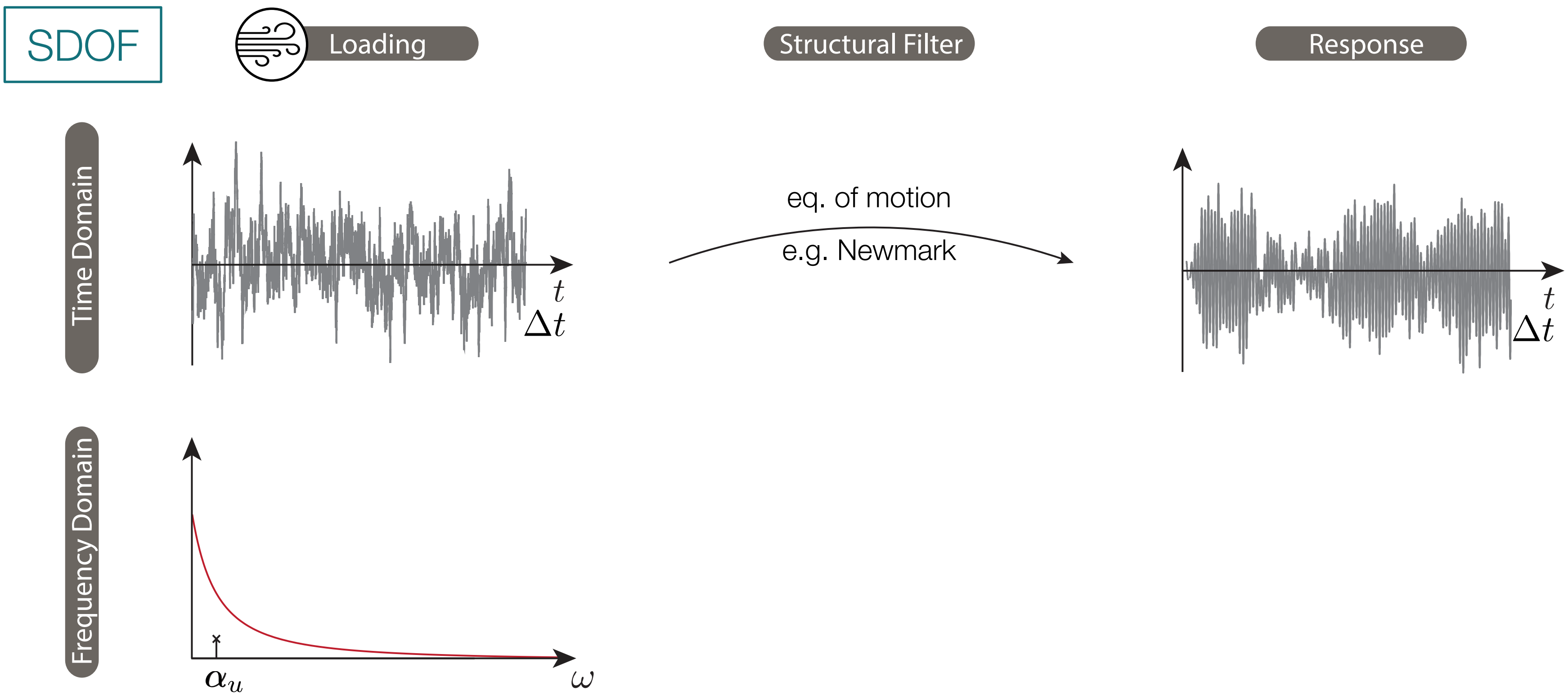
Possibilities for Crossing Large and Deep Straits

Usual analysis methods are slow due to the timescales separation.

SDOF

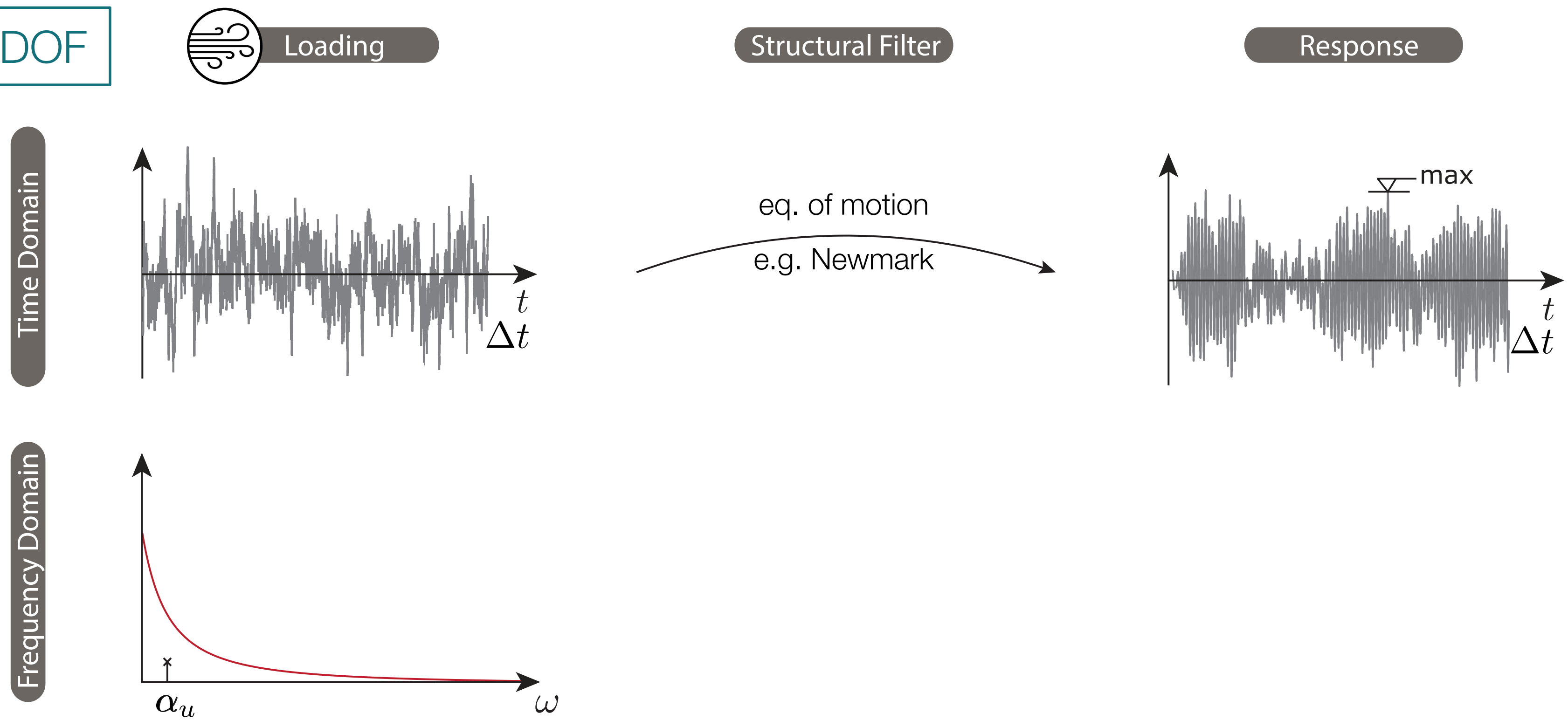


Usual analysis methods are slow due to the timescales separation.

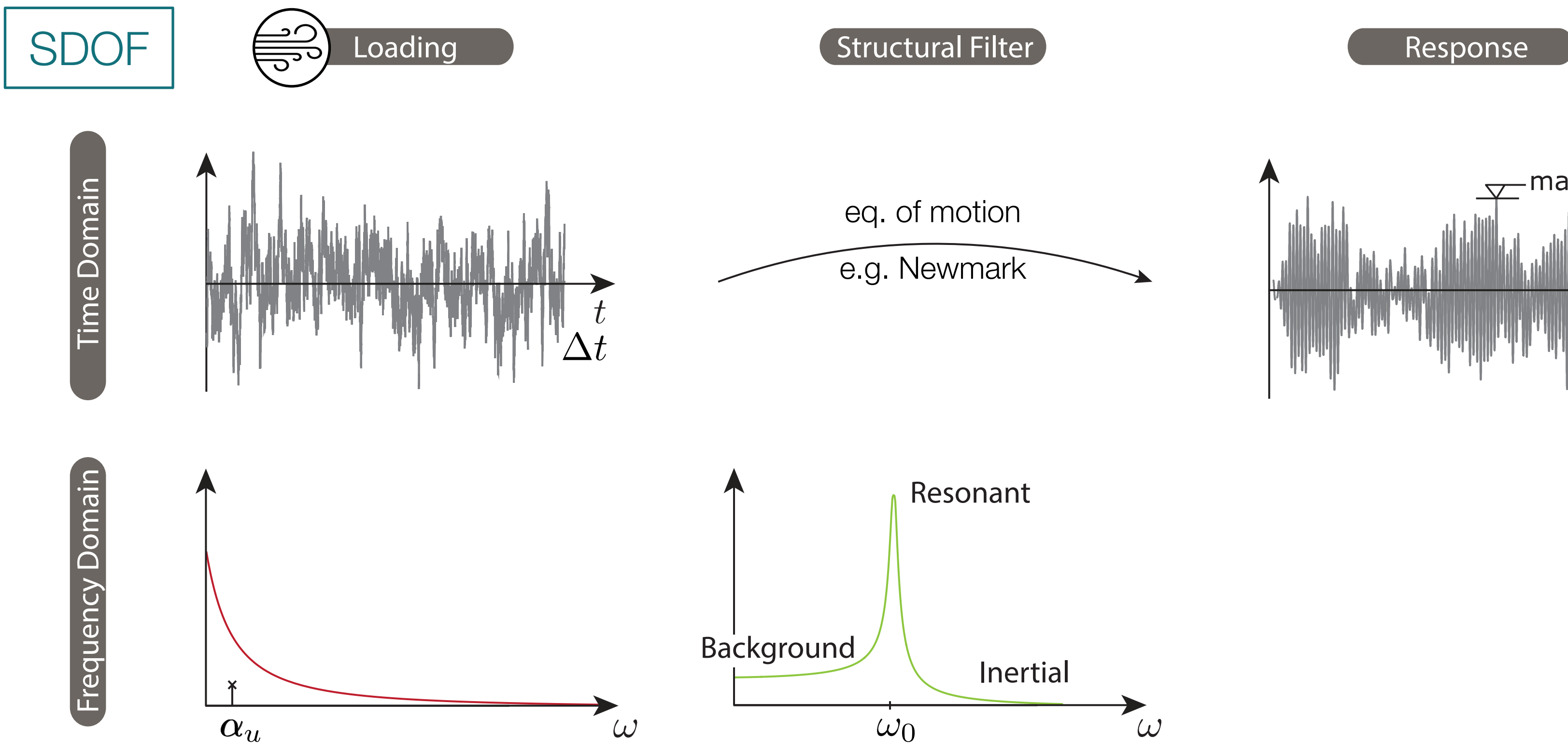


Usual analysis methods are slow due to the timescales separation.

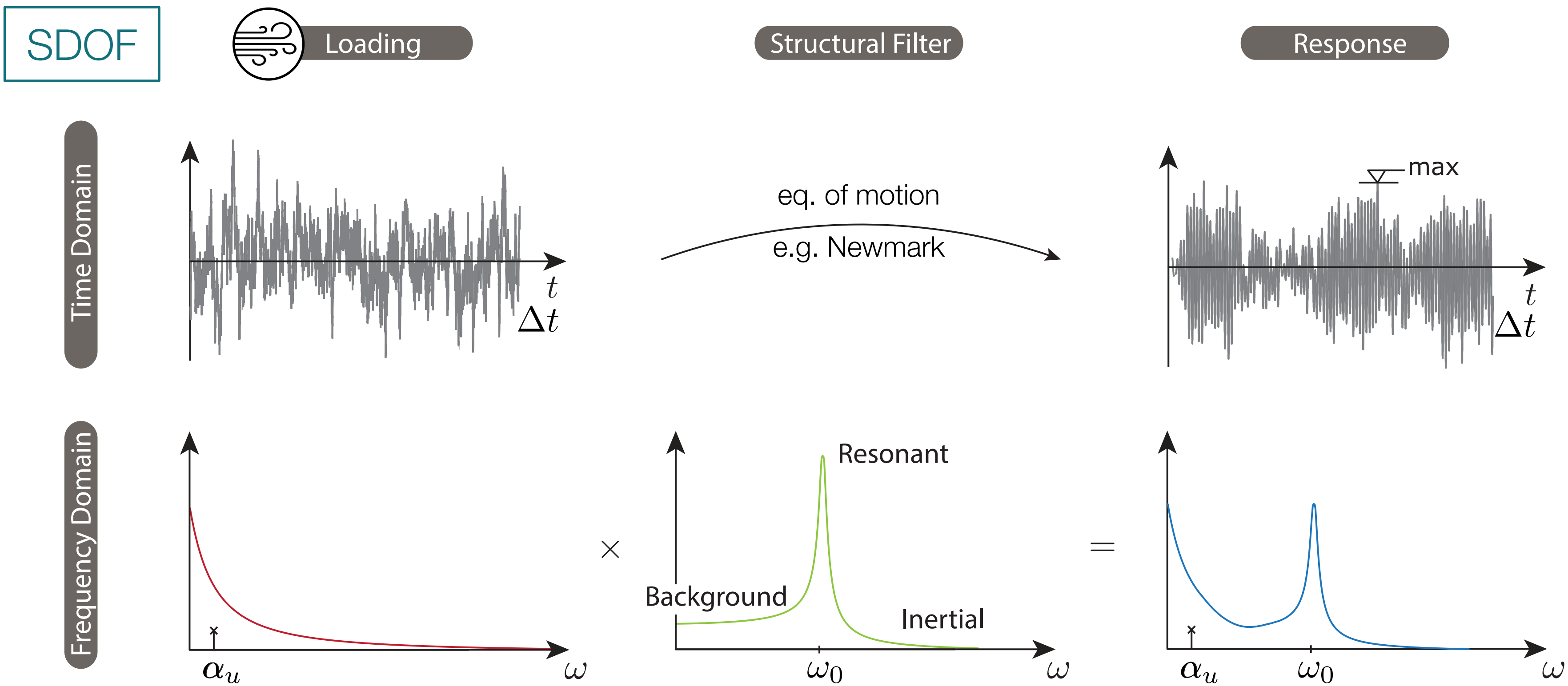
SDOF



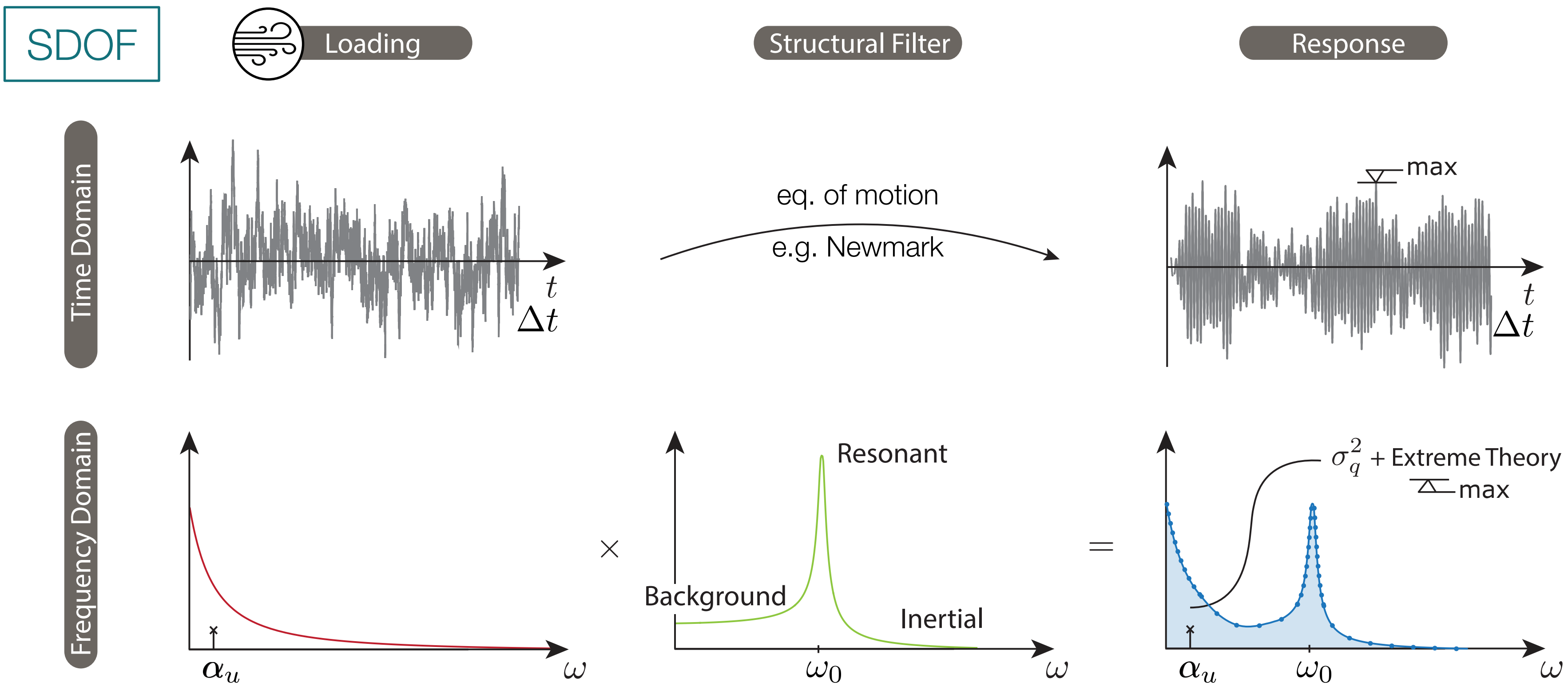
Usual analysis methods are slow due to the timescales separation.



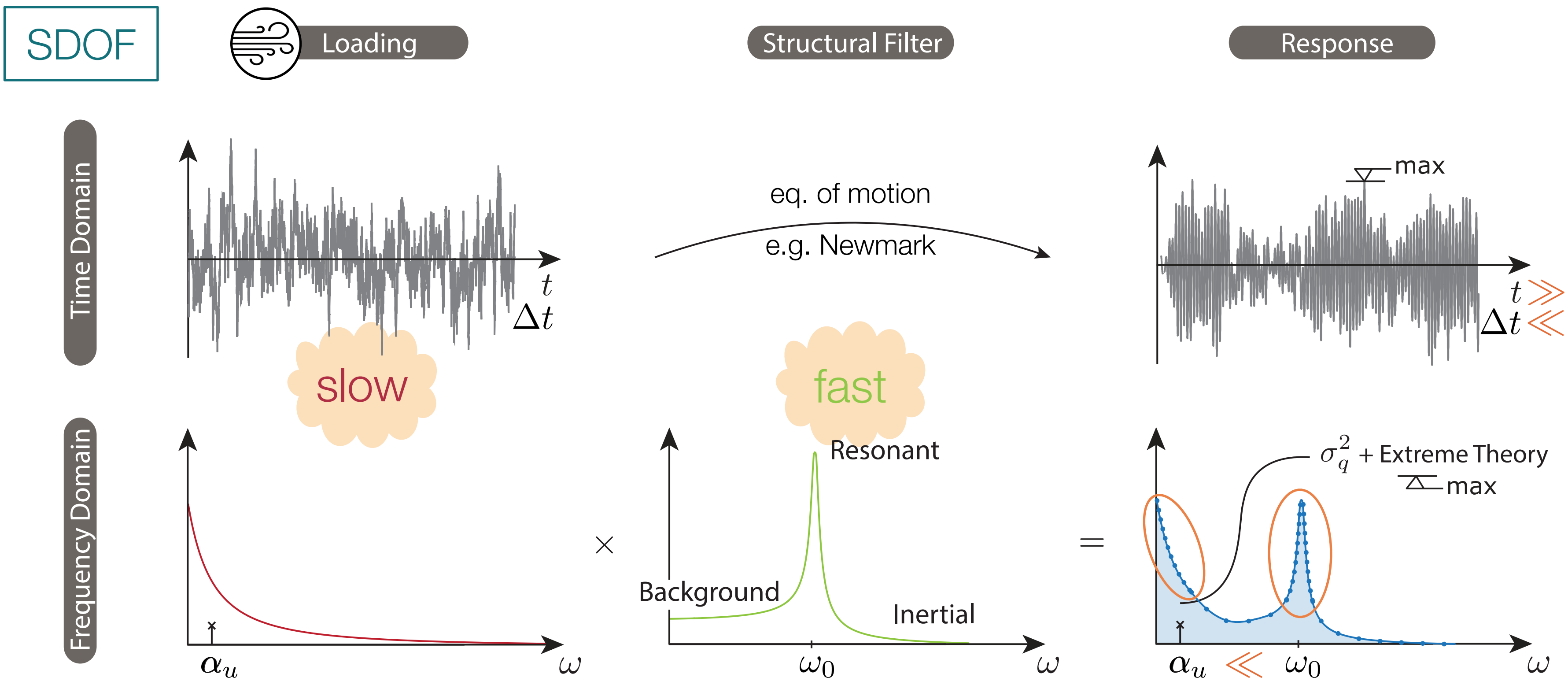
Usual analysis methods are slow due to the timescales separation.



Usual analysis methods are slow due to the timescales separation.

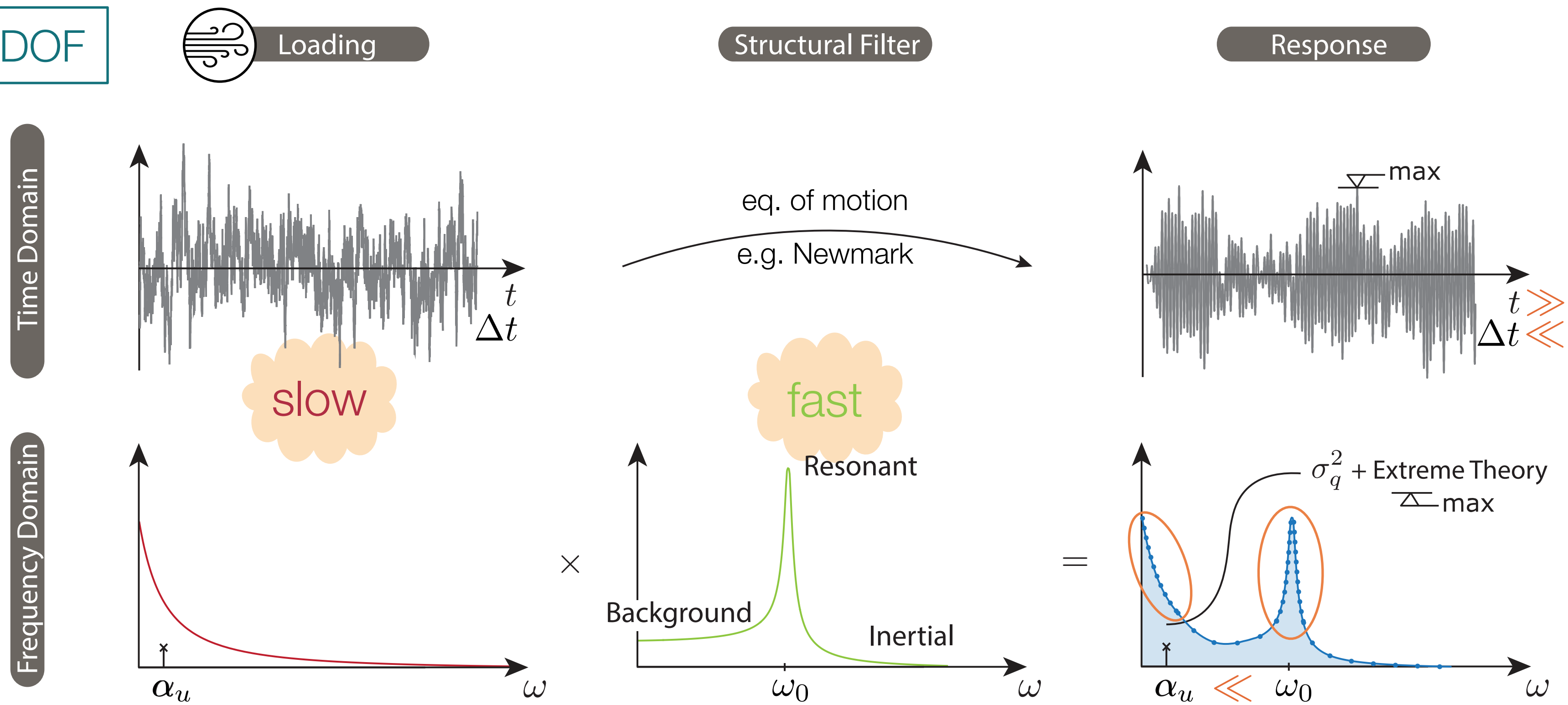


Usual analysis methods are slow due to the timescales separation.



Usual analysis methods are slow due to the timescales separation.

SDOF



MDOF

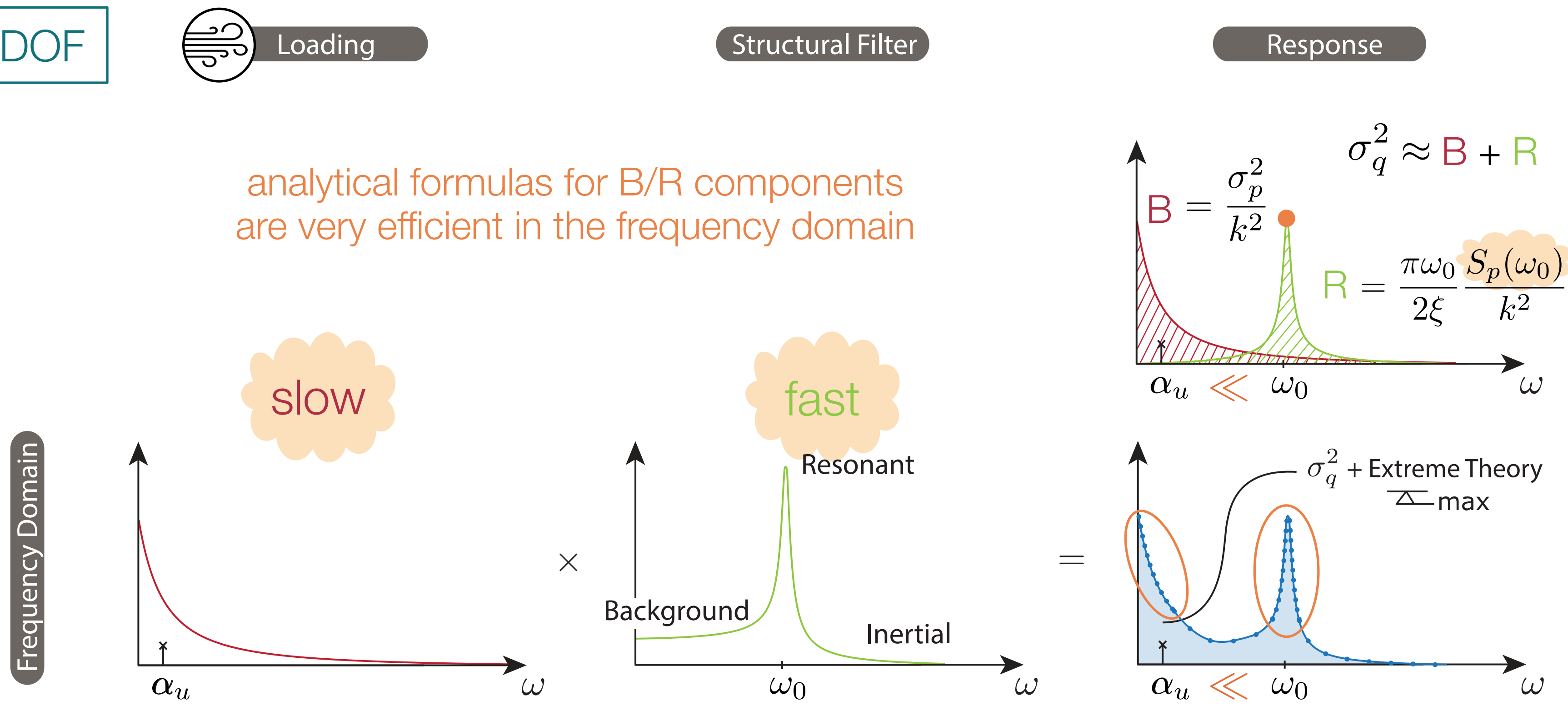
projection of a full matrix

inversion of a full matrix

at every point

Usual analysis methods are slow due to the timescales separation.

SDOF



MDOF

projection
of a
full matrix

inversion
of a
full matrix

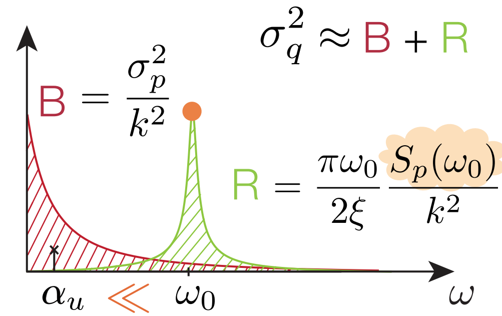
at every point

Usual analysis methods are slow due to the timescales separation.

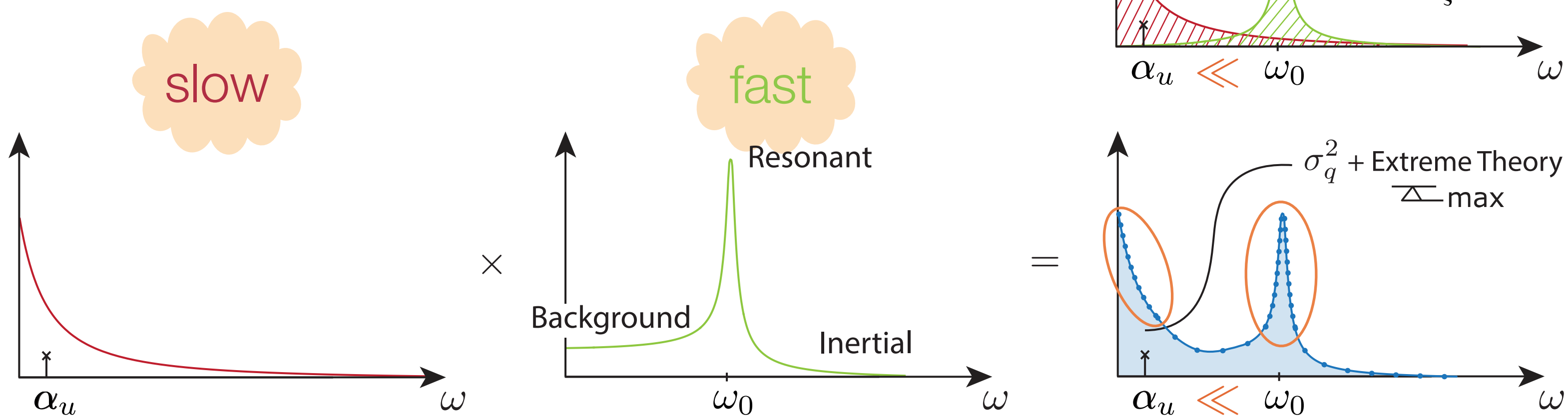
SDOF

Loading — Structural Filter — Response

analytical formulas for B/R components are very efficient in the frequency domain



Frequency Domain



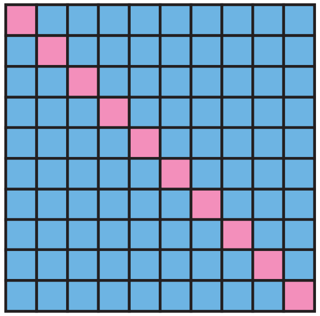
MDOF

projection of a full matrix

inversion of a DIAGONAL matrix

at ONE point/MODE if the modal responses are decoupled

The nodal variances can be computed efficiently with a complete quadratic combination scheme.

$\boldsymbol{\Sigma}_{\mathbf{q}} =$  $\approx \boldsymbol{\Sigma}_{\mathbf{B}} + \boldsymbol{\Sigma}_{\mathbf{R}}$

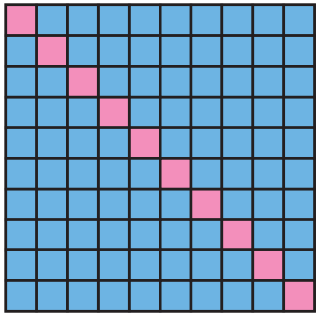
SRSS

CQC

$$\sigma_{x,i}^2 = \sum_{\substack{m=1 \\ n=m}}^{M} \phi_{im}^2 \sigma_{q,m}^2 + \sum_{m=1}^{M} \sum_{\substack{n=1 \\ n \neq m}}^{M} \phi_{im} \phi_{in} \sigma_{q,mn}$$

The nodal variances can be computed efficiently with a complete quadratic combination scheme.

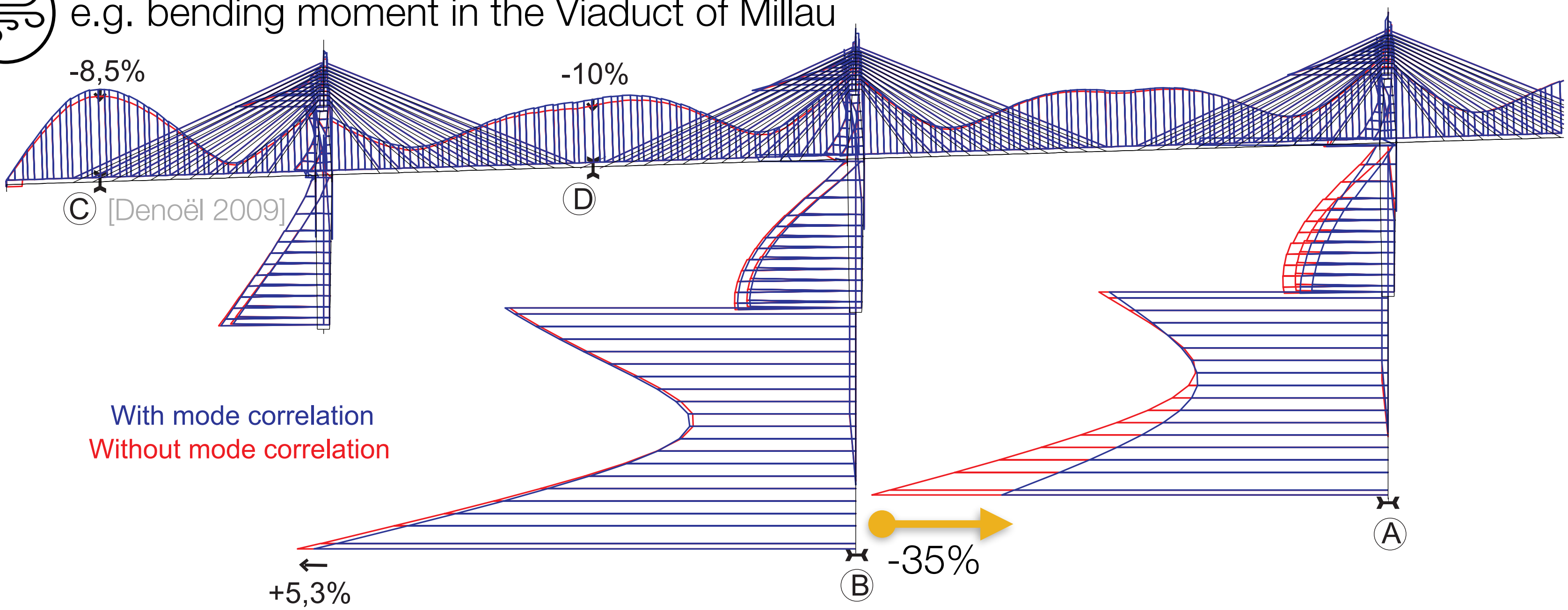
$$\mathbf{\Sigma_q} = \text{[matrix]} \approx \mathbf{\Sigma_B} + \mathbf{\Sigma_R}$$



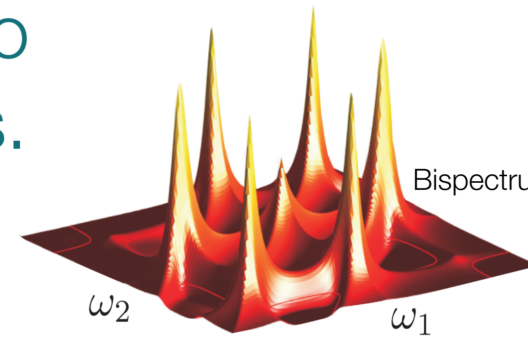
SRSS / CQC

$$\sigma_{x,i}^2 = \sum_{\substack{m=1\\n=m}}^{M} \phi_{im}^2 \sigma_{q,m}^2 + \sum_{m=1}^{M} \sum_{\substack{n=1\\n\neq m}}^{M} \phi_{im}\phi_{in}\sigma_{q,mn}$$

e.g. bending moment in the Viaduct of Millau



The extension at third order gave birth to the Multiple Timescale Spectral Analysis.



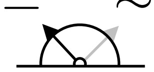
- It is mathematically justified with the perturbation theory.
- It is based on the existence of sharp and distinct peaks.
- It generalizes B/R to crossed and higher-order statistics.
- It provides simple and efficient analytical approximations.

