THE ADJUSTMENT FOR THE INTERVALLING EFFECT BIAS IN BETA
A BROADER AND MULTIPERIOD TEST

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1 Introduction

The number of empirical studies using daily equity prices and returns has been increasing for some years. Daily data becomes more and more used in research dealing, for example, with event studies and tests of efficiency. This is however not without problems: the substantial information gained from the use of daily data is indeed counterbalanced by the apparition of some statistical and structural problems. So Brown and Warner (1985) called attention to potential important problems which arise when using daily data. Among these are the bias in the Market Model beta and the existence of serial dependence in the returns. The impact of the length of the differencing interval used to measure the returns on the estimated betas was first shown by Pogue and Solnik (1974). Using samples from seven European countries, including Belgium, they found that the daily beta estimators depend on the length of the differencing interval. Subsequently Levhari and Levy (1977) developed a model explaining this so-called intervalling effect. But because of its assumption of independence in the rates of return, this model is not valid for short differencing intervals. In 1980, Hawawini established the relationship between the beta and the length of the differencing interval when returns are defined as continuously compounded returns. Besides, Hawawini's model formulation clearly shows that the betas also depend on the index specification. This dependence was shown later by Saniga and al. (1981) for the US and by Corhay (1988) for Belgium.

In 1980 Cohen, Hawawini, Maier, Schwartz and Whitcomb (CHMSW) ascribed the intervalling effect bias in beta to the friction in the trading process. Infrequent trading, or more generally, delays in the adjustment of a security price to a change in information induces cross serial correlation in the security returns and subsequently autocorrelation in the market index returns. According to CHMSW's theory the expected magnitude of the price-adjustment delays is inversely related to the thinness of the securities: thinner securities have greater adjustment delays than frequently traded securities, they also have a downward bias in their betas while relatively frequently traded securities have an upward bias. Naturally, as the differencing interval used to measure the returns is increased, the adjustment delays vanish and the beta of a security approaches an asymptotic value which is an estimate of the true security beta.

The first models dealing with the intervalling effect bias in beta were presented by Scholes and Williams (1977) and Dimson (1979). Both imputed the intervalling effect bias in beta to infrequent trading. Their models consist in regressing observed returns against synchronous or lagged market returns in order to avoid the problem of serial correlation, and then in aggregating the slope coefficients obtained to calculate a beta estimate. Beyond the difficulty of obtaining high synchronous data, these models do not take account of the thinness. In 1983 CHMSW presented two procedures for correcting the empirical estimates of beta for the intervalling effect. Their first procedure consists in estimating an asymptotic beta by increasing the length of the differencing interval, and the second one consists in adjusting the beta estimates by cross-sectional differences in the intervalling effect. A proxy variable for the security thinness, the market value of the outstanding shares, is used to determine the cross-sectional differences. These procedures were tested by CHMSW (1983) and Fung, Schwartz and Whitcomb (FSW) (1985) on samples of 50 American and 52 French securities respectively. The results of these two studies support CHMSW's theory while another study of McInish and Wood (1986), using intermediate results of CHMSW, does not plead in favour of it.

The purpose of this paper is to test CHMSW's adjustment procedures on a comprehensive sample of Belgian securities of the spot market of the Brussels Stock Exchange (BSE). The analysis will be replicated for three time periods and for various proxy variables of the thinness. Two market indexes, an equally weighted and a value weighted, will also be used in the tests. The performance of CHMSW's procedures will then be compared to that of the Scholes and Williams adjustment method.

In addition a preliminary test will be carried on the relationship between the intervalling effect and the size or small firm effect.

The Belgian case is interesting for various reasons. First because Pogue and Solnik (1974) showed on a small sample of Belgian securities that the daily beta estimates are very sensitive to the length of the differencing interval. The second reason concerns the microstructure of the spot market of the BSE. Its trading process is indeed characterized by some peculiar elements which have an impact on the security price adjustment delay. On the one hand some elements tend to decrease the adjustment delay. So, the BSE spot market is, like the Paris Bourse, a periodic call market where all spot securities are traded at the same time once a day. There is no dealer but a trading advantage is given to some brokers who act as specialists. On the other hand, the price adjustment delay is lengthened because the spot market is mainly a written batch trading system. Traders who are on the floor cannot always modify their orders. After a trial price is given the orders can be modified, but only if they tend to absorb the excess demand or supply of shares determined by the clearing price of the auction. Another positive factor on the price adjustment delay are the maximum price-change limits of the security prices. The maximum change in the price allowed for a security on the spot market varies between 2 to 10 percent according to the degree of trading of the security in general. If the price change generated by the clearing price of the auction is beyond the specified limit, no trade or a partial trade is allowed and the price is quoted at the previous day's price plus the maximum price-change limit unless there are orders that tend to absorb the excess demand or supply. All of these particularities of the BSE microstructure affect the price adjustment delay and hence the intervalling effect.

2 The Sample and the Test Methodology

2.1 The Sample

The data base consists in the daily returns of 250 domestic securities traded on the spot market of the Brussels Stock Exchange (BSE)[1]. The time period covered is from January 1977 to December 1985. The returns, 2213 for the whole period, are continuously compounded returns $[\ln(P_t/P_{t-1})]$; they are corrected for all capital adjustments and they incorporate dividends. Alongside the returns,

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[1] This represents approximately the complete spot market of the BSE. Some securities have been deleted since they were traded less than twenty times during the whole period of study.

the data base also includes the market value of the outstanding shares of the securities as well as their volume of trading. An equally weighted index and a value weighted index, both calculated on the 250 securities of the data base, are used as market indexes.

The total 9-year period is divided into three three-year subperiods of 738 (1977 to 1979), 735 (1980 to 1982) and 740 daily returns (1983 to 1985). In order to avoid data problems due to the listing and delisting of securities, I have selected the securities on the basis of their continuous presence on a whole subperiod. Therefore the number of securities for the three subperiods is respectively reduced to 153, 180 and 164 securities.

2.2 Test Methodology

In 1983 CHMSW developed two procedures for obtaining unbiased beta estimates. Both procedures assume that the true returns are generated by the Market Model,

$$\tilde{R}_{it} = \alpha_i + \beta_i \tilde{R}_{mt} + \tilde{\varepsilon}_{it} \tag{1}$$

and that, as the differencing interval used to calculate the returns is lengthened, an ordinary least square security beta asymptotically approaches its true value.

$$\beta_i^* = \lim_{L \to \infty} \beta_i(L) \tag{2}$$

where $\beta_i(L)$ is an ordinary least square estimator of beta for a differencing interval of length L.

Their first adjustment method for the intervalling effect bias in beta consists in computing the estimated asymptotic beta β_i^* by increasing the length of the differencing interval. In their second method an inferred beta is estimated by adjusting the OLS security betas by cross sectional differences in the intervalling effect. Both methods of adjustment need a two-step procedure. The asymptotic betas as well as the intervalling effect on the OLS betas are estimated using a two-pass regression analysis. A third pass regression is then run to estimate the inferred asymptotic betas.

In this paper, CHMSW's adjustment methods for the intervalling effect bias in betas are tested using a comprehensive sample of daily returns from the BSE. Two market indexes, an equally weighted and a value weighted, are also used and the analysis is replicated for each of the three-year subperiods.

The beta of each security i is first estimated for a set of differencing interval lengths L by a first pass regression or Market Model,

$$R_{iLt} = {}_1\hat{\alpha}_{iL} + {}_1\hat{\beta}_{iL} R_{mLt} + {}_1\varepsilon_{iLt} \qquad (3)$$

where R_{iLt} and R_{mLt} are respectively the returns of security i and the market index, measured as the continuously compounded returns, over a differencing interval of L days, L varying from one day to thirty days. The prescript 1 in equation (3) indicates that the parameters are estimated by a first pass regression.

Besides, Corhay (1988) showed that the estimated Belgian security betas exhibit a seasonal pattern, especially when an equally weighted index is used. In order to correct the estimated beta of equation (3) for this seasonality, I ran the first pass regression L times for an interval length of L. The regression is run a first time with the complete series, then the first daily return is deleted and the regression is run again with the remaining observations and so forth until it is run L times.[2]

$$R_{iLt} = {}_1\hat{\alpha}_{iLn} + {}_1\hat{\beta}_{iLn} R_{mLt} + {}_1\varepsilon_{iLt} \quad (4)$$

for t=1+n-1,...,N and n=1,L.

An average beta for each interval length is then calculated:

$$ {}_1\bar{\bar{\beta}}_{iL} = \sum_{n=1}^{L} {}_1\hat{\beta}_{iLn} \Big/ L \quad (5)$$

The intervalling effect on the beta of each security, as well as its estimated asymptotic beta, are then measured using a second pass regression. The average estimated security betas[3] of equation (5) are regressed against a function of the inverse of the length of the differencing intervals.

$$ {}_1\bar{\bar{\beta}}_{iL} = {}_2\hat{\alpha}_i + {}_2\hat{\beta}_i f(L^{-n}) + {}_2\varepsilon_{iL} \quad (6)$$

The parameter ${}_2\hat{\alpha}_i$ is the estimate of the asymptotic beta coefficient β_i^* and ${}_2\hat{\beta}_i$ is a measure of the intervalling effect on a security beta. The prescript 2 denotes the second pass regression parameters. Any inverse function of L which converges to an asymptote can be used in equation (6). In their study on US data

[2] Deleting L-1 daily returns from the series decreases by a maximum of one the number of returns of interval length L. The number of observations in the first pass regression is given in the following table for some differencing interval lengths.

Period	L=1	L=2	L=5	L=10	L=20	L=30
77-79	738	368 to 369	147	72 to 73	35 to 36	23 to 24
80-82	735	367	146 to 147	72 to 73	35 to 36	23 to 24
83-85	740	369 to 370	147 to 148	73 to 74	36 to 37	23 to 24

The largest number in the columns gives the number of returns of differencing interval length L for the complete series, and the second one, when it is different, the number of returns after deleting L-1 daily observations.

[3] Subsequently I will use equally the terms "average estimated betas" and "estimated betas" to refer to the average estimated beta.

CHMSW simply used the inverse of the length of the differencing interval L^{-n}. In their work on French data FSW used two other additional functions, $\ln(1+L^{-n})$ and $\exp(-L^{-n})$. In their case, the function $\ln(1+L^{-n})$ fits best the data. Like FSW, I tested various inverse functions of L, that is, L^{-n}, $\log(1+L^{-n})$, $\log(1+L)^{-n}$ and $\exp(-L^{-n})$, and I also found that the function $\ln(1+L^{-n})$ fits best the Belgian data. The value of n is determined experimentally. It must be positive and the value chosen must produce the best linear fit between ${}_1\hat{\beta}_{iL}$ and $f(L^{-n})$. Values of n between 0.1 and 2.0 were tested and on average the best linear fit was obtained for n=1.0. Besides, I did not use all thirty differencing interval lengths in the second pass regression in order to limit the weight given to the large differencing interval lengths in the regression and to concentrate on the intervalling effect[4]. To this end the second pass regressions were run only with 16 different interval lengths, i.e. L=1,...,6,8,10,12,14,15,16,18,20,25,30.

