Equations for hydraulic conductivity estimation from particle size distribution: A dimensional analysis

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Key Points:

- Dimensional analysis is employed in the analysis of the relationship between hydraulic conductivity and particle size distribution
- New equations are proposed for hydraulic conductivity estimation by basic soil gradation parameters
- The new method improves the estimation of hydraulic conductivity significantly comparing to classic equations

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Equations for hydraulic conductivity estimation from particle size distribution: A dimensional analysis

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Abstract Estimating hydraulic conductivity from particle size distribution (PSD) is an important issue for various engineering problems. Classical models such as Hazen model, Beyer model, and Kozeny-Carman model usually regard the grain diameter at 10% passing (d_{10}) as an effective grain size and the effects of particle size uniformity (in Beyer model) or porosity (in Kozeny-Carman model) are sometimes embedded. This technical note applies the dimensional analysis (Buckingham's $\prod$ theorem) to analyze the relationship between hydraulic conductivity and particle size distribution (PSD). The porosity is regarded as a dependent variable on the grain size distribution in unconsolidated conditions. It indicates that the coefficient of grain size uniformity and a dimensionless group representing the gravity effect, which is proportional to the mean grain volume, are the main two determinative parameters for estimating hydraulic conductivity. Regression analysis is then carried out on a database comprising 431 samples collected from different depositional environments and new equations are developed for hydraulic conductivity estimation. The new equation, validated in specimens beyond the database, shows an improved prediction comparing to using the classic models.

1. Introduction

Understanding the hydraulic conductivity of a geomaterial, noted as K [m/s], is crucial for various aspects of hydrogeology and geotechnical engineering, such as for modeling the groundwater flow. It is recognized that the hydraulic conductivity of a sediment sample is related to its pore structure, which is difficult to measure. Pore structure is however intrinsically dependent on soil particle size distribution (PSD), and estimating hydraulic conductivity from empirical equations based PSD is still widely adopted in the nowadays [*Song et al.*, 2009; *Vienken and Dietrich*, 2011; *Lu et al.*, 2012; *Riva et al.*, 2014]. The earliest empirical equation of estimation hydraulic conductivity from particle size dates back to the work of *Hazen* [1892], in which the hydraulic conductivity was expressed proportional to the squared grain size at 10% passing (noted as d_{10}, unit in meter to obtain K in m/s). By considering unit consistency, it may be written as:

$$K=C_H\frac{g}{v}d_{10}^2 \tag{1}$$

where g is the gravitational acceleration [m/s^2] and ν is the fluid kinematic viscosity [m^2/s] ($\nu=0.89\times10^{-6}$m^2/s at 25°C for water). C_H is a unitless coefficient about 6.54×10^{-4} [*Harleman et al.*, 1963]. Other typical equations to estimate hydraulic conductivity from grain size, but considering also the effects of particle size uniformity and porosity have been also developed later. The equation of *Beyer* [1964], for instance, is mostly known to account the effect of particle size uniformity:

$$K=C_B\frac{g}{v}\log\left(\frac{500}{C_u}\right)d_{10}^2 \tag{2}$$

in which the coefficient C_B is 6×10^{-4} (unitless) and the coefficient of uniformity C_u (unitless) is the ratio of grain size at 60% passing and grain size at 10% passing ($C_u=\frac{d_{60}}{d_{10}}$). Furthermore, the Kozeny-Carman model [*Kozeny*, 1927, 1953, *Carman*, 1937, 1956] is another classical model to embed the porosity effect:

$$K = C_K \frac{g}{v} \frac{n^3}{(1-n)^2} d_{10}^2 \tag{3}$$

where the coefficient C_K is $1/180$ determined by flow in capillary tubes or beds of spheres and n is the porosity.