SIMPLICITY

$\rightarrow$ 1 + 1 = 2

ACCURACY

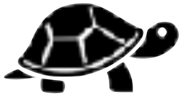
$=$ $\rightarrow$



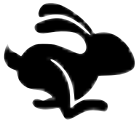
$\simeq$ $\approx$

$\varepsilon \ll 1$

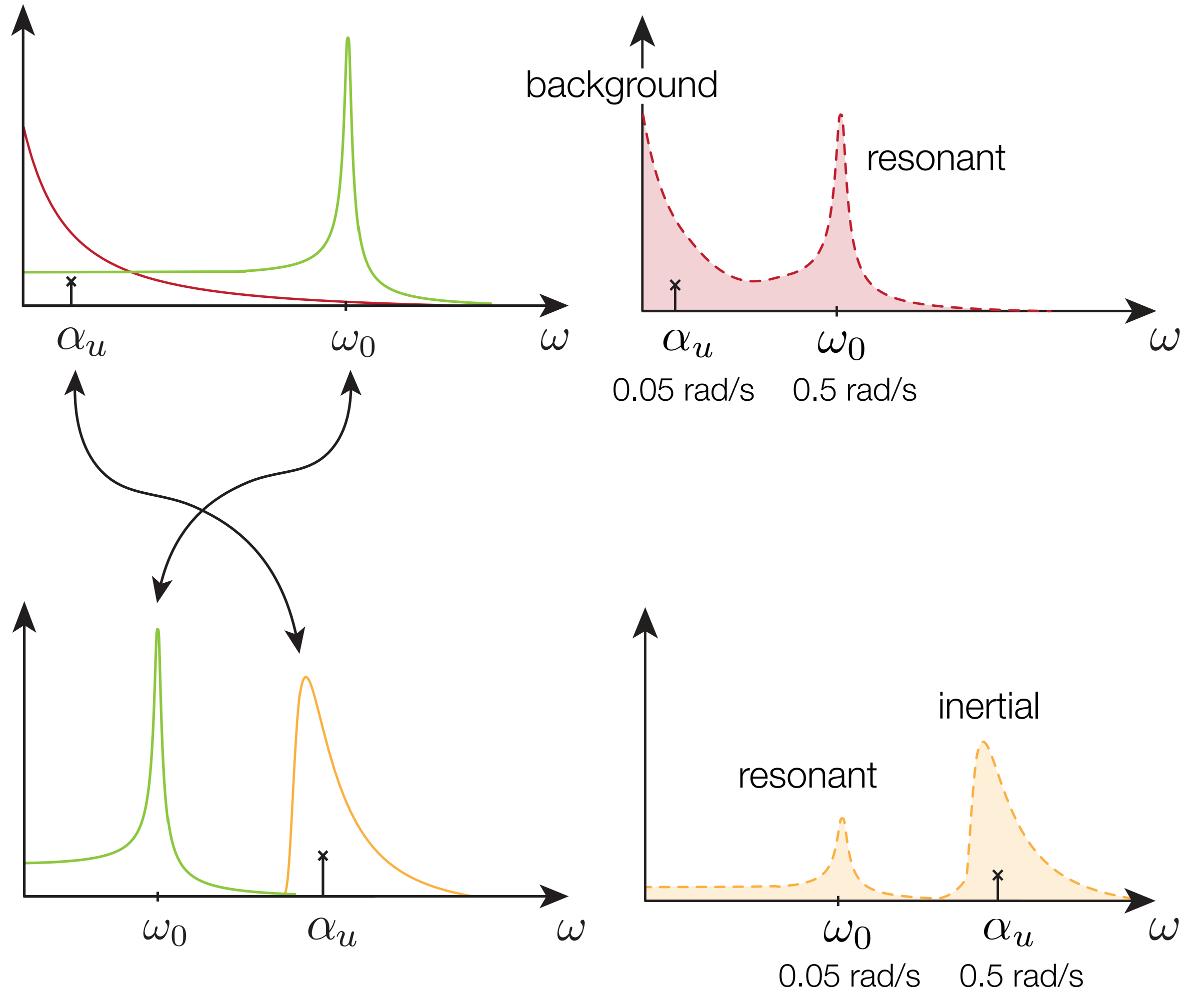
RAPIDITY



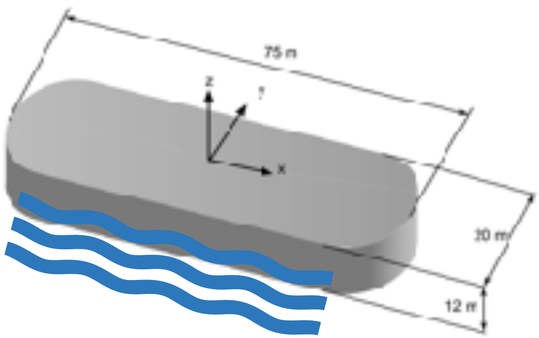
$\rightarrow$



What if the excitation triggers the inertial regime ?

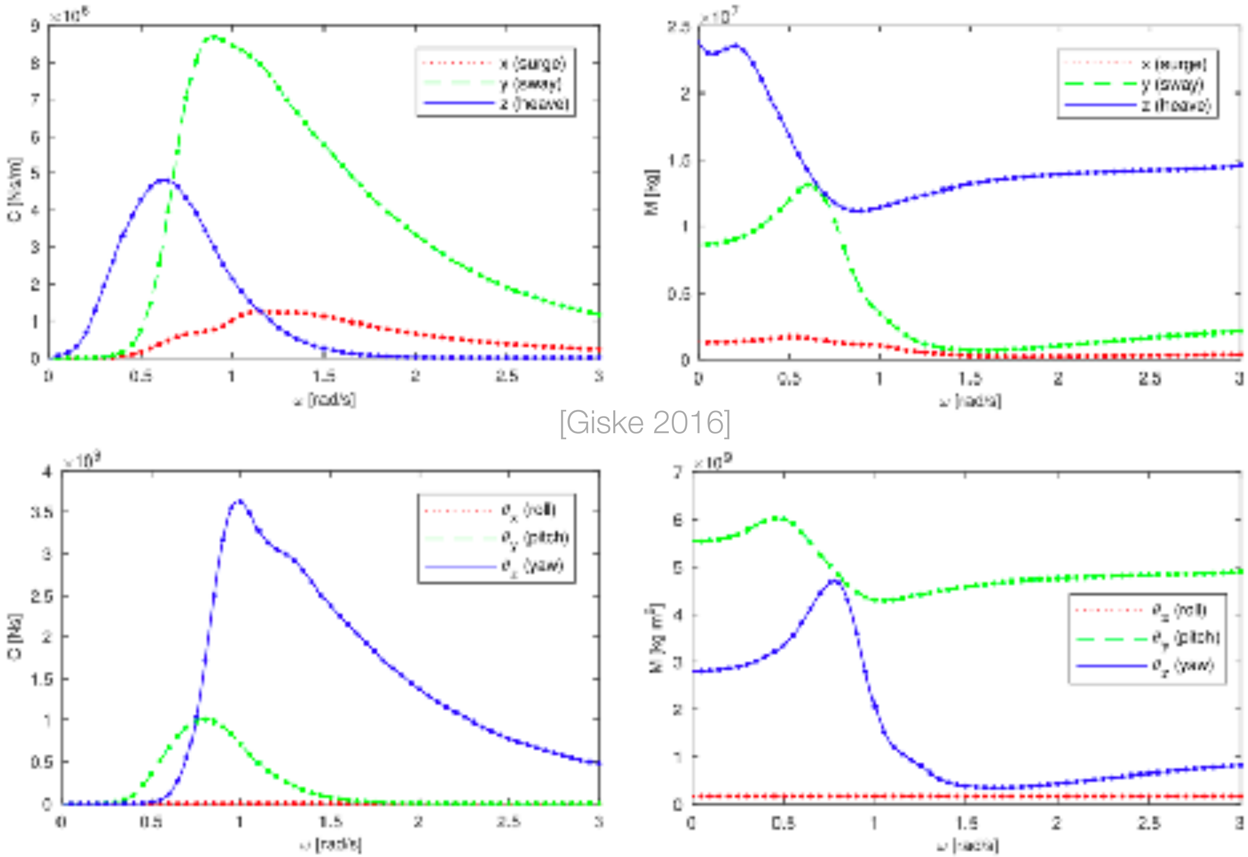


What if the hydroelastic effects are considered ?



typical added damping

typical added mass



[Giske 2016]

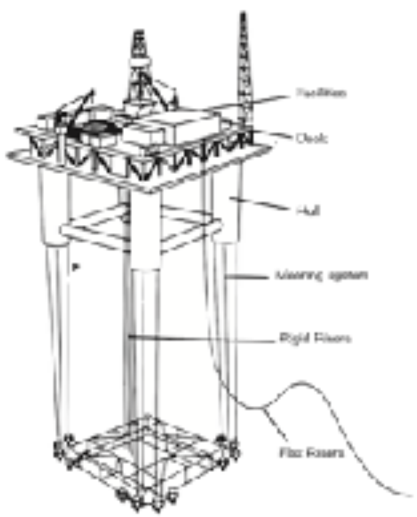
neither classical, nor negligible

and also frequency-dependent

SDOF

What about the inertial component ?

1) Theoretical Novelties
2) Minimalistic Example



MDOF

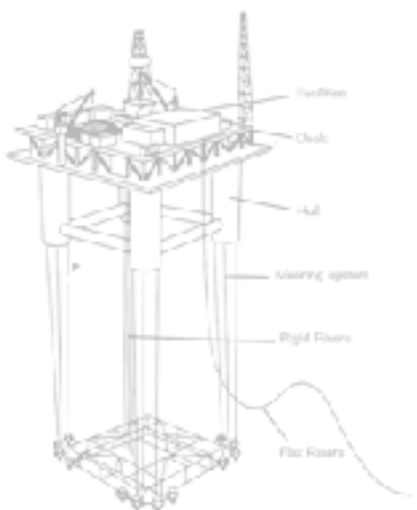
What about the hydroelastic effects ?

3) Theoretical Novelties
4) Realistic Applications

SDOF

What about the inertial component ?

1) Theoretical Novelties
2) Minimalistic Example

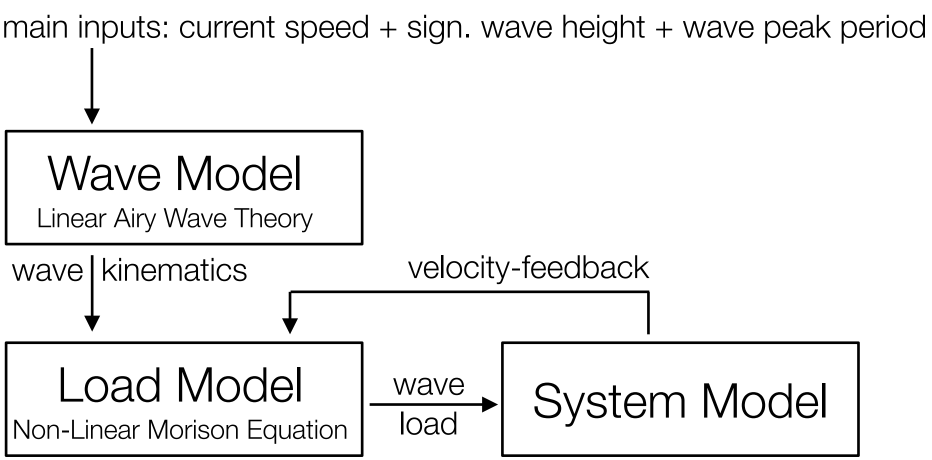


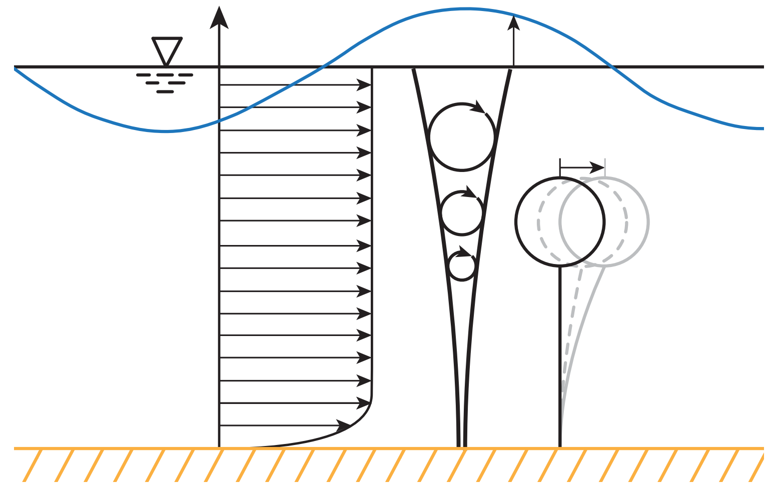
MDOF

What about the hydroelastic effects ?

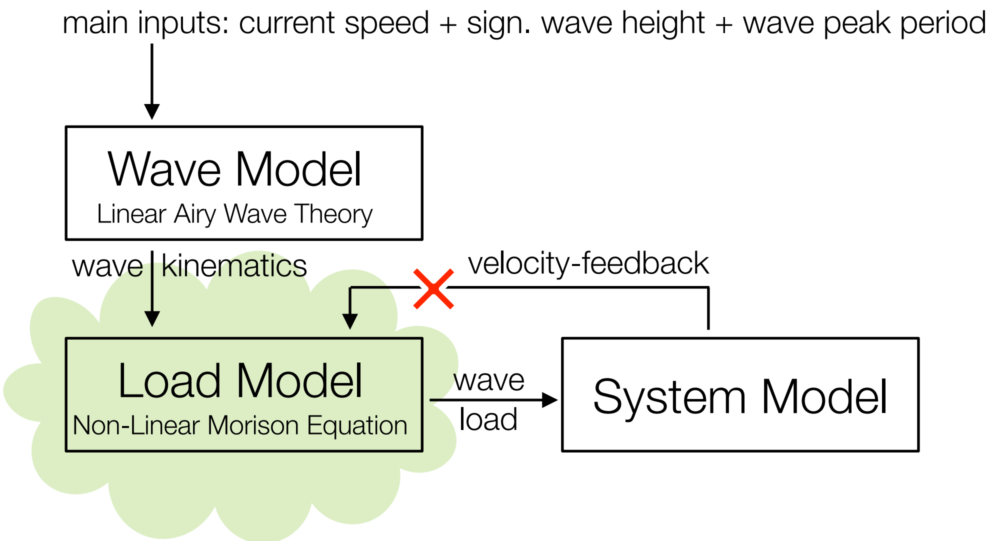
3) Theoretical Novelties
4) Realistic Applications

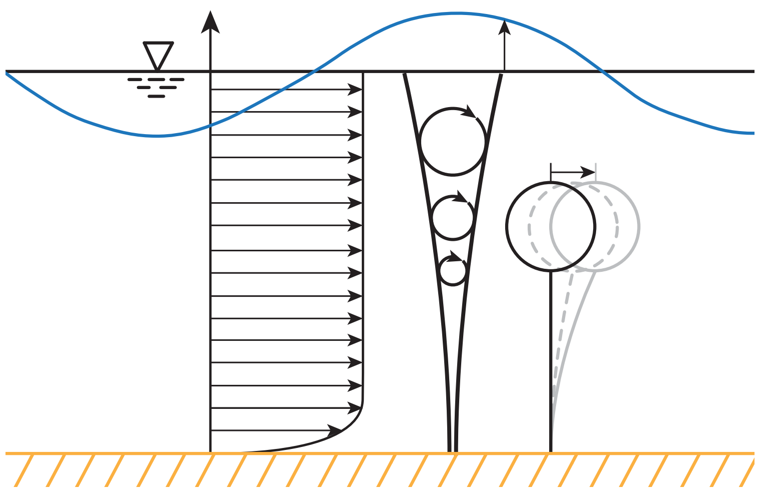
The loading process is quadratized to express its spectrum more easily.

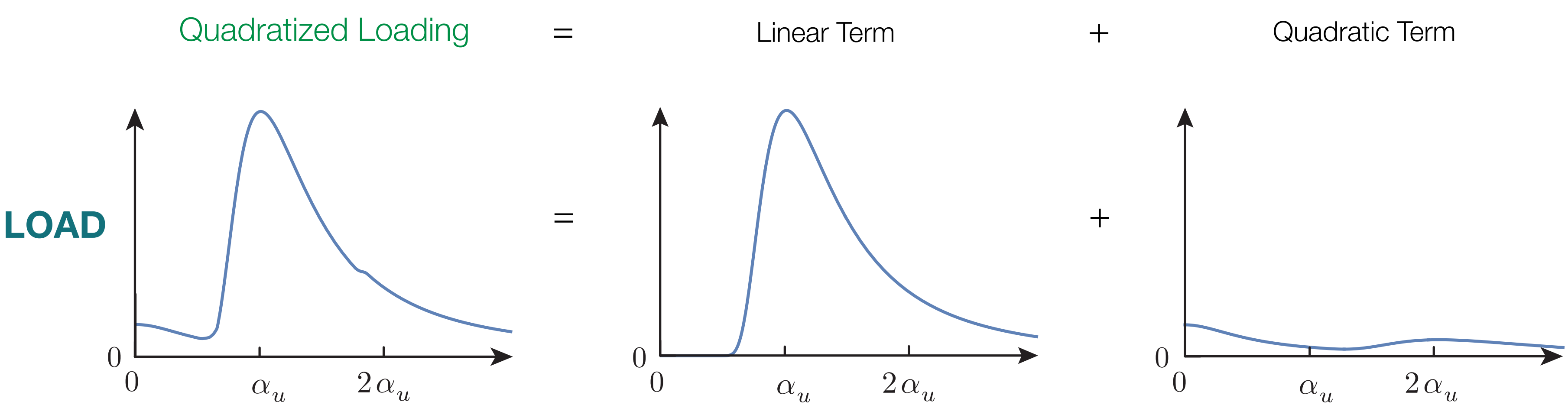




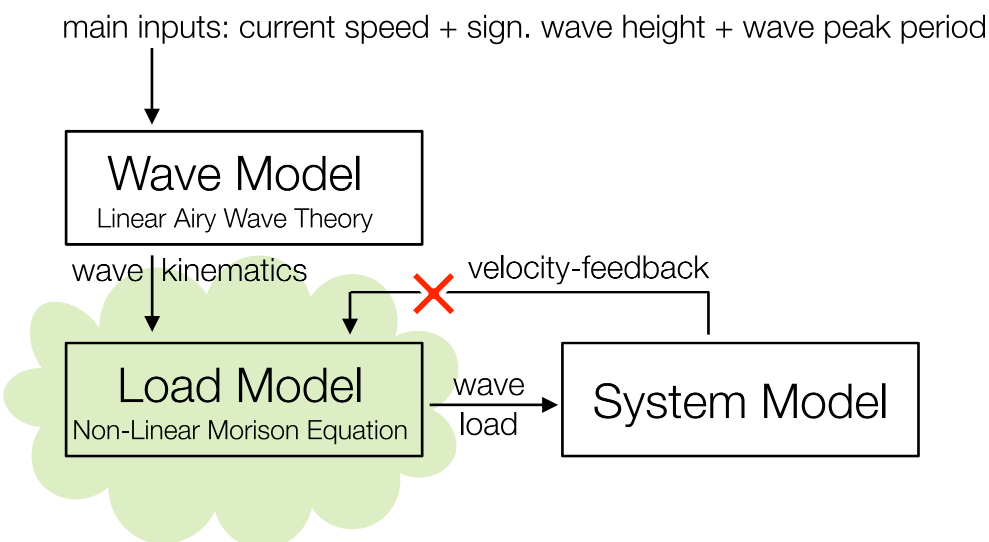
The loading process is quadratized to express its spectrum more easily.

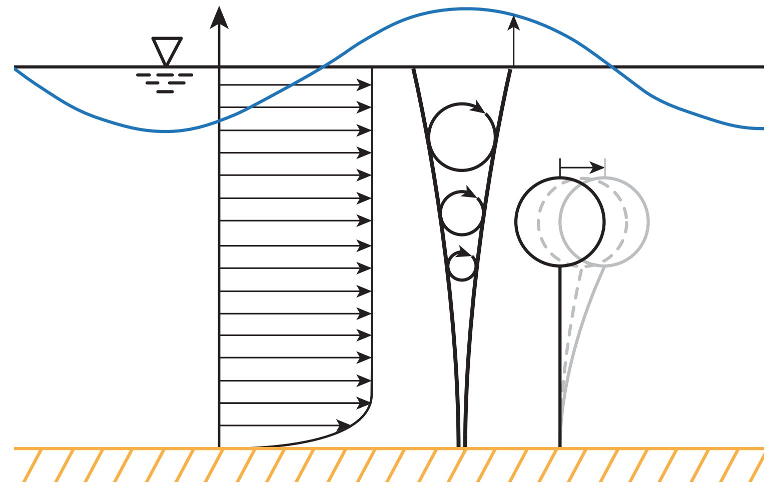


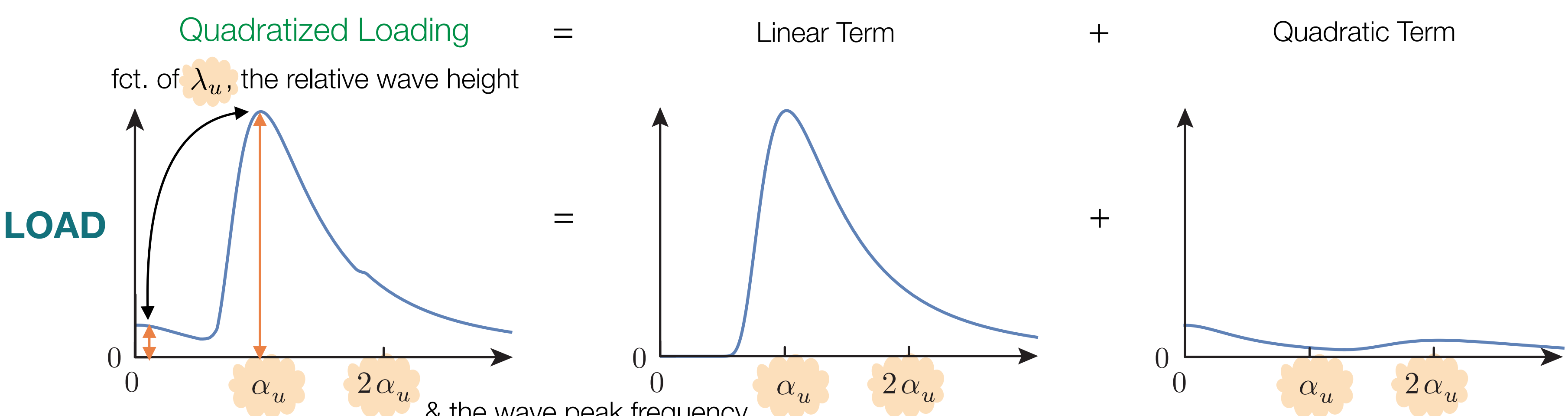




The loading process is quadratized to express its spectrum more easily.



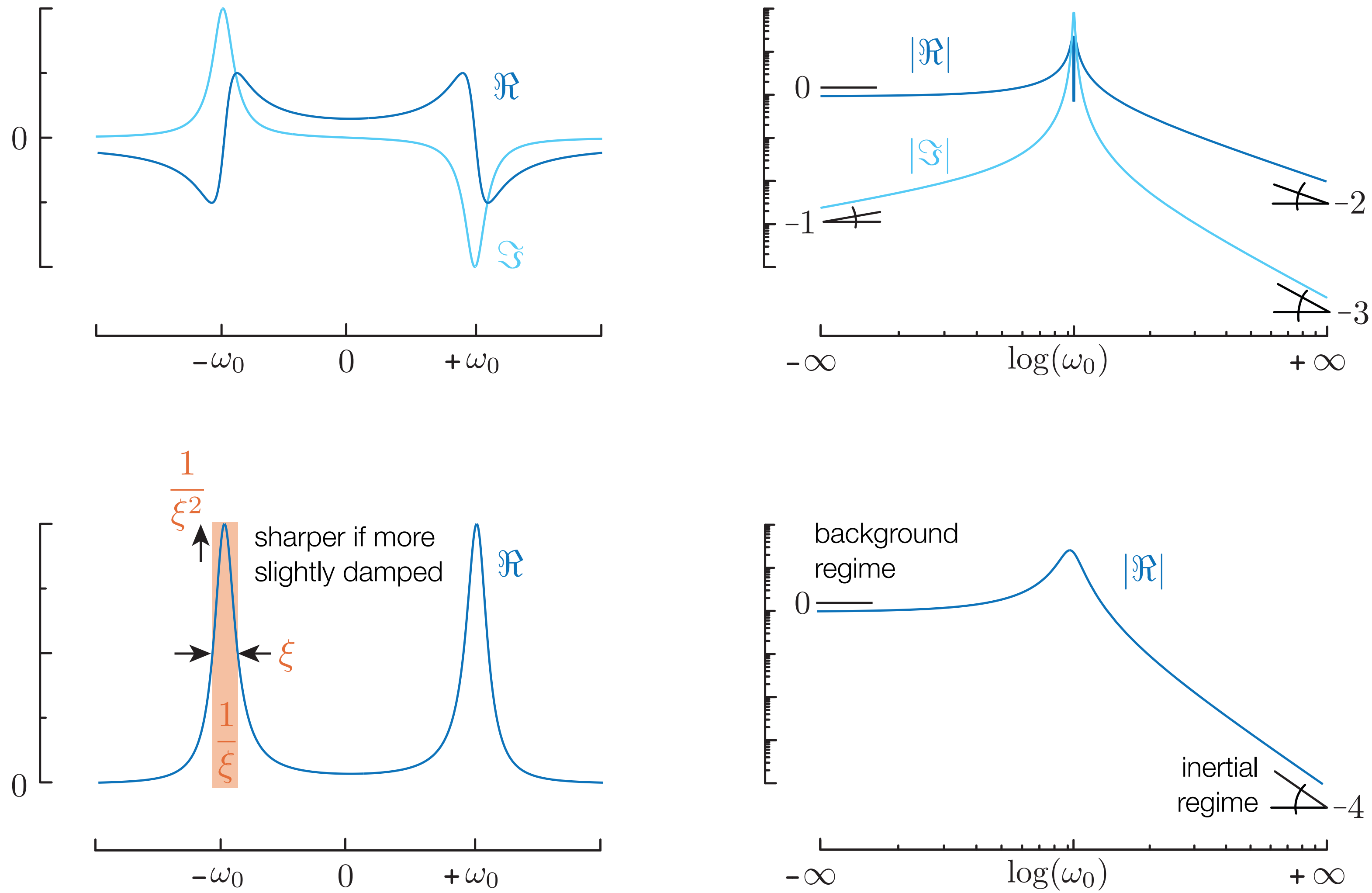




The structural kernel is characterized by four poles + two monomial trends.

FRF

|FRF|^2



Multiple Timescale Spectral Analysis
Objective : find a unified formulation.

"slow" load

"fast" load

LOADING

x

KERNEL

=

RESPONSE

Multiple Timescale Spectral Analysis
Objective : find a unified formulation.

"slow" load

"fast" load

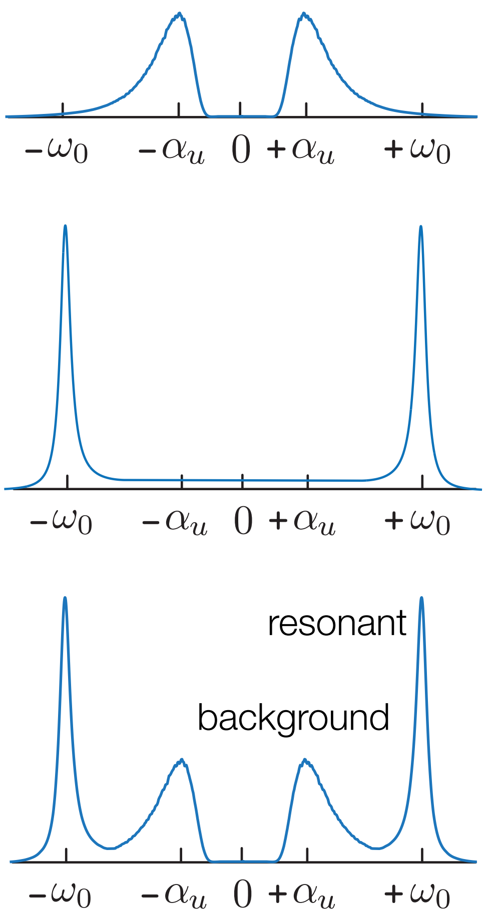
LOADING

x

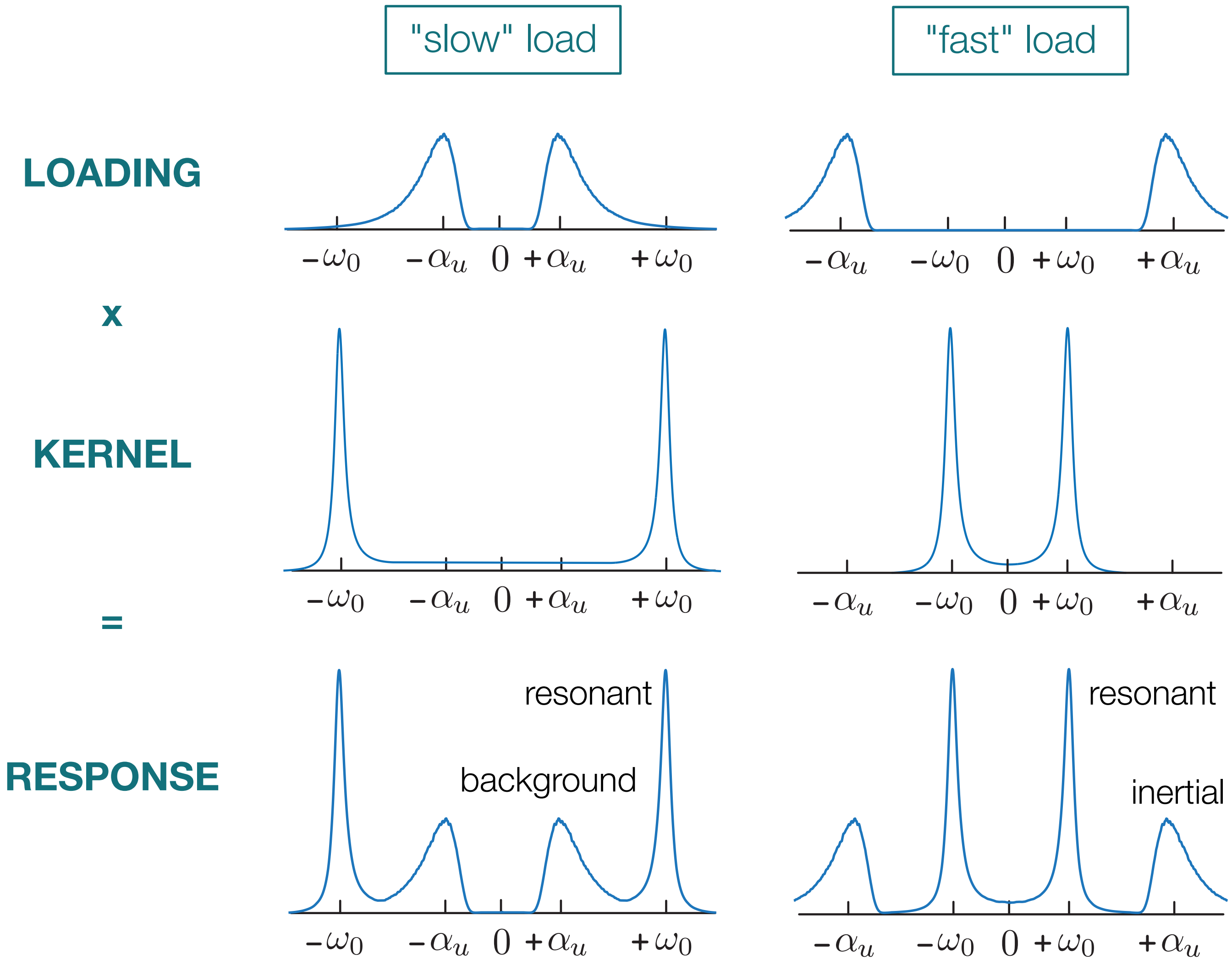
KERNEL

=

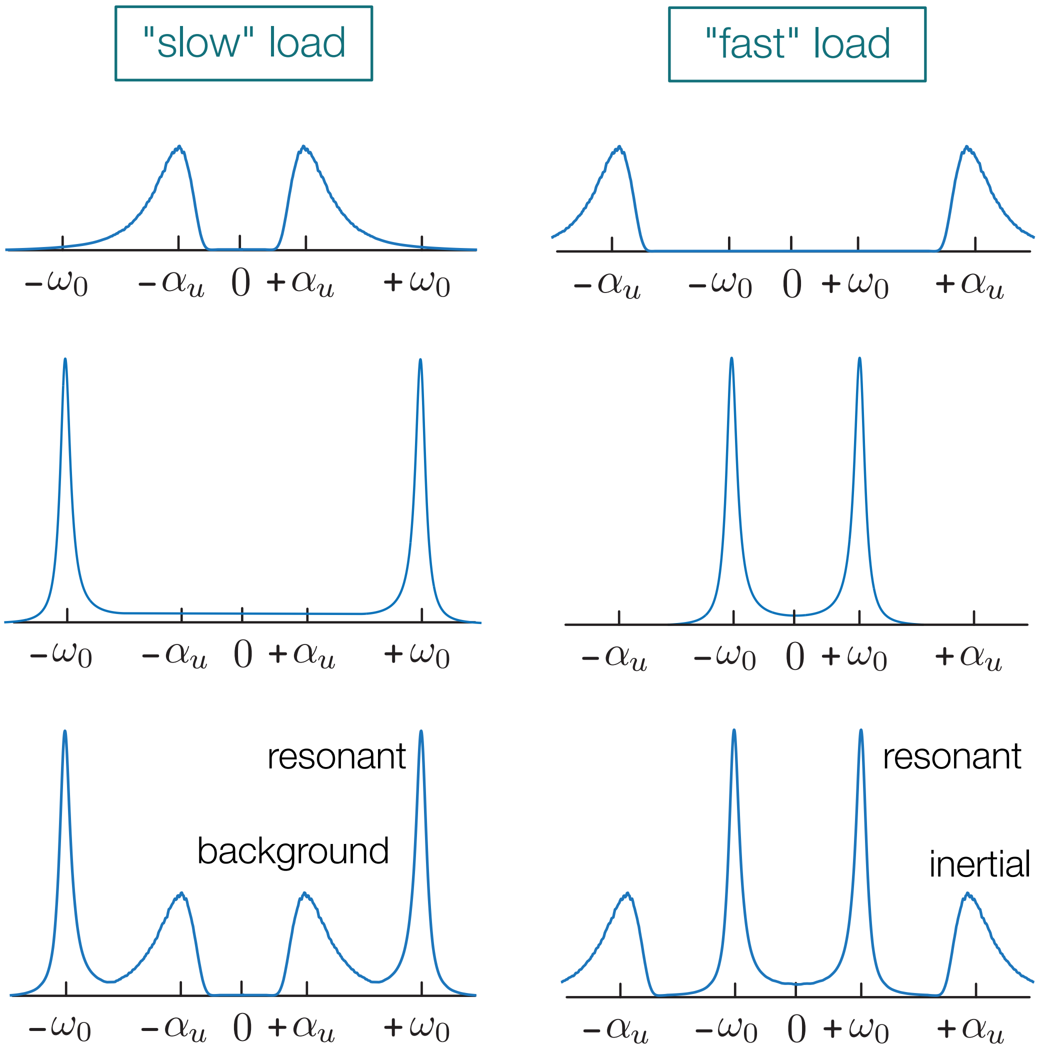
RESPONSE



Multiple Timescale Spectral Analysis
Objective : find a unified formulation.



Multiple Timescale Spectral Analysis
Objective : find a unified formulation.



LOADING

x

KERNEL

=

RESPONSE

IF

cst across both peaks of the structural kernel

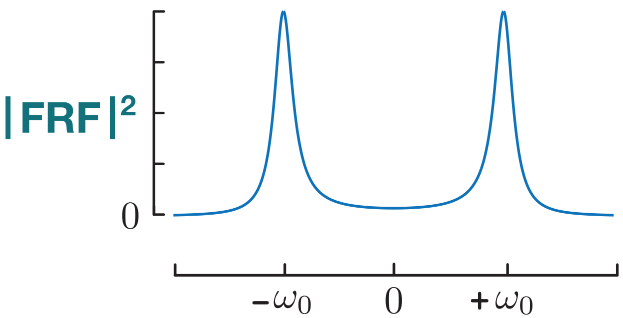
&

natural frequency far from peak frequency

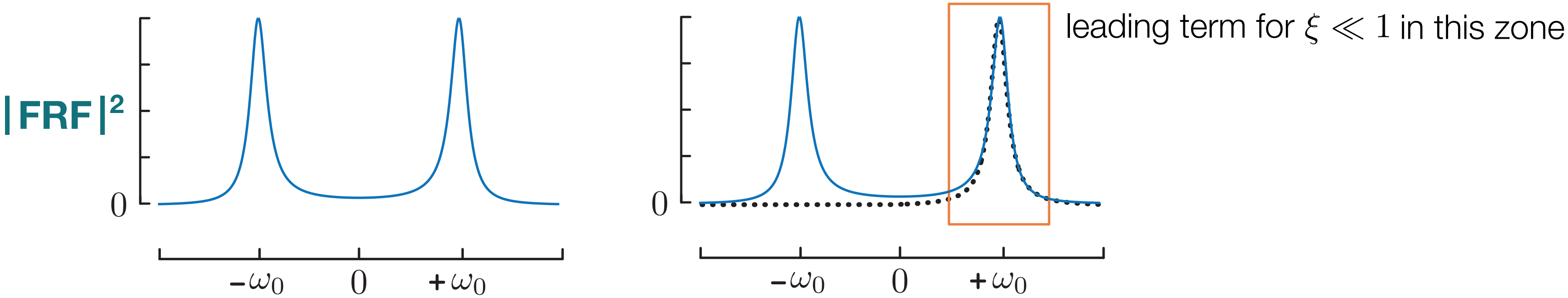
THEN

sharp & distinct peaks thus OK to use MTSA

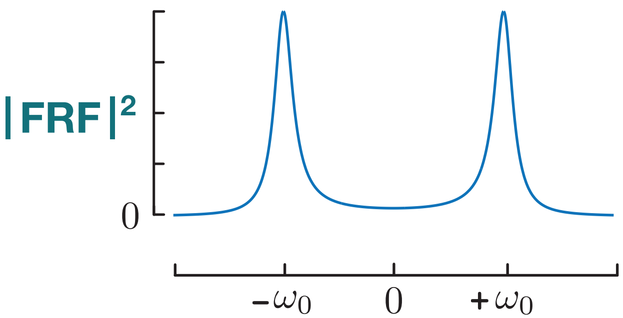
It is usual to approximate locally both peaks in the kernel to separate them.