In turn, the cross sectional differences in the intervalling effect are determined by a third pass regression which provides a relationship between the intervalling effect on the betas of the securities and their thinness,

$$ {}_2\hat{\beta}_i = {}_3\hat{\alpha} + {}_3\hat{\beta}\, f_i + {}_3\varepsilon_i \qquad (7) $$

where f_i is a proxy variable for thinness of security i. The prescript 3 identifies the regression parameters estimated by the third pass regression. Various proxies of the thinness can be chosen in equation (7). The market value of the outstanding shares

[4]The reasons are that when the differencing interval is lengthened, on the one hand the OLS security betas generally converge quickly to an asymptotic value, and on the other hand the quality of the beta estimates decreases because of the limited number of observations.

of the securities is usually chosen as the proxy variable for thinness[5]. In this study the tests were carried out with four different independent variables: the market value, the volume of trading, the market value of the volume of trading and the ratio volume of trading to the number of outstanding shares, which can be considered as a measure of the degree of rotation of the shares. The market value of a security is measured at the midpoint of a subperiod. It is the natural logarithm of the value, in million of Belgian francs, of outstanding shares of the security. The other three proxy variables are calculated on a whole subperiod. The volume of trading and its market value are also expressed in million of Belgian francs.

These cross sectional differences in the intervalling effect are then used to determine an inferred asymptotic beta ${}_2\hat{\alpha}_i^*$ which is obtained by substituting equation (7) in equation (6) and solving it for ${}_2\hat{\alpha}_i$. In order to avoid biases in the results due to the use of the same data in the two equations (6) and (7), the Lachenbruch method is used.[6] The following third pass regression is run for each security,

$$ {}_2\hat{\beta}_k = {}_3\hat{\alpha}_k + {}_3\hat{\beta}_k f_k + {}_3\varepsilon_k \qquad k=1,..n \;\; k\neq i \qquad (8) $$

Replacing ${}_2\hat{\beta}_k$ in (6) gives the security inferred asymptotic beta

$$ {}_2\hat{\alpha}_i^* = {}_1\hat{\beta}_{iL} - \left({}_3\hat{\alpha}_k + {}_3\hat{\beta}_k f_k \right) f(L^{-n}) \qquad (9) $$

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5 CHMSW also used the volume of trading as proxy variable for the thinness in the third pass regression. They found similar but less significant results.

6 The Lachenbruch method consists in using a sequence of hold-out samples to estimate the parameters of the third pass regression. Each security is in turn deleted from the sample to estimate its corresponding third pass regression parameters.

In order to test the quality of the adjustment of the inferred asymptotic betas, I calculated the mean square errors of the estimated betas ${}_1\bar{\hat{\beta}}_{iL}$ and inferred asymptotic betas ${}_2\hat{\alpha}_i^*$ from the estimated asymptotic betas ${}_2\hat{\alpha}_i$,

$$MSE(\hat{\beta}) = \sum_{i=1}^{s} (\hat{\beta} - {}_2\hat{\alpha}_i)^2 \Big/ s \qquad (10)$$

where s is the number of securities of a subperiod and $\hat{\beta}$ is respectively equal to ${}_1\bar{\hat{\beta}}_{iL}$ and ${}_2\hat{\alpha}_i^*$. Afterwards I compared these MSE to the mean square errors of the beta estimator $\hat{\beta}_{iL}^{sw}$ of Scholes and Williams,

$$\hat{\beta}_{iL}^{sw} = \frac{\beta_{iL}^{-} + \beta_{iL} + \beta_{iL}^{+}}{1 + 2\,\rho_{mL}} \qquad (11)$$

where β_{iL}^{-} and β_{iL}^{+} are the estimated beta coefficients when the returns, measured on a differencing interval of length L, are respectively lagged and leaded; $\dot{\rho}_{mL}$ is the first order autocorrelation coefficient of the market index returns.

3 The Empirical Results

Given the number of securities, the number of proxy variables for thinness and the number of periods in the study, the individual security results cannot be presented in this paper[7]. Results are generally presented for 10 portfolios and the sample as a whole. The portfolios are equally weighted portfolios and they are constructed on the basis of the proxy variables. The number of

7 The individual security results for all tables are available on request.

securities in each portfolio for the subperiods is given in the tables[8].

3.1 Relationship Between the Proxy Variables and Thinness

CHMSW's procedure to estimate the inferred asymptotic betas of the securities is based on the choice of a proxy variable for the thinness of the securities. The four variables I use in this study, market value of the securities, volume of their transaction and its market value as well as the degree of rotation of their shares, are related to their thinness. On the one hand one expects indeed that larger firms, having a larger volume of transaction and whose public has generally more information, have a shorter delay in their price adjustment than smaller firms. On the other hand trading firm shares having a high degree of rotation certainly present some advantages to the investor who can more easily and more quickly dispose of the shares. The relationship between the first three proxy variables and the degree of rotation of the shares is more difficult to establish since one can find, for example, small firms having a high degree of rotation of their shares and large firms having a low degree of rotation of their shares. This reflection is confirmed by the correlation coefficients between the proxy variables in table 1. The correlation between the first three variables is high whatever the subperiod concerned, and it is negative and low between the degree of rotation of the shares and the three former variables. These coefficients let suppose, and this will be confirmed later by the

[8] The number of securities in a portfolio for a particular subperiod is equal to the larger integer of the division of the number of securities by the number of portfolios. If there is a remainder it is allocated to the first and the tenth portfolios.

results, that the adjustment for the intervalling effect bias will be quite similar for the market value and the two volume variables.

Table 1 : Correlation Coefficients Between the Thinness Proxy Variables

	1977-1979 (1)	(2)	(3)	(4)	1980-1982 (1)	(2)	(3)	(4)	1983-1985 (1)	(2)	(3)	(4)
(1)	1.00	.64	.78	-.29	1.00	.54	.74	-.17	1.00	.57	.69	-.22
(2)	.64	1.00	.78	-.20	.54	1.00	.79	-.09	.57	1.00	.73	-.07
(3)	.78	.78	1.00	-.22	.74	.79	1.00	-.08	.69	.73	1.00	-.10
(4)	-.29	-.20	-.22	1.00	-.17	-.09	-.08	1.00	-.25	-.07	-.10	1.00

(1) Market value of the outstanding shares
(2) Volume of trading
(3) Market value of the volume of trading
(4) Volume of trading/number of outstanding shares

As the results of the third pass regression are generally more significant and more consistent across the periods when the market value is used as the independent variable, tables with portfolios formed on the basis of this proxy variables will only appear in the paper.[9]

3.2 First Pass Regression Results

The results of the first pass regression for the three subperiods are presented in the tables 2a, 2b, and 2c respectively. The values of the average betas ${}_1\bar{\hat{\beta}}_{iL}$, the standard deviation and the average t test value of the ${}_1\hat{\beta}_{iLn}$ and the average R-square are summarized in the tables for the ten market value formed portfolios as well as for the whole sample, and for some lengths of the differencing interval.[10]

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[9] Tables for all proxy variables are available on request.

[10] The results for the other values of L are consistent with the values of tables 2a, 2b and 2c. The complete tables can be obtained on request.

There is no intervalling effect on the whole sample when the equally weighted index is used, and its average beta is always close to one. This is because this sample represents almost approximately the entire spot equity market of the BSE. Consequently the whole sample can be considered as the market itself. Considering the average betas of the size portfolios there is an intervalling effect, but its magnitude and above all its direction vary according to the period considered. Roughly, the intervalling effect is inversely related to the market value for the periods 80-82 and 83-83, and it is the reverse for the first period. When the value weighted index is used it can be observed that the average betas increase for the whole sample. The average percentage of increase, all firms included, is about 129%, 109% and 75% for the three subperiods. The direction of the intervalling effect is the same for each size portfolio, but its magnitude seems to be highly inversely related to the firm size. The increase varies between 0.5% and 25% for the largest market value portfolio to a value between 388% and 542% for the smallest market value portfolio. However these percentages must be taken with caution. It is indeed worth mentioning that these numbers are average numbers and that some securities, especially of the small market value portfolios, have negative betas for small differencing interval lengths. In addition the number of observations in the regression rapidly decreases when the differencing interval is lengthened. This also explains to a certain extent the pattern of the average t-test coefficients which are decreasing with the interval length and which are generally not significant for the small market value portfolios.

Table 2a : First Pass Regression Statistics : Period 1977-1979.