However, these different models are developed based on a limited number of samples and may therefore not be applied to all conditions. It should also be noted that the equations above are mostly empirical, and the physical background is insufficiently considered. It is better to use physically independent variables in the empirical equations (such as that soil porosity may be related to PSD and grain shape), and not only d_{10} but also the mean size of a PSD may also affect the hydraulic conductivity value. *Rosas et al.* [2014] tested 431 samples (0.05 mm $< d_{10} <$ 0.83 mm, 0.09 mm $< d_{60} <$ 4.29 mm, 1.3 $< C_u <$ 18.3) from global sites with different depositional environment based on the constant water head method. The results are normalized by using the kinematic viscosity of water at 25°C ($\nu = 0.89 \times 10^{-6} \mathrm{m}^2/\mathrm{s}$). These results can be regarded as a database highly representative for the analysis of the relationship between PSD and hydraulic conductivity. For the purpose to consider the physical basis, the classic dimensional analysis (also known as Buckingham's $\prod$ theorem, *Buckingham* [1914]), could be a useful tool to investigate an hypothetical physical relationship. Indeed, this methodology reduces the number of parameters, the reduction being equal to the rank of the so-called dimensional matrix (see next section). By using the dimensionless groups, it can simplify the regression analysis of an experiment data set [*Vignaux*, 1992]. In this technical note, we will carry out a dimensional analysis to explore the relationship between PSD and hydraulic conductivity and therefore clarify the controlling parameters. In particular, the effects of mean grain size, grain size uniformity, and porosity will be integrated in the analysis. Then, a regression analysis will be conducted on the database (including 431 samples) to obtain novel equations toward hydraulic conductivity estimation. The new equation will then be benchmarked thanks to experimental results beyond the database to prove its applicability.

2. Dimensional Analysis of the Relationship Between PSD and Hydraulic Conductivity

2.1. Principle of Dimensional Analysis

Dimensional analysis, namely Buckingham's Π theorem, is raised by *Buckingham* [1914]. Its principle is to describe a physical relationship between independent (dimensional) parameters with dimensionless groups (also called PI-terms or nondimensional numbers) built from these variables. Beside a reduction of the number of controlling parameters and helping the regression analysis [*Vignaux*, 1992], this approach also define similarity laws used to study the scale effect of two geometrically similar systems [*Cheng and Cheng*, 2004].

More particularly, let us consider a set of n fundamentally independent variables $X_1, X_2, \ldots, X_n$. A dimensionless group or Π term can be constructed as a product of powers of $X_1, X_2, \ldots, X_n$ with adequate exponents $s_1, s_2, \ldots, s_n$:

$$\prod = X_1^{s_1} X_2^{s_2} \cdots X_n^{s_n} \tag{4}$$

Since Π must have no dimension, these exponents are linked by dimensional equations (or dimensional constraints): for each base dimension involved in the physical problem such as mass (M), length (L), time (T), etc., a dimensional equation can be written in the following form:

$$\sum d_{ij} s_j = 0 \tag{5}$$

where d_{ij} is the exponent of dimension i in the parameter X_j. These d_{ij} coefficients define the so-called dimensional matrix with p rows (p is the number of base dimensions, for instance, three in a mechanical problem driven by mass, length, and time) and n columns (n is the number of physical parameters).

Let us now recall the results of the Buckingham's theorem: the n dimensional parameters are linked by k dimensional equations, where k is the rank of the dimensional matrix (so $k \leq p$), which reduces the total number of independent quantity terms to $i = n - k$. Moreover, the theorem also states that any physical relationship f linking a dependent dimensional variable X_0 and n dependent dimensional parameters such as below:

$$X_0 = f(X_1, X_2, \cdots, X_n) \tag{6}$$

can be captured by another mathematical relationship F linking the $i+1$ dimensionless terms as:

$$\Pi_0 = F(\Pi_1, \Pi_2, \cdots, \Pi_i) \tag{7}$$

where Π_0 is the dependent dimensionless term.

2.2. Analysis of the Relationship Based on a Large Data Set

We can now apply the dimensional analysis method to the study of the relationship between hydraulic conductivity and grain size distribution. Physically, we can deduce that the hydraulic conductivity K[m/s] of any granular soil is related to the particle size distribution (PSD), the liquid kinematic viscosity ν[m^2/s], the effect from gravity (related to the gravitational acceleration g[m/s^2]), the material porosity n[[–], ggvvvbrain shape, and grain roughness (the grain shape and roughness effects are not considered in this study as they arecomplicated to be quantified in a large database). Following *Wang et al*. [2017], PSD effect can be modeled with three basic parameters: the relative mean grain size (could be quantified by the grain size at 50% or 60% passing, i.e., d_{50} or d_{60}), $C_u = \frac{d_{60}}{d_{10}}$ (quantifying the grain size uniformity or grain size polydispersity) and the coefficient of curvature $C_c = \frac{(d_{30})^2}{d_{10}d_{60}}$. We use d_{60} instead of d_{50} to evaluate the relative mean size because d_{60} is already involved in the calculation of $C_u$and it will benefit the model simplification afterward. Consequently, the relationship f between the seven-dimensional parameters can be written as:

$$K = f(g, \nu, d_{60}, C_u, C_c, n) \tag{8}$$

In dimensional analysis, the variables on the right side of the above equation are assumed to be independent from one another. However, it can be imaged that the porosity n of a granular material may relate to PSD, especially to the grain size uniformity C_u. Indeed, pores between large grains will be filled with finer particles if the grain size polydispersity is high. Accordingly, *Vukovic and Soro* [1992] found that the porosity n is indeed a function of C_u empirically given by:

$$n = \alpha\left(1 + \beta^{C_u}\right) \tag{9}$$

where α and β are unitless constants. This relationship is graphically shown in Figure 1 for the 431 unlithified sediment samples of Rosas data base [*Rosas et al*., 2014] and regressed to fit equation (9), which leads to fitting parameters $\alpha=0.2$ and $\beta=0.93$. The fitted line represents the porosity deduced from grain size uniformity, which can be referred as the intrinsic porosity. However, we can see that the measured porosity has a variation around the intrinsic porosity; it will be considered in a latter part.

In the first step, it is assumed that the variation of the porosity around its intrinsic value do not affect the hydraulic conductivity estimation. The number of independent parameters in equation (8) can therefore be reduced from 8 to 7. The fundamental dimensions are length (L) and time (T) and the symbol – means dimensionless. In this particular case, the number of degrees of freedom in equation (8) can be reduced from 7 to 5 (i.e., the rank k is equal to 2). Additionally, as the coefficient of curvature C_c is not provided in the database of *Rosas et al*. [2014] (d_{30} is not available), we do not consider here the effect of this parameter. The remaining five variables may then be grouped in three-dimensional groups, written as follows thanks to dimensional analysis:

$$\frac{K}{d_{60}^{-1}\nu} = F\left(\frac{g}{d_{60}^{-3}\nu^2}, C_u\right) \tag{10}$$

in which $\Pi_0 = \frac{K}{d_{60}^{-1}\nu}$ is a dimensionless hydraulic conductivity, $\Pi_1 = \frac{g}{d_{60}^{-3}\nu^2}$ as a normalized gravity effect and $\Pi_2 = C_u$ representing the grain size polydispersity effect (C_u is already dimensionless!). By assuming constant temperature, ν does not change for a given liquid. Since g is also a constant, $\frac{g}{d_{60}^{-3}\nu^2}$ actually represents the effect of mean particle size (or mean particle volume as the exponent is in the third order).

To go one step further, let us remind that the set of dimensionless groups are linearly independent, which means any dimensionless group can be replaced by a product of powers of the existing dimensionless

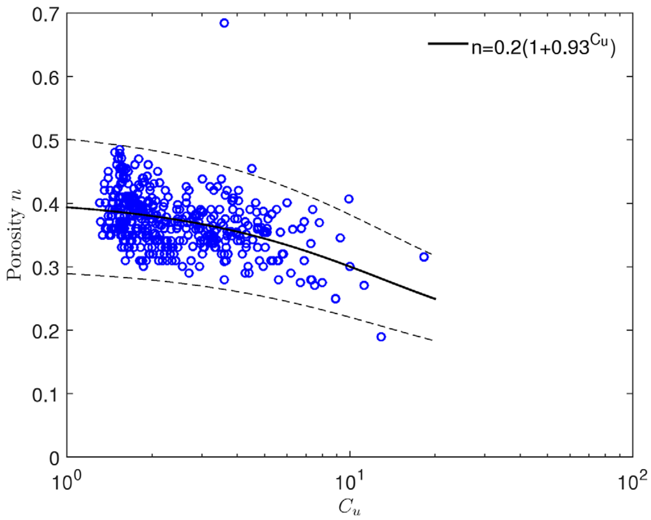

Figure 1. Relationship between particle size uniformity and porosity.

groups. Let us then chose to replace Π_0 by Π_0/Π_1. Consequently, the previous equation can be simplified into:

$$K^*=\frac{\Pi_0}{\Pi_1}=\frac{K\nu}{gd_{60}^2}\approx F'(C_u) \qquad (11)$$

where $F'(C_u)$ is a function of C_u. The relationship between K^* and C_u is presented in a semilogarithm plot in Figure 2a for the 431 sediment samples. It can be seen that K^* is a significantly high value for a small value of C_u (which means the PSD is narrow and grain size polydispersity is low). When the grain size polydispersity is larger (higher C_u), the sample porosity is relatively lower and K^* is decreased. The relationship may then be fitted by the following equation:

$$K^*=C_1{C_u}^a \qquad (12)$$

where C_1 and a are fitting parameters. The best fitted a by the least square method is about -2.2. The Hazen type equation can be recovered by taking $a=-2$ and the best coefficient for $a=-2$ is $C_1=1.52\times10^{-3}$. Then, the hydraulic conductivity is:

$$K=C_1{C_u}^a\frac{g}{\nu}d_{60}^2\approx C_1\frac{g}{\nu}d_{10}^2 \qquad (13)$$

This also suggests that taking $C_1=1.52\times10^{-3}$ instead of the Hazen value $C_H=6.54\times10^{-4}$ may be more appropriate for sediments in different depositional environments, according to the regression analysis on the 431 samples.