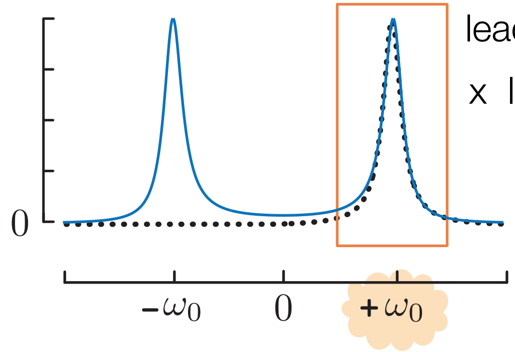


It is usual to approximate locally both peaks in the kernel to separate them.



It is usual to approximate locally both peaks in the kernel to separate them.

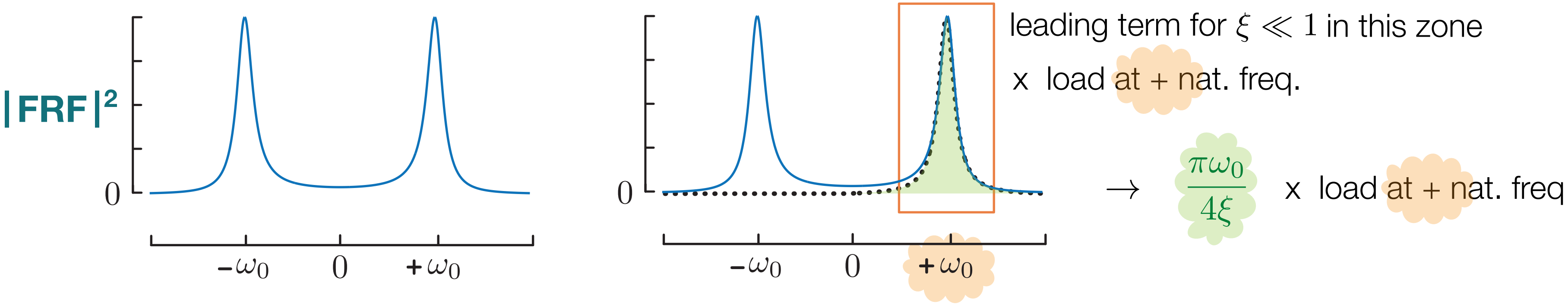




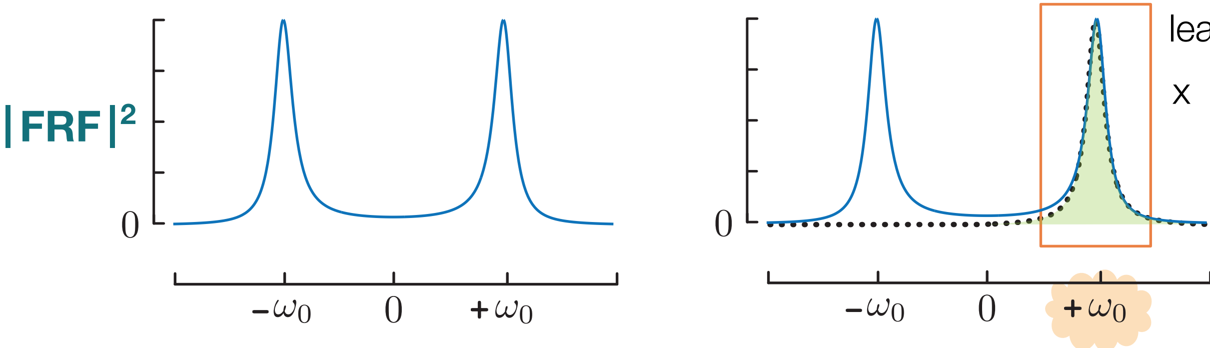
leading term for $\xi \ll 1$ in this zone

x load at + nat. freq.

It is usual to approximate locally both peaks in the kernel to separate them.



It is usual to approximate locally both peaks in the kernel to separate them.



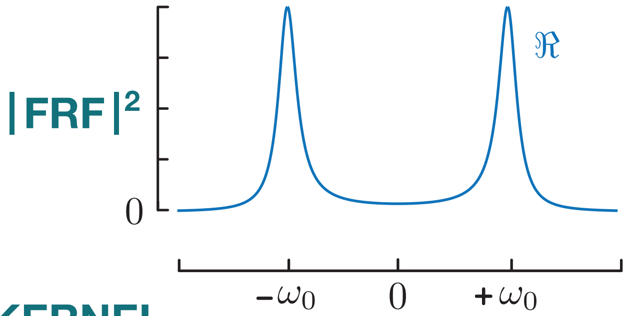
leading term for $\xi \ll 1$ in this zone

x load at + nat. freq.

$\rightarrow \dfrac{\pi\omega_0}{4\xi}$ x load at + nat. freq.

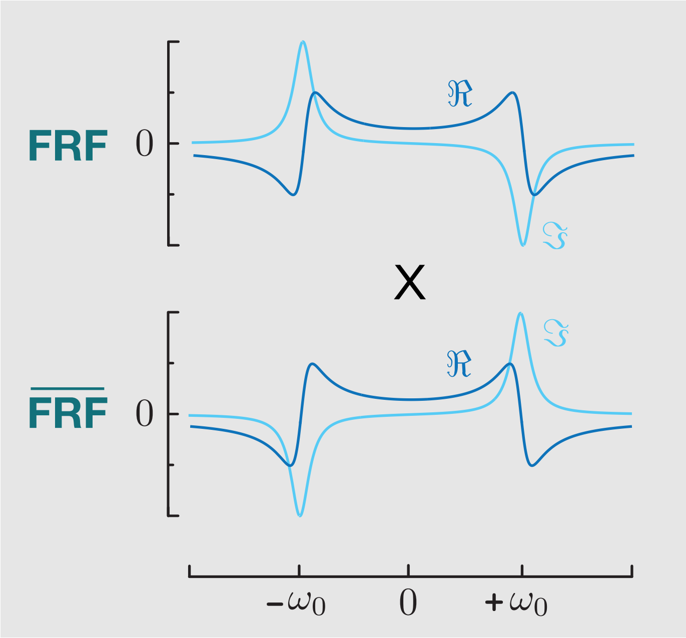
x 2 due to symmetry

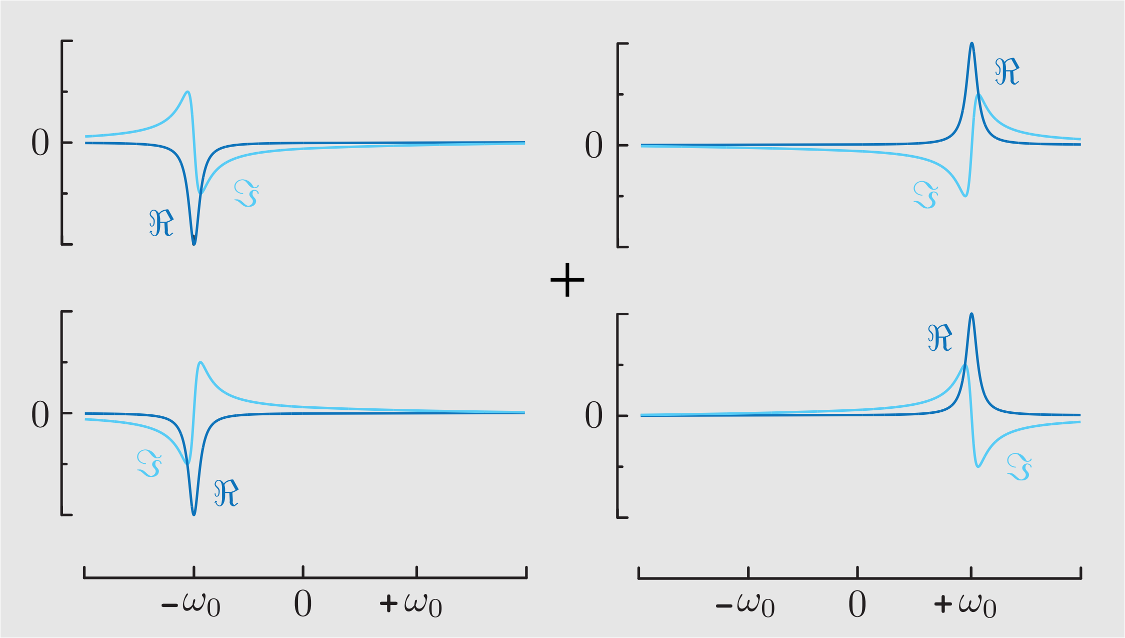
A partial fraction expansion of the kernel can separate the peaks in an exact way.



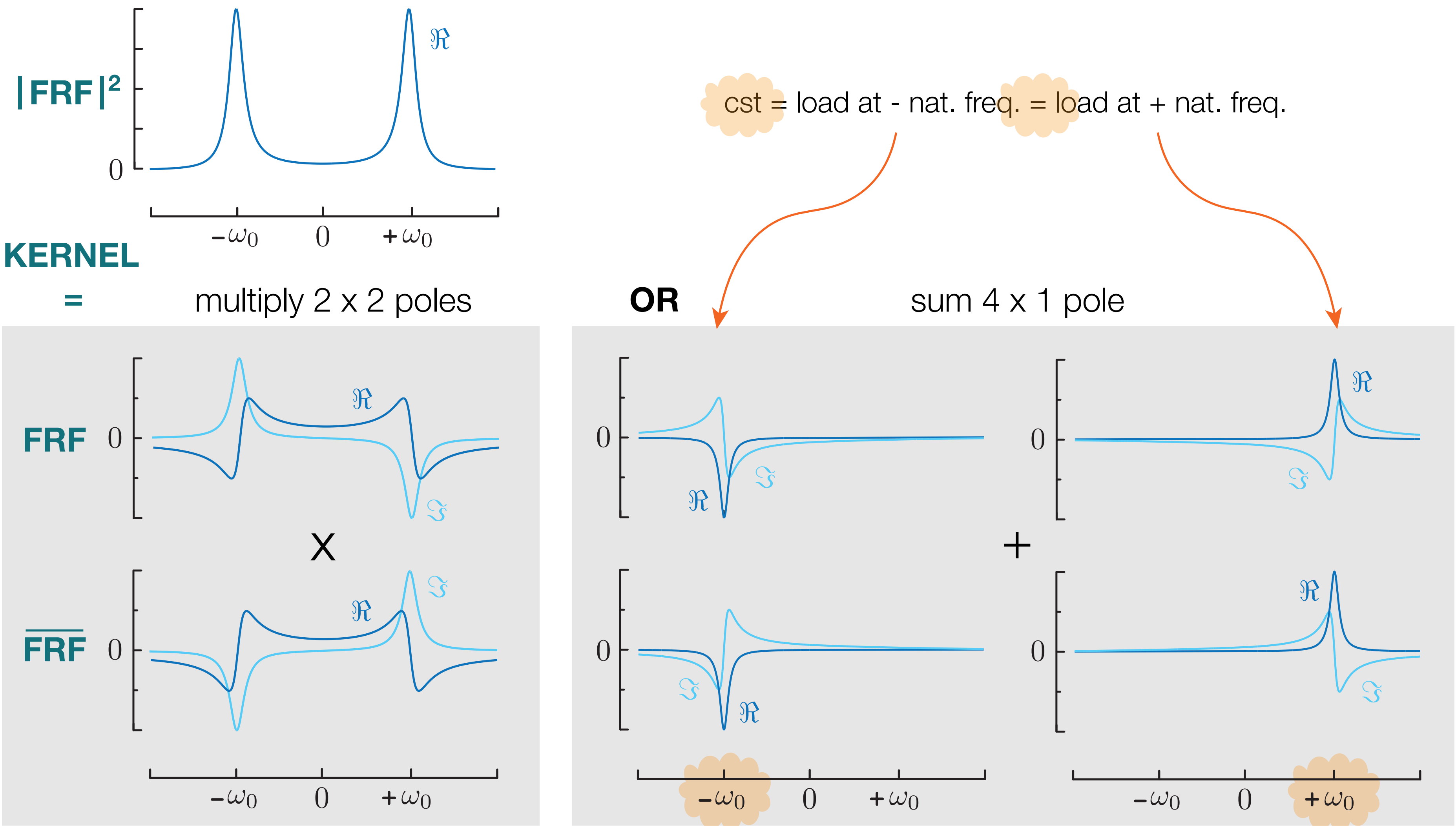
KERNEL

= multiply 2 x 2 poles **OR** sum 4 x 1 pole

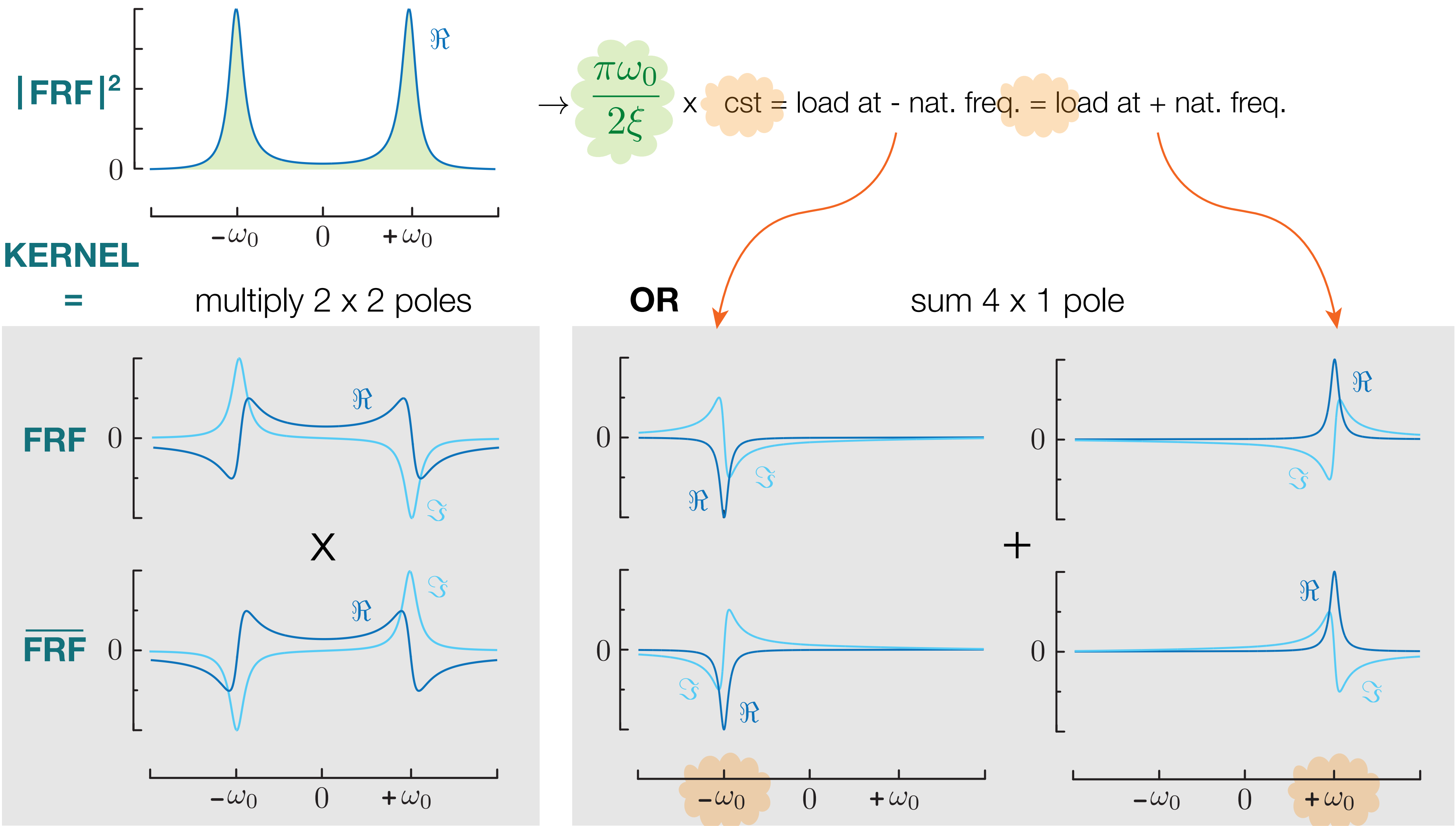




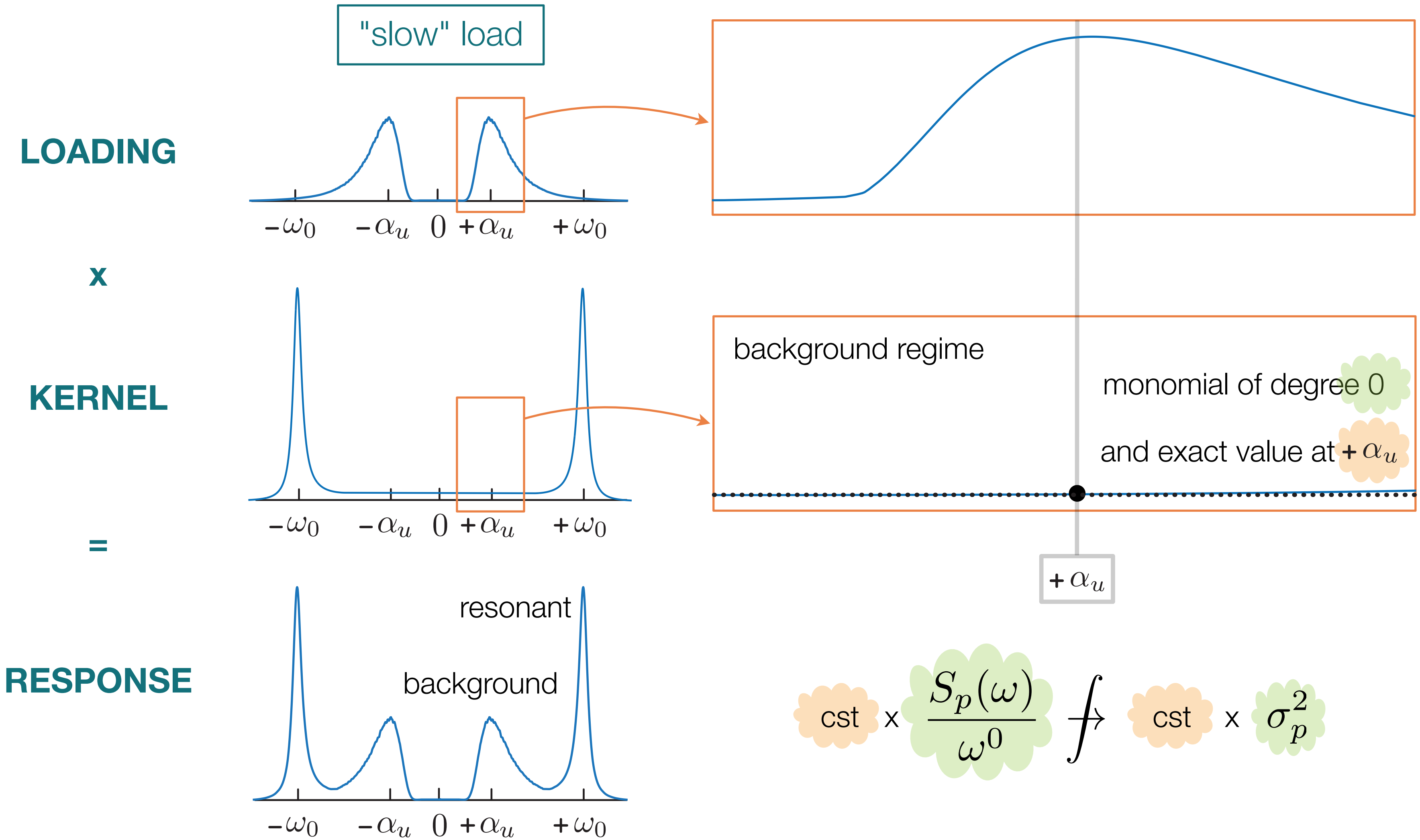
A partial fraction expansion of the kernel can separate the peaks in an exact way.



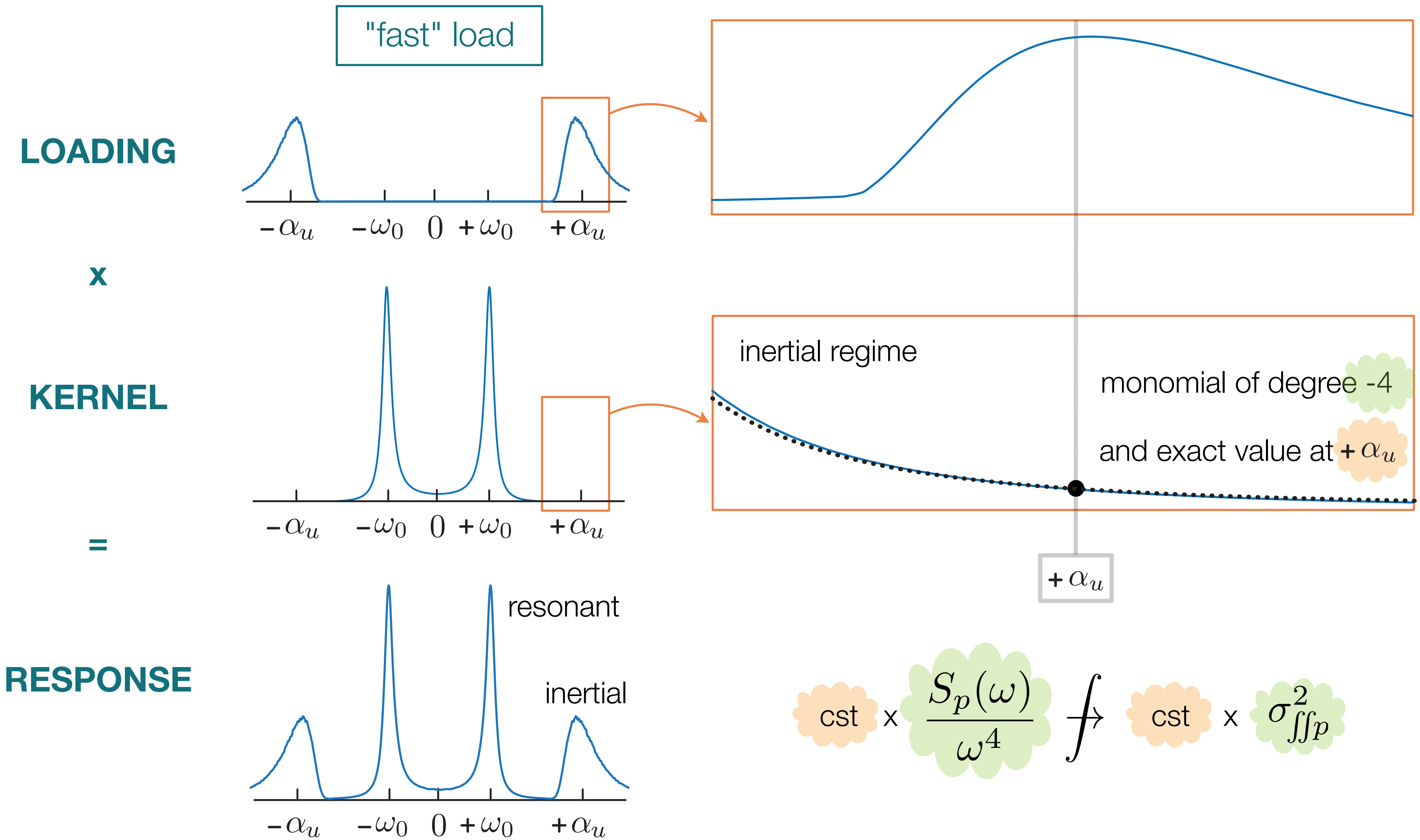
A partial fraction expansion of the kernel can separate the peaks in an exact way.



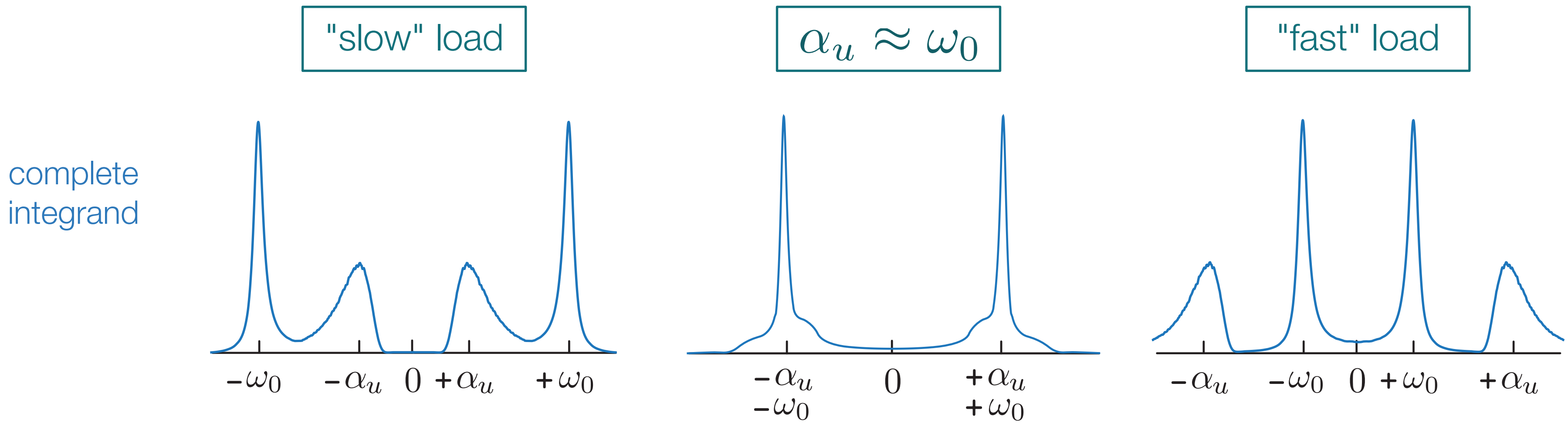
The loading components differ by the degree of the monomial trends.



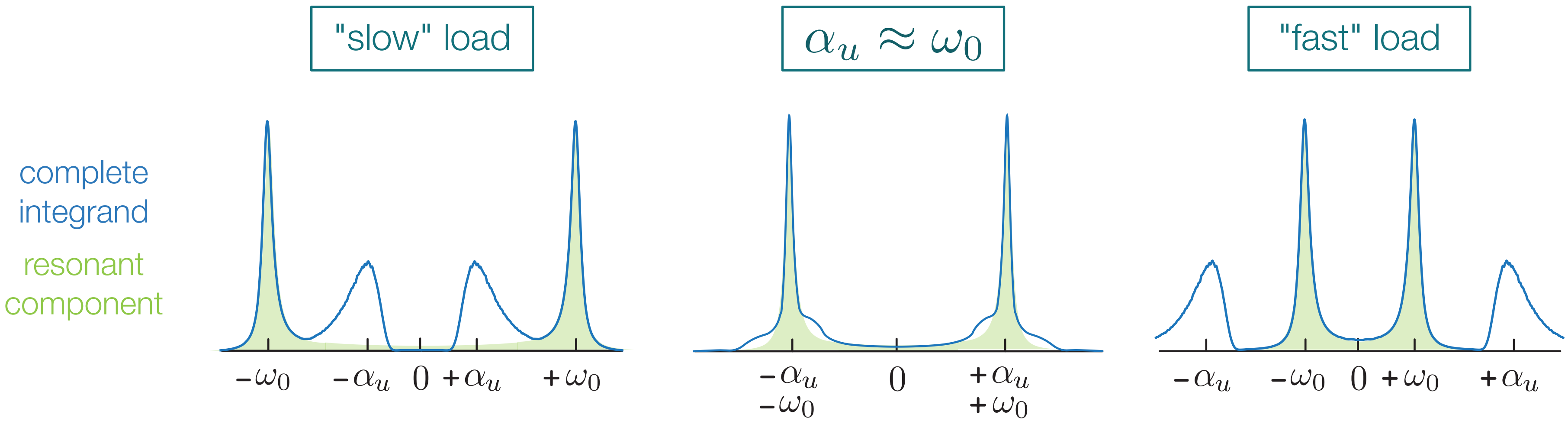
The loading components differ by the degree of the monomial trends.



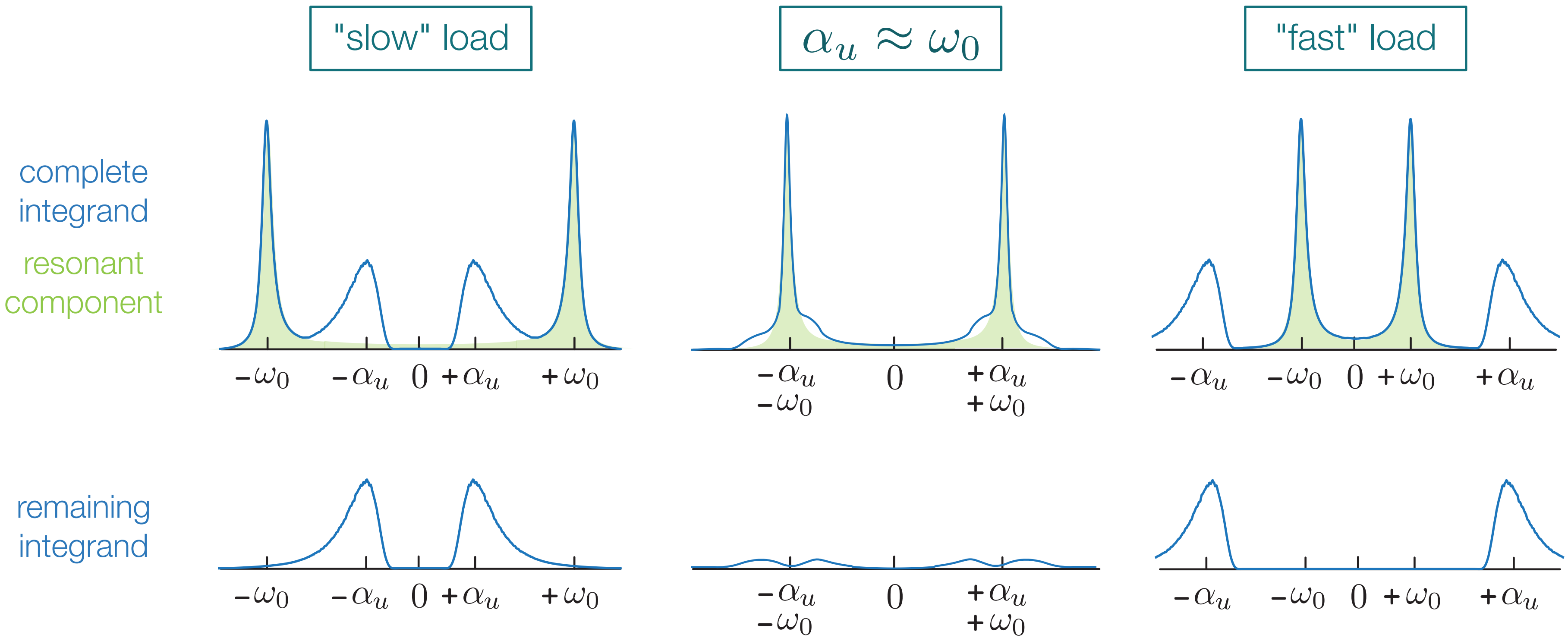
The transition background-inertial is ensured by the multiplicative factor.



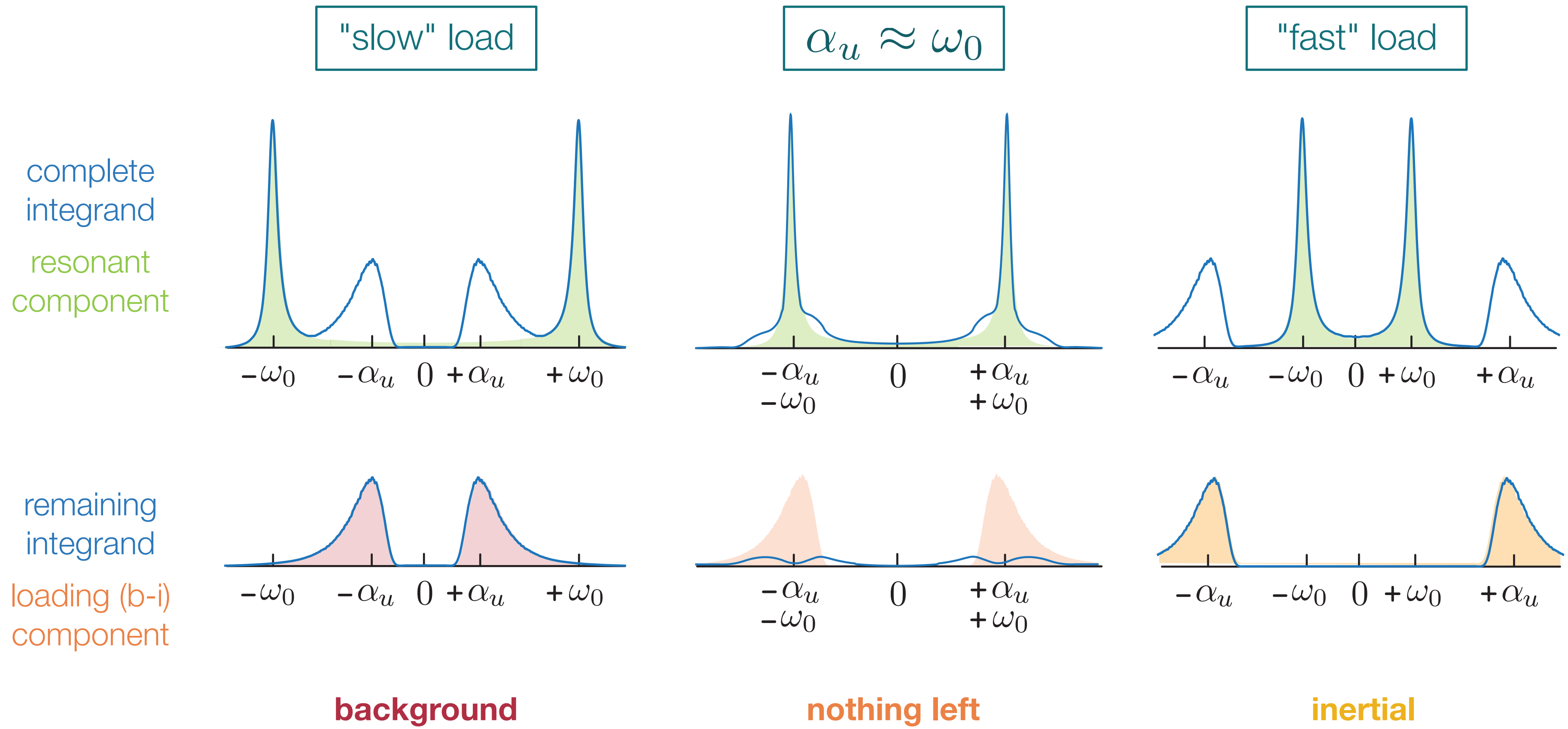
The transition background-inertial is ensured by the multiplicative factor.



The transition background-inertial is ensured by the multiplicative factor.



The transition background-inertial is ensured by the multiplicative factor.



The transition background-inertial is ensured by the multiplicative factor.