Obs.		Mark.Value		Equally weighted market index L=1	2	3	5	10	15	20	25	30	Value weighted market index 1	2	3	5	10	15	20	25	30
1	17	16.30	(1)	0.724	0.898	0.944	0.970	0.973	0.956	0.939	0.918	0.900	0.822	0.859	0.879	0.910	0.947	0.974	0.997	1.012	1.030
			(2)	0.00	0.15	0.13	0.17	0.11	0.11	0.13	0.14	0.16	0.00	0.05	0.09	0.10	0.11	0.13	0.14	0.15	0.19
			(3)	4.87	4.86	4.69	4.37	3.83	3.61	3.41	3.22	3.05	9.67	8.00	7.09	6.00	4.90	4.52	4.21	3.89	3.66
			(4)	0.04	0.06	0.09	0.12	0.17	0.22	0.25	0.27	0.29	0.12	0.15	0.17	0.20	0.25	0.30	0.33	0.35	0.36
2	15	15.02	(1)	1.013	1.084	1.101	1.081	1.038	1.032	1.030	1.034	1.045	0.607	0.706	0.784	0.878	0.946	0.972	0.991	1.039	1.078
			(2)	0.00	0.13	0.16	0.20	0.15	0.17	0.21	0.23	0.24	0.00	0.10	0.11	0.15	0.15	0.19	0.22	0.26	0.29
			(3)	4.26	3.89	3.65	3.33	2.92	2.84	2.74	2.64	2.57	4.54	4.30	4.10	3.90	3.51	3.30	3.07	2.93	2.75
			(4)	0.03	0.05	0.06	0.08	0.11	0.15	0.18	0.20	0.23	0.03	0.06	0.07	0.10	0.15	0.19	0.21	0.23	0.25
3	15	13.99	(1)	1.156	1.172	1.153	1.128	1.088	1.074	1.049	1.013	0.979	0.459	0.573	0.642	0.717	0.803	0.854	0.885	0.895	0.927
			(2)	0.00	0.27	0.27	0.23	0.19	0.18	0.19	0.21	0.25	0.00	0.11	0.15	0.13	0.15	0.19	0.24	0.28	0.29
			(3)	4.51	3.90	3.56	3.19	2.76	2.61	2.50	2.33	2.19	3.06	3.12	3.03	2.80	2.54	2.40	2.25	2.08	1.98
			(4)	0.04	0.05	0.06	0.08	0.11	0.14	0.17	0.19	0.20	0.02	0.03	0.05	0.06	0.10	0.12	0.14	0.15	0.16
4	15	13.55	(1)	1.185	1.163	1.143	1.114	1.127	1.127	1.146	1.183	1.224	0.497	0.610	0.658	0.731	0.828	0.863	0.919	0.984	1.063
			(2)	0.00	0.27	0.23	0.20	0.21	0.19	0.22	0.26	0.26	0.00	0.10	0.16	0.18	0.21	0.24	0.28	0.29	0.33
			(3)	4.20	3.62	3.31	3.01	2.80	2.69	2.64	2.61	2.61	2.96	3.02	2.85	2.74	2.56	2.42	2.35	2.28	2.25
			(4)	0.03	0.04	0.05	0.07	0.11	0.14	0.17	0.20	0.23	0.02	0.03	0.04	0.06	0.09	0.12	0.14	0.17	0.19
5	15	13.06	(1)	1.361	1.422	1.439	1.430	1.374	1.348	1.328	1.306	1.283	0.456	0.599	0.673	0.756	0.829	0.915	0.982	1.030	1.069
			(2)	0.00	0.21	0.30	0.26	0.21	0.23	0.28	0.28	0.28	0.00	0.15	0.15	0.19	0.21	0.21	0.33	0.35	0.37
			(3)	4.09	3.69	3.50	3.18	2.74	2.59	2.44	2.29	2.15	2.23	2.44	2.41	2.28	2.00	1.96	1.87	1.79	1.69
			(4)	0.03	0.04	0.06	0.07	0.11	0.14	0.16	0.18	0.19	0.01	0.02	0.03	0.04	0.07	0.09	0.11	0.12	0.13
6	15	12.53	(1)	0.953	1.109	1.145	1.168	1.164	1.147	1.129	1.110	1.112	0.231	0.363	0.452	0.557	0.645	0.700	0.750	0.786	0.835
			(2)	0.00	0.25	0.30	0.26	0.25	0.26	0.26	0.33	0.33	0.00	0.15	0.17	0.20	0.25	0.30	0.33	0.41	0.40
			(3)	2.74	2.68	2.55	2.46	2.29	2.25	2.21	2.13	2.06	1.13	1.50	1.65	1.77	1.72	1.72	1.70	1.65	1.63
			(4)	0.01	0.02	0.03	0.04	0.07	0.11	0.13	0.15	0.17	0.00	0.01	0.01	0.03	0.05	0.07	0.09	0.10	0.12
7	15	12.16	(1)	0.861	0.795	0.810	0.820	0.851	0.861	0.885	0.899	0.905	0.186	0.239	0.289	0.377	0.496	0.569	0.659	0.716	0.763
			(2)	0.00	0.28	0.29	0.19	0.16	0.19	0.26	0.22	0.32	0.00	0.09	0.15	0.20	0.19	0.22	0.32	0.29	0.43
			(3)	3.24	2.45	2.25	2.06	1.87	1.83	1.83	1.79	1.73	1.15	1.18	1.26	1.38	1.38	1.44	1.54	1.52	1.50
			(4)	0.02	0.02	0.03	0.04	0.06	0.09	0.11	0.13	0.14	0.00	0.01	0.01	0.02	0.04	0.06	0.08	0.10	0.11
8	15	11.53	(1)	0.800	0.716	0.668	0.635	0.679	0.681	0.663	0.646	0.625	0.152	0.149	0.188	0.214	0.304	0.367	0.396	0.405	0.409
			(2)	0.00	0.19	0.20	0.19	0.18	0.23	0.25	0.27	0.25	0.00	0.12	0.17	0.19	0.21	0.22	0.30	0.33	0.32
			(3)	2.77	2.09	1.81	1.61	1.56	1.51	1.46	1.41	1.36	0.96	0.75	0.82	0.79	0.93	1.03	1.03	1.00	0.96
			(4)	0.01	0.01	0.02	0.02	0.05	0.07	0.08	0.09	0.10	0.00	0.00	0.01	0.01	0.03	0.04	0.06	0.07	0.07
9	15	10.89	(1)	1.021	1.077	1.095	1.044	0.950	0.928	0.945	0.963	0.963	0.146	0.280	0.362	0.454	0.498	0.563	0.669	0.755	0.824
			(2)	0.00	0.22	0.21	0.25	0.20	0.23	0.26	0.23	0.35	0.00	0.18	0.19	0.20	0.24	0.26	0.30	0.33	0.44
			(3)	3.17	2.76	2.58	2.29	1.96	1.87	1.84	1.81	1.75	0.76	1.18	1.32	1.41	1.31	1.34	1.44	1.49	1.49
			(4)	0.02	0.02	0.03	0.04	0.07	0.09	0.11	0.13	0.14	0.00	0.01	0.01	0.02	0.04	0.06	0.08	0.10	0.11
10	16	10.07	(1)	1.178	1.000	0.945	0.907	0.823	0.784	0.759	0.742	0.720	0.090	0.129	0.184	0.266	0.325	0.386	0.425	0.444	0.440
			(2)	0.00	0.37	0.31	0.28	0.25	0.23	0.29	0.29	0.31	0.00	0.16	0.16	0.23	0.27	0.30	0.36	0.40	0.49
			(3)	3.16	2.23	1.90	1.60	1.24	1.11	1.02	0.94	0.89	0.44	0.49	0.59	0.66	0.59	0.61	0.61	0.59	0.54
			(4)	0.02	0.02	0.02	0.02	0.04	0.05	0.06	0.07	0.07	0.00	0.00	0.00	0.01	0.02	0.03	0.04	0.04	0.05
	153	12.94	(1)	1.022	1.041	1.042	1.028	1.005	0.992	0.985	0.979	0.973	0.369	0.454	0.514	0.588	0.664	0.717	0.768	0.807	0.844

(1) average ${}_1\bar{\beta}_{iL}$ (2) average $\sigma({}_1\hat{\beta}_{iL})$ (3) average $t({}_1\bar{\beta}_{iL})$, significant values at the 5% level are underlined (4) average R2

Table 2b : First Pass Regression Statistics : Period 1980-1982.

Obs.		Mark.Value		EW 1	EW 2	EW 3	EW 5	EW 10	EW 15	EW 20	EW 25	EW 30	VW 1	VW 2	VW 3	VW 5	VW 10	VW 15	VW 20	VW 25	VW 30
				Equally weighted market index									Value weighted market index								
1	18	15.96	(1)	1.394	1.501	1.520	1.494	1.423	1.380	1.340	1.306	1.277	0.928	0.967	0.990	1.004	1.005	1.010	1.012	1.022	1.027
			(2)	0.00	0.10	0.13	0.10	0.14	0.13	0.14	0.15	0.16	0.00	0.04	0.07	0.08	0.08	0.10	0.11	0.11	0.12
			(3)	7.24	6.87	6.57	6.14	5.53	5.10	4.78	4.51	4.26	11.46	9.89	9.17	8.27	7.13	6.51	5.93	5.45	5.08
			(4)	0.07	0.12	0.15	0.21	0.30	0.35	0.39	0.41	0.43	0.16	0.22	0.26	0.32	0.40	0.46	0.48	0.50	0.51
2	18	14.58	(1)	1.596	1.670	1.660	1.615	1.517	1.452	1.407	1.363	1.328	0.734	0.809	0.852	0.908	0.943	0.958	0.972	0.985	0.987
			(2)	0.00	0.12	0.12	0.12	0.15	0.15	0.17	0.17	0.17	0.00	0.07	0.07	0.08	0.11	0.12	0.14	0.14	0.15
			(3)	7.65	7.20	6.82	6.26	5.54	5.07	4.73	4.43	4.22	7.63	7.29	6.97	6.54	5.82	5.37	4.99	4.62	4.32
			(4)	0.08	0.13	0.17	0.22	0.30	0.35	0.38	0.41	0.42	0.08	0.14	0.18	0.23	0.32	0.37	0.40	0.42	0.43
3	18	13.66	(1)	1.304	1.350	1.336	1.309	1.238	1.199	1.160	1.126	1.102	0.491	0.572	0.622	0.684	0.737	0.760	0.782	0.798	0.810
			(2)	0.00	0.12	0.13	0.13	0.14	0.17	0.16	0.17	0.18	0.00	0.09	0.09	0.11	0.12	0.14	0.16	0.15	0.16
			(3)	5.23	4.94	4.71	4.48	4.06	3.81	3.57	3.35	3.21	4.27	4.37	4.35	4.27	3.99	3.77	3.57	3.31	3.17
			(4)	0.04	0.07	0.09	0.13	0.20	0.24	0.27	0.29	0.31	0.03	0.06	0.09	0.13	0.19	0.24	0.27	0.28	0.30
4	18	12.94	(1)	1.409	1.437	1.430	1.415	1.345	1.285	1.240	1.199	1.162	0.372	0.450	0.508	0.594	0.701	0.723	0.746	0.769	0.770
			(2)	0.00	0.14	0.17	0.17	0.17	0.20	0.16	0.23	0.21	0.00	0.10	0.14	0.15	0.15	0.19	0.18	0.23	0.21
			(3)	4.66	4.38	4.18	3.99	3.71	3.46	3.24	3.05	2.86	2.66	2.88	2.96	3.07	3.09	2.94	2.86	2.73	2.60
			(4)	0.03	0.06	0.08	0.11	0.18	0.22	0.25	0.27	0.29	0.02	0.04	0.05	0.08	0.14	0.17	0.21	0.23	0.25
5	18	12.39	(1)	1.070	1.090	1.087	1.115	1.170	1.204	1.214	1.225	1.236	0.273	0.307	0.351	0.421	0.556	0.634	0.697	0.762	0.807
			(2)	0.00	0.09	0.14	0.13	0.15	0.18	0.18	0.19	0.20	0.00	0.05	0.11	0.10	0.12	0.17	0.17	0.18	0.17
			(3)	4.12	3.74	3.55	3.48	3.54	3.52	3.42	3.31	3.22	2.22	2.12	2.19	2.31	2.64	2.74	2.76	2.75	2.73
			(4)	0.03	0.04	0.06	0.09	0.16	0.22	0.25	0.28	0.31	0.01	0.02	0.02	0.04	0.10	0.15	0.18	0.22	0.25
6	18	11.96	(1)	0.709	0.673	0.685	0.702	0.709	0.724	0.726	0.734	0.744	0.122	0.175	0.216	0.277	0.331	0.377	0.410	0.455	0.490
			(2)	0.00	0.08	0.12	0.13	0.16	0.15	0.16	0.18	0.17	0.00	0.07	0.08	0.10	0.13	0.14	0.15	0.16	0.14
			(3)	2.49	2.06	2.00	1.96	1.89	1.84	1.74	1.65	1.60	0.83	1.08	1.24	1.38	1.41	1.47	1.42	1.41	1.41
			(4)	0.01	0.02	0.03	0.04	0.07	0.09	0.11	0.12	0.14	0.01	0.01	0.01	0.02	0.05	0.07	0.09	0.11	0.12
7	18	11.51	(1)	0.860	0.831	0.840	0.854	0.891	0.898	0.912	0.926	0.928	0.142	0.181	0.228	0.300	0.399	0.444	0.495	0.554	0.582
			(2)	0.00	0.13	0.09	0.13	0.16	0.16	0.18	0.19	0.21	0.00	0.07	0.08	0.11	0.15	0.16	0.19	0.17	0.21
			(3)	3.23	2.81	2.68	2.56	2.56	2.50	2.52	2.52	2.52	1.20	1.34	1.47	1.64	1.85	1.89	2.02	2.11	2.16
			(4)	0.02	0.03	0.04	0.06	0.11	0.15	0.19	0.22	0.24	0.01	0.01	0.02	0.03	0.07	0.11	0.14	0.18	0.21
8	18	10.91	(1)	0.534	0.523	0.531	0.562	0.587	0.617	0.646	0.665	0.690	0.065	0.086	0.114	0.153	0.221	0.270	0.328	0.382	0.431
			(2)	0.00	0.10	0.12	0.13	0.16	0.16	0.18	0.19	0.20	0.00	0.05	0.06	0.08	0.12	0.13	0.15	0.16	0.17
			(3)	2.29	1.96	1.88	1.88	1.91	1.97	2.05	2.07	2.07	0.65	0.71	0.82	0.95	1.16	1.26	1.43	1.53	1.61
			(4)	0.01	0.02	0.02	0.04	0.07	0.10	0.13	0.16	0.18	0.00	0.00	0.01	0.02	0.04	0.06	0.08	0.11	0.14
9	18	10.46	(1)	0.775	0.757	0.763	0.771	0.799	0.841	0.892	0.935	0.970	0.110	0.162	0.203	0.251	0.327	0.395	0.463	0.542	0.597
			(2)	0.00	0.19	0.12	0.12	0.16	0.17	0.23	0.22	0.18	0.00	0.08	0.08	0.08	0.14	0.15	0.19	0.20	0.18
			(3)	3.02	2.56	2.39	2.21	2.14	2.14	2.17	2.19	2.20	1.03	1.21	1.33	1.39	1.50	1.58	1.69	1.79	1.86
			(4)	0.01	0.02	0.03	0.04	0.08	0.11	0.14	0.17	0.20	0.00	0.01	0.01	0.02	0.05	0.07	0.09	0.12	0.15
10	18	9.42	(1)	0.612	0.541	0.554	0.595	0.679	0.724	0.754	0.778	0.800	0.064	0.079	0.102	0.160	0.272	0.312	0.348	0.385	0.411
			(2)	0.00	0.09	0.11	0.13	0.17	0.16	0.20	0.19	0.22	0.00	0.07	0.08	0.11	0.15	0.14	0.21	0.18	0.20
			(3)	2.09	1.66	1.61	1.60	1.67	1.68	1.67	1.66	1.65	0.52	0.54	0.61	0.80	1.08	1.07	1.08	1.08	1.09
			(4)	0.01	0.01	0.02	0.03	0.06	0.09	0.11	0.13	0.15	0.00	0.01	0.01	0.01	0.03	0.04	0.06	0.08	0.09
	180	12.38	(1)	1.026	1.037	1.041	1.043	1.036	1.032	1.029	1.026	1.024	0.330	0.379	0.419	0.475	0.549	0.588	0.625	0.665	0.691

(1) average ${}_1\bar{b}_{iL}$ (2) average $S({}_1\hat{b}_{iL})$ (3) average $t({}_1\bar{b}_{iL})$, significant values at the 5% level are underlined (4) average R2

Table 2c : First Pass Regression Statistics : Period 1983-1985.

	Obs.	Mark. Value		Equally weighted market index									Value weighted market index								
				1	2	3	5	10	15	20	25	30	1	2	3	5	10	15	20	25	30
1	18	16.65	(1)	1.582	1.598	1.547	1.423	1.314	1.281	1.272	1.277	1.276	0.987	1.017	1.024	1.019	1.001	0.992	0.992	0.993	0.991
			(2)	0.00	0.08	0.08	0.10	0.11	0.11	0.14	0.16	0.19	0.00	0.07	0.06	0.08	0.09	0.08	0.09	0.12	0.12
			(3)	9.44	8.38	7.62	6.55	5.59	5.09	4.78	4.55	4.27	12.84	10.97	9.81	8.19	6.76	6.16	5.83	5.54	5.23
			(4)	0.12	0.17	0.20	0.23	0.31	0.35	0.39	0.42	0.44	0.19	0.25	0.29	0.32	0.39	0.44	0.48	0.51	0.53
2	16	15.58	(1)	1.498	1.515	1.501	1.426	1.342	1.303	1.273	1.259	1.243	0.790	0.847	0.891	0.939	0.982	0.976	0.963	0.958	0.951
			(2)	0.00	0.10	0.10	0.11	0.13	0.13	0.17	0.14	0.20	0.00	0.04	0.06	0.09	0.10	0.10	0.13	0.11	0.14
			(3)	7.90	7.22	6.76	5.97	5.19	4.74	4.42	4.18	3.92	8.42	7.78	7.32	6.46	5.67	5.24	4.92	4.68	4.47
			(4)	0.08	0.13	0.16	0.20	0.28	0.33	0.36	0.39	0.40	0.10	0.15	0.19	0.23	0.31	0.37	0.41	0.44	0.46
3	16	14.63	(1)	1.289	1.289	1.240	1.166	1.071	1.034	1.012	1.006	1.001	0.616	0.690	0.717	0.754	0.774	0.771	0.759	0.756	0.755
			(2)	0.00	0.10	0.09	0.13	0.12	0.16	0.17	0.17	0.22	0.00	0.06	0.09	0.10	0.10	0.13	0.12	0.13	0.17
			(3)	6.08	5.64	5.24	4.60	3.94	3.61	3.42	3.28	3.17	5.78	5.72	5.45	4.86	4.24	3.97	3.77	3.61	3.51
			(4)	0.06	0.09	0.11	0.14	0.19	0.23	0.26	0.29	0.31	0.06	0.10	0.12	0.15	0.21	0.26	0.30	0.33	0.35
4	16	14.00	(1)	1.063	1.137	1.167	1.198	1.242	1.265	1.267	1.281	1.296	0.358	0.459	0.530	0.640	0.776	0.827	0.846	0.872	0.892
			(2)	0.00	0.08	0.10	0.13	0.14	0.15	0.15	0.20	0.20	0.00	0.06	0.07	0.10	0.12	0.11	0.12	0.14	0.15
			(3)	4.77	4.70	4.59	4.45	4.28	4.12	3.93	3.86	3.79	3.19	3.55	3.69	3.80	3.85	3.79	3.68	3.67	3.63
			(4)	0.03	0.06	0.08	0.13	0.21	0.27	0.31	0.35	0.38	0.02	0.04	0.06	0.10	0.18	0.24	0.28	0.33	0.36
5	16	13.44	(1)	1.185	1.211	1.207	1.197	1.209	1.231	1.250	1.244	1.246	0.367	0.457	0.512	0.591	0.680	0.718	0.736	0.745	0.749
			(2)	0.00	0.08	0.13	0.18	0.20	0.22	0.24	0.27	0.27	0.00	0.07	0.09	0.13	0.16	0.18	0.16	0.18	0.19
			(3)	4.33	3.97	3.76	3.48	3.29	3.20	3.09	2.93	2.80	2.63	2.79	2.83	2.76	2.68	2.63	2.55	2.42	2.30
			(4)	0.03	0.05	0.06	0.08	0.14	0.19	0.22	0.24	0.26	0.02	0.03	0.04	0.06	0.10	0.14	0.17	0.19	0.20
6	16	12.97	(1)	0.755	0.769	0.784	0.839	0.929	0.958	0.953	0.952	0.962	0.203	0.228	0.269	0.365	0.535	0.580	0.593	0.599	0.608
			(2)	0.00	0.09	0.17	0.15	0.18	0.18	0.18	0.18	0.24	0.00	0.05	0.12	0.12	0.14	0.14	0.15	0.14	0.19
			(3)	3.07	2.89	2.81	2.83	2.98	2.91	2.83	2.72	2.68	1.67	1.65	1.77	2.04	2.50	2.51	2.49	2.40	2.40
			(4)	0.01	0.03	0.04	0.06	0.12	0.16	0.19	0.22	0.24	0.01	0.01	0.02	0.04	0.09	0.13	0.16	0.18	0.20
7	16	12.40	(1)	0.751	0.786	0.806	0.860	0.913	0.943	0.955	0.961	0.967	0.164	0.214	0.247	0.347	0.479	0.506	0.514	0.523	0.532
			(2)	0.00	0.15	0.10	0.17	0.17	0.19	0.20	0.20	0.27	0.00	0.08	0.06	0.12	0.13	0.15	0.16	0.15	0.20
			(3)	3.13	2.90	2.79	2.75	2.66	2.53	2.39	2.31	2.21	1.37	1.50	1.54	1.80	2.02	1.90	1.81	1.73	1.66
			(4)	0.02	0.03	0.04	0.06	0.10	0.13	0.15	0.17	0.19	0.01	0.01	0.02	0.03	0.06	0.08	0.10	0.11	0.12
8	16	12.00	(1)	0.555	0.537	0.562	0.629	0.672	0.715	0.779	0.802	0.822	0.095	0.135	0.177	0.259	0.336	0.384	0.442	0.466	0.481
			(2)	0.00	0.09	0.13	0.16	0.17	0.21	0.21	0.30	0.33	0.00	0.07	0.08	0.13	0.13	0.17	0.16	0.21	0.22
			(3)	2.06	1.83	1.81	1.85	1.79	1.78	1.84	1.83	1.83	0.77	0.90	1.04	1.25	1.33	1.36	1.47	1.48	1.46
			(4)	0.01	0.01	0.02	0.03	0.06	0.09	0.12	0.14	0.16	0.00	0.01	0.01	0.02	0.04	0.06	0.09	0.11	0.12
9	16	11.39	(1)	0.629	0.644	0.681	0.716	0.767	0.816	0.853	0.868	0.870	0.154	0.184	0.216	0.279	0.370	0.404	0.435	0.455	0.455
			(2)	0.00	0.12	0.14	0.15	0.17	0.20	0.24	0.28	0.30	0.00	0.10	0.08	0.11	0.13	0.16	0.17	0.21	0.21
			(3)	2.42	2.20	2.17	2.09	2.03	2.05	2.04	2.05	2.00	1.27	1.27	1.29	1.36	1.45	1.46	1.47	1.50	1.47
			(4)	0.01	0.02	0.02	0.03	0.06	0.09	0.12	0.15	0.16	0.00	0.01	0.01	0.02	0.04	0.06	0.08	0.10	0.11
10	18	10.22	(1)	0.586	0.500	0.494	0.541	0.524	0.468	0.444	0.453	0.454	0.045	0.058	0.086	0.155	0.228	0.235	0.231	0.247	0.247
			(2)	0.00	0.10	0.14	0.19	0.23	0.30	0.29	0.35	0.39	0.00	0.09	0.11	0.15	0.18	0.22	0.20	0.26	0.27
			(3)	1.87	1.48	1.39	1.43	1.28	1.12	1.00	1.00	0.97	0.31	0.38	0.49	0.72	0.86	0.84	0.78	0.80	0.77
			(4)	0.01	0.01	0.01	0.03	0.04	0.06	0.07	0.08	0.09	0.00	0.00	0.01	0.01	0.03	0.04	0.05	0.06	0.07
	164	13.33	(1)	0.992	1.000	0.999	0.999	0.996	0.998	1.002	1.007	1.010	0.381	0.431	0.469	0.536	0.616	0.639	0.650	0.660	0.665

(1) average $_1\bar{\beta}_{IL}$ (2) average $\sigma(_1\hat{\beta}_{iL})$ (3) average $t(_1\bar{\beta}_{IL})$, significant values at the 5% level are underlined (4) average $R2$

In a paper Hawawini (1980) demonstrated that R^2 increase with the length of the differencing interval and that the intervalling effect is generally positive and stronger on the R^2 than on the beta. It can be observed in the tables that it is indeed the case for the large market value portfolios, whatever index is used. Their R^2, which is always very low and even sometimes equal to zero for one or two days's intervals, grows very quickly. The unsignificant values of R^2 for the small market value portfolios confirm however that some of the securities of these portfolios actually have a negative beta[11].