"slow" load	$\alpha_u \approx \omega_0$	"fast" load

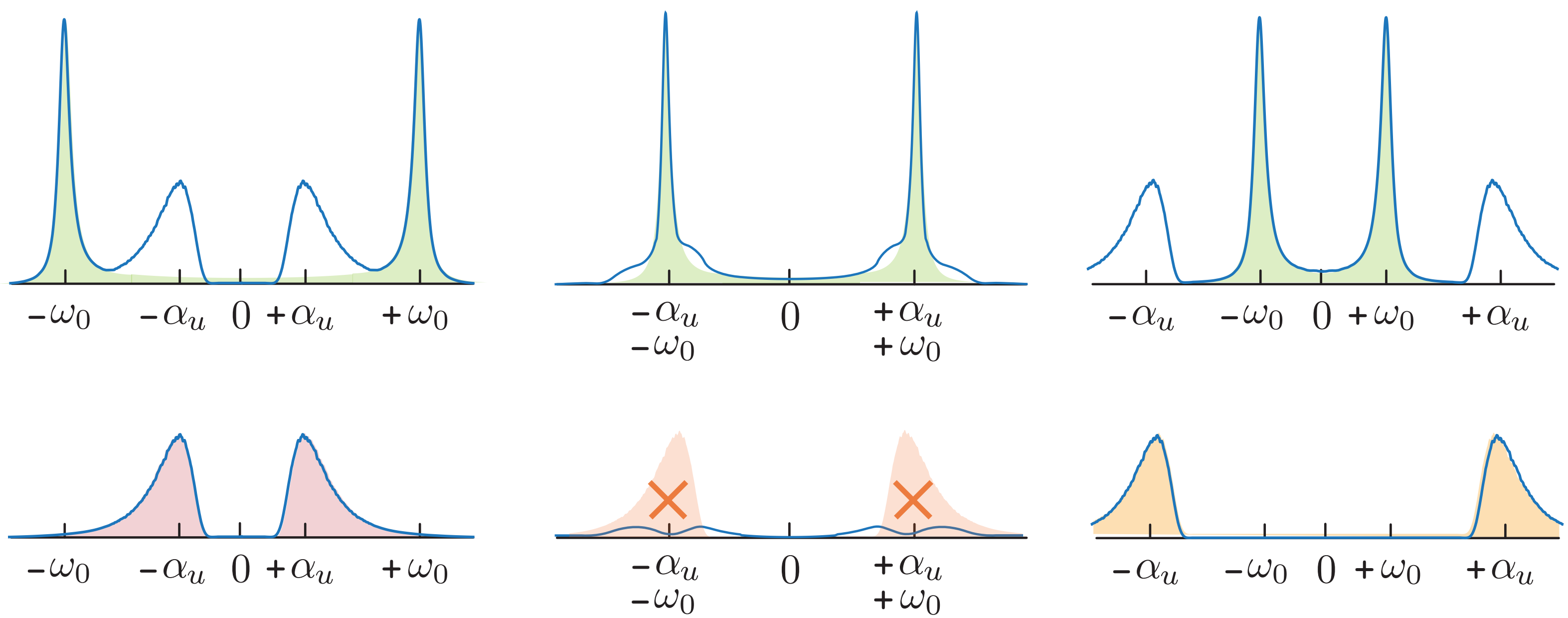
complete integrand

resonant component

remaining integrand

loading (b-i) component

multiplied by

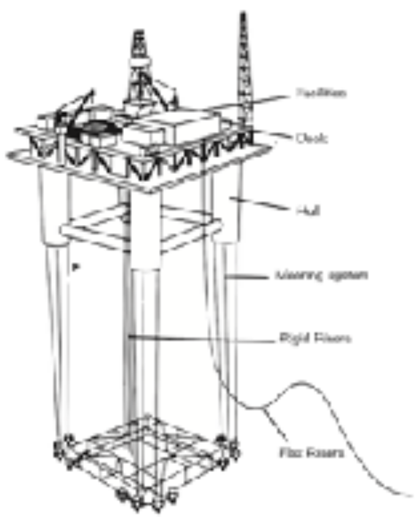


	background	**nothing left**	**inertial**
$\left[1 - \frac{S_p(\omega_0)}{S_p(\alpha_u)}\right]$ =	1	0	1
because	$S_p(\alpha_u) \gg S_p(\omega_0)$	$S_p(\alpha_u) \approx S_p(\omega_0)$	$S_p(\alpha_u) \gg S_p(\omega_0)$

SDOF

What about the inertial component ?

1) Theoretical Novelties
2) Minimalistic Example



MDOF

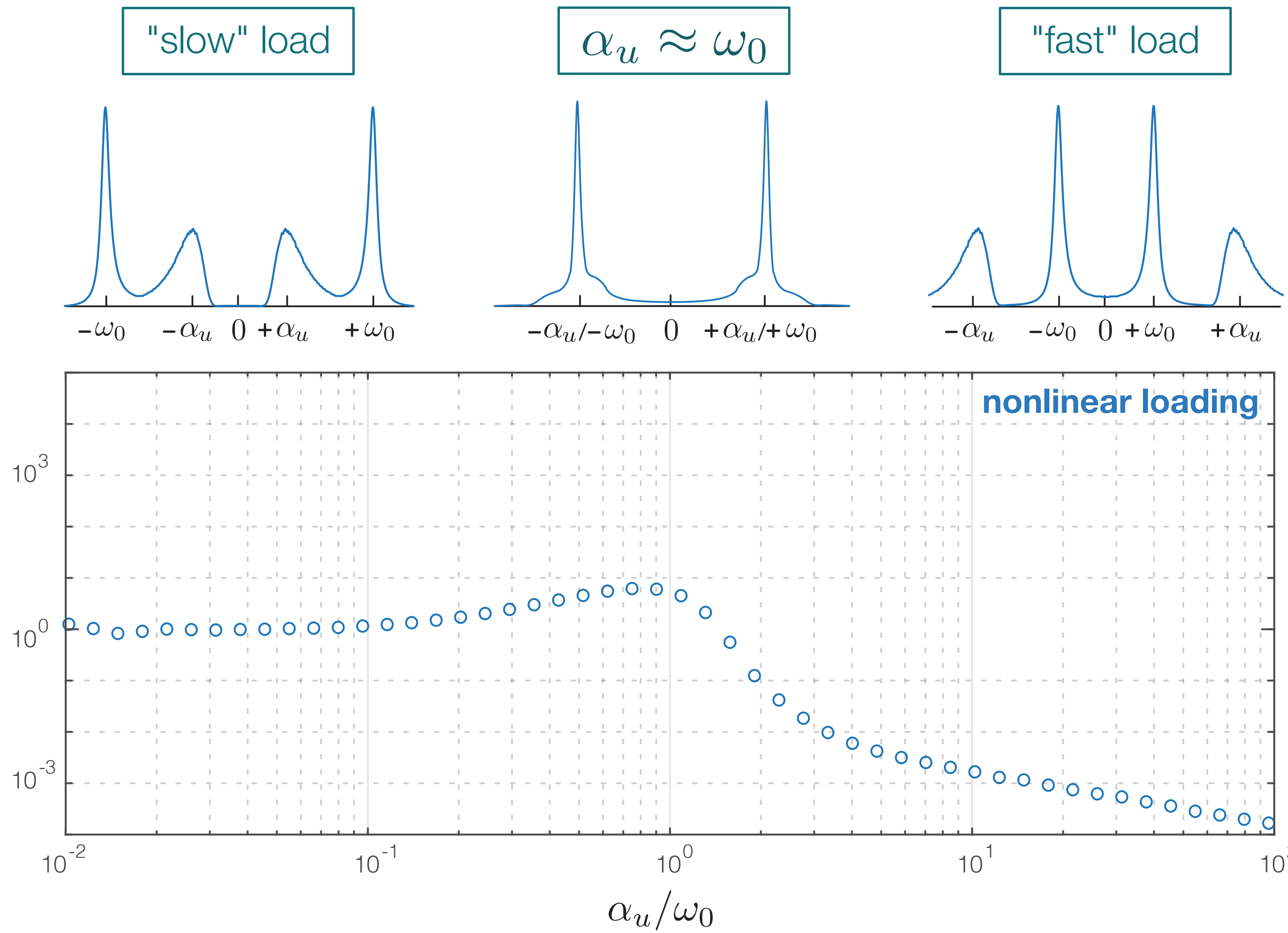
What about the hydroelastic effects ?

3) Theoretical Novelties
4) Realistic Applications

Reference and decomposition for 3 types of loading process.

RESPONSE

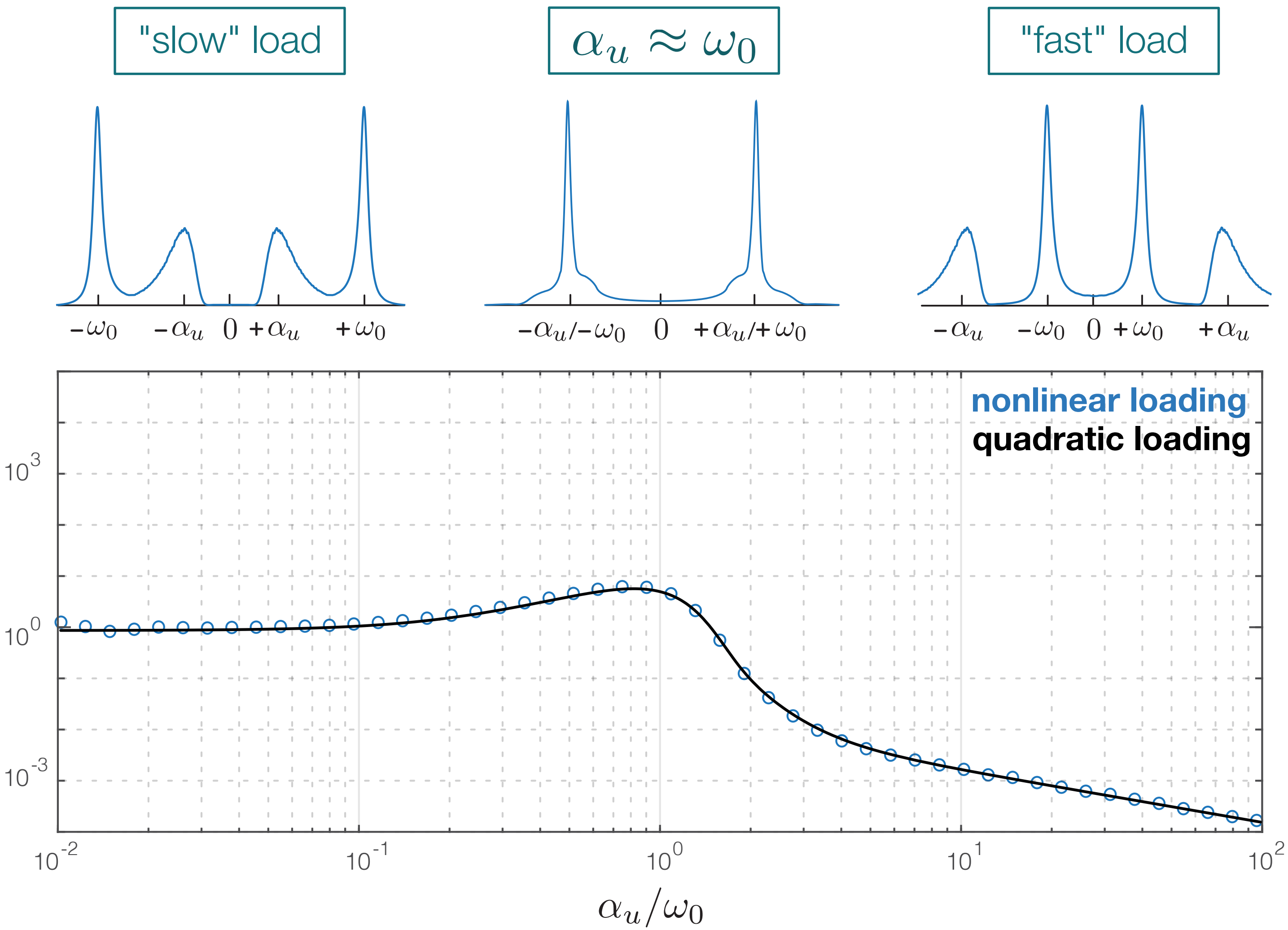
VARIANCE



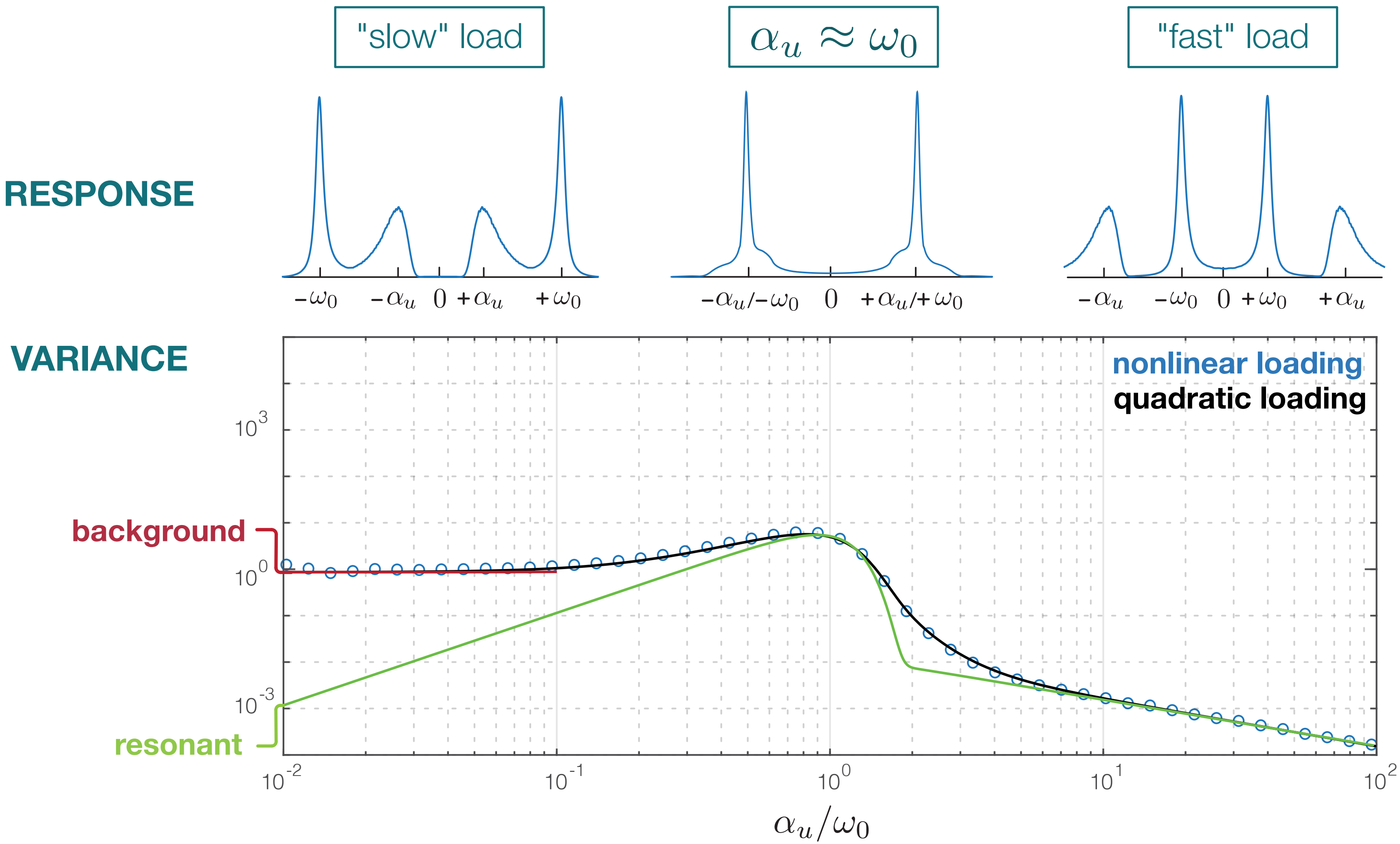
Reference and decomposition for 3 types of loading process.

RESPONSE

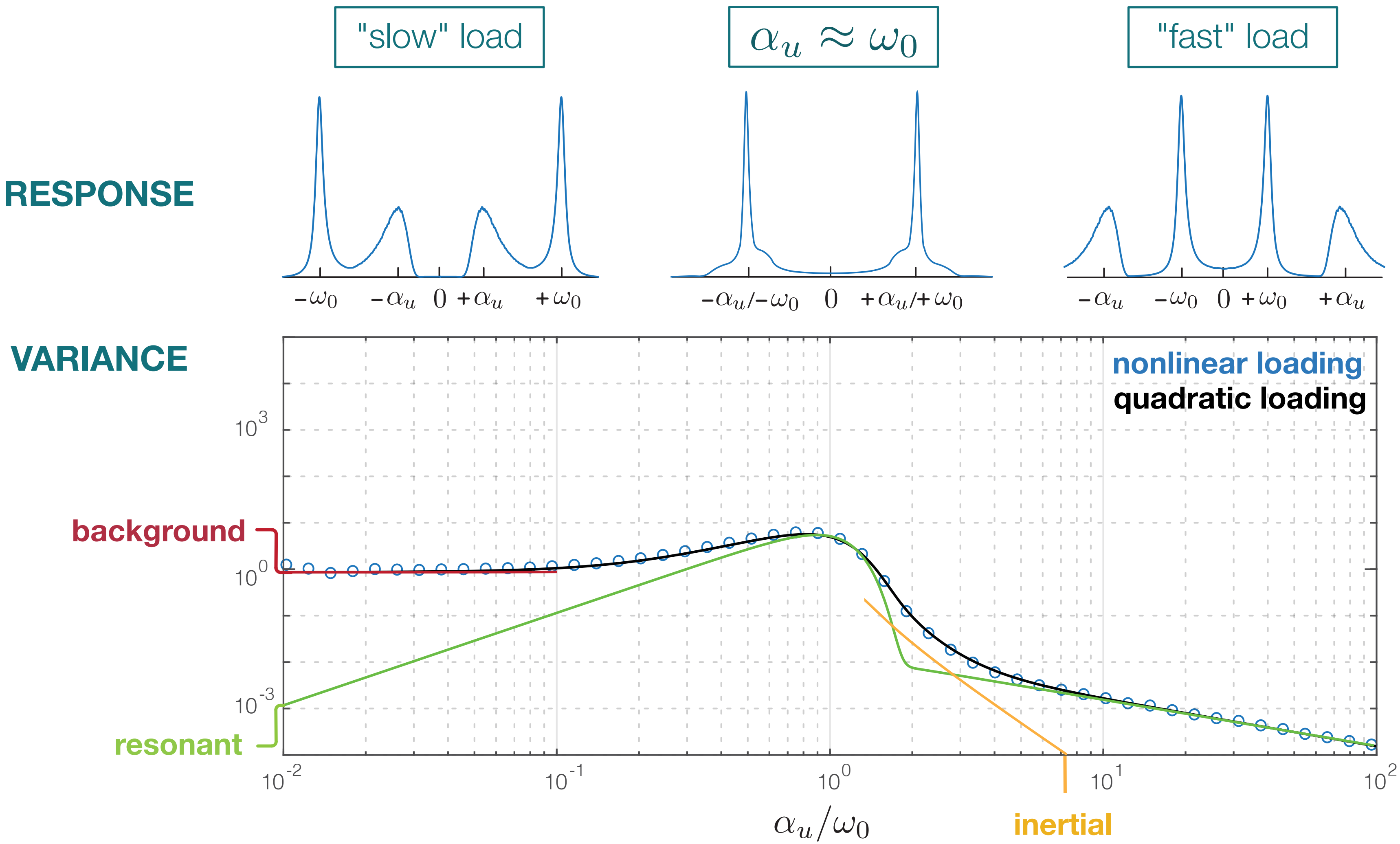
VARIANCE



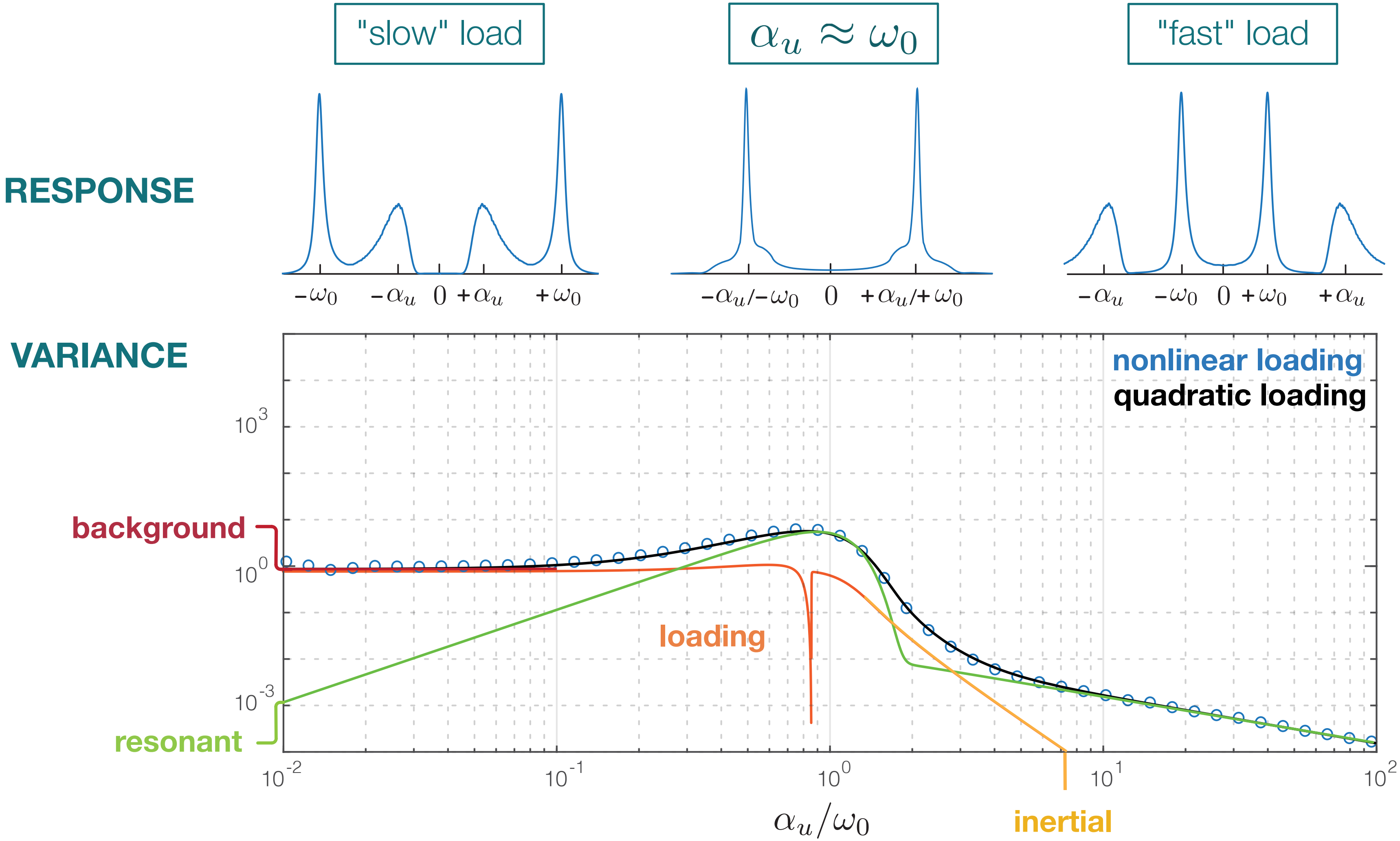
Reference and decomposition for 3 types of loading process.



Reference and decomposition for 3 types of loading process.

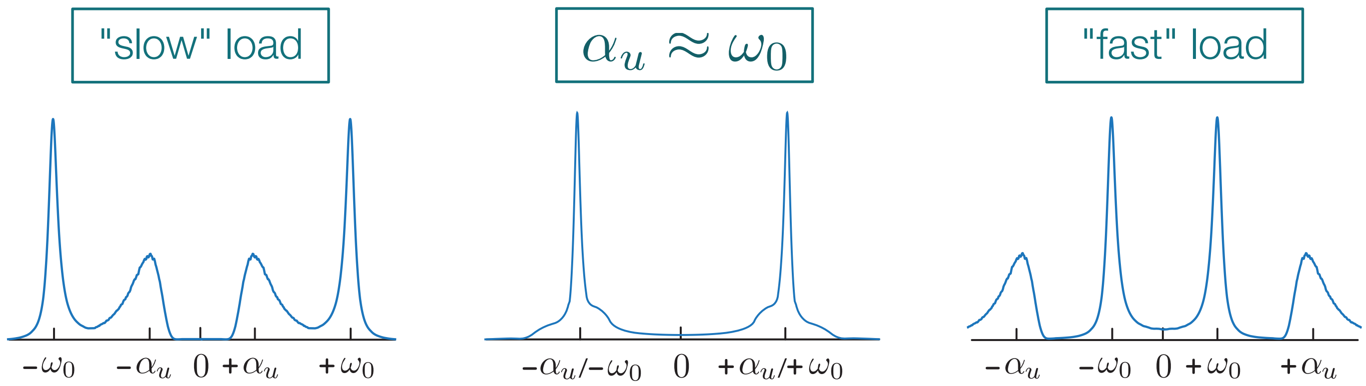


Reference and decomposition for 3 types of loading process.

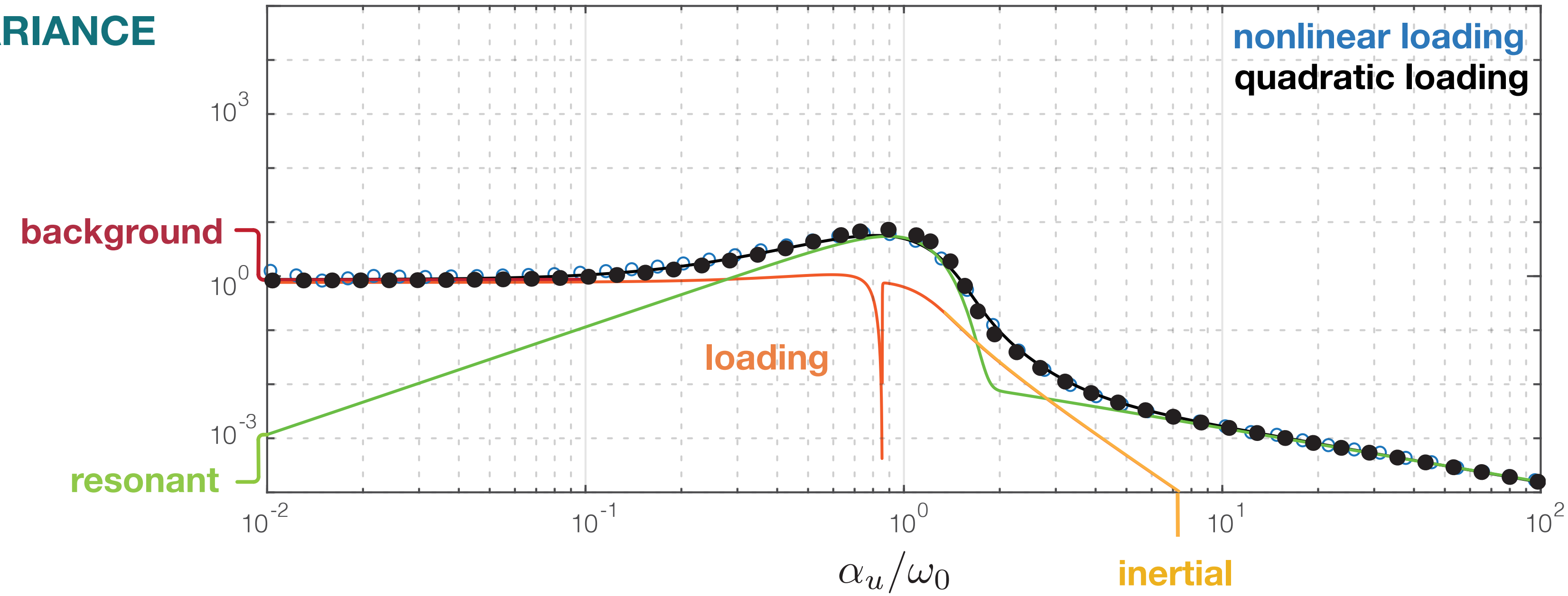


Reference and decomposition for 3 types of loading process.

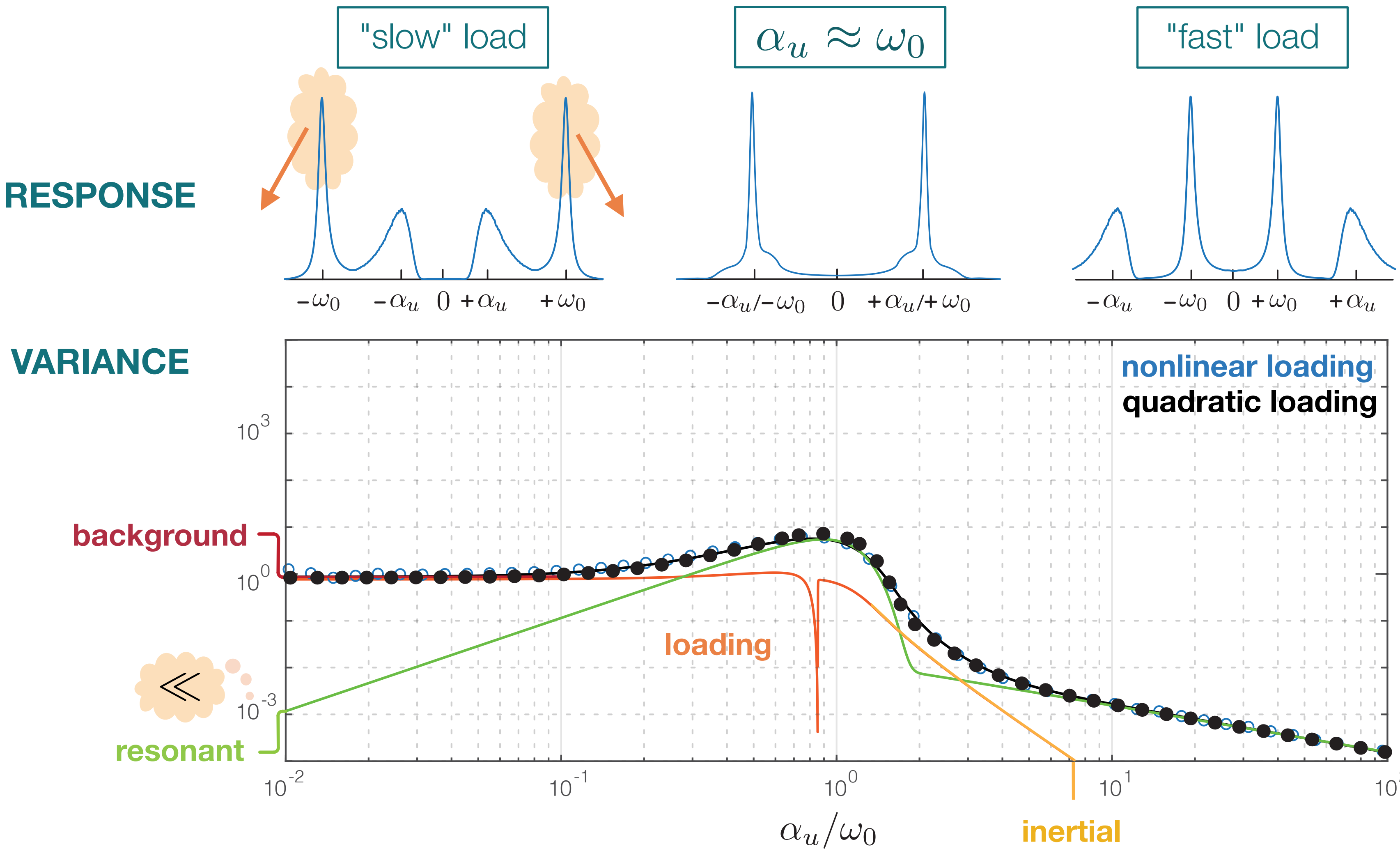
RESPONSE



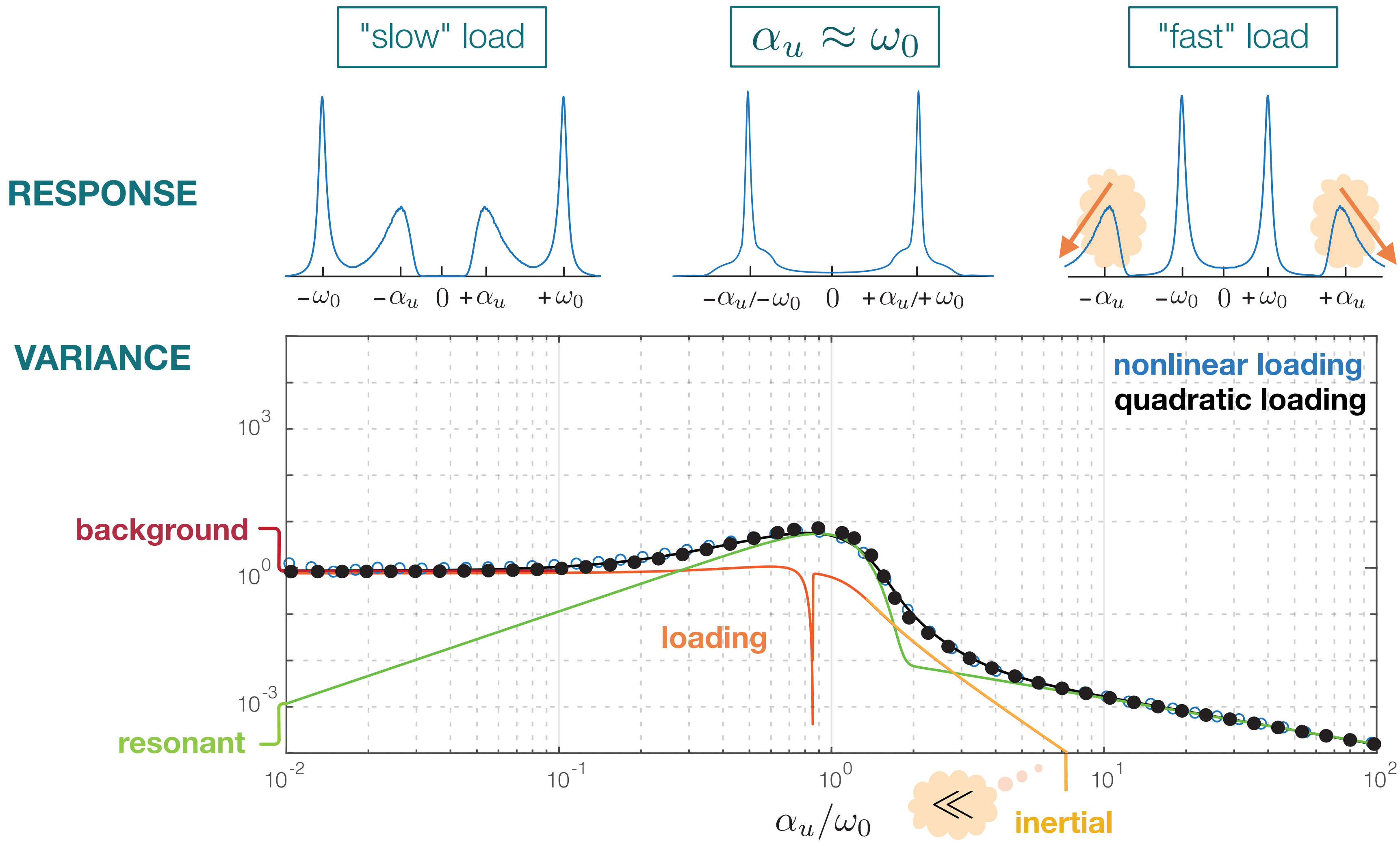
VARIANCE



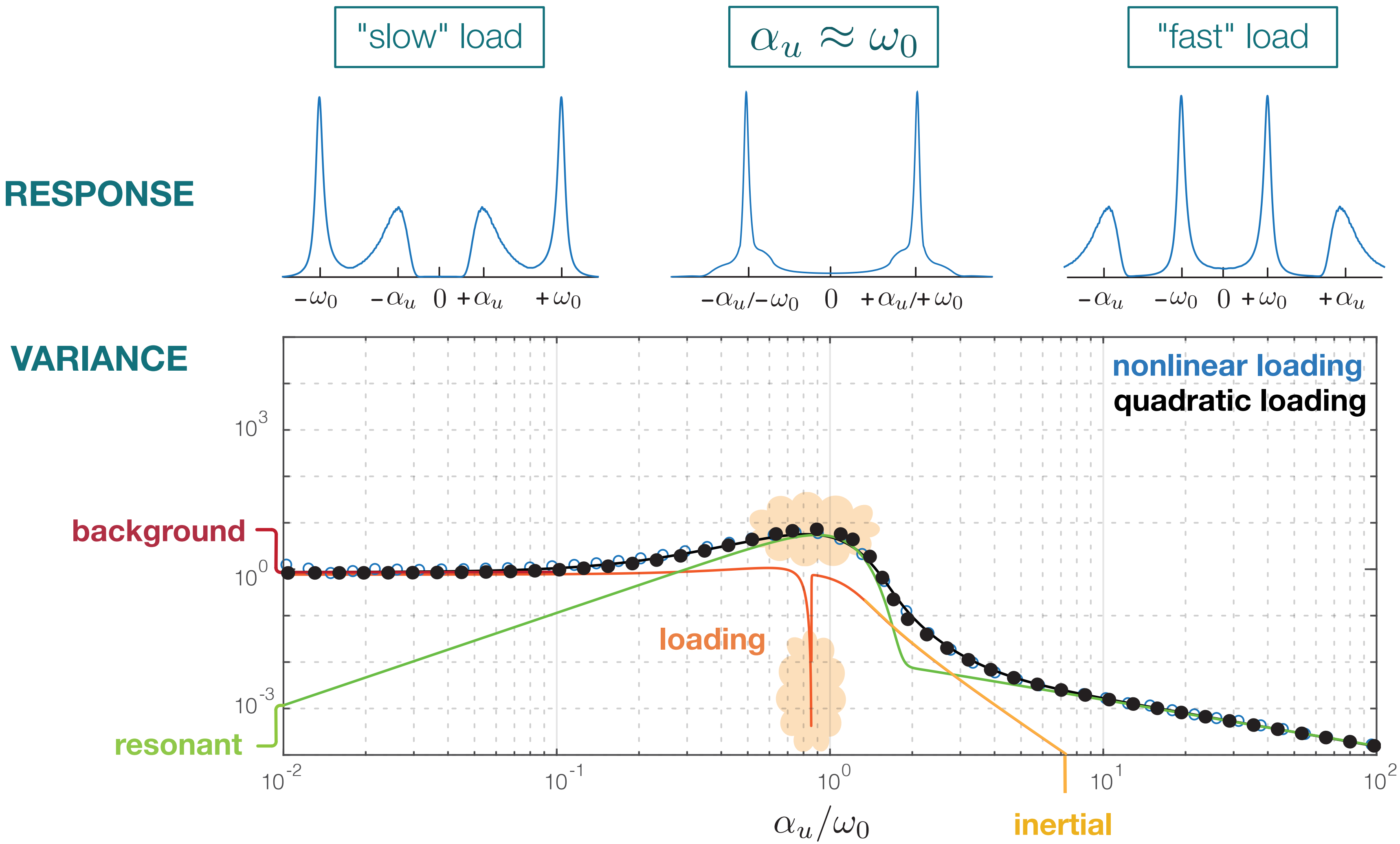
Reference and decomposition for 3 types of loading process.



Reference and decomposition for 3 types of loading process.