Looking at the $\sigma({}_1\hat{\beta}_{1L})$ of tables 2a, 2b and 2c it can be observed that the fluctuations in the betas, as shown by Corhay (1988) for a smaller sample of Belgian securities, are quite strong in all size portfolios and for both indexes[12]. However small firm portfolios have larger fluctuations than large firm portfolios, and the fluctuations are weaker for low values of L and stronger for high values of L when the value weighted index is used rather than when the equally weighted index is used. The main conclusion is that the method of adjustment for the fluctuations in the betas used in this study certainly improves the quality of the estimates of the systematic risk for a given differencing interval length.

Another interesting feature of the results of the first pass regression is that they invalidate to a certain extent Levhari and

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[11] According to Hawawini a weak intervalling effect on the R^2 may be the result of a negative systematic risk, a negative intervalling effect or a negative autocorrelation in the market index returns. In an unpublished paper Corhay (1988) showed that the Belgian market index autocorrelation is, as expected, positive for both indexes. As the growth of the betas of the small market value portfolios is very strong for all periods, except the first one when the equally weighted index is used, the low R^2 is due to the presence of negative betas in these portfolios.

[12] The zero values of $\sigma({}_1\hat{\beta}_{1L})$ are due to the fact that there cannot be any fluctuation for a one day differencing interval.

Levy's hypothesis. Using monthly rates of return, Levhari and Levy proved and showed that when the differencing interval is lengthened, security betas greater than one continue to increase and security betas lower than one continue to decrease. Looking at the pattern of the portfolio betas for large L in the tables, one can see that such a behaviour really appears only 11 times out of 60. Our results tend more to confirm the asymptotic behaviour of the security betas as it is demonstrated by Hawawini (1980) and CHMSW (1983).

3.3 Second Pass Regression Results

The equation (6) has been estimated for values of n varying from 0.1 to 2.0 and for the following set of inverse functions of L:

$$f(L^{-n}) = L^{-n}$$
$$f(L^{-n}) = \log(1+L^{-n})$$
$$f(L^{-n}) = \log(1+L)^{-n}$$
$$f(L^{-n}) = \exp(-L^{-n})$$
$$L=1,...,5,6,8,10,12,14,16,18,20,25,30$$

Like the French case, the best linear fit between ${}_1\hat{\beta}_{iL}$ and $f(L^{-n})$ was on average obtained for the function $\log(1+L^{-n})$ with n=1.0.

The average results of the second pass regression for the ten size portfolios are presented in table 3. They corroborate the observations made for the first pass regression results. When the equally weighted index is used, few portfolio ${}_2\hat{\beta}_i$ are on average statistically significant, and their sign across the portfolios for the first period is not always consistent. According to CHMSW's model, firms traded less often than the average should have a negative intervalling effect, and inversely, firms traded more often should have a positive intervalling effect. Looking at the

Table 3 : Second Pass Regressions Statistics

Period	Port.	Sec.	M.V.	Equally Weighted Index						Value Weighted Index					
				${}_2\hat{\alpha}$	$t({}_2\hat{\alpha})$	${}_2\hat{\beta}$	$t({}_2\hat{\beta})$	R^2	DW	${}_2\hat{\alpha}$	$t({}_2\hat{\alpha})$	${}_2\hat{\beta}$	$t({}_2\hat{\beta})$	R^2	DW
1977-1979	1	17	16.30	0.98015	70.32	-0.27303	-6.56	0.496	0.674	0.99083	52.35	-0.29790	-3.39	0.638	0.448
	2	15	15.03	1.04767	39.35	0.02539	-2.00	0.583	0.635	1.02087	38.58	-0.67876	-5.88	0.589	0.582
	3	15	14.00	1.05383	40.68	0.22574	3.60	0.531	0.568	0.88940	45.99	-0.71044	-8.34	0.621	0.646
	4	15	13.55	1.13723	42.06	0.03935	-0.09	0.318	0.613	0.92443	31.93	-0.73361	-6.69	0.669	0.646
	5	15	13.07	1.35340	48.70	0.11703	0.62	0.598	0.894	0.97069	25.93	-0.86229	-5.36	0.580	0.462
	6	15	12.53	1.16881	40.17	-0.22250	-2.29	0.493	0.752	0.75663	24.61	-0.87442	-6.82	0.703	0.591
	7	15	12.16	0.86472	28.03	-0.08507	-0.16	0.595	0.557	0.62360	15.86	-0.83168	-4.41	0.585	0.489
	8	15	11.53	0.64224	22.07	0.18069	2.12	0.468	0.679	0.36918	9.45	-0.44658	-2.49	0.557	0.635
	9	15	10.90	0.95127	25.12	0.21772	0.86	0.590	0.500	0.64972	15.17	-0.84878	-4.48	0.523	0.427
	10	16	10.08	0.74458	24.61	0.65717	2.75	0.605	0.762	0.41083	14.23	-0.58389	-3.81	0.552	0.635
		153	12.94	0.99257	38.44	0.08725	-0.18	0.528	0.664	0.76134	27.65	-0.68107	-5.14	0.602	0.555
1980-1982	1	18	15.96	1.38177	62.14	0.18599	0.58	0.415	0.585	1.02170	89.44	-0.12993	-3.86	0.698	0.605
	2	18	14.58	1.44436	52.30	0.41916	2.98	0.444	0.534	0.98231	69.64	-0.38924	-6.18	0.637	0.463
	3	18	13.66	1.18809	59.12	0.30331	0.53	0.598	0.524	0.79049	54.33	-0.48593	-8.04	0.660	0.595
	4	18	12.94	1.27464	42.22	0.33674	0.06	0.504	0.512	0.75536	37.61	-0.65751	-6.82	0.671	0.483
	5	18	12.39	1.20658	61.45	-0.26928	-2.22	0.480	0.501	0.66838	23.04	-0.77578	-5.97	0.639	0.506
	6	18	11.96	0.72269	36.13	-0.06397	0.48	0.417	0.498	0.40847	13.46	-0.51060	-3.73	0.539	0.500
	7	18	11.51	0.90419	49.35	-0.12261	-0.60	0.576	0.654	0.48452	18.94	-0.64288	-4.64	0.578	0.467
	8	18	10.91	0.62839	31.81	-0.21177	2.53	0.587	0.635	0.30271	11.05	-0.47167	-2.95	0.447	0.436
	9	18	10.46	0.86024	26.40	-0.21848	-0.15	0.404	0.431	0.43940	17.67	-0.62181	-4.52	0.580	0.430
	10	18	9.42	0.72526	22.80	-0.32348	-0.49	0.621	0.575	0.33301	12.23	-0.53715	-2.43	0.518	0.528
		180	12.38	1.03361	44.37	0.00356	0.37	0.505	0.545	0.61863	34.74	-0.52224	-4.91	0.597	0.501
1983-1985	1	18	16.65	1.26956	65.47	0.60964	3.84	0.697	0.809	0.99899	94.20	0.01765	-1.54	0.595	0.562
	2	16	15.58	1.28974	61.64	0.44221	4.85	0.599	0.617	0.98867	81.84	-0.29883	-3.56	0.559	0.624
	3	16	14.63	1.02188	54.26	0.51757	4.17	0.629	0.781	0.78483	70.38	-0.23134	-4.54	0.543	0.794
	4	16	14.00	1.27970	69.28	-0.34329	-3.32	0.549	0.688	0.86071	46.86	-0.86921	-9.74	0.848	0.585
	5	16	13.44	1.23456	55.11	-0.08588	-1.59	0.480	0.605	0.74536	38.26	-0.63460	-6.85	0.629	0.582
	6	16	12.98	0.95551	39.28	-0.38563	-3.19	0.489	0.564	0.59057	25.72	-0.74488	-6.63	0.714	0.658
	7	16	12.40	0.95371	39.92	-0.36251	-3.06	0.430	0.492	0.52495	26.96	-0.65713	-6.62	0.677	0.561
	8	16	12.00	0.74329	25.07	-0.39976	-2.28	0.490	0.505	0.41900	14.63	-0.59961	-3.71	0.488	0.539
	9	16	11.39	0.83070	32.40	-0.38624	-3.27	0.506	0.479	0.42506	26.53	-0.50847	-6.23	0.601	0.532
	10	18	10.22	0.47190	24.78	0.16248	-0.45	0.675	0.657	0.24701	8.57	-0.37219	-3.08	0.544	0.608
		164	13.33	1.00177	46.68	-0.01316	-0.38	0.558	0.622	0.65764	43.59	-0.48223	-5.18	0.618	0.604

(1) Average t test values which are significant at the 5% level are underlined

second and third periods, one can effectively see that most of the large (small) size portfolio have an average positive (negative) intervalling effect. As for the average intervalling effect estimates of the portfolios when the value weighted index is used, all of them, but one for the largest size portfolio in the third period, are negative and their average t-test values are also statistically significant. This is not surprising given the weight of the few big securities which are highly traded in the value weighted index. Nearly all stocks, and consequently portfolios, have a lower trading activity than the average market. The unsignificant and positive value of ${}_{2}\hat{\beta}_{i}$ for the largest size portfolio in the 1983-1985 period is probably due to the fact that this period saw, for various reasons, an important additional activity on the BSE which has been mainly concentrated on the largest firms securities.

As for the other regression parameters, let us note that all portfolio estimated asymptotic betas have a significant average t-test value. Besides, the low value of all Durbin Watson test coefficients indicates the presence of some autocorrelation in the second pass regression. In order to eliminate this autocorrelation I also tested forms of the second pass regression which account for it, but it did not bring substantial improvement in the results.[13]

[13] The presence of autocorrelation in the second pass regressions can also be explained to a certain extent by the level of autocorrelation in the regression dependent variable. It can be shown (see Pindyck and Rubinfeld, 1986) that the Durbin Watson test is approximately equal to:

$$DW = 2 - 2\,\frac{\mathrm{cov}(\tilde{\varepsilon}_t,\tilde{\varepsilon}_{t-1}) + \rho(\beta-\hat{\beta})\ \mathrm{var}(x)}{\mathrm{var}(\tilde{\varepsilon})_t + (\beta-\hat{\beta})^2\ \mathrm{var}(x)}$$

where ρ is the autocorrelation of the independent variable x. If $\hat{\beta}$ is not identically equal to its true parameter β and if the autocorrelation coefficient of the independent variable is large, 0.98 in our case, the DW coefficient will be lower.

3.4 Third Pass Regression Results

The third pass regressions were run for each security using the Lachenbruch hold-out sample technique. As the results for the ten portfolios are very similar, differences between the average estimated parameters of the portfolios generally never exceed 5 percent, the presentation of all results is not useful. Only the estimated parameters of the third pass regression for the whole sample and for the four proxy variables of the thinness are presented in table 4.

The relationship between the market value and the intervalling effect is not surprising for the second and third period. The regression F test is significant, and its two parameters are also significantly different from zero. The intercept is negative and the slope is positive. This means that the small market value securities have a negative intervalling effect, their beta increases as the differencing interval is lengthened, while securities with a market value greater than the average have a positive intervalling effect. However, as in the French case, the t statistics are less conclusive than those of CHMSW on the US data. Besides our R^2 are very small, so the security market value only explains ten percent of the variance of the intervalling effect in the best situation, that is, for the 1980-1982 period and for the equally weighted index. As far as the first period is concerned, it appears, as the previous tables seem to suggest, that the relationship thinness-intervalling effect is reversed when the equally weighted index is used and that the regression is not at all significant when the value weighted index is used.

Table 4: Third Pass Regression Statistics

Period	Var.	Index	$_3\hat{\alpha}$	$t(_3\hat{\alpha})$	$_3\hat{\beta}$	$t(_3\hat{\beta})$	R^2	F	DW
1977-1979	(1)	(a)	1.3458	2.58	-0.0973	-2.44	0.038	5.945	1.861
		(b)	-1.1283	-2.79	0.0346	1.16	0.008	1.245	2.041
	(2)	(a)	0.0981	0.16	-0.0010	-0.02	0.000	0.000	1.774
		(b)	-1.2442	-2.68	0.0544	1.22	0.010	1.489	2.091
	(3)	(a)	3.2449	3.36	-0.1791	-3.28	0.067	10.749	1.880
		(b)	-1.3806	-1.81	0.0397	0.92	0.006	0.847	2.050
	(4)	(a)	0.0183	0.16	0.4457	0.80	0.004	0.637	1.789
		(b)	-0.7149	-8.13	0.2187	0.51	0.002	0.262	2.050
1980-1982	(1)	(a)	-1.4648	-4.10	0.1186	4.16	0.089	17.291	1.560
		(b)	-1.3555	-4.94	0.0673	3.07	0.050	9.446	1.539
	(2)	(a)	-1.7671	-4.95	0.1730	5.01	0.124	25.130	1.582
		(b)	-0.7154	-2.50	0.0189	0.68	0.003	0.465	1.577
	(3)	(a)	-1.8124	-3.23	0.1056	3.26	0.056	10.598	1.591
		(b)	-1.0798	-2.50	0.0324	1.30	0.009	1.682	1.555
	(4)	(a)	0.2083	2.67	-1.2329	-3.69	0.071	13.629	1.629
		(b)	-0.3093	-5.46	-1.2822	-5.29	0.136	28.010	1.661
1983-1985	(1)	(a)	-1.7267	-4.29	0.1285	4.30	0.102	18.468	2.008
		(b)	-1.3920	-4.81	0.0682	3.18	0.059	10.104	1.791
	(2)	(a)	-2.0055	-4.94	0.1841	4.95	0.132	24.542	1.958
		(b)	-1.4742	-4.99	0.0916	3.39	0.066	11.517	1.791
	(3)	(a)	-1.8302	-2.77	0.0983	2.76	0.045	7.620	2.002
		(b)	-1.3904	-2.97	0.0491	1.95	0.023	3.786	1.817
	(4)	(a)	0.2164	2.28	-0.9731	-3.10	0.056	9.595	2.003
		(b)	-0.3699	-5.49	-0.4760	-2.13	0.027	4.535	1.827

(1) Market value of the outstanding shares
(2) Volume of trading
(3) Market value of the volume of trading
(4) Volume of trading/number of outstanding shares
(a) Equally weighted index
(b) Value weighted index

Concerning the other thinness proxy variables, the results for the volume of trading as well as for its value are consistent with those for the market value, although the tests are less often conclusive. Finally the sign of the estimated slope for the second and third periods is reversed , as expected, when the independent variable in the third pass regression is the rotation of the shares. Its statistical tests are also less conclusive.

The inferred asymptotic betas of the security are then calculated using the equation (9). Their average value for the ten size portfolios are reported in table 5. A look at this table shows that the adjustment for the intervalling effect bias is only partial since some intervalling effect bias obviously subsist in the inferred betas.

3.5 Comparison and Test of the Adjustment Methods

The mean square errors (MSE) of the estimated betas $\bar{\hat{\beta}}_{iL}$, inferred asymptotic betas ${}_2\hat{\alpha}_i^*$ and of the Scholes and Williams estimator $\hat{\beta}_{iL}^{sw}$ from the estimated asymptotic betas ${}_2\hat{\alpha}_i$, are reported in table 6. The MSE of the inferred asymptotic beta for a one day differencing interval is lower than the MSE of the unadjusted betas in all cases but one, i.e. for the second period and the equally weighted index. The difference is however always tiny for the equally weighted index, showing that the adjustment for the intervalling effect is on average not conclusive when this index is used. In this respect, it is worth noting that the adjustment method of Scholes Williams gives a better estimate. When the value weighted index is used the inferred asymptotic beta has a lower MSE, and it slightly supersedes the Scholes Williams's estimator. Unfortunately the situation is reversed for differencing interval

Table 5 : Inferred Betas

Period	Port.	Sec	L=1	2	3	5	10	15	20	25	30	1	2	3	5	10	15	20	25	30
1977-1979	1	17	0.891	0.996	1.013	1.013	0.996	0.971	0.951	0.927	0.908	1.222	1.093	1.046	1.016	1.002	1.011	1.025	1.035	1.049
	2	15	1.094	1.131	1.134	1.102	1.049	1.040	1.036	1.039	1.049	1.028	0.953	0.959	0.989	1.004	1.011	1.021	1.063	1.098
	3	15	1.168	1.179	1.158	1.131	1.089	1.076	1.050	1.014	0.979	0.906	0.835	0.827	0.834	0.865	0.896	0.916	0.920	0.948
	4	15	1.166	1.152	1.135	1.109	1.124	1.125	1.144	1.182	1.223	0.954	0.877	0.847	0.851	0.890	0.906	0.951	1.010	1.084
	5	15	1.309	1.392	1.417	1.416	1.367	1.343	1.325	1.303	1.281	0.924	0.873	0.867	0.879	0.894	0.958	1.015	1.056	1.091
	6	15	0.863	1.057	1.108	1.144	1.152	1.138	1.123	1.105	1.107	0.712	0.644	0.652	0.683	0.711	0.745	0.784	0.813	0.858
	7	15	0.747	0.728	0.763	0.790	0.835	0.850	0.877	0.893	0.900	0.677	0.526	0.492	0.505	0.563	0.614	0.694	0.744	0.786
	8	15	0.644	0.625	0.604	0.594	0.658	0.666	0.652	0.637	0.617	0.660	0.446	0.399	0.347	0.373	0.414	0.432	0.434	0.433
	9	15	0.822	0.960	1.013	0.992	0.922	0.910	0.931	0.952	0.953	0.666	0.584	0.578	0.591	0.570	0.611	0.705	0.785	0.848
	10	16	0.932	0.855	0.843	0.842	0.789	0.761	0.742	0.728	0.708	0.638	0.450	0.412	0.410	0.400	0.437	0.464	0.475	0.466
		153	0.962	1.006	1.018	1.012	0.997	0.986	0.981	0.976	0.970	0.842	0.731	0.710	0.713	0.729	0.762	0.801	0.834	0.866
1980-1982	1	18	1.093	1.325	1.395	1.415	1.382	1.352	1.318	1.289	1.263	1.128	1.084	1.073	1.057	1.033	1.028	1.026	1.033	1.037
	2	18	1.414	1.564	1.584	1.567	1.492	1.435	1.395	1.352	1.319	0.993	0.960	0.960	0.976	0.978	0.982	0.991	0.999	0.999
	3	18	1.197	1.287	1.291	1.281	1.224	1.189	1.153	1.120	1.097	0.793	0.748	0.747	0.764	0.778	0.788	0.804	0.815	0.824
	4	18	1.362	1.409	1.411	1.403	1.338	1.280	1.236	1.196	1.160	0.707	0.646	0.648	0.682	0.747	0.754	0.770	0.788	0.786
	5	18	1.065	1.087	1.085	1.114	1.170	1.204	1.213	1.225	1.236	0.634	0.518	0.501	0.516	0.606	0.667	0.723	0.782	0.824
	6	18	0.741	0.692	0.699	0.710	0.713	0.727	0.729	0.736	0.746	0.504	0.399	0.375	0.378	0.384	0.412	0.437	0.476	0.508
	7	18	0.929	0.871	0.869	0.872	0.900	0.905	0.917	0.930	0.932	0.544	0.416	0.395	0.406	0.454	0.481	0.523	0.576	0.601
	8	18	0.652	0.592	0.580	0.593	0.604	0.628	0.654	0.671	0.696	0.496	0.338	0.293	0.267	0.280	0.310	0.359	0.406	0.452
	9	18	0.930	0.847	0.827	0.812	0.820	0.856	0.903	0.943	0.977	0.561	0.426	0.391	0.370	0.389	0.437	0.495	0.567	0.619
	10	18	0.852	0.692	0.654	0.658	0.712	0.746	0.771	0.792	0.811	0.565	0.373	0.310	0.292	0.341	0.359	0.383	0.413	0.434
		180	1.023	1.036	1.039	1.042	1.035	1.032	1.029	1.025	1.024	0.693	0.591	0.569	0.571	0.599	0.622	0.651	0.686	0.708
1983-1985	1	18	1.299	1.433	1.429	1.349	1.276	1.255	1.252	1.261	1.262	1.171	1.124	1.100	1.057	1.026	1.009	1.005	1.004	1.000
	2	16	1.308	1.404	1.422	1.376	1.316	1.285	1.260	1.248	1.234	1.018	0.981	0.986	0.999	1.013	0.997	0.979	0.971	0.962
	3	16	1.185	1.228	1.197	1.139	1.057	1.024	1.005	1.001	0.996	0.890	0.850	0.831	0.826	0.812	0.797	0.778	0.772	0.768
	4	16	1.010	1.106	1.145	1.185	1.235	1.260	1.264	1.278	1.293	0.659	0.634	0.655	0.719	0.817	0.855	0.867	0.889	0.906
	5	16	1.184	1.210	1.207	1.197	1.209	1.231	1.250	1.244	1.246	0.695	0.649	0.649	0.678	0.725	0.749	0.759	0.763	0.765
	6	16	0.794	0.792	0.801	0.849	0.934	0.961	0.956	0.955	0.963	0.553	0.432	0.414	0.457	0.583	0.613	0.617	0.618	0.624
	7	16	0.842	0.839	0.843	0.884	0.926	0.952	0.962	0.966	0.971	0.541	0.435	0.404	0.446	0.531	0.541	0.541	0.545	0.550
	8	16	0.681	0.611	0.614	0.662	0.689	0.727	0.788	0.809	0.828	0.492	0.367	0.342	0.364	0.390	0.421	0.470	0.488	0.500
	9	16	0.810	0.750	0.756	0.764	0.792	0.833	0.865	0.878	0.879	0.581	0.434	0.393	0.392	0.429	0.444	0.465	0.479	0.476
	10	18	0.881	0.672	0.616	0.619	0.564	0.496	0.465	0.469	0.468	0.531	0.343	0.288	0.283	0.295	0.280	0.265	0.274	0.270
		164	1.002	1.006	1.004	1.002	0.998	0.999	1.003	1.007	1.011	0.717	0.627	0.608	0.624	0.662	0.670	0.674	0.679	0.681