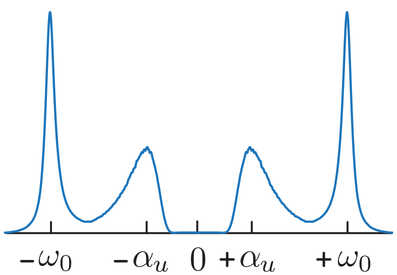
Reference and decomposition for 3 types of loading process.

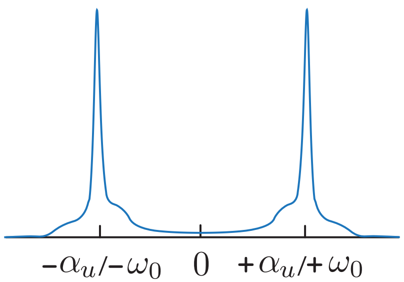


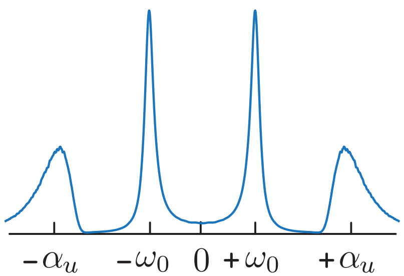
The resonant term increases with the relative wave height.

"slow" load | $\alpha_u \approx \omega_0$ | "fast" load

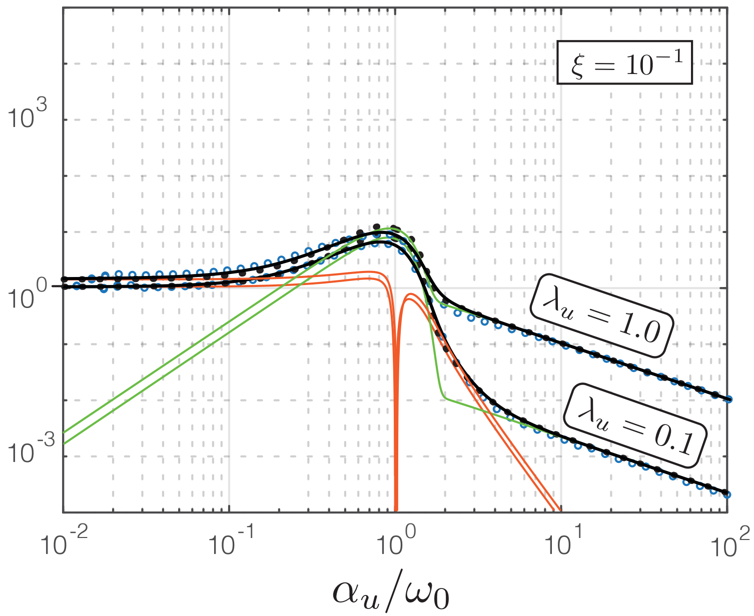
RESPONSE



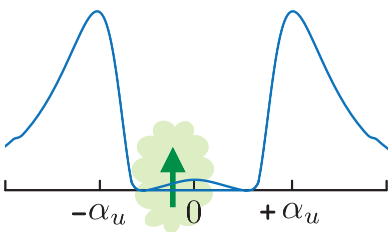




VARIANCE

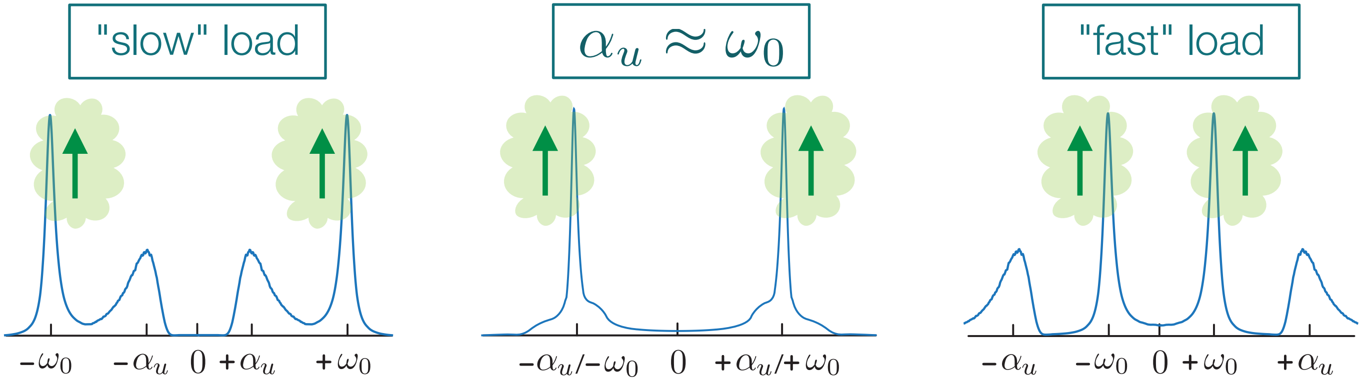


Quadratized Loading

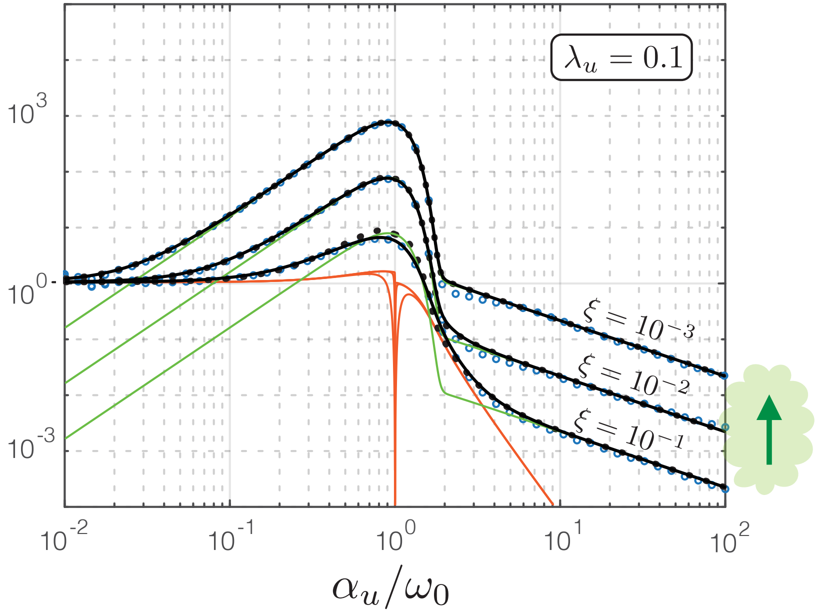


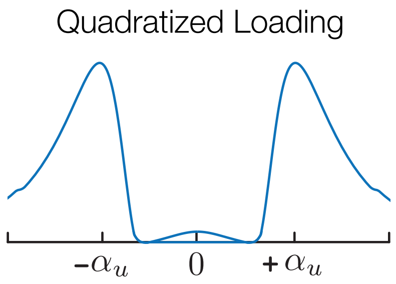
The resonant term increases with smaller damping ratios.

RESPONSE



VARIANCE





The resonant component disappears if the loading is inertial and linearized.

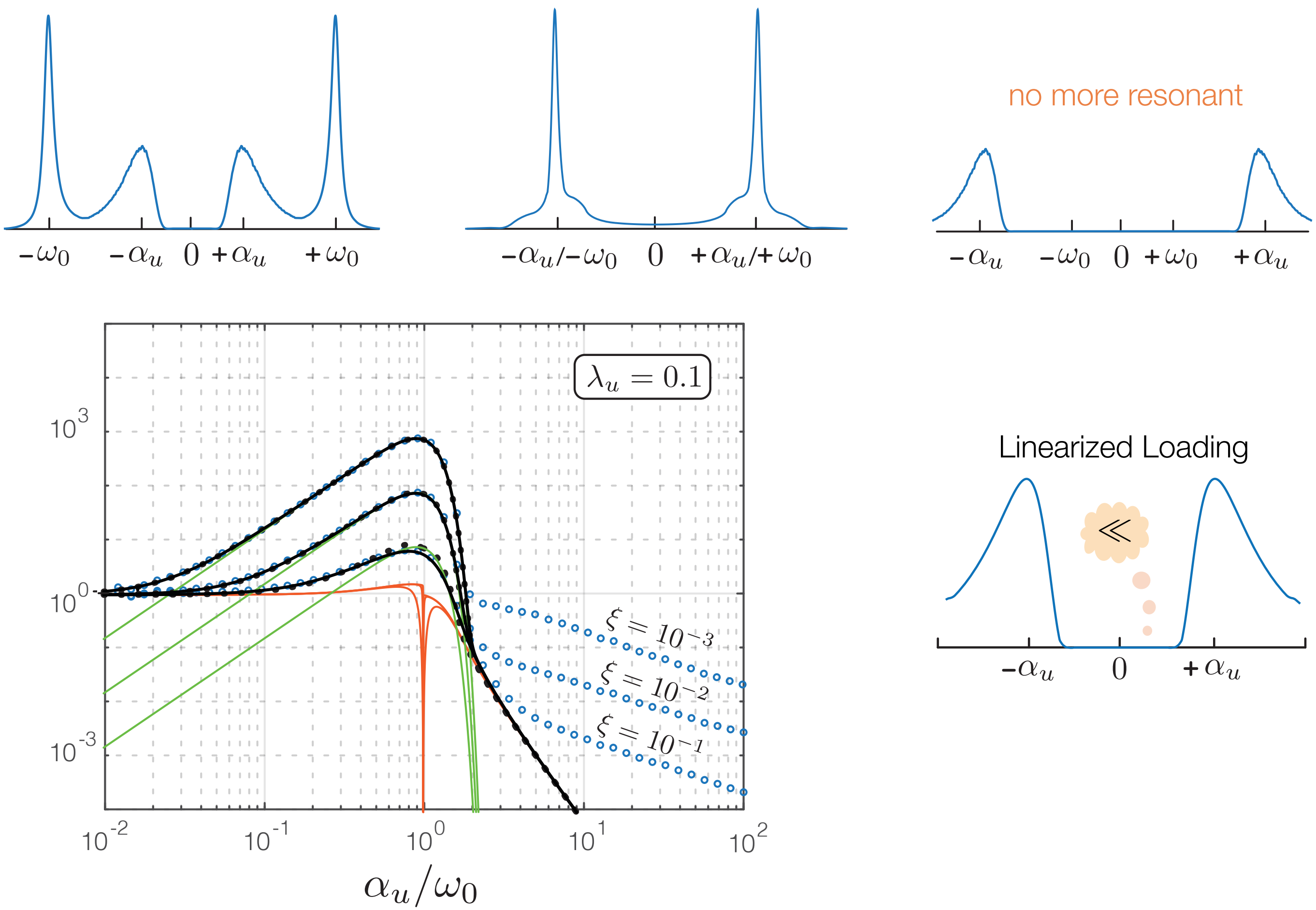
"slow" load

$\alpha_u \approx \omega_0$

"fast" load

RESPONSE

VARIANCE

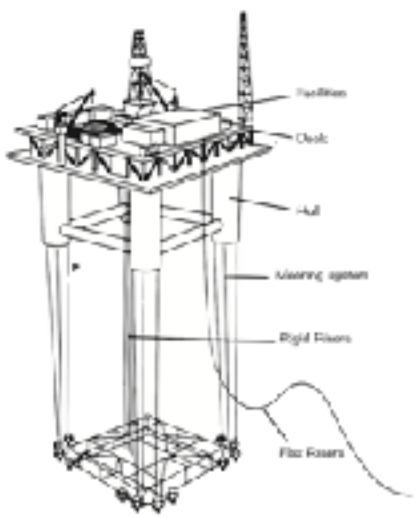


SDOF

What about the inertial component ?

1) Theoretical Novelties

2) Minimalistic Example



MDOF

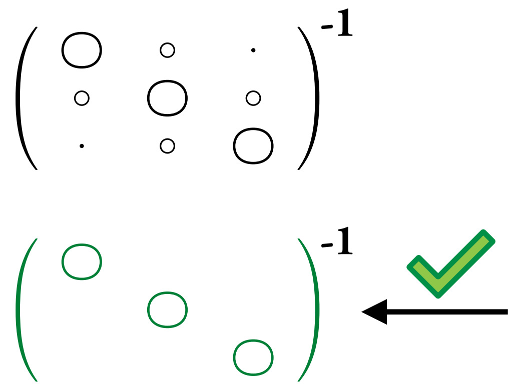
What about the hydroelastic effects ?

3) Theoretical Novelties

4) Realistic Applications

The FRF matrix can be computed once a diagonal matrix is inverted.

equations	adv. + inc.	(FRF)	hypotheses
MCK nodal			
	truncation projection		
MCK modal			damping {classical or negligible}



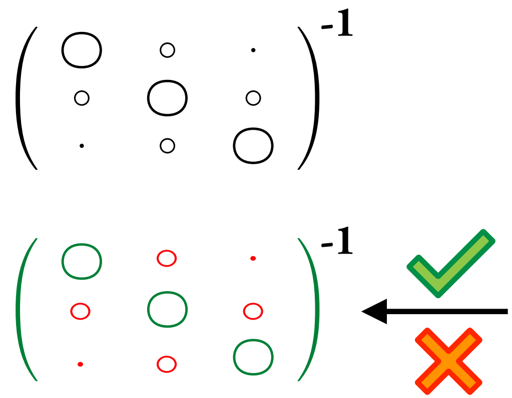
The FRF matrix can be computed once a diagonal matrix is inverted.

equations | adv. + inc. | **(FRF)** | hypotheses

MCK nodal

truncation
projection

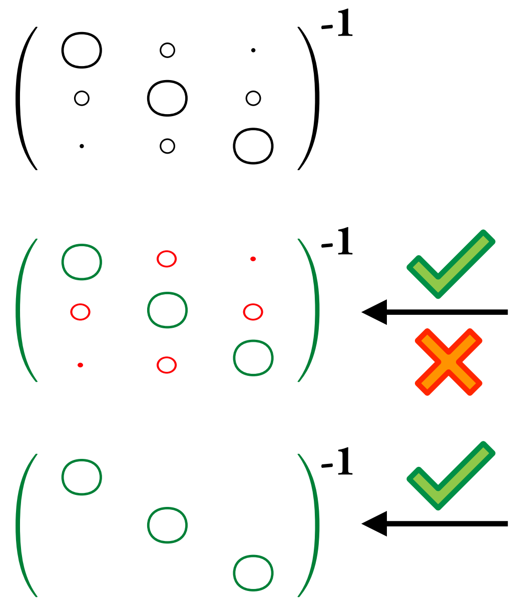
MCK modal



damping { classical or negligible

The FRF matrix can be computed once a diagonal matrix is inverted.

equations	adv. + inc.	(FRF)	hypotheses
MCK nodal			
	truncation projection		
MCK modal			damping { classical or negligible
	standard EP doubled size		
SSF modal State-Space Formulation			freq. dep. negligible



The FRF matrix can be computed once a diagonal matrix is inverted.

equations | adv. + inc. | (FRF) | hypotheses

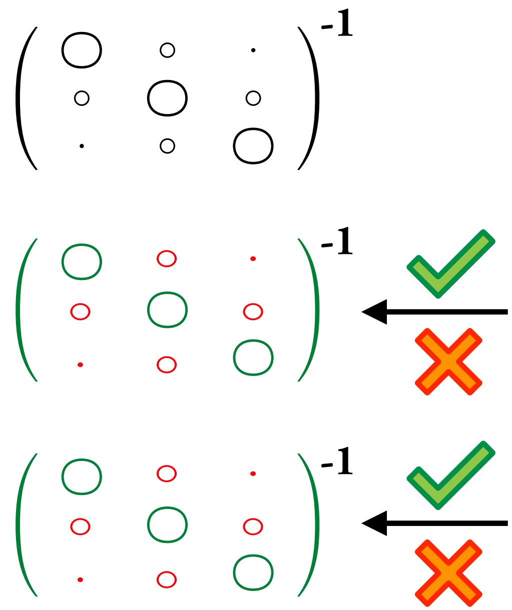
MCK nodal

truncation
projection

MCK modal

standard EP
doubled size

SSF modal
State-Space Formulation



damping — classical or negligible

freq. dep. — negligible

The FRF matrix can be computed once a diagonal matrix is inverted.

equations	adv. + inc.	(FRF)	hypotheses

MCK nodal

truncation
projection

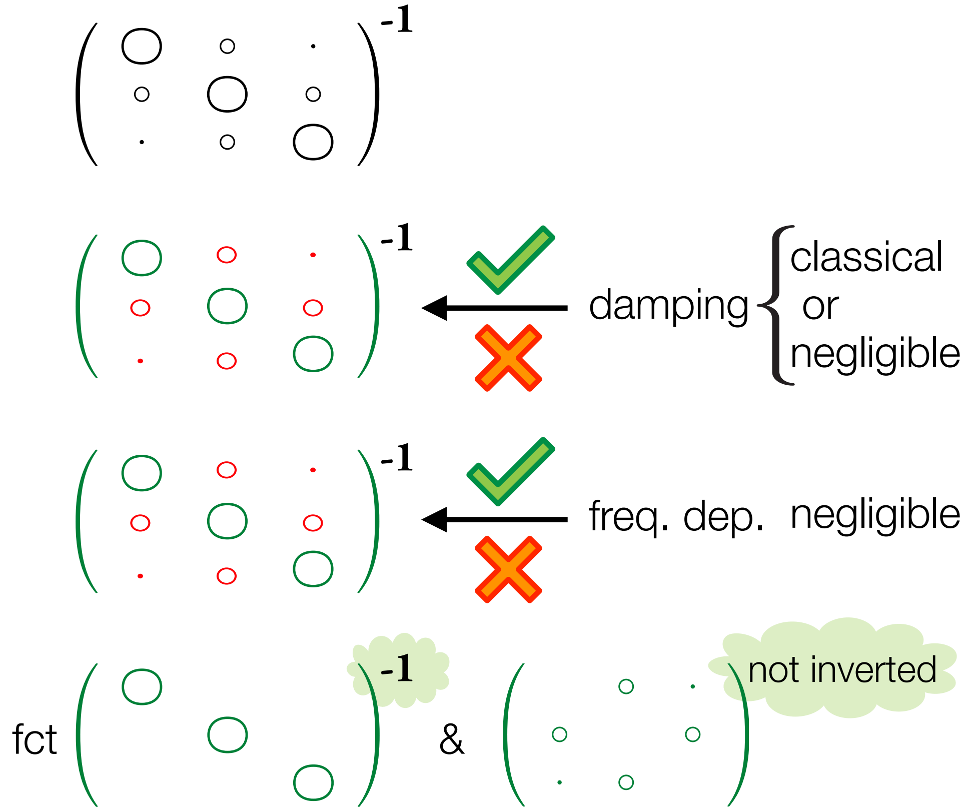
MCK modal

standard EP
doubled size

SSF modal
State-Space Formulation

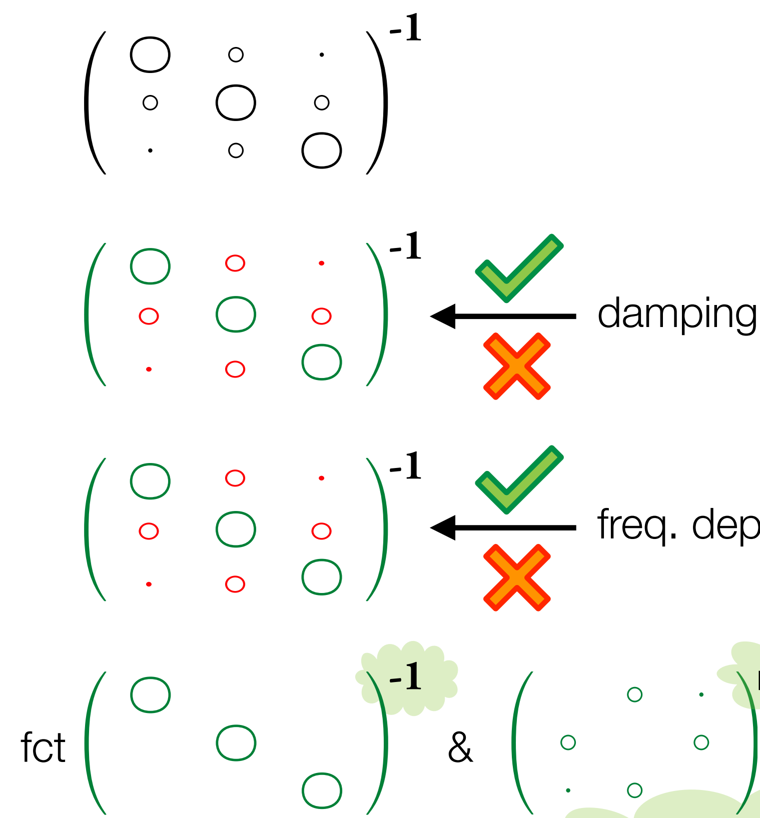
infinite series expansion
1st + 2nd if small coupling

SSF + ASE
Asymptotic Series Expansion



The FRF matrix can be computed once a diagonal matrix is inverted.

equations | adv. + inc. | (FRF) | hypotheses



MCK nodal

truncation
projection

MCK modal — damping: classical or negligible

standard EP
doubled size

SSF modal
State-Space Formulation — freq. dep. negligible

infinite series expansion
1st + 2nd if small coupling

SSF + ASE
Asymptotic Series Expansion — not inverted; freq. dep. restricted

The FRF matrix can be computed once a diagonal matrix is inverted.

equations	adv. + inc.	(FRF)	hypotheses

MCK nodal

truncation
projection

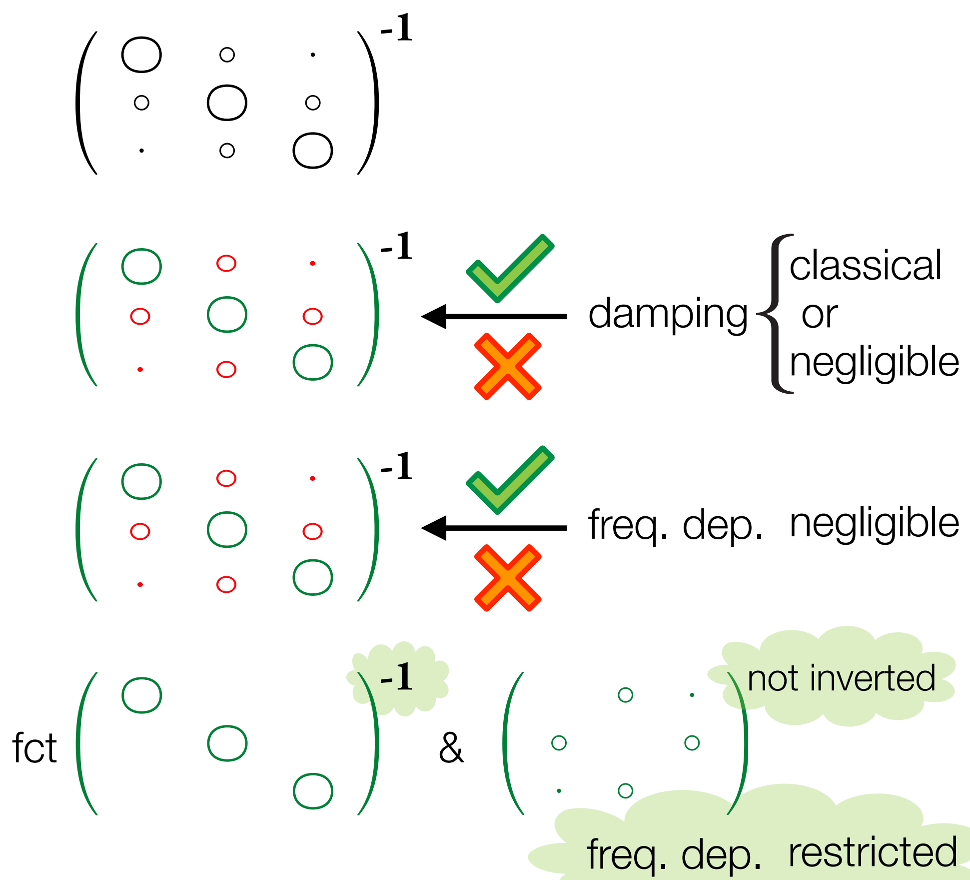
MCK modal

standard EP
doubled size

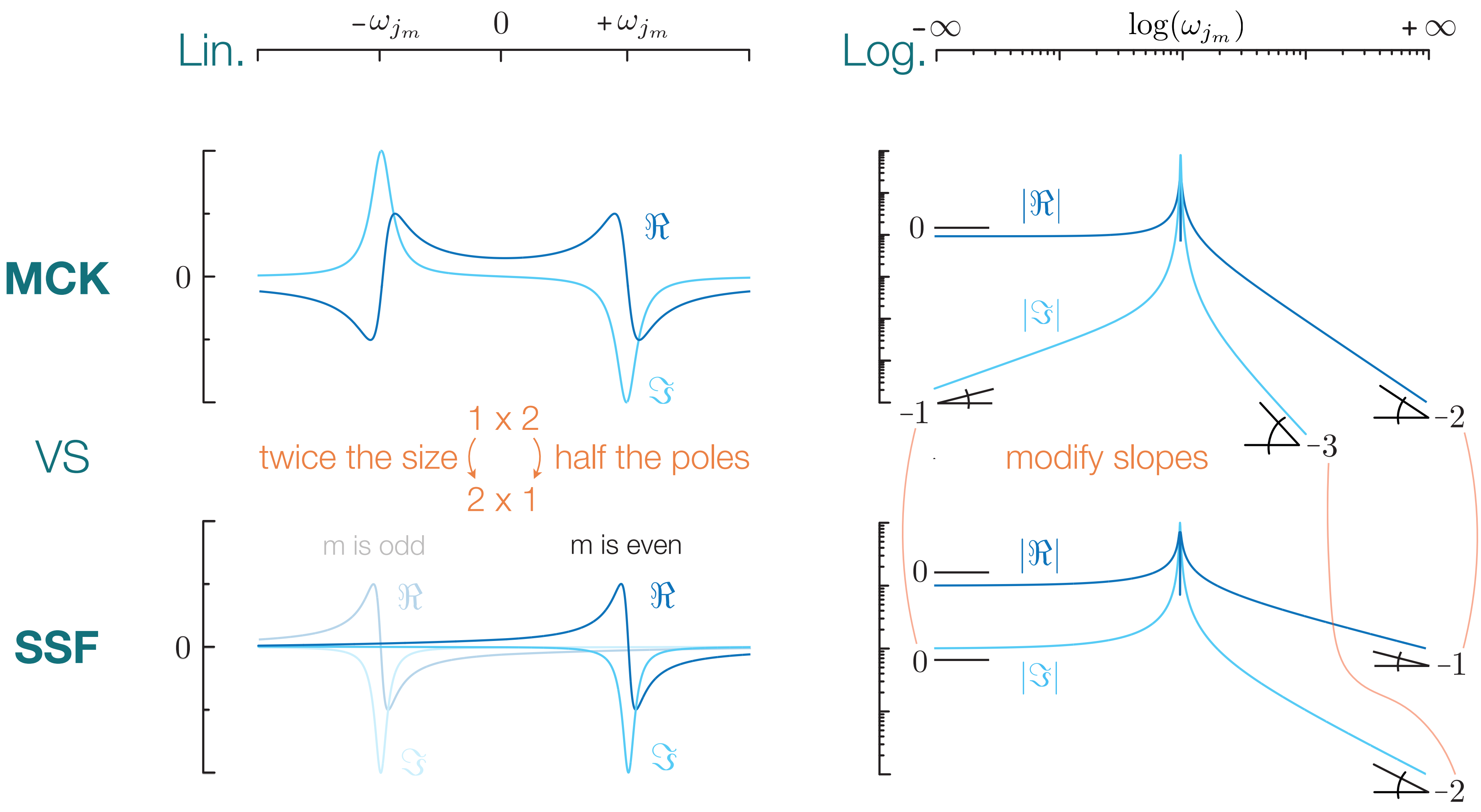
Scenario 1
SSF modal
State-Space Formulation

infinite series expansion
1st + 2nd if small coupling

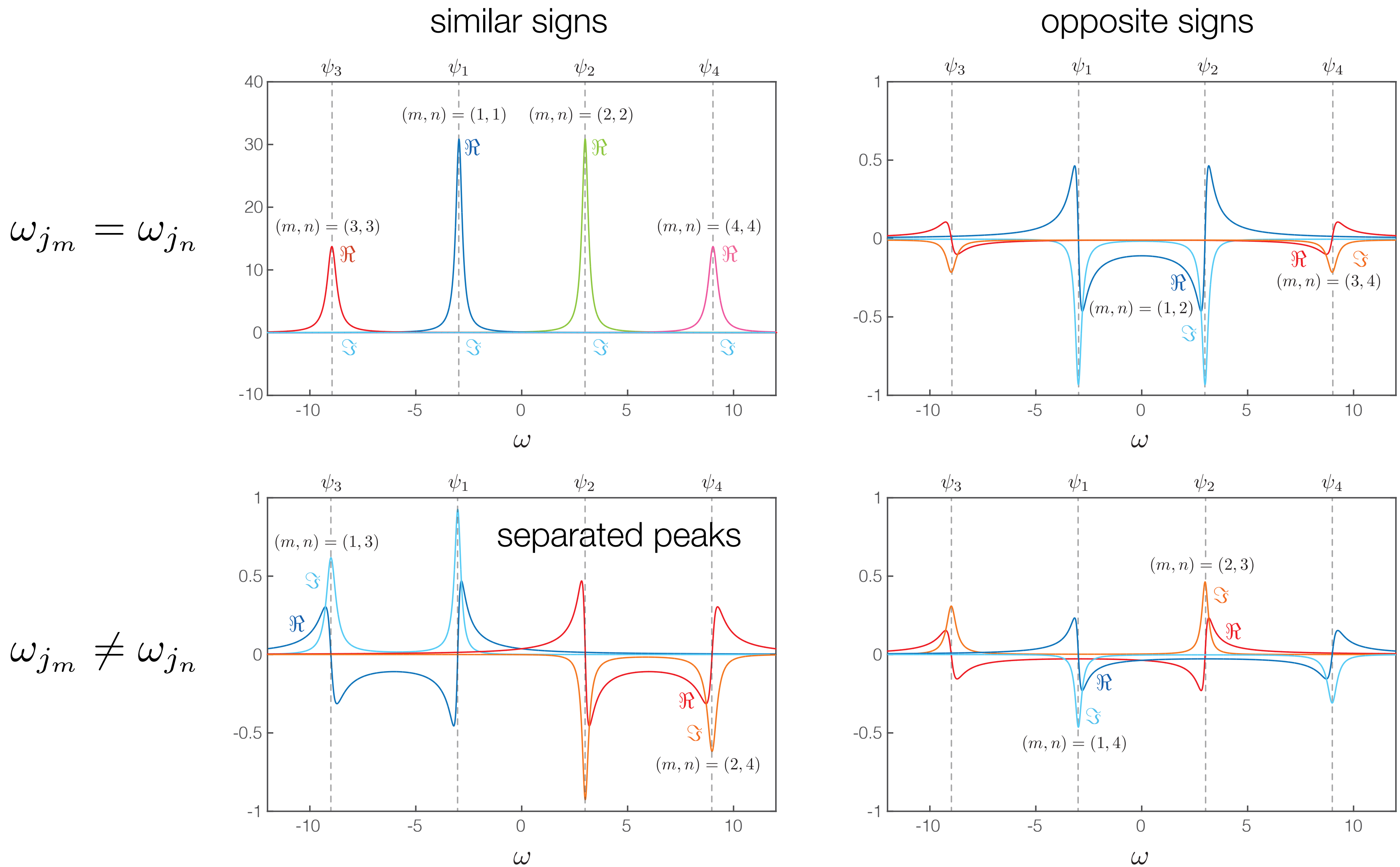
Scenario 2
SSF + ASE
Asymptotic Series Expansion



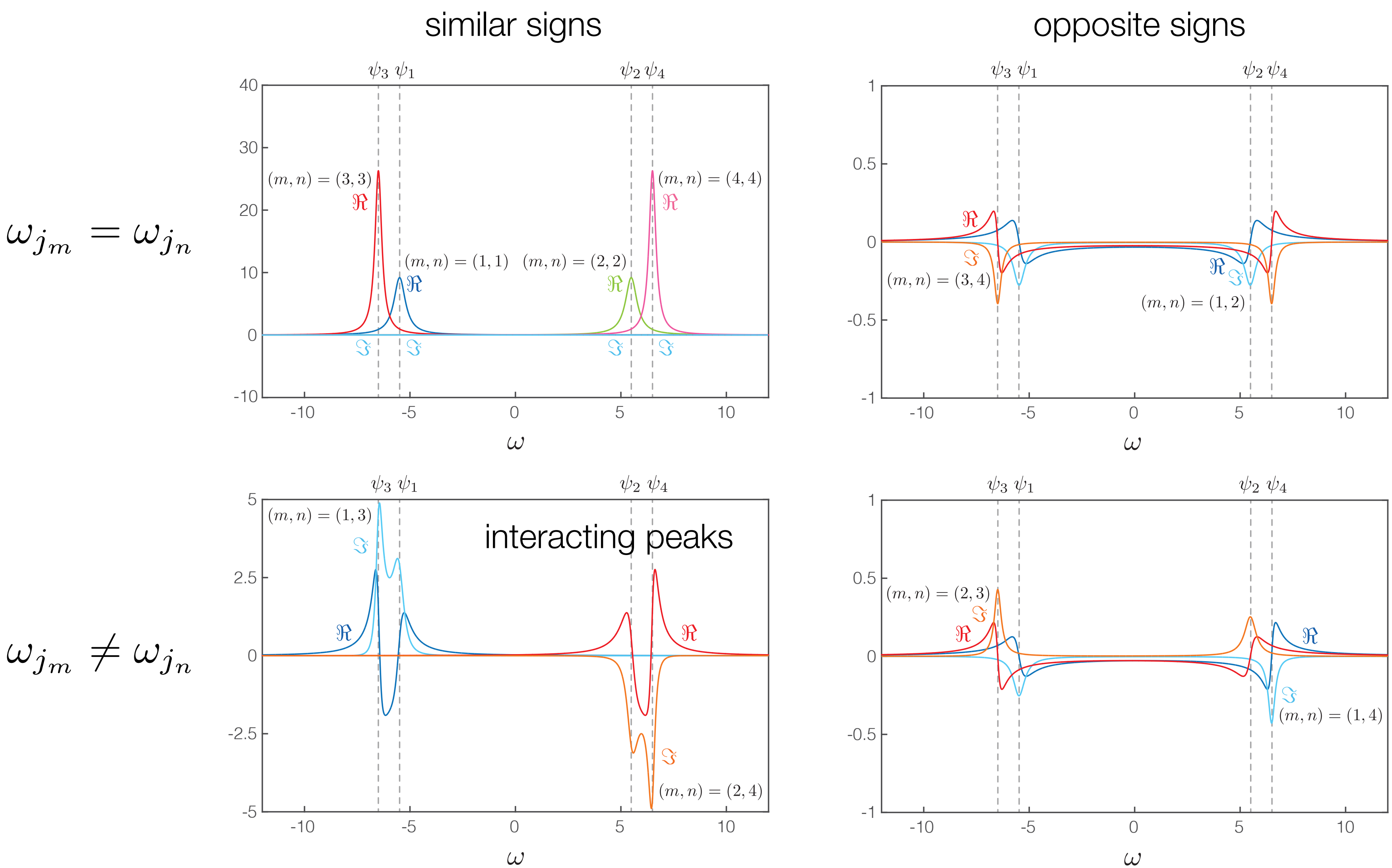
The FRFs exhibit half the poles and monomial trends of different slopes.



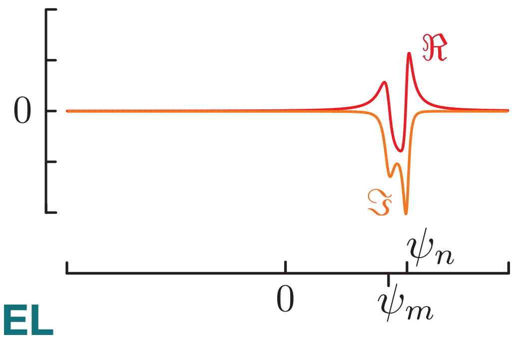
The kernel is complex-valued and the eigenfrequencies form conjugate pairs.



The kernel is complex-valued and the eigenfrequencies form conjugate pairs.

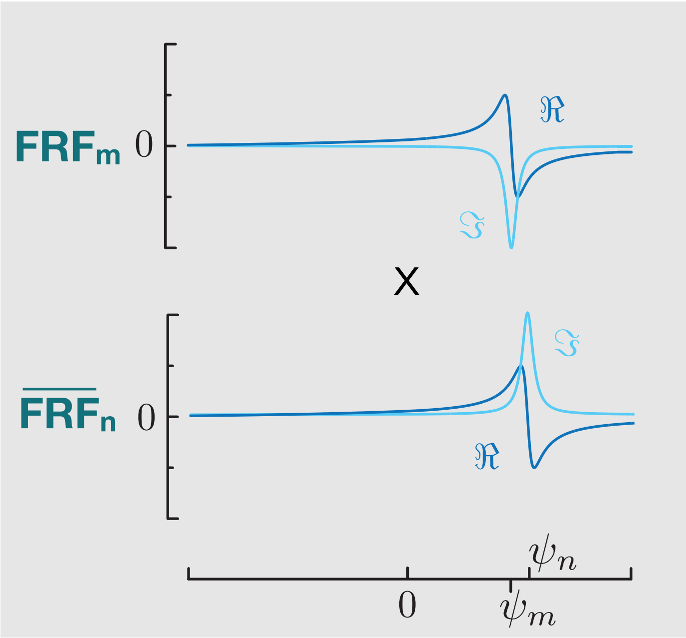


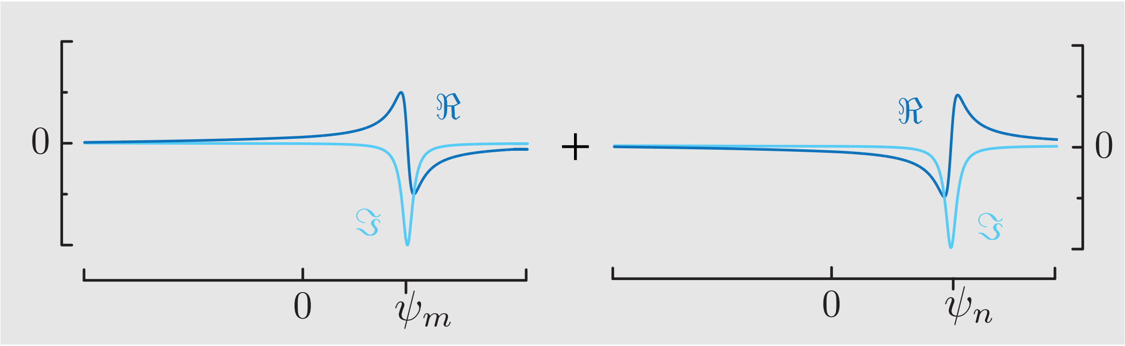
A partial fraction expansion of the kernel can still separate the peaks in an exact way.



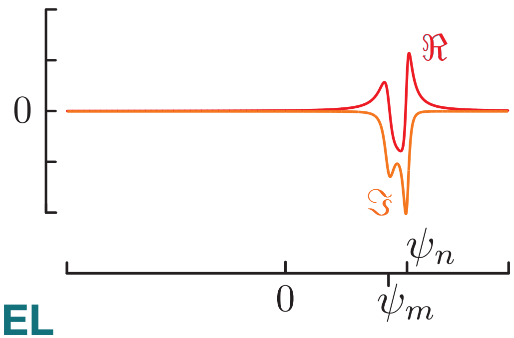
KERNEL

= multiply 2 x 1 pole **OR** sum 2 x 1 pole



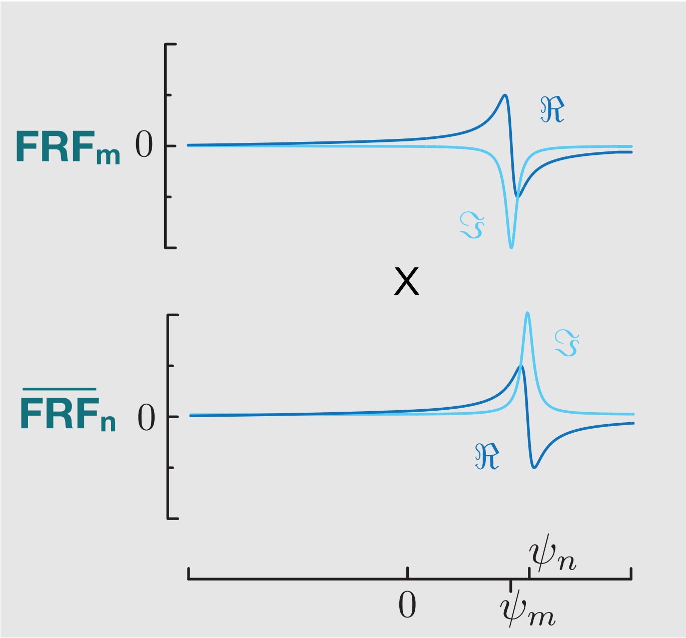


A partial fraction expansion of the kernel can still separate the peaks in an exact way.

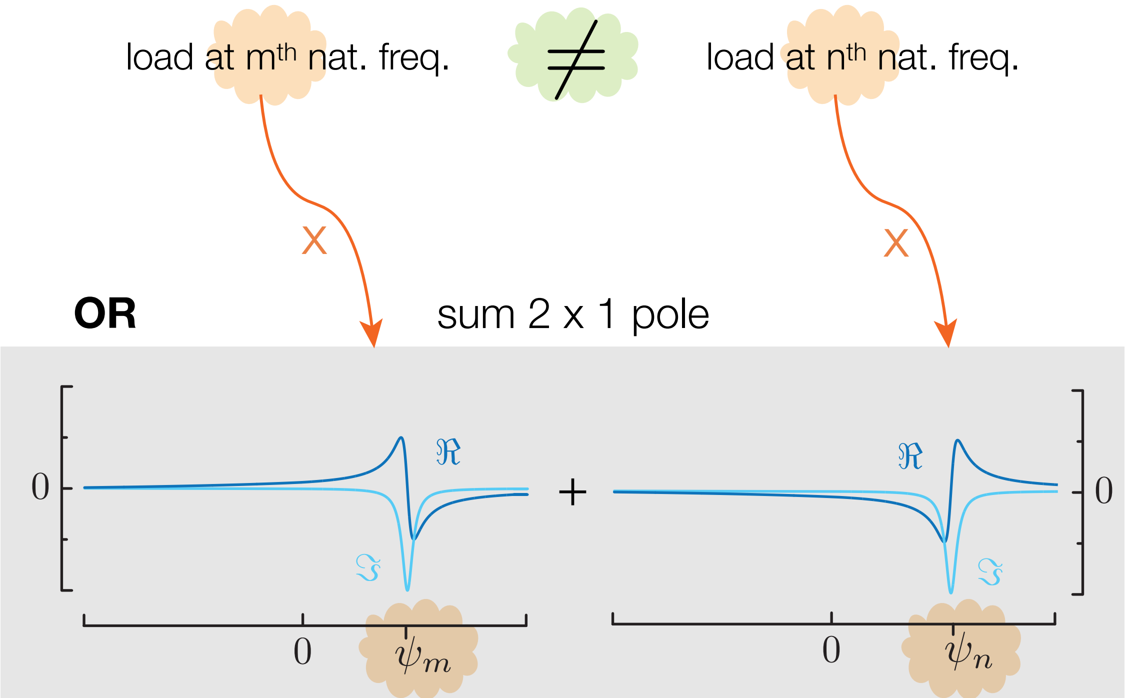


KERNEL

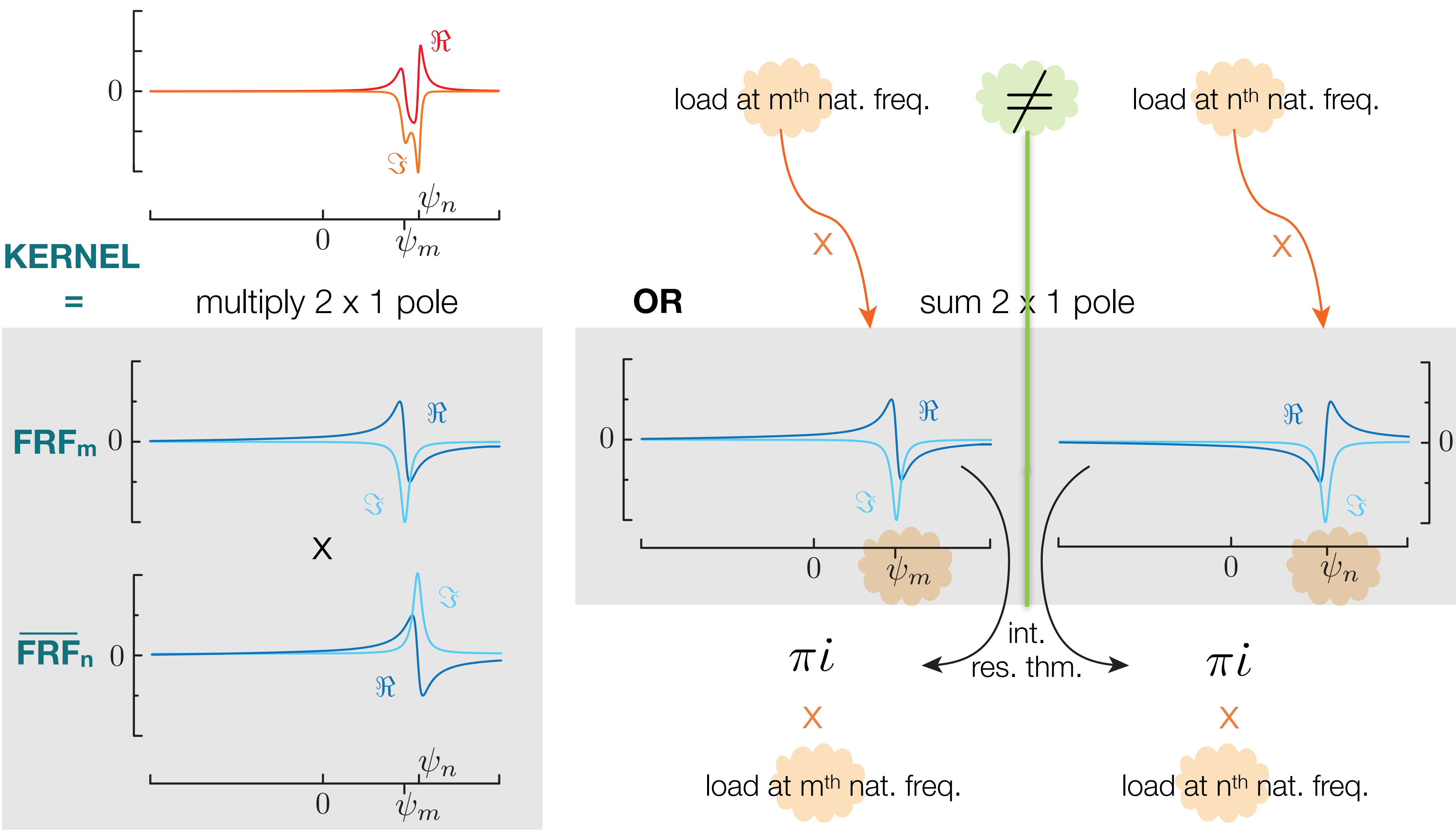
= multiply 2 x 1 pole



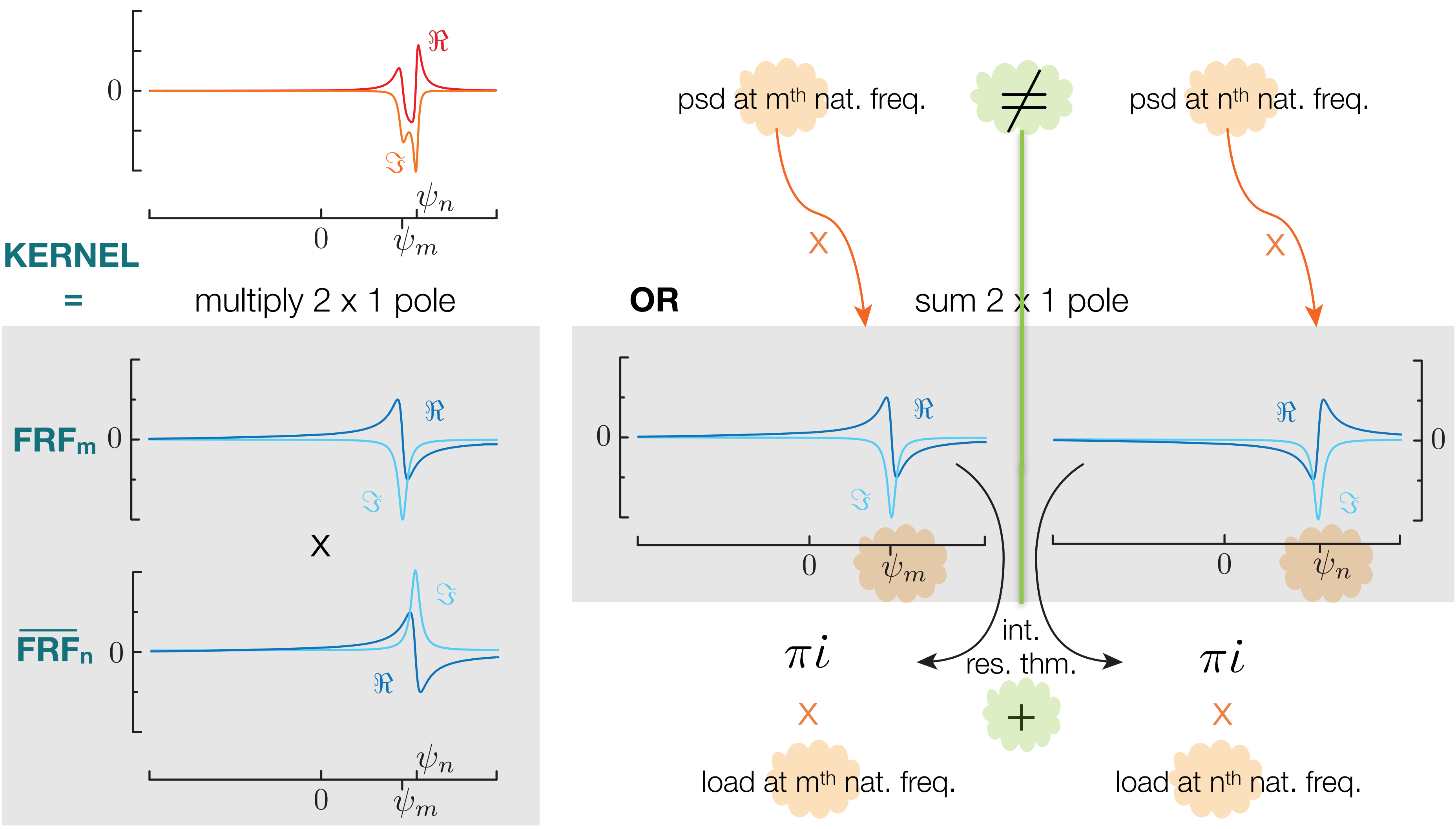
OR sum 2 x 1 pole



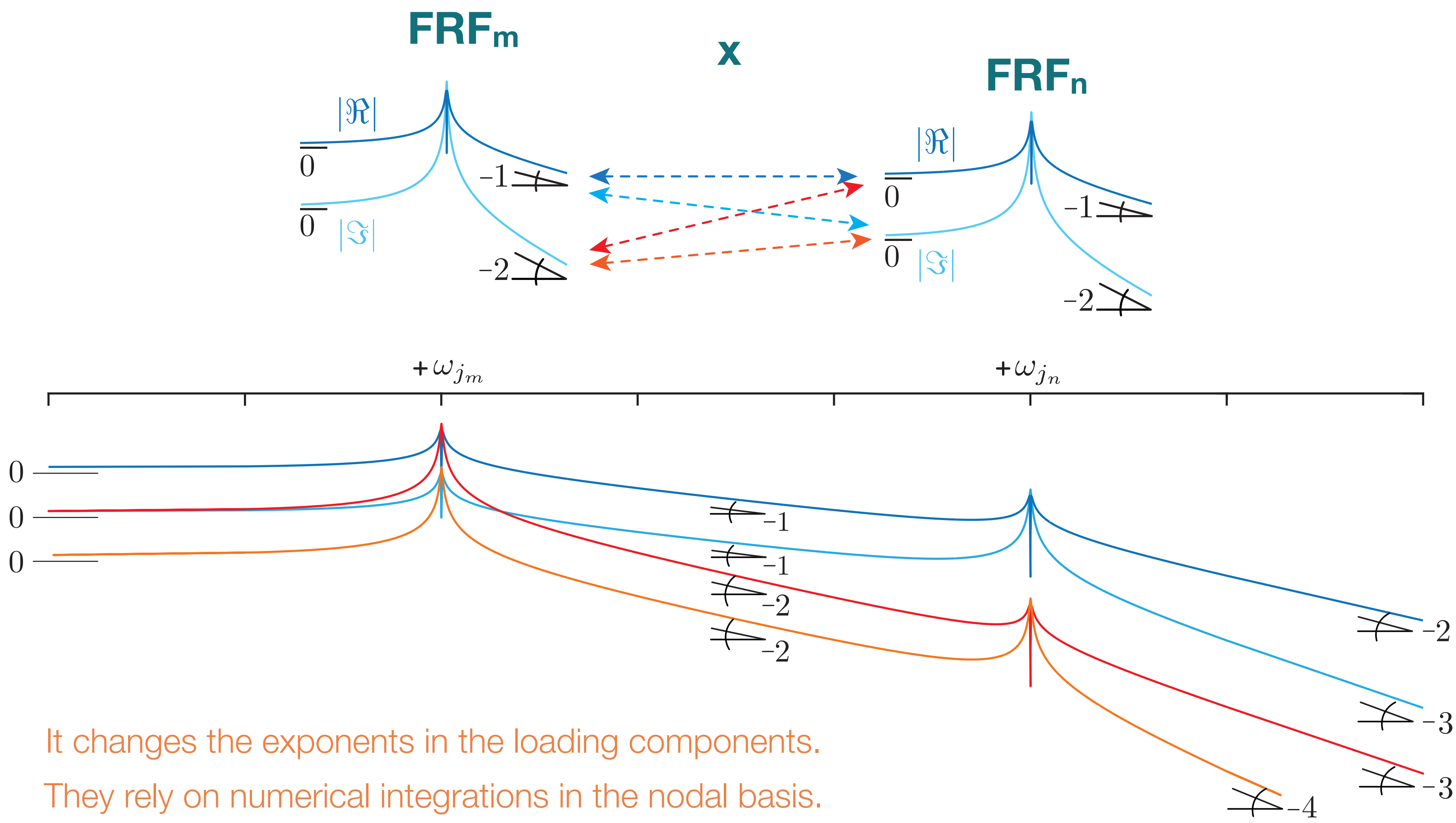
A partial fraction expansion of the kernel can still separate the peaks in an exact way.



A partial fraction expansion of the kernel can still separate the peaks in an exact way.



The monomial trends in the kernel depend on the pairs of FRFs parts.

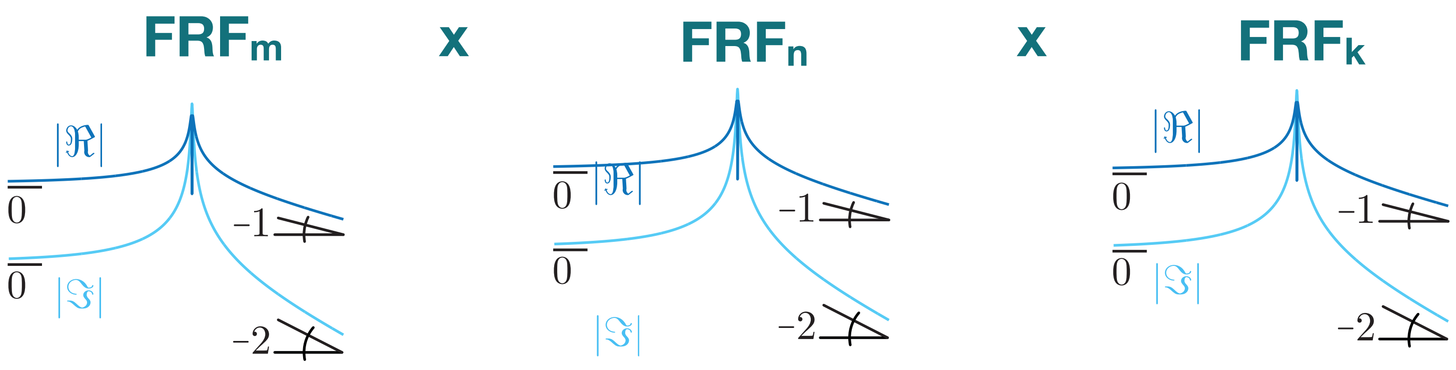


It changes the exponents in the loading components.

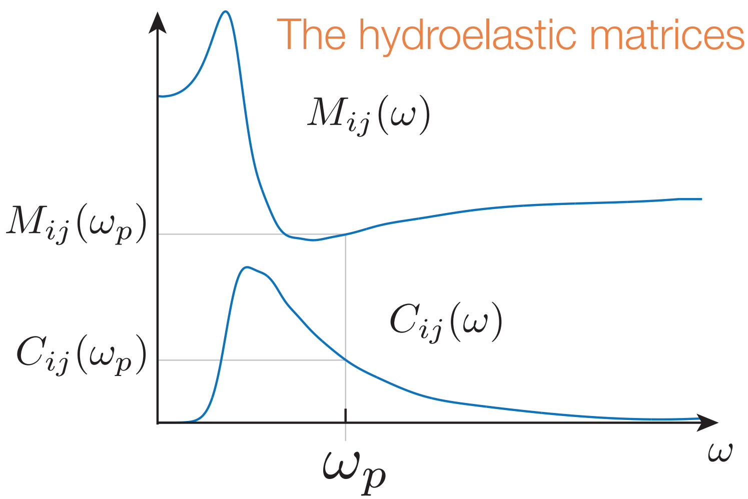
They rely on numerical integrations in the nodal basis.

A few modifications are needed if the properties are frequency-dependent.

The structural kernel has three poles and steeper slopes at first order.



The hydroelastic matrices are considered as constant across the peaks.



Use values at:

- all of the natural frequencies (as for the PSDs of loading)
- the peak frequency of waves (as for the structural kernel)

Released hypotheses

Geuzaine

Denoël

Davenport

SDOF
B + R

MDOF
B + R

BOTH
R + I

matched
loading
component

damping
non
classical

frequency
dependent
properties

along the thesis

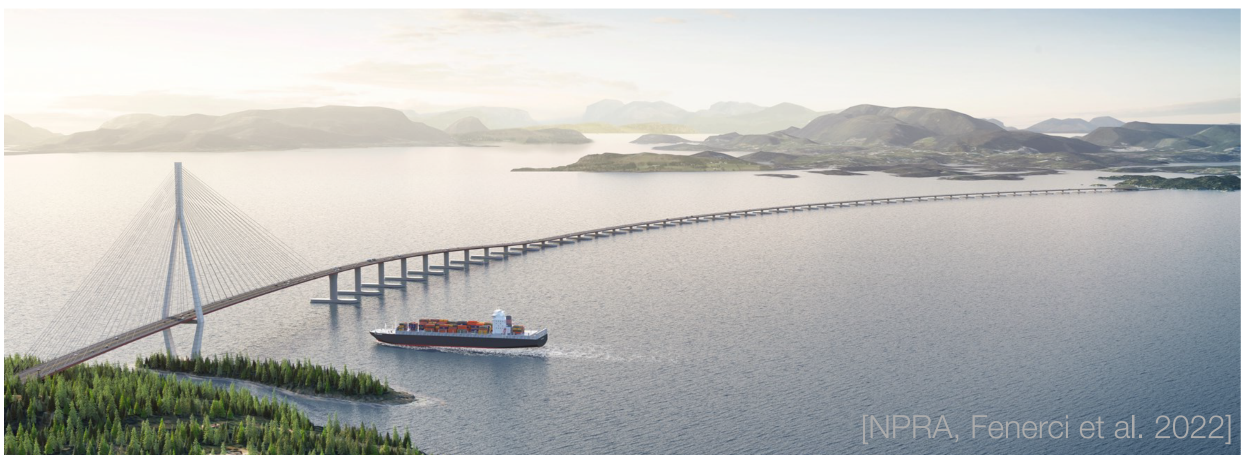
Released hypotheses with a consistent degeneration.

Realistic Applications

A simplified 2D model inspired by the Bergsøysund Bridge in Norway

1

2



A 3D model of the future Bjørnafjorden Bridge, to be build in Norway

Realistic Applications

1

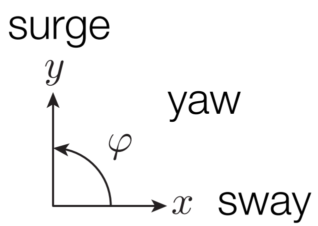
A simplified 2D model inspired by the Bergsøysund Bridge in Norway

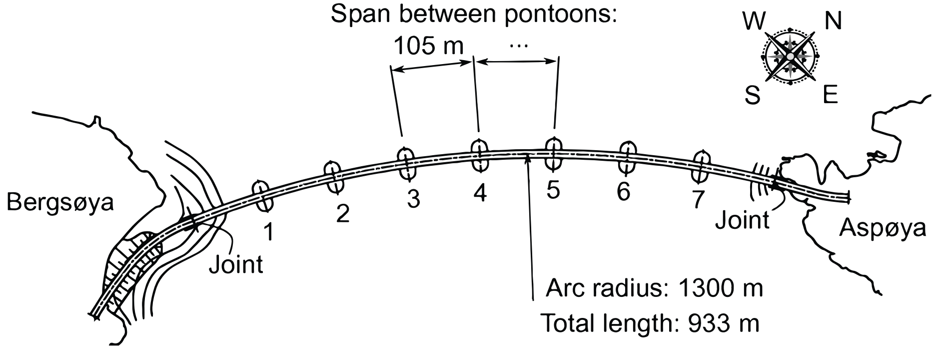
2

A 3D model of the future Bjørnafjorden Bridge, to be build in Norway

Parameters of the Model

- deep water and short-crested waves
- absence of cables > linear behaviour
- a significant wave height of 2,4 m
- a wave peak frequency of 2,2 rad/s
- thus, a wave peak period of 2,85 s





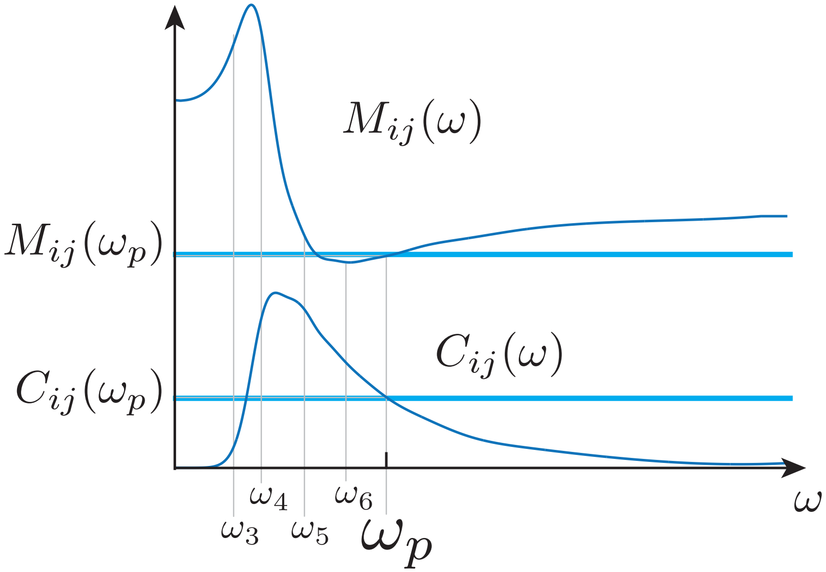
Modal Analysis Results

Scenario 1

freq. dep. is negligible
use constant properties

Scenario 2

freq. dep. is just limited
use freq. dep. properties



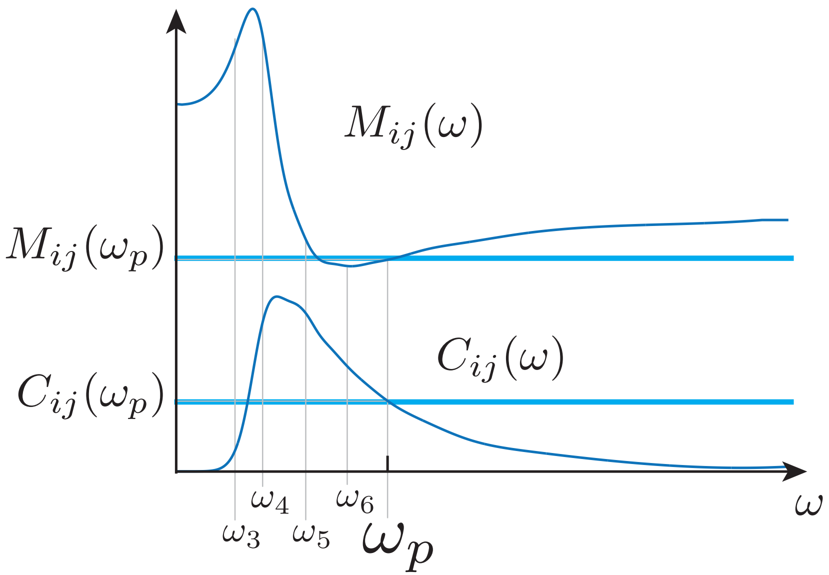
Modal Analysis Results

Scenario 1

freq. dep. is negligible
use constant properties

Scenario 2

freq. dep. is just limited
use freq. dep. properties



	Scenario 1 overestimated	Scenario 2	Mode Shape
j_m	ω_{j_m}	ω_{j_m}	
Unit	[rad/s]	[rad/s]	
1	0.27	0.25	
2	0.42	0.38	
3	0.68	0.62	
4	0.88	0.78	
5	1.32	1.23	
6	1.77	1.75	
7	2.07	2.06	
8	2.73	2.68	
9	3.21	3.12	
10	3.75	3.59	
11	4.32	4.08	
12	4.85	4.54	
13	5.33	4.96	
14	5.38	4.87	
15	5.77	5.28	

closest to ω_p

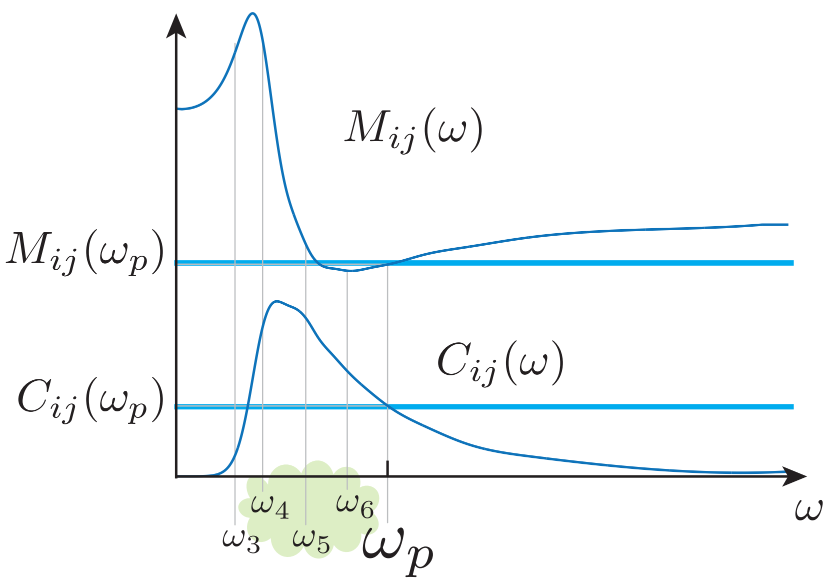
Modal Analysis Results

Scenario 1

freq. dep. is negligible
use constant properties

Scenario 2

freq. dep. is just limited
use freq. dep. properties

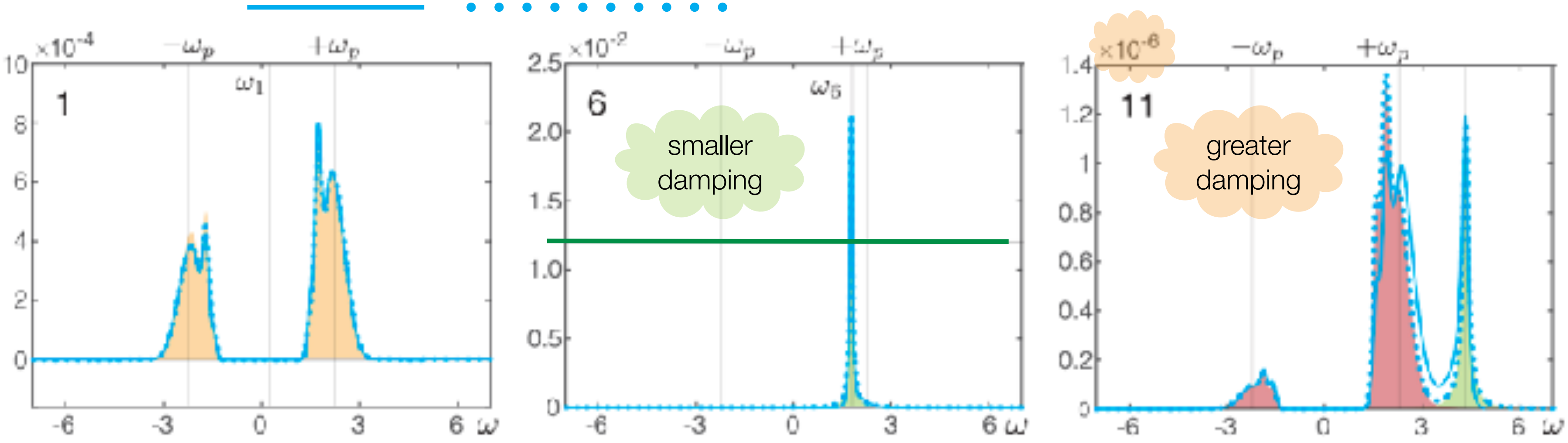


	Scenario 1 overestimated		Scenario 2	Mode Shape
f_m	ξ_{f_m}		ξ_{f_m}	
Unit	[%]		[%]	
1	16.5		0.01	
2	10.1		0.12	
3	5.93		1.08	
4	5.27		2.43	
5	3.64	except for	5.05	
6	2.79		3.86	
7	2.19		2.43	
8	5.58		4.13	
9	2.84		1.16	
10	2.65		0.71	
11	2.51		0.43	
12	2.38		0.25	
13	2.28		0.24	
14	2.56		0.15	
15	2.38		0.18	