Table 6 : Mean Square Errors from the Estimated Asymptotic Beta

	1977-1979						1980-1982						1983-1985					
	Equally Weighted Index			Value Weighted Index			Equally Weighted Index			Value weighted Index			Equally Weighted Index			Value Weighted Index		
	${}_1\bar{\hat{\beta}}_{iL}$	${}_2\hat{\alpha}^*_i$	β^{SW}_{iL}	${}_1\bar{\hat{\beta}}_{iL}$	${}_2\hat{\alpha}^*_i$	β^{SW}_{iL}	${}_1\bar{\hat{\beta}}_{iL}$	${}_2\hat{\alpha}^*_i$	β^{SW}_{iL}	${}_1\bar{\hat{\beta}}_{iL}$	${}_2\hat{\alpha}^*_i$	β^{SW}_{iL}	${}_1\bar{\hat{\beta}}_{iL}$	${}_2\hat{\alpha}^*_i$	β^{SW}_{iL}	${}_1\bar{\hat{\beta}}_{iL}$	${}_2\hat{\alpha}^*_i$	β^{SW}_{iL}
1	0.380	0.370	0.148	0.302	0.161	0.166	0.178	0.183	0.121	0.175	0.098	0.110	0.192	0.186	0.126	0.160	0.088	0.103
2	0.170	0.167	0.067	0.208	0.117	0.072	0.129	0.117	0.058	0.129	0.070	0.051	0.139	0.124	0.047	0.120	0.067	0.038
3	0.122	0.121	0.032	0.151	0.094	0.032	0.098	0.087	0.026	0.098	0.058	0.022	0.097	0.087	0.021	0.089	0.053	0.013
4	0.085	0.084	0.017	0.112	0.075	0.021	0.074	0.066	0.014	0.073	0.046	0.011	0.066	0.061	0.011	0.063	0.040	0.008
5	0.063	0.062	0.012	0.083	0.056	0.017	0.055	0.049	0.007	0.055	0.035	0.007	0.046	0.043	0.007	0.043	0.029	0.007
6	0.045	0.045	0.009	0.064	0.043	0.013	0.040	0.036	0.004	0.042	0.027	0.005	0.034	0.032	0.005	0.030	0.020	0.006
7	0.032	0.032	0.009	0.048	0.032	0.013	0.031	0.028	0.003	0.031	0.020	0.004	0.025	0.024	0.004	0.020	0.014	0.005
8	0.024	0.024	0.010	0.036	0.024	0.017	0.024	0.021	0.006	0.024	0.015	0.005	0.018	0.018	0.005	0.014	0.010	0.005
9	0.018	0.018	0.014	0.029	0.019	0.026	0.018	0.016	0.011	0.018	0.011	0.010	0.014	0.014	0.009	0.010	0.008	0.008
10	0.013	0.013	0.021	0.024	0.015	0.035	0.013	0.012	0.020	0.013	0.008	0.022	0.011	0.011	0.013	0.007	0.006	0.011
11	0.010	0.010	0.031	0.019	0.012	0.049	0.010	0.009	0.031	0.010	0.006	0.039	0.008	0.008	0.019	0.005	0.004	0.014
12	0.008	0.008	0.043	0.014	0.009	0.075	0.007	0.006	0.044	0.008	0.005	0.063	0.005	0.006	0.025	0.003	0.003	0.017
13	0.006	0.006	0.055	0.011	0.007	0.102	0.004	0.004	0.059	0.006	0.003	0.094	0.004	0.004	0.031	0.003	0.002	0.020
14	0.005	0.005	0.070	0.008	0.005	0.142	0.003	0.003	0.072	0.004	0.002	0.124	0.003	0.003	0.037	0.002	0.001	0.024
15	0.004	0.004	0.086	0.005	0.003	0.189	0.002	0.002	0.082	0.002	0.001	0.147	0.002	0.002	0.044	0.001	0.001	0.027
16	0.004	0.004	0.103	0.003	0.003	0.250	0.002	0.002	0.088	0.002	0.001	0.160	0.001	0.001	0.052	0.001	0.001	0.030
17	0.004	0.004	0.120	0.003	0.003	0.341	0.002	0.002	0.092	0.001	0.001	0.178	0.001	0.001	0.063	0.001	0.001	0.035
18	0.005	0.005	0.141	0.003	0.004	0.443	0.002	0.002	0.095	0.001	0.001	0.185	0.002	0.002	0.078	0.001	0.001	0.041
19	0.005	0.005	0.162	0.003	0.004	0.581	0.003	0.003	0.095	0.001	0.002	0.170	0.002	0.002	0.095	0.001	0.002	0.049
20	0.007	0.007	0.184	0.004	0.005	0.745	0.004	0.005	0.094	0.002	0.003	0.161	0.004	0.004	0.113	0.002	0.002	0.058
21	0.009	0.009	0.202	0.005	0.007	0.917	0.006	0.007	0.092	0.003	0.005	0.156	0.004	0.004	0.135	0.002	0.003	0.068
22	0.013	0.013	0.217	0.007	0.009	1.013	0.007	0.008	0.090	0.005	0.007	0.149	0.005	0.005	0.159	0.003	0.003	0.083
23	0.013	0.013	0.235	0.009	0.012	1.186	0.010	0.011	0.088	0.008	0.010	0.140	0.005	0.006	0.191	0.004	0.004	0.097
24	0.016	0.016	0.261	0.014	0.017	1.392	0.013	0.014	0.088	0.011	0.014	0.137	0.007	0.007	0.215	0.004	0.005	0.111
25	0.020	0.020	0.266	0.018	0.021	1.238	0.015	0.016	0.088	0.013	0.016	0.135	0.008	0.008	0.246	0.005	0.006	0.130
26	0.021	0.021	0.287	0.019	0.023	1.390	0.019	0.020	0.091	0.017	0.020	0.136	0.010	0.010	0.289	0.006	0.006	0.157
27	0.027	0.027	0.312	0.032	0.036	1.676	0.022	0.023	0.094	0.021	0.024	0.142	0.010	0.011	0.326	0.006	0.007	0.178
28	0.029	0.029	0.315	0.028	0.032	1.565	0.026	0.027	0.097	0.024	0.027	0.146	0.013	0.014	0.386	0.008	0.008	0.206
29	0.034	0.034	0.341	0.041	0.045	1.996	0.032	0.033	0.102	0.031	0.034	0.148	0.013	0.013	0.413	0.008	0.008	0.234
30	0.036	0.036	0.362	0.042	0.046	1.648	0.031	0.032	0.107	0.030	0.033	0.160	0.015	0.015	0.475	0.009	0.009	0.271

lengths of two to more or less nine days. The MSE of the Scholes Williams's estimate at first decreases quicker than the inferred beta MSE, and then increases again for longer intervals. Another interesting feature of table 6 is that it clearly shows the fast convergence of the betas, adjusted or unadjusted. The minimum MSE is indeed always reached for approximately 15 days differencing intervals.

4 Intervalling Effect and Firm Size Effect

A lot of researchers have been concerned for some years with the existence of market anomalies in the security returns. Systematic and persistent deviations from equity pricing as predicted, for example, by the Capital Asset Pricing Model (CAPM) are indeed observed in almost all security markets[14]. The choice of the market value as a proxy variable for the thinness of a security and the relationship between intervalling effect bias in beta and thinness would suggest that the intervalling effect could explain the size effect discovered by Banz (1981). Are the abnormal risk adjusted returns of the small firms due to a bias in the estimation of the beta? In a paper Roll (1981) concluded that trading infrequency, which induces some autocorrelation in the market indexes, seems to be a powerful cause of bias in the systematic risk estimation and, according to CHMSW, the adjustment for the intervalling effect bias should be more important for small firms than for large firms.

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[14] The number of papers dealing with market anomalies is very large. For an introduction the reader is invited to consult Hawawini (1984) and Dimson (1988).

Following CHMSW I tested three regressions with the natural logarithm of the market value of the firms as the independent variable in order to test the relationship between the intervalling effect adjustment and the firm size. The dependent variables are respectively the one day interval unadjusted beta $_1\hat{\beta}_{11}$, the asymptotic beta estimate $_2\hat{\alpha}_1$ and the adjustment ratio $(_1\hat{\beta}_{11}-_2\hat{\alpha}_1)/_2\hat{\alpha}_1$. A fourth regression with Scholes-Williams's estimate as the dependent variable was also tested to compare. Results of these regressions are reported in table 7. The slope coefficients of the unadjusted betas are all, but for the first period and the equally weighted index, significantly positive. Their corresponding F test coefficients are also significant and their R^2 are quite high. This indicates clearly, especially when the value weighted index is used, that one day interval betas are related to the market value. For small differencing intervals, small firms have lower betas than larger firms. If the differencing interval is lengthened, or in other words, if the asymptotic beta estimate is the dependent variable, the relationship is still positive and significant. After adjustment for the intervalling effect bias, small firms have still lower betas than larger firms. The slope coefficients are however smaller and their t-test coefficients as well as the regression F test value are also less significant . This suggests that only part of the size effect is explained by the intervalling effect[15]. The results for the third regression with the adjustment ratio as

[15] A second reason might suggest that the size effect cannot be totally explained by the intervalling effect. Hawawini, Michel and Corhay showed indeed that there is a size effect in the Belgian security returns when a monthly differencing interval is used. As the intervalling effect bias in beta tends to disappear quickly when the differencing interval is lengthened, we should expect almost no size effect in monthly returns if the size effect is only caused by an intervallingeffect.

Table 7 : Intervalling Effect and Firm Size

$$\gamma = \gamma_0 + \gamma_1 \ln(MV)$$

Period	γ	γ_0	$t(\gamma_0)$	γ_1	$t(\gamma_1)$	R^2	F test	DW
1977-1979	(1) (a)	1.3244	3.07	-0.0234	-0.71	0.003	0.50	1.660
	(b)	-1.2541	-6.89	0.1254	9.01	0.349	81.11	1.941
	(2) (a)	0.3224	0.87	0.0518	1.83	0.022	3.34	1.932
	(b)	-0.6508	-2.34	0.1091	5.14	0.149	26.41	1.879
	(3) (a)	-0.1307	-0.03	0.0188	0.06	0.000	0.00	2.037
	(b)	-1.7383	-1.60	0.0998	1.20	0.010	1.45	2.065
	(4) (a)	0.5401	1.38	0.0395	1.32	0.011	1.74	1.881
	(b)	-1.1292	-5.18	0.1270	7.62	0.278	58.06	2.025
1980-1982	(1) (a)	-0.9215	-3.13	0.1573	6.69	0.201	44.79	1.871
	(b)	-1.4842	-10.59	0.1466	13.09	0.491	171.46	1.720
	(2) (a)	-0.4169	-1.36	0.1172	4.80	0.114	23.01	1.644
	(b)	-0.8522	-4.48	0.1188	7.82	0.256	61.22	1.704
	(3) (a)	-0.3361	-0.37	0.0291	0.40	0.001	0.16	1.670
	(b)	-1.0280	-1.07	0.0418	0.55	0.002	0.30	2.186
	(4) (a)	-1.3739	-4.45	0.1952	7.92	0.261	62.80	1.761
	(b)	-1.4940	-9.89	0.1535	12.73	0.477	162.07	1.779
1983-1985	(1) (a)	-1.3794	-5.34	0.1778	9.27	0.346	85.89	1.982
	(b)	-1.6884	-11.23	0.1552	13.91	0.544	193.58	1.988
	(2) (a)	-0.5462	-2.33	0.1161	6.68	0.216	44.68	1.599
	(b)	-0.9970	-6.21	0.1241	10.41	0.401	108.41	1.606
	(3) (a)	7.3172	1.55	-0.4874	-1.39	0.012	1.92	2.026
	(b)	-1.5289	-2.93	0.0851	2.20	0.029	4.82	2.091
	(4) (a)	-1.5805	-6.26	0.1939	10.35	0.398	107.21	1.982
	(b)	-1.6941	-10.88	0.1617	13.99	0.547	195.66	2.182

(1) $\gamma = {}_1\hat{\beta}_{i1}$

(2) $\gamma = {}_2\hat{\alpha}_i$

(3) $\gamma = ({}_1\hat{\beta}_{i1} - {}_2\hat{\alpha}_1)/{}_2\hat{\alpha}_1$

(4) $\gamma = \hat{\beta}^{sw}_{1L}$

(a) Equally Weighted Index

(b) Value Weighted Index

Statistical test values which are significant at the 5% level are underlined.

dependent variable show indeed the partial improvement of the relationship between the betas and the firm size due to the adjustment. As expected, all slope coefficients of the adjustment ratio, but one for the equally weighted index, are positive. Only one out of these five coefficients has however a significant value and all five regression R^2 are negligible[16]. It seems therefore that the adjustment for the intervalling effect bias in the Belgian firm betas effectively decreases the magnitude of the size effect. This impact on the size effect is however less conclusive than for the American sample. In their study CHMSW found a significant impact of the adjustment on the relationship between the beta and the size of the firms.

When Scholes-Williams's estimate is used as the dependent variable, the regression results are very similar to those of the regression with the unadjusted betas. This demonstrates that Scholes-Williams's adjustment, more than the asymptotic beta procedure, is certainly not a solution to the small firm effect.

5 Conclusion

The purpose of this paper was to test CHMSW's adjustment procedure for the intervalling effect bias in beta on a comprehensive security sample of the Belgian equity market where the impact of price-adjustment delays on the beta is important. Two

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[16] The unsignificant value of the statistical tests as well as the low value of the regression R^2 can, to a certain extent, be explained. The size effect is mainly a small firm effect, that is, the firms with the smallest market value have on average a higher risk adjusted return than the firms with the largest market value. Therefore the size effect mainly concerns firms with extreme market values. Using a comprehensive sample of Belgian securities with a large range of different market values, there is no reason to expect a relationship between the adjustment for the intervalling effect bias in the betas of the middle size firms and their market value.

market indexes, one equally and one value weighted, were used and the study was carried out on three adjacent periods of three years.

The adjustment procedure of CHMSW includes three steps which lead to the determination of an asymptotic beta and an inferred asymptotic beta. The first step consists in the calculation of the unadjusted betas for various differencing interval lengths. As the tests showed that the betas exhibit some seasonality, an appropriate correction has also been applied to them. In the second step the asymptotic beta as well as a measure of the intervalling effect were derived. To this end various functions of the inverse of the differencing interval length were tested. A third step then allowed the estimation of the inferred asymptotic beta by adjusting the unadjusted betas for cross-sectional differences in the intervalling effect. In this step four different measures related to the market value of the security and its volume of trading were also tested as proxy variables for thinness. As in the French and American studies, the market value of the outstanding shares gave the best results.

The beta values for the various differencing intervals show that the intervalling effect bias is present in the security betas and that it tends to disappear when the the differencing interval used to measure the returns increases. However its direction and magnitude depend on the choice of the index. It is indeed important, especially for small market value securities when the value weighted index is used. Considering this latter index, portfolios betas generally increase with the length of the differencing interval. The results are also consistent across the periods. As for the equally weighted index there is some inconsistency as during the second and third periods, large market value portfolio betas tend to decrease while small market value portfolio betas tend to increase. The opposite is observed for the first period.

Like for the American and French samples, the cross-sectional adjustment procedure allows the correction of the intervalling effect bias in the Belgian security betas. The adjustment is however not substantial and the tests are less conclusive. First, the relationship between intervalling effect and thinness had the expected direction and its regression tests were significant for the second and the third periods only. Unfortunately, all third pass regression R-squares are very low. Secondly, there are few differences between the value of the MSE of the inferred asymptotic betas compared to the estimated asymptotic beta and those of the unadjusted betas for the equally weighted index. When the value weighted index is used the value of the MSE of the inferred betas is largely inferior. It is however hardly lower than the MSE value of the Scholes-Williams's estimate for a one day differencing interval. Considering longer differencing intervals, the estimate of Scholes-Williams supersedes the inferred asymptotic beta. As the inferred asymptotic beta needs more calculation to be estimated than Scholes-Williams's does, one could argue the advantage of the inferred beta, even for a one day differencing interval.

As far as the relationship between beta and size of the firms is concerned, I showed that the adjustment for the intervalling effect can decrease the magnitude of the small firm effect without explaining it. It is however not true for all periods when the equally weighted index is used.

6 References

1) Banz R., 'The Relationship Between Return and Market Value of Common Stocks', *Journal of Financial Economics*, 9, 1981, 3-18.

2) Brown S.J. and J.B. Warner, 'Using Daily Stock Returns, the Case of Event Studies', *Journal of Financial Economics*, No.14, 1985,3-31.

3) Cohen K.J., G.A. Hawawini, S.F. Maier, R,A. Schwartz and D.K. Whitcomb, 'Implications of Microstructure Theory for Empirical Research on Stock Price Behavior', *The Journal of Finance*, Vol.35, No.2, May 1980, 249-257.

4) Cohen K.J., G.A. Hawawini, S.F. Maier, R,A. Schwartz and D.K. Whitcomb, 'Estimating and Adjusting for the Intervalling Effect Bias in Beta', *Management Science*, 29, 1983, 135-148.

5) Cohen K.J., G.A. Hawawini, S.F. Maier, R,A. Schwartz and D.K. Whitcomb, 'Friction in the Trading Process and the Estimation of Systematic Risk', *Journal of Financial Economics*, 12, 1983, 263-278.

6) Cohen K.J., S.F. Maier, R.A. Schwartz and D.K. Whitcomb, *The Microstructure of Securities Markets*, Prentice-Hall, 1986, 254 pages.

7) Corhay A.H., 'Market Index Specification and Estimation of Risk: Some Empirical Results', Unpublished Paper, 1988.

8) Dimson E., 'Risk Measurement when Shares are Subject to Infrequent Trading', *Journal of Financial Economics*, 7, 1979, 197-226.

9) Dimson E., *Stock Market Anomalies*, Cambridge University Press, 1988, 295 pages.

10) Fowler D.J. and C.H. Rorke, 'Risk Measurement when Shares are Subject to Infrequent Trading: Comment', *Journal of Financial Economics*, 12, 1983, 279-283.

11) Fung W.K.H., R.A. Schwartz and D.K. Whitcomb, 'Adjusting for the Intervalling Effect Bias in Beta: A test Using Paris Bourse Data', *Journal of Banking and Finance*, 9, 1985, 443-460.

12) Hawawini G.A., 'Intertemporal Cross-Dependence in Securities Daily Returns and the Short-Run Intervalling Effect on Systematic Risk', *Journal of Financial and Quantitative Analysis*, Vol.15, No.1, March 1980, 139-149.
13) Hawawini G.A., *European Equity Markets: Price Behavior and Efficiency*, 1984, Monograph Series in Finance and Economics, No.4-5, 180 pages.
14) Hawawini G.A., P.A. Michel and A.H. Corhay, 'The Pricing of Common Stocks on the Brussels stock Exchange: A Re-Examination of the Evidence', Working paper, Insead, 1986.
15) Hellinckx B., *Bien Investir en Actions*, 1984, 158 pages.
16) Lachenbruch P.A., 'An Almost Unbiased Method of Obtaining Confidence Intervals for the Probability of Misclassification in Discriminant Analysis', *Biometrika*, 23, 1967, 639-645.
17) Levhari D. and H. Levy, 'The Capital Asset Pricing Model and the Investment Horizon', *The Review of Economics and Statistics*, Vol.59, 1977, 92-104.
18) McInish T.H. and R.A. Wood, 'Adjusting for Beta Bias: An Assessment of Alternate Techniques: A Note', *The Journal of Finance*, Vol.41, No.1, March 1986, 277-286.
19) Pindyck R.S. and D.L. Rubinfeld, *Econometric Models and Economic Forecasts*, McGraw-Hill Book Company, 1986, 630 pages.
20) Pogue G.A. and B.H. Solnik, 'The Market Model Applied to European Common Stocks: Some Empirical Results', *Journal of Financial and Quantitative Analysis*, December 1974, 917-944.
21) Roll R., 'A Possible Explanation of the Small Firm Effect', *The Journal of Finance*, Vol.36, No.4, September 1981, 879-888.
22) Saniga E.M., T.H. McInish and B.K. Gouldey, 'The Effect of Differencing Interval Length on Beta', *The Journal of Financial Research*, Vol.4, No.2, 1981, 121-135.
23) Scholes M. and J. Williams, 'Estimating Betas from Nonsynchronous Data', *Journal of Financial Economics*, 5, 1977, 309-327.
24) Stonham P., *Major Stock Market of Europe*, Gower Publishing Company, 1982.