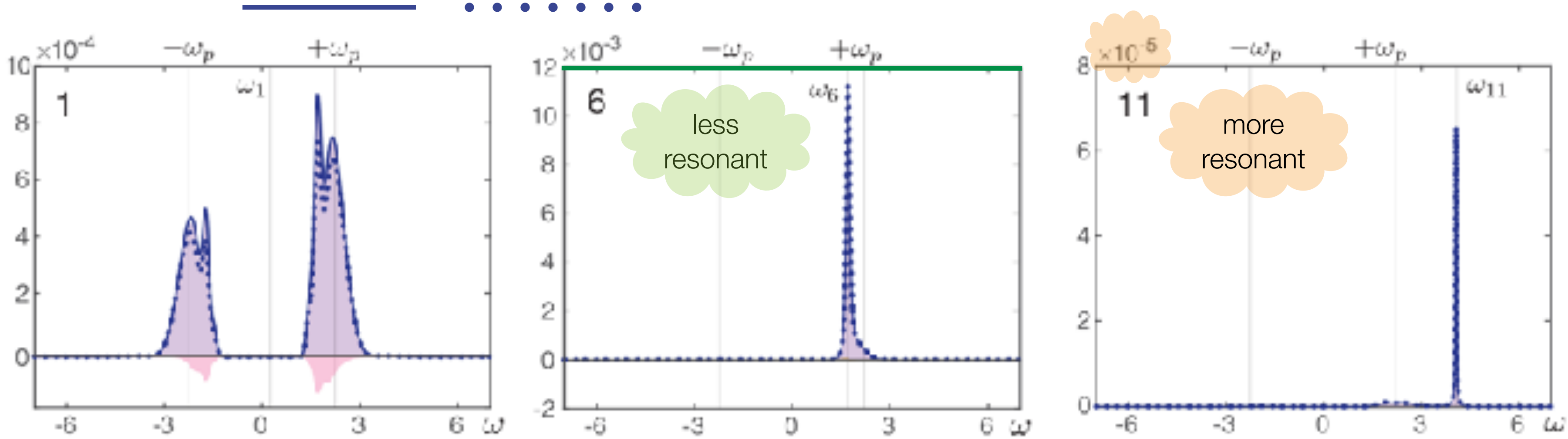
Modal Auto-PSDs

resonant
inertial
mixed
background
leading order
second order

Scenario 1 - reference + decomposition



Scenario 2 - reference + expansion

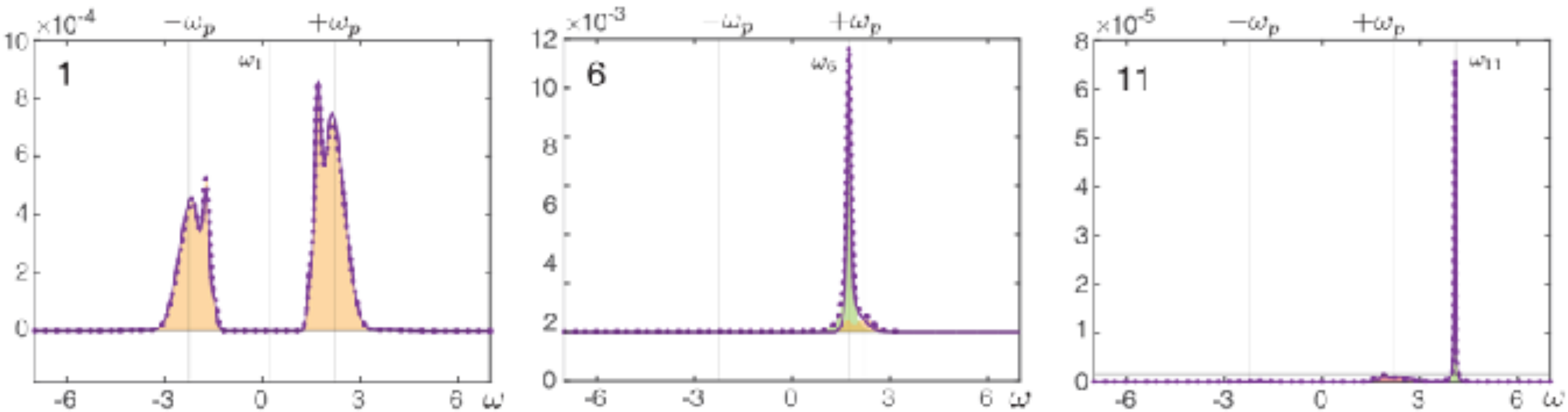


Modal Auto-PSDs

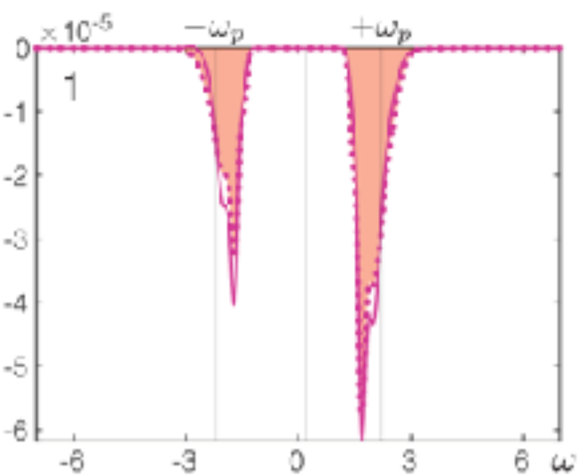
resonant
inertial
mixed

background
leading order
second order

Scenario 2 - leading order term + decomposition

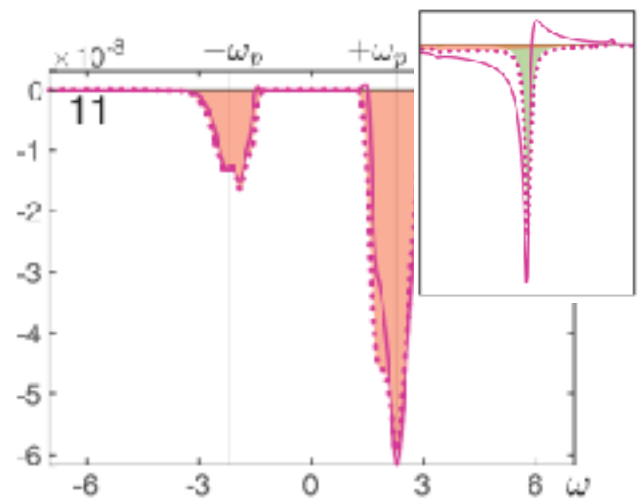


Scenario 2 - second order term + decomposition

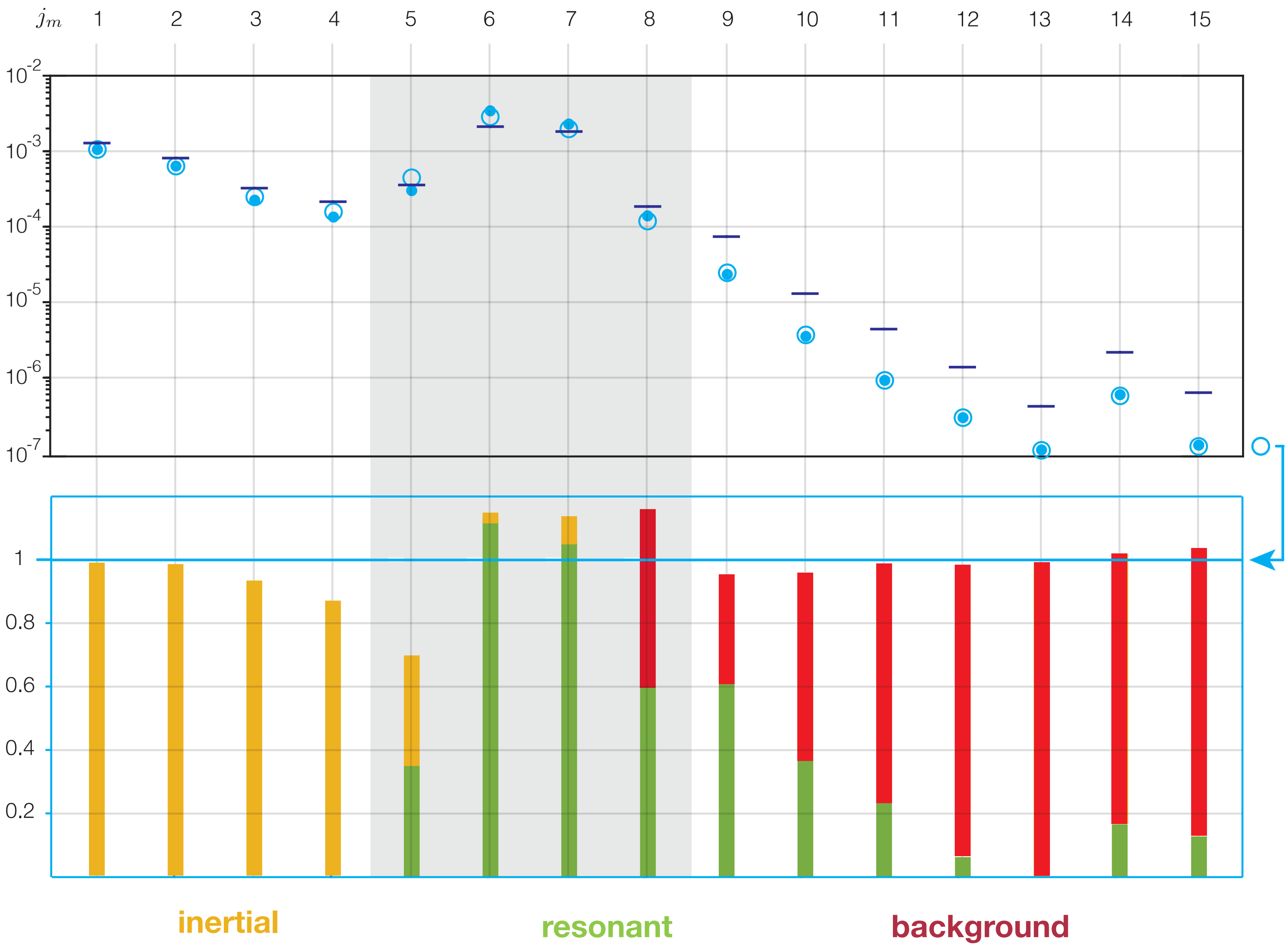


6

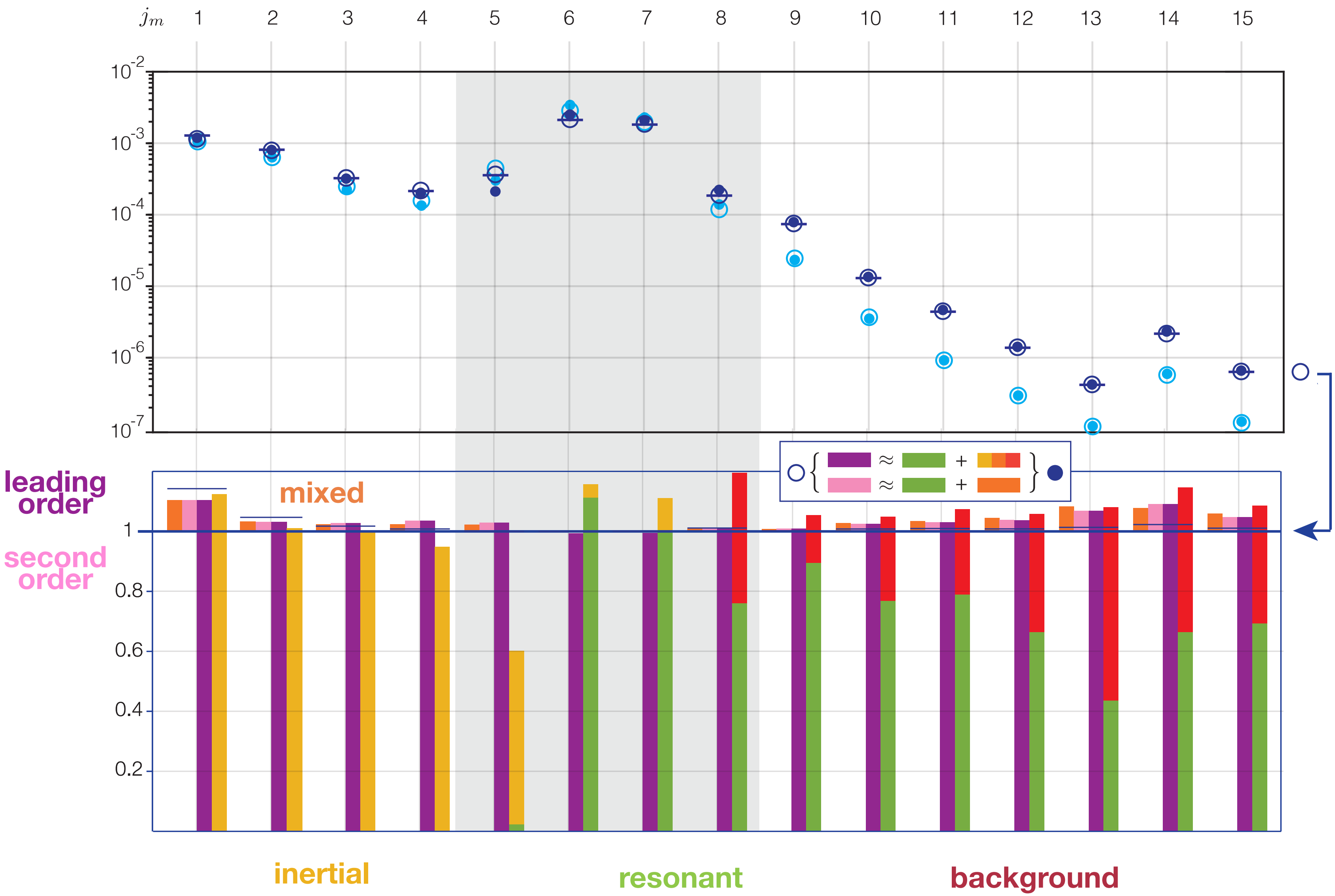
almost nothing



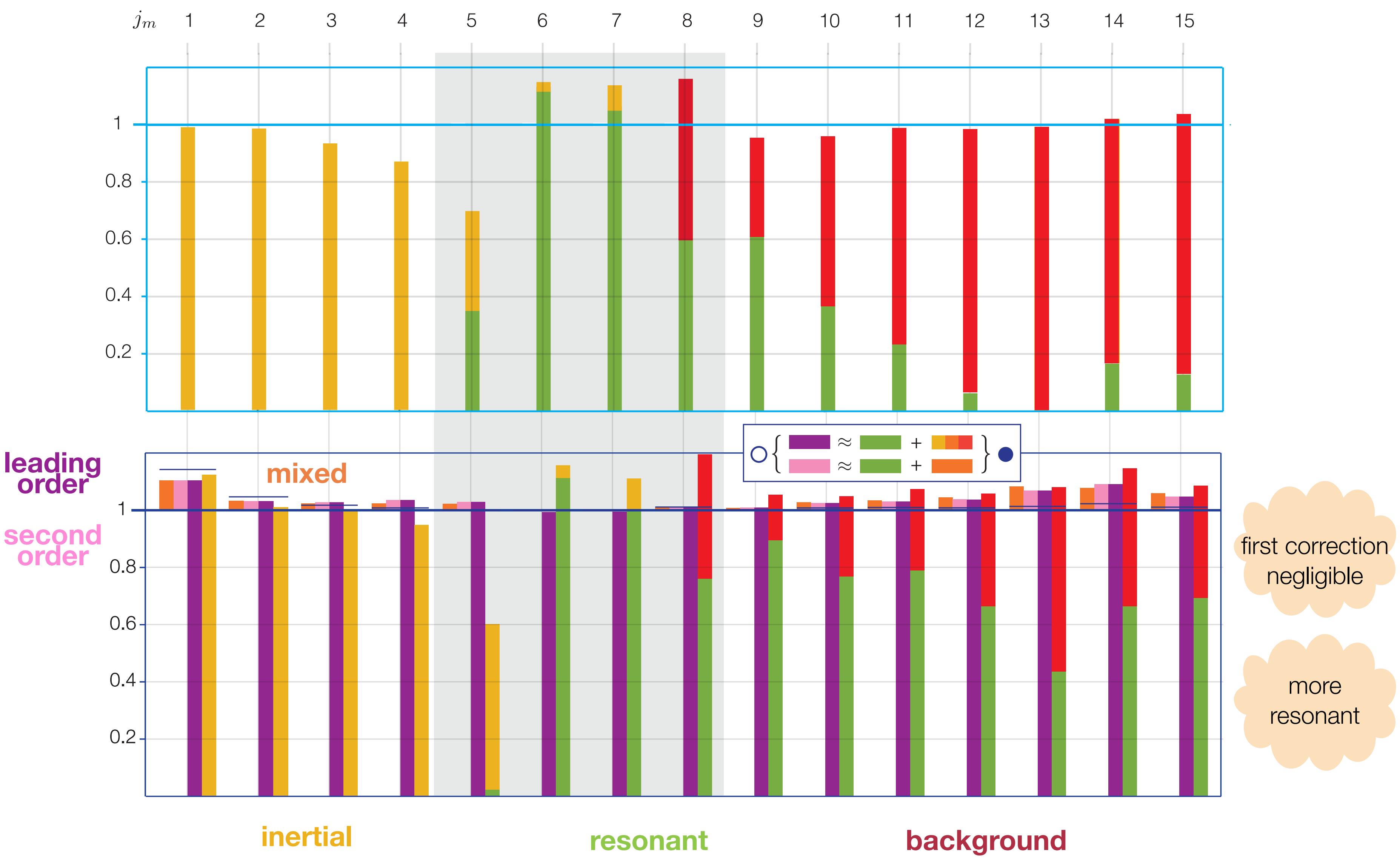
Modal Variances - **Scenario 1**



Modal Variances - **Scenario 2**



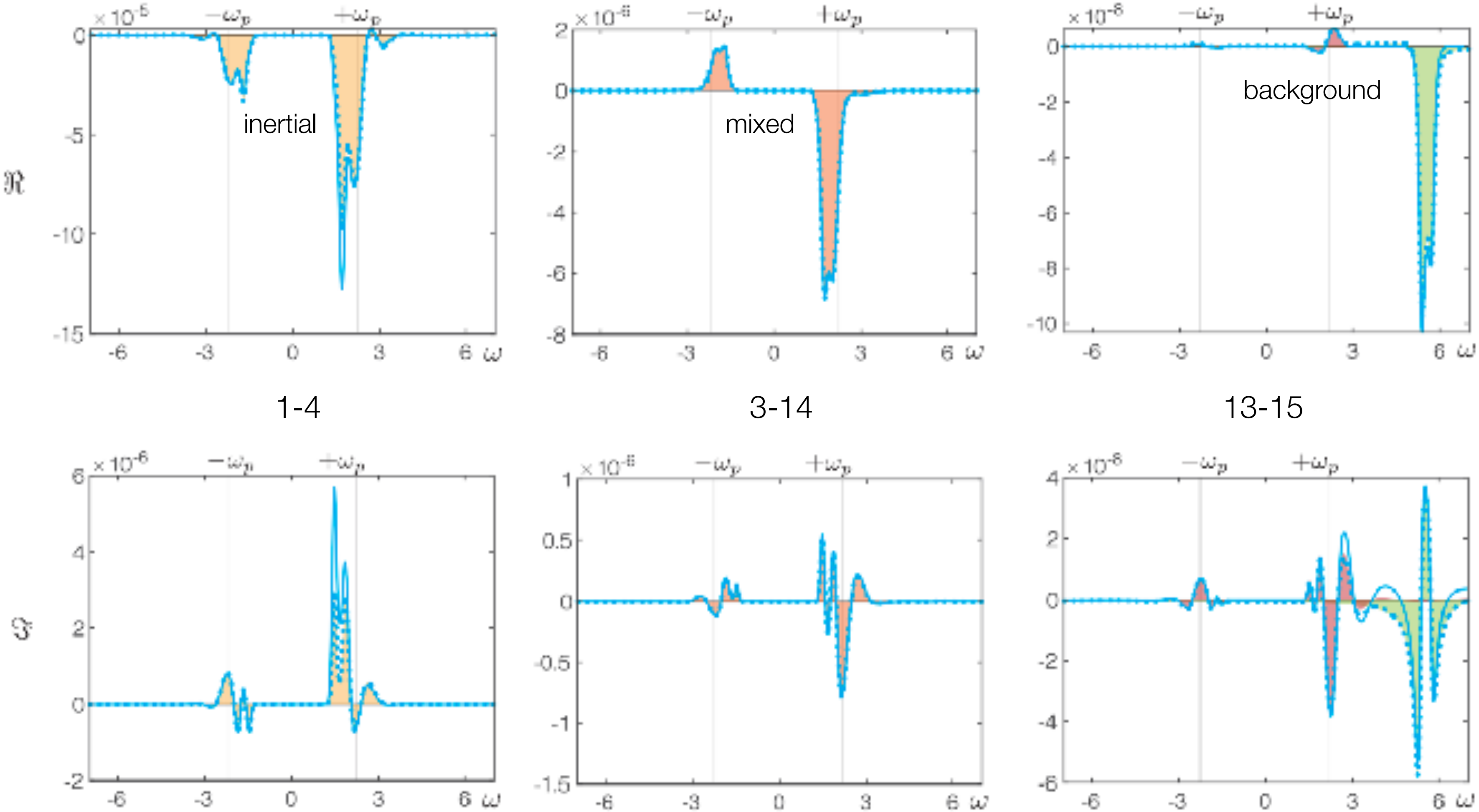
Modal Variances - Comparison



Modal Cross-PSDs

Scenario 1 - reference + decomposition

resonant — inertial — mixed — background — leading order — second order

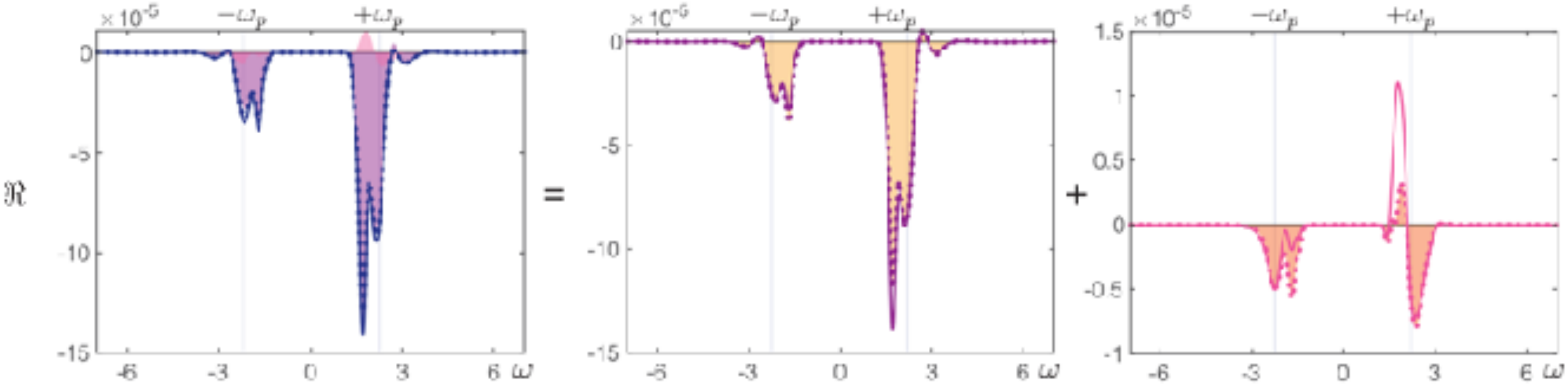


Modal Cross-PSDs

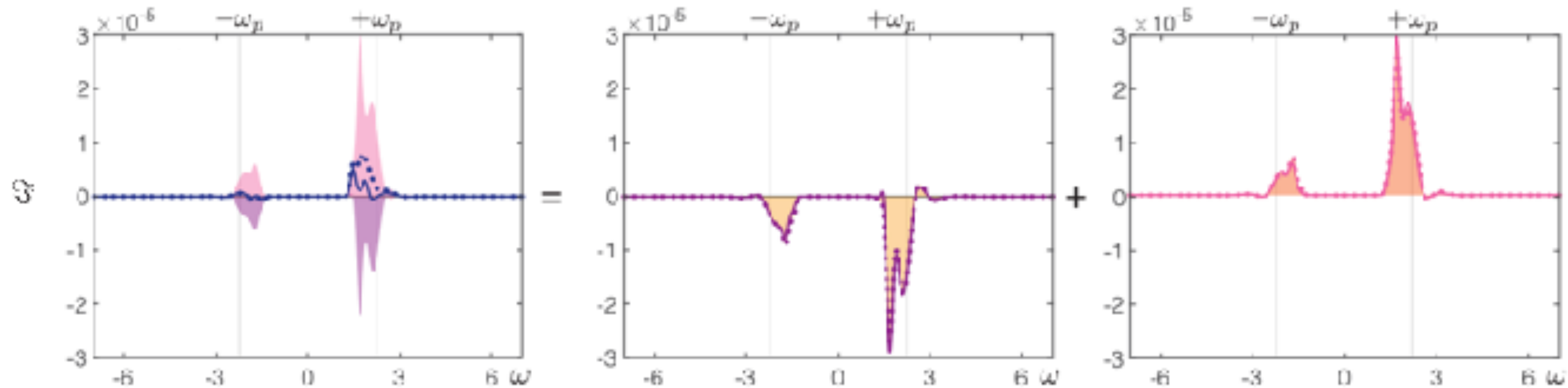
resonant
inertial
mixed

background
leading order
second order

Scenario 2 - reference + expansion + decomposition



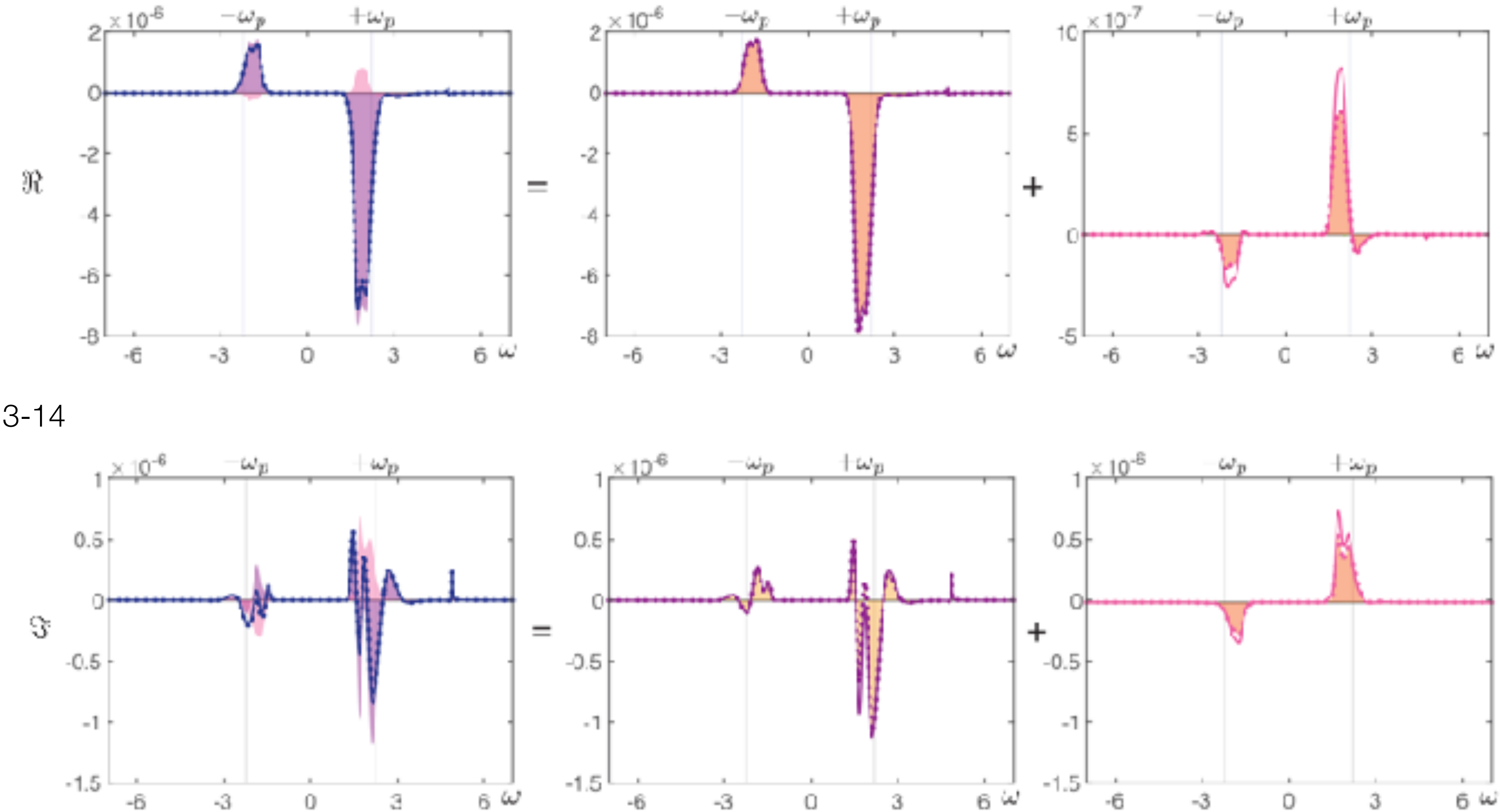
1-4



Modal Cross-PSDs

resonant
inertial
mixed
background
leading order
second order

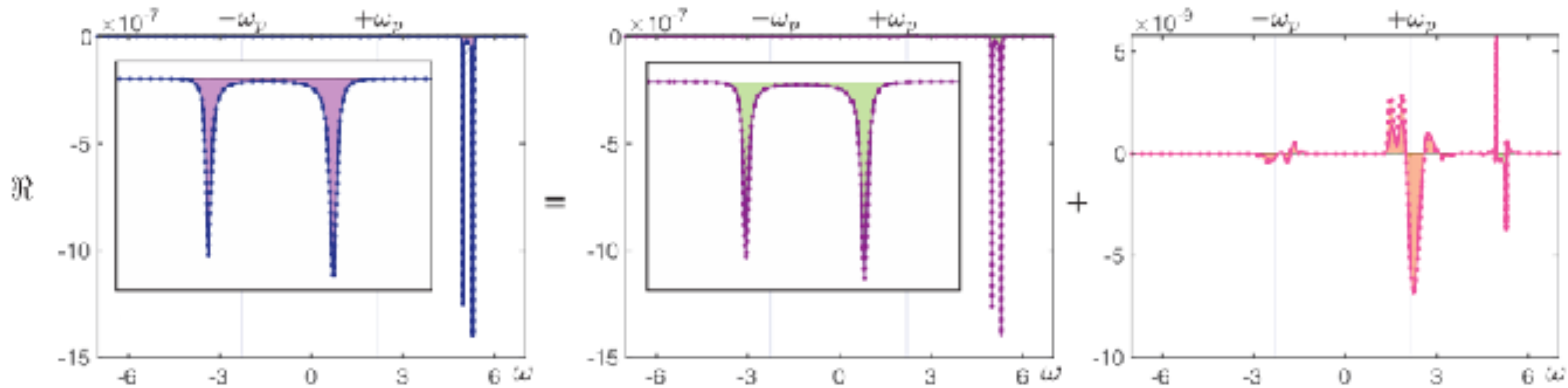
Scenario 2 - reference + expansion + decomposition



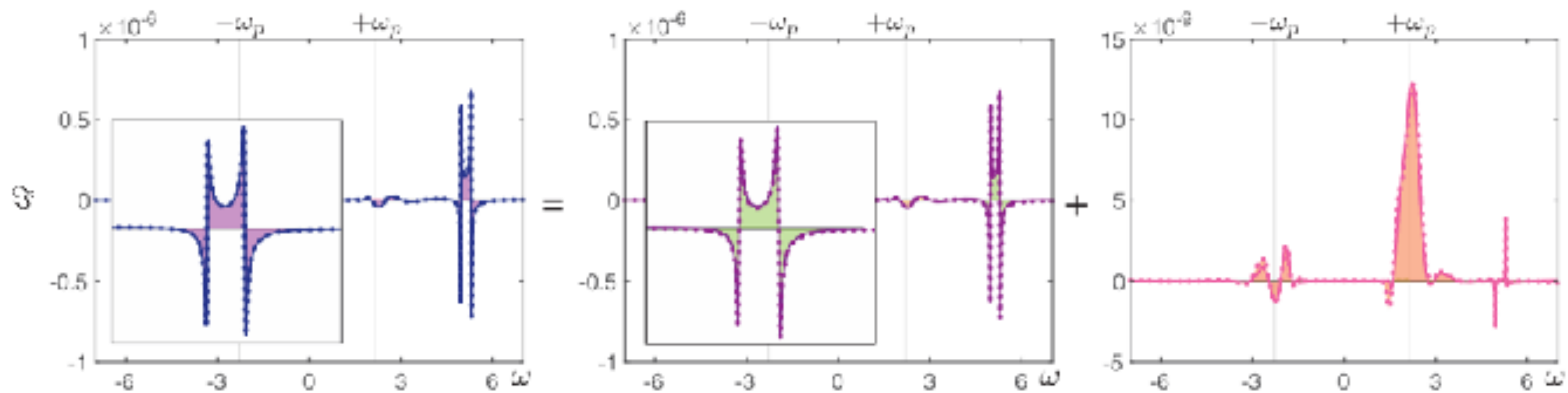
Modal Cross-PSDs

Scenario 2 - reference + expansion + decomposition

resonant background
inertial leading order
mixed second order



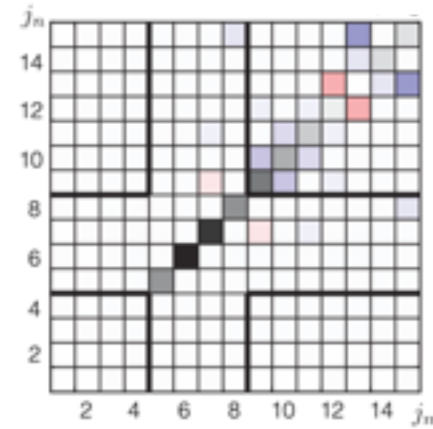
13-15



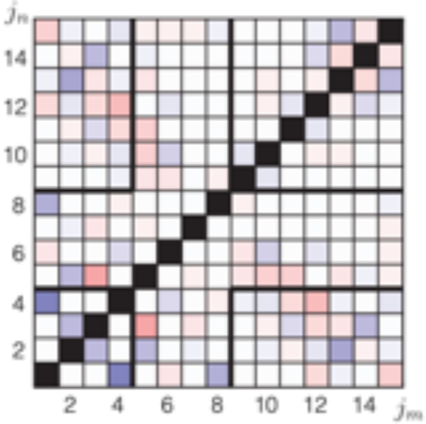
Modal Correlations

Scenario 1 - reference + approximation + decomposition

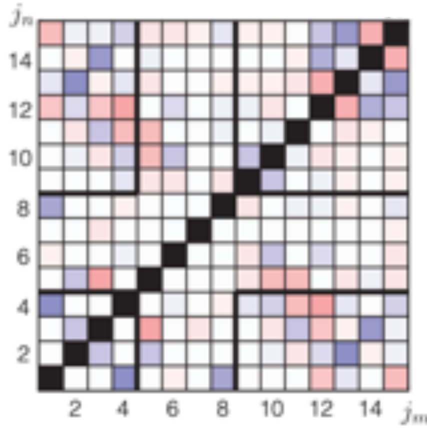
Resonant Component



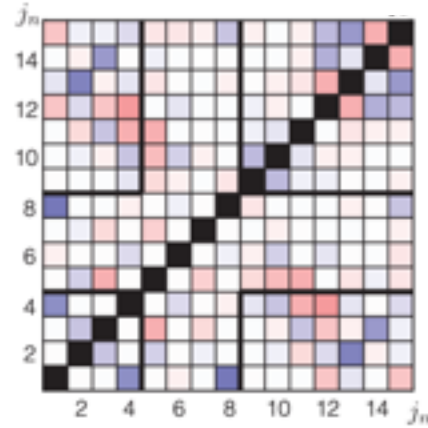
Reference



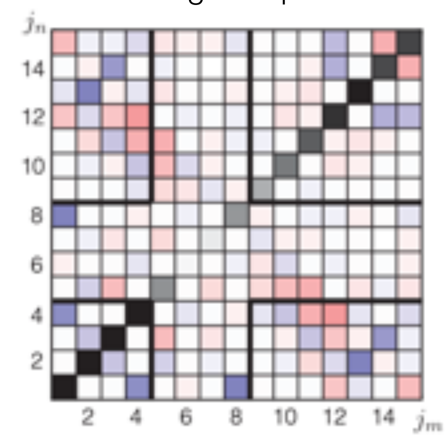
Decoupling Approximation



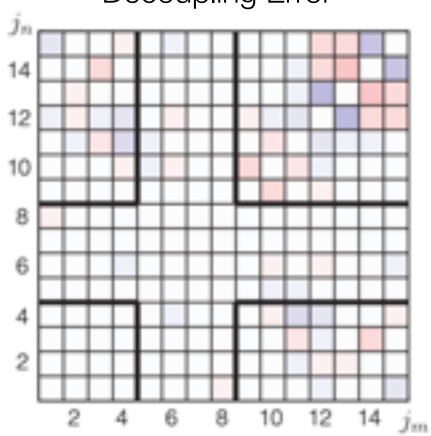
MTSA Decomposition



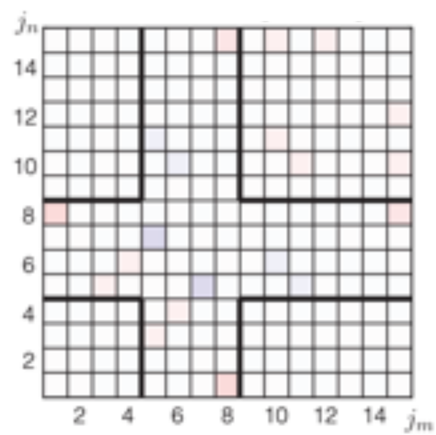
Loading Component



Decoupling Error



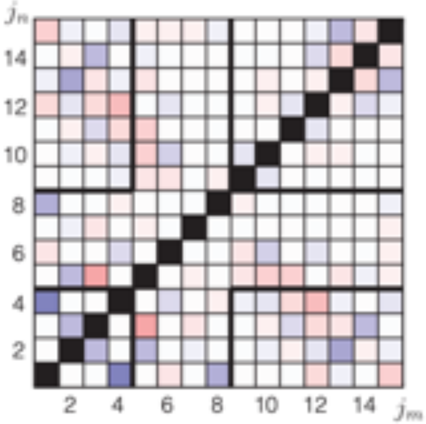
MTSA Error



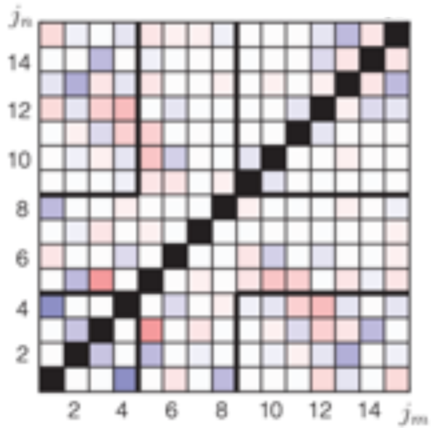
Modal Correlations

Scenario 2 - reference + approximation + decomposition

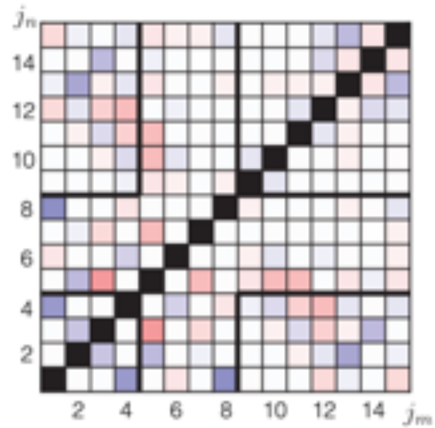
Reference



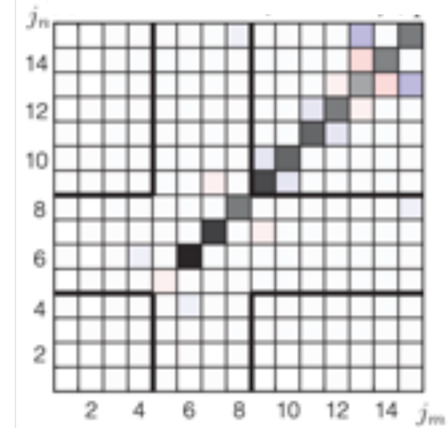
Decoupling Approximation



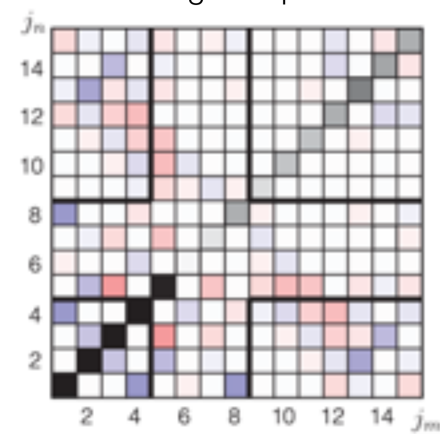
MTSA Decomposition



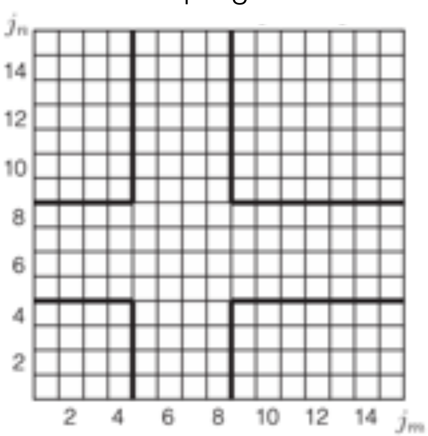
Resonant Component



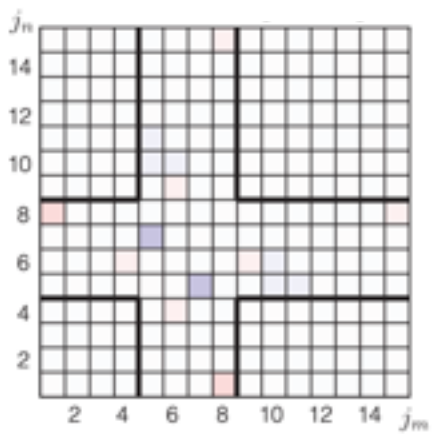
Loading Component



Decoupling Error



MTSA Error

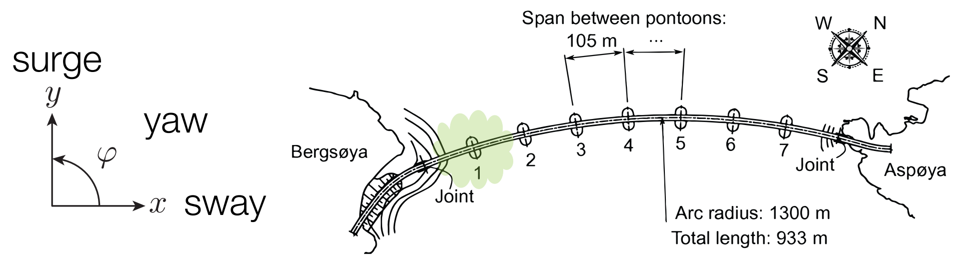


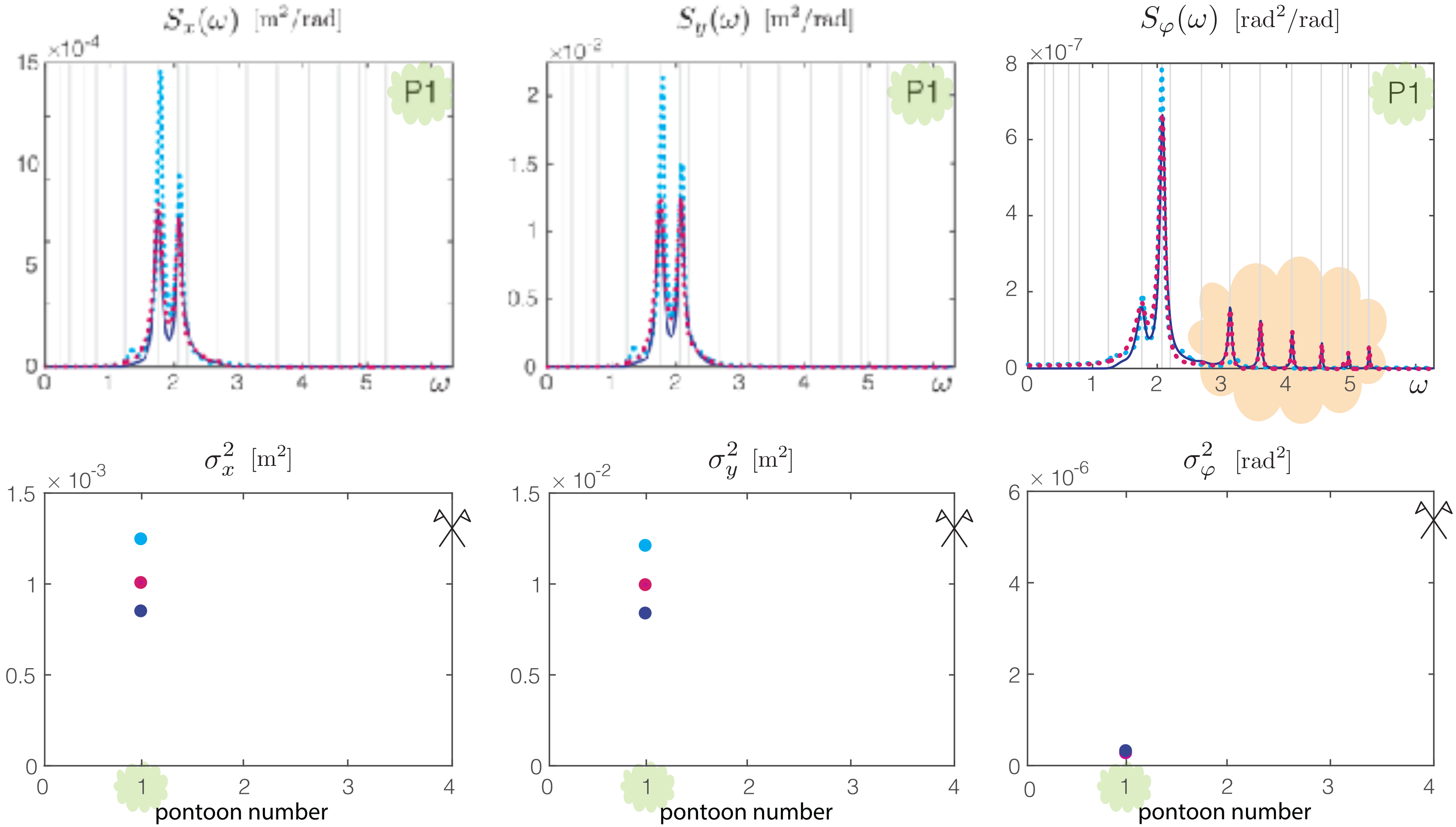
less error

more resonant

Nodal Recombinations

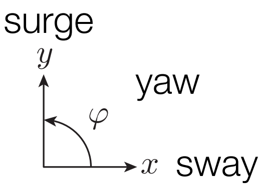
REFERENCE - **MTSA 1** - **MTSA 2**

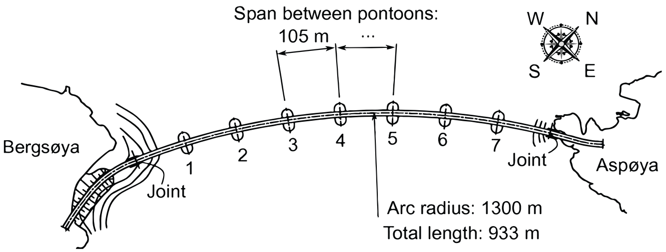


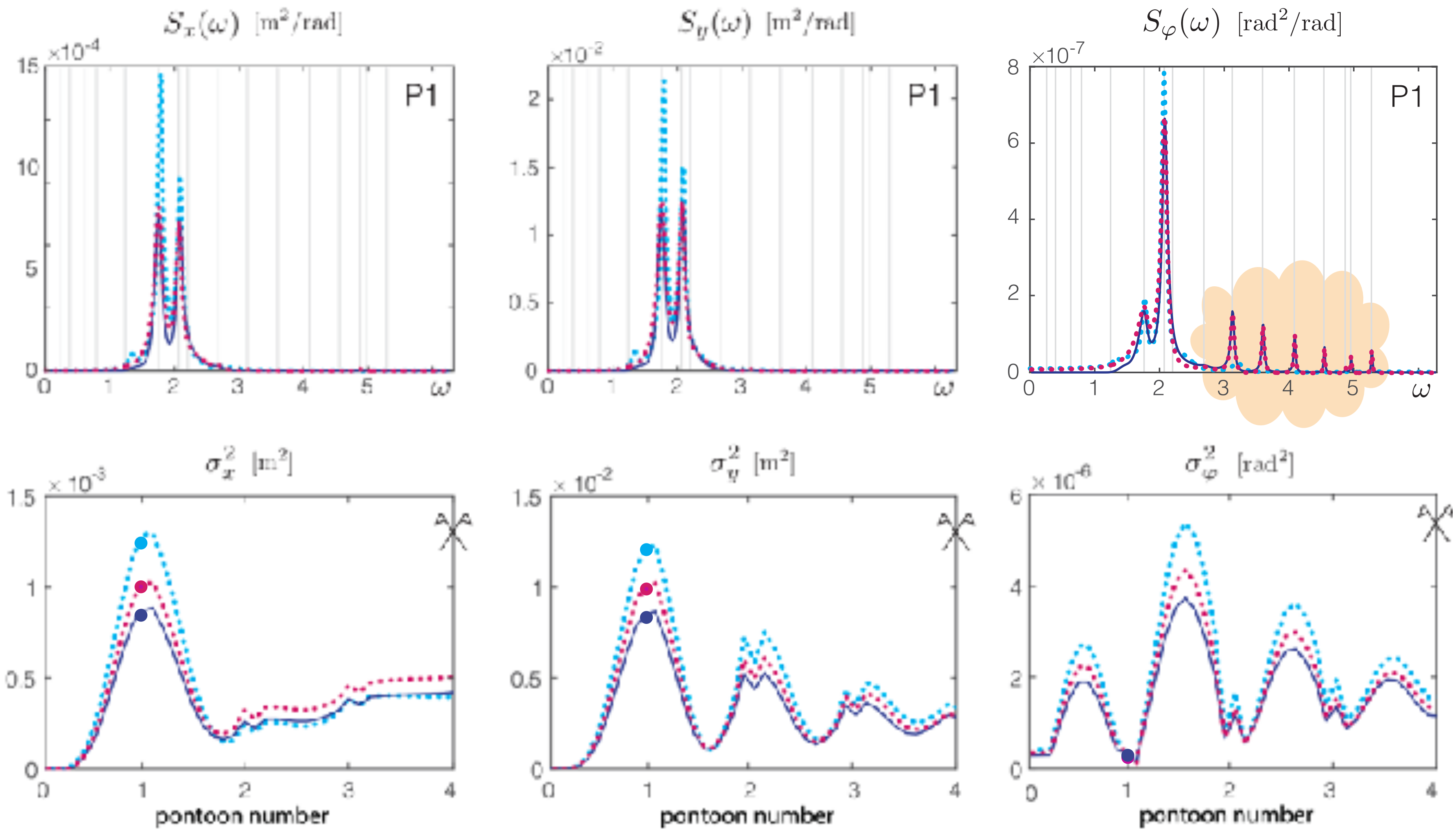


Nodal Recombinations

REFERENCE - **MTSA 1** - **MTSA 2**

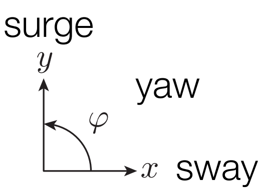


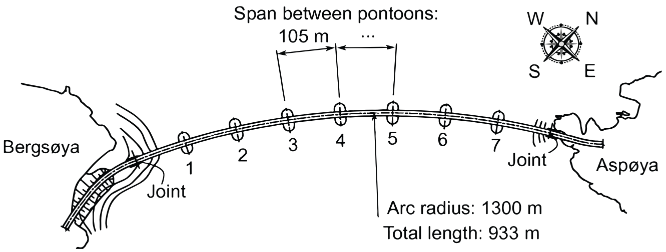


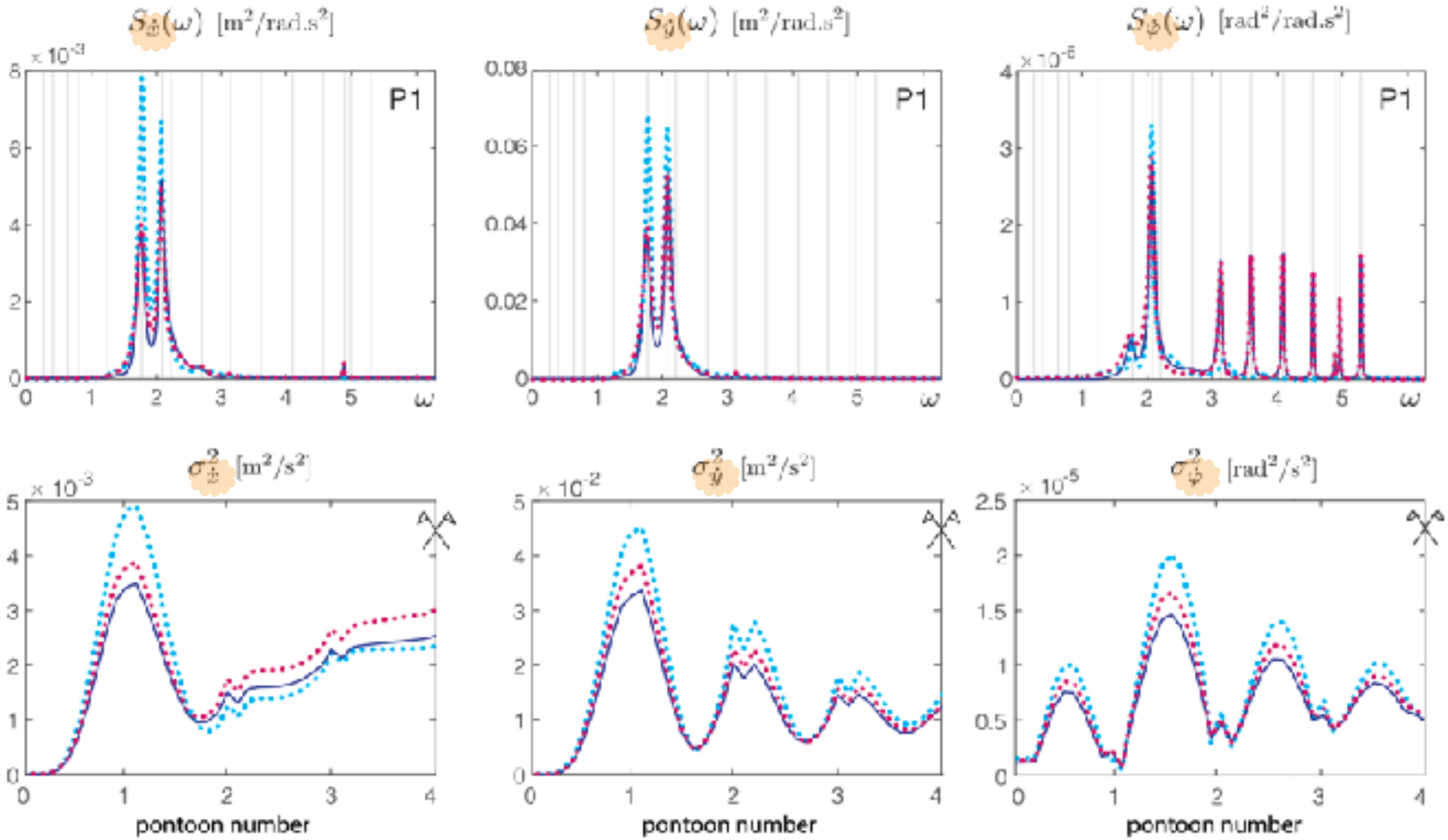


Nodal Recombinations

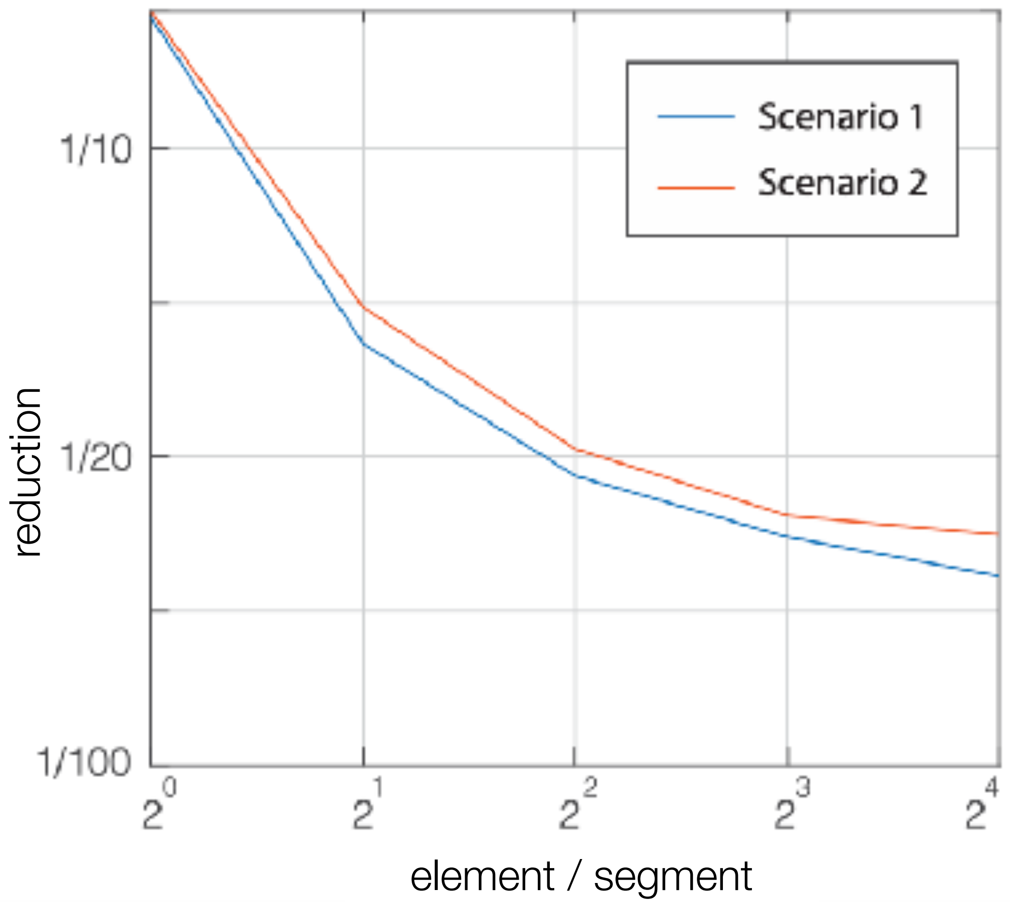
REFERENCE - **MTSA 1** - **MTSA 2**







Gain in CPU time thanks to MTSA increases with the number of dofs.

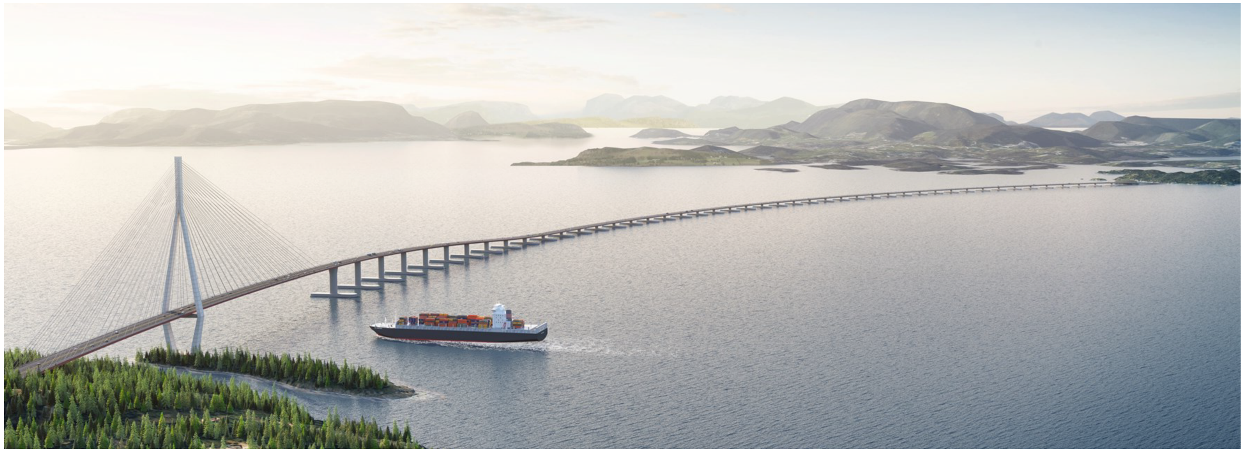


Realistic Applications

A simplified 2D model inspired by the Bergsøysund Bridge in Norway

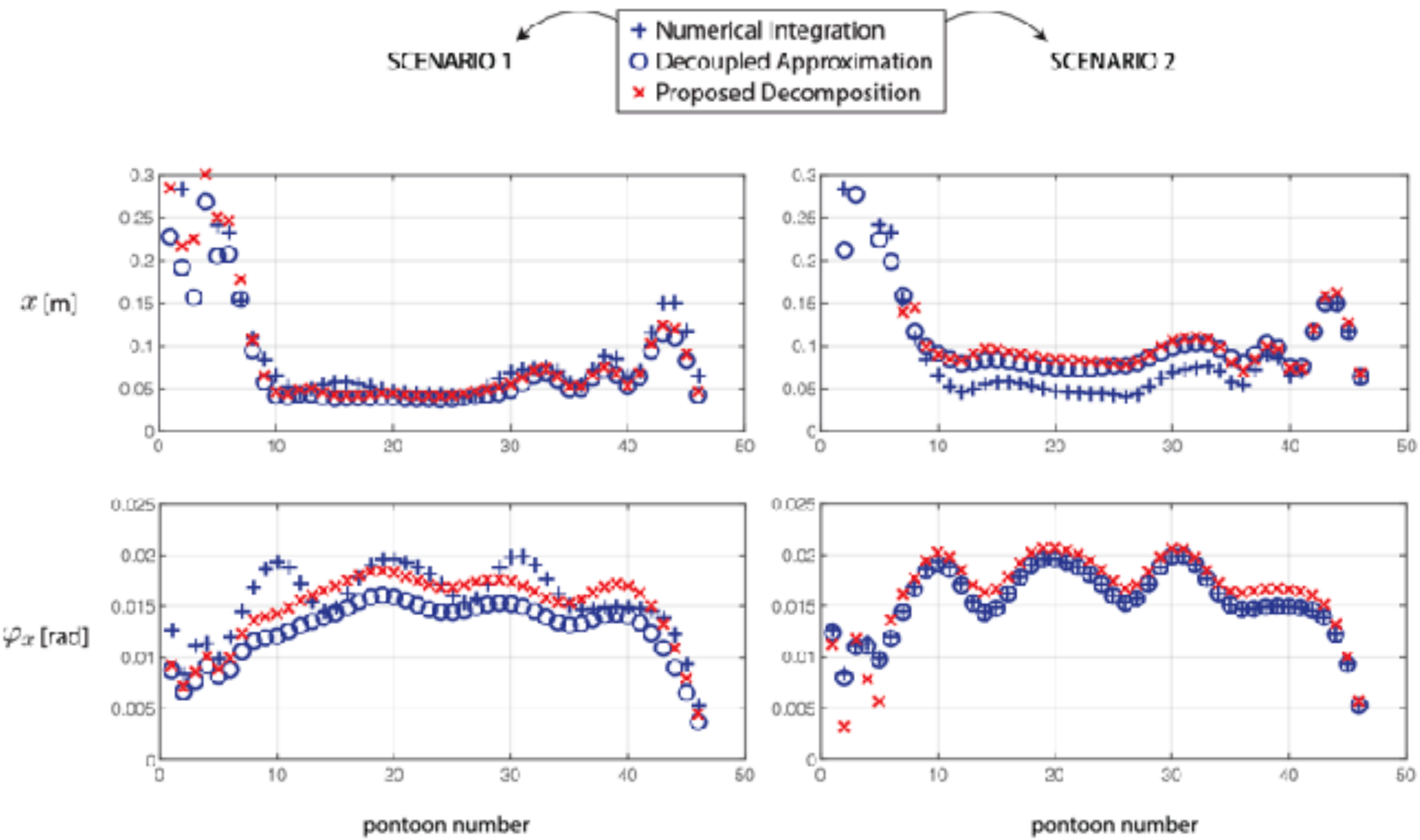
1

2

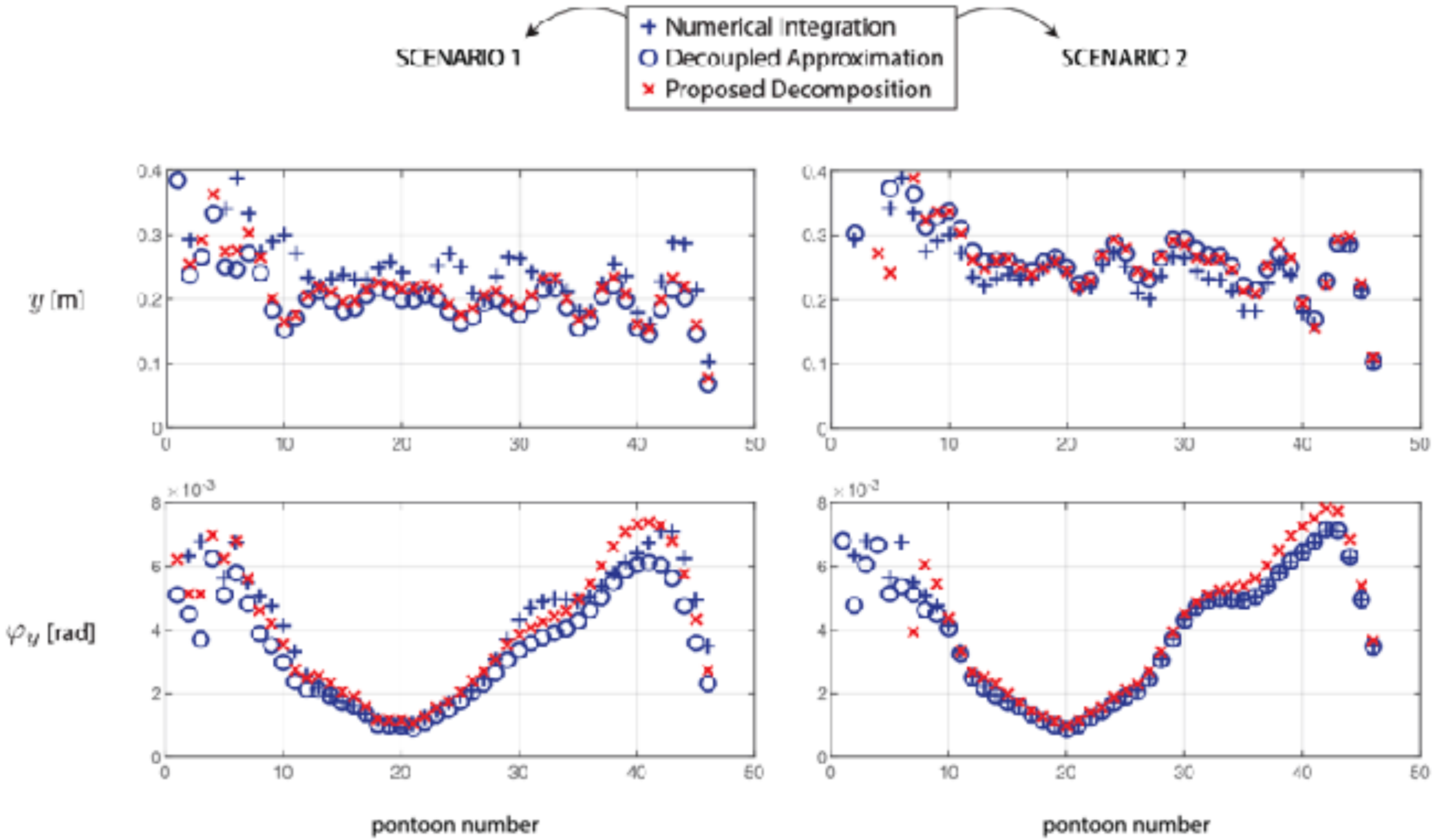


A 3D model of the future Bjørnafjorden Bridge, to be build in Norway

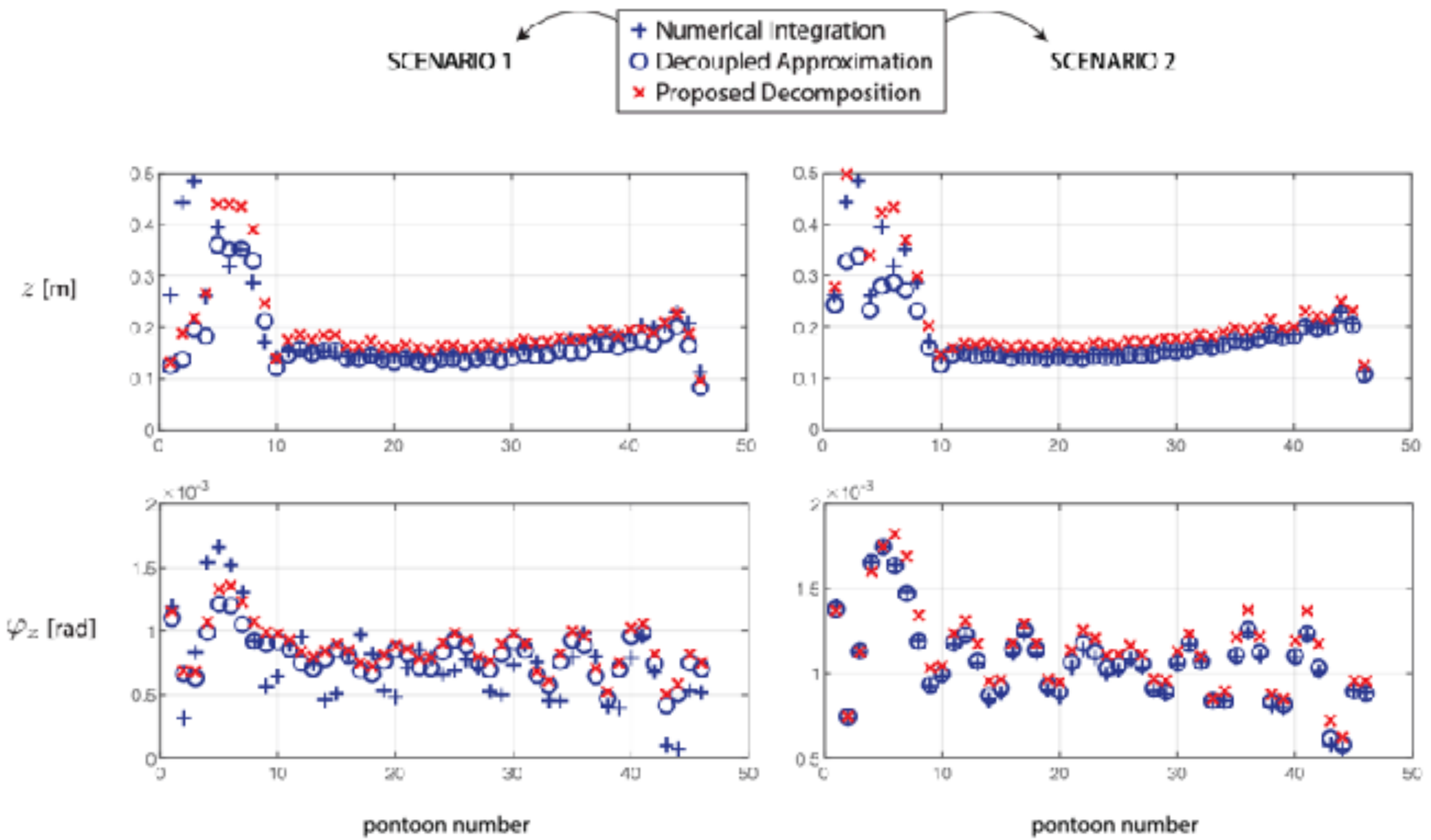
Results in a Nutshell



Results in a Nutshell



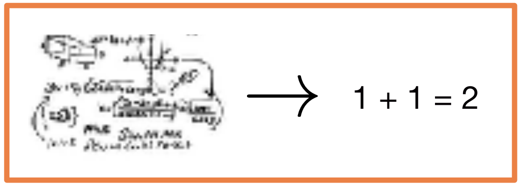
Results in a Nutshell



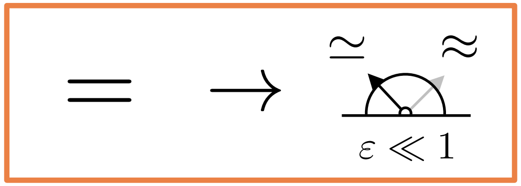
Wave-loaded floating structures can be analyzed 10-100x faster.

- background, inertial & matching
- non-classical damping matrices
- frequency-dependent properties
- partial fraction expansion of FRF
- avoid to project/invert matrices

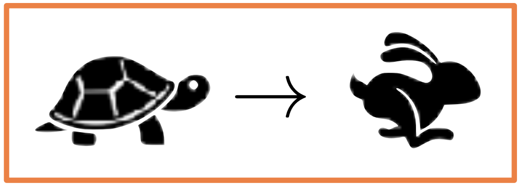
SIMPLICITY



ACCURACY



RAPIDITY

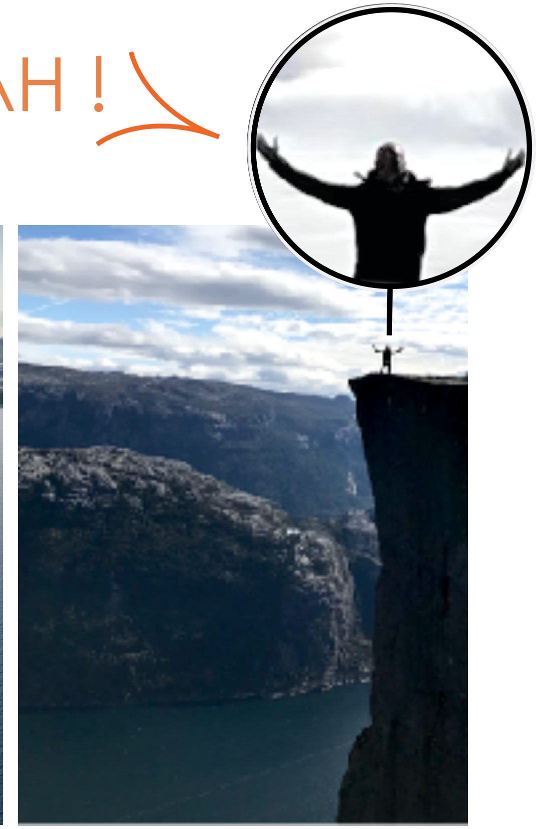


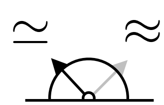
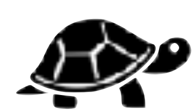
AT SECOND ORDER, BUT NOT COMPLETELY AT THIRD ORDER... YET!

Wave-loaded floating structures can be analyzed 10-100x faster.

YEAH !

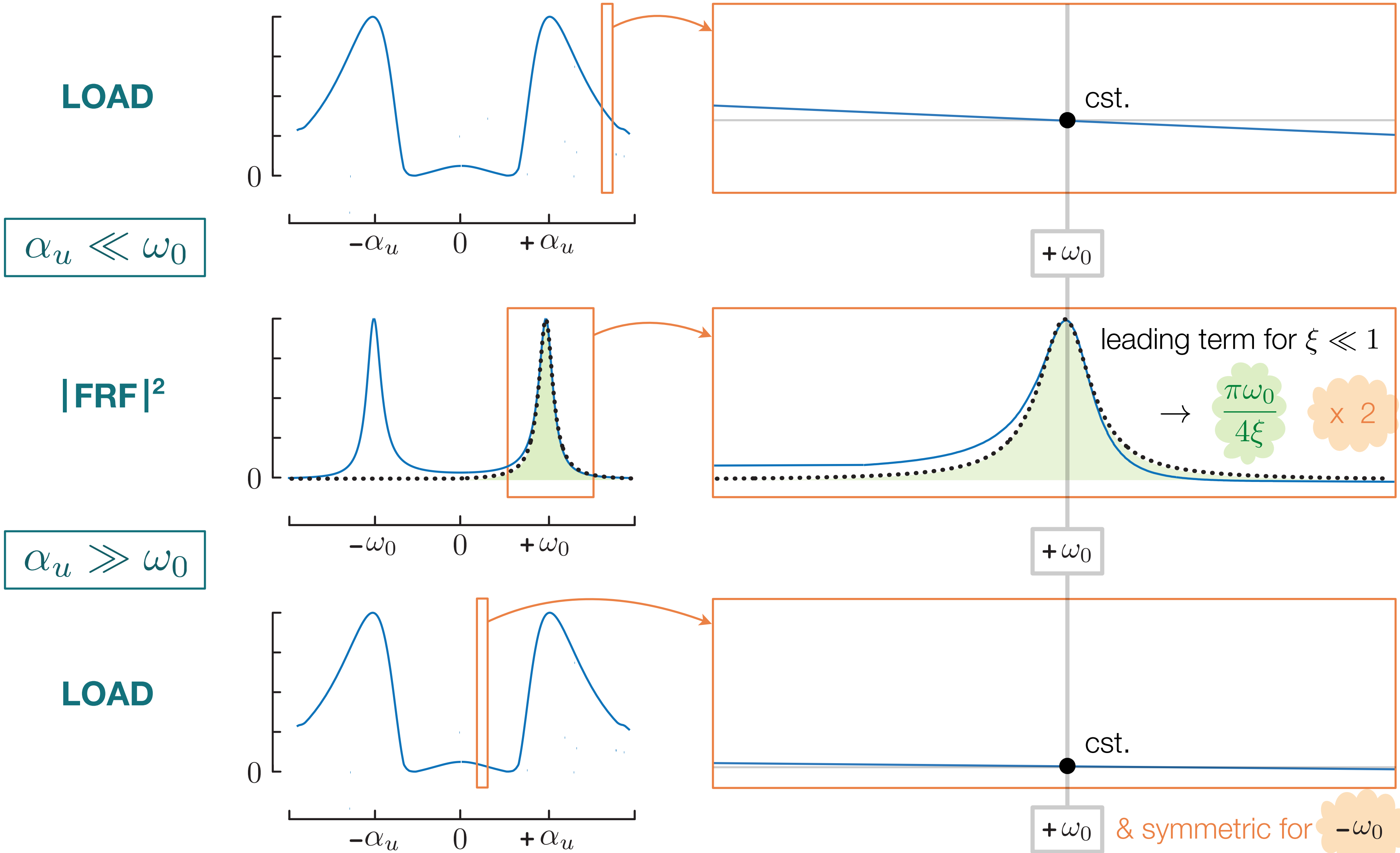
- background, inertial & matching
- non-classical damping matrices
- frequency-dependent properties
- partial fraction expansion of FRF
- avoid to project/invert matrices



SIMPLICITY	ACCURACY	RAPIDITY
 → 1 + 1 = 2	= →  $\varepsilon \ll 1$	 → 

AT SECOND ORDER, BUT NOT COMPLETELY AT THIRD ORDER... YET!

For the resonant component, it is usual to approximate each peak in the kernel.



The loading components differ by the degree of the monomial trends.