One step further, the effect from $\frac{g}{d_{60}^{-3}\nu^2}$ now should be unraveled. In Figure 2b, the ratio between the measured hydraulic conductivity ($K_{measure}$) from the Rosas' data set (measured by a standard constant head permeameter following *Wenzel* [1942] and ASTM standard D2434–68, *American Society for Testing and Materials* [2006]) and the hydraulic conductivity predicted by equation (13) ($K_{predict}$) are depicted versus $\frac{gd_{60}^3}{\nu^2}$ for the 431 sediments in a semilogarithm plot. It can be seen that for relatively large mean grain size (large $\frac{gd_{60}^3}{\nu^2}$), equation (13) overestimates the hydraulic conductivity value and for finer materials equation (13) may have

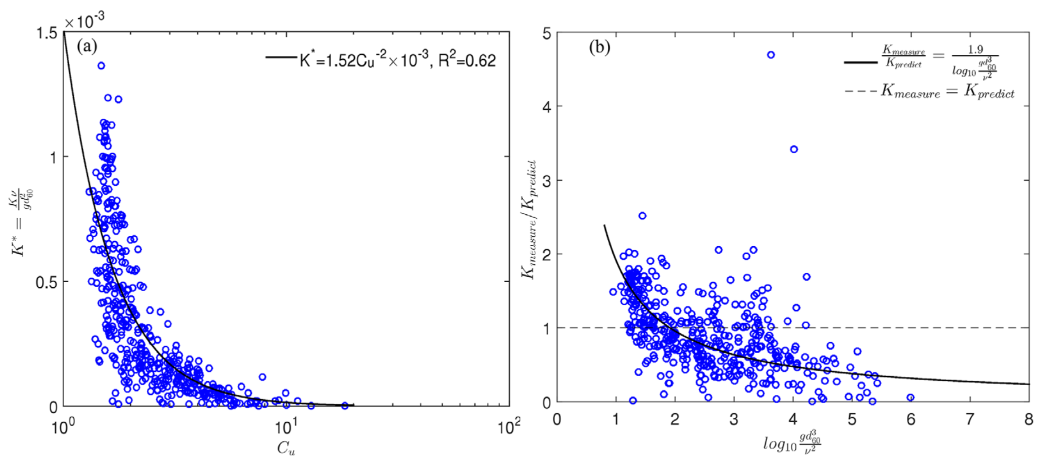

Figure 2. Regression analysis on the two independent dimensionless variables. (a) Effect of C_u on K^*; (b) Correction of $\frac{gd_{60}^3}{\nu^2}$.

underestimation. The ratio of $K_{measure}/K_{predict}$ is inversely proportional to $\log_{10}\frac{gd_{60}^3}{\nu^2}$ and may be expressed as the following form:

$$\frac{K_{measure}}{K_{predict}}=C_2\left(\log_{10}\frac{gd_{60}^3}{\nu^2}\right)^{-1} \tag{14}$$

where C_2 is estimated to be equal to 1.9 by fitting the database of 431 sediments. Then taking the correction from equation (14) on equation (13), K can be expressed as:

$$K=C_W C_u{}^a \frac{g}{\nu} d_{60}^2\left(\log_{10}\frac{gd_{60}^3}{\nu^2}\right)^{-1} \approx C_W \frac{g}{\nu} d_{10}^2\left(\log_{10}\frac{gd_{60}^3}{\nu^2}\right)^{-1} \tag{15}$$

where $C_W=C_1C_2=2.9\times10^{-3}$ and a ≈ -2. In the above equation, the effect of particle size polydispersity C_u and the effect from the mean grain size d_{60} are already embedded with the effect of porosity regarded as a dependent parameter on PSD. In reality, as can be seen in Figure 1, the porosity of a sediment sample may be varied around the theoretical correlation due to the particle arrangement and stress state. Then a further correction may be considered by accounting the effect of the porosity variation. A parameter Δn, as the difference between the measured porosity and the intrinsic porosity in equation (9), is adopted to correct the dimensionless hydraulic conductivity K^*. To keep the model simple Δn may be assumed to be a linear relationship with the K^* value. Therefore, another possible form of the hydraulic conductivity equation could be:

$$K=\left(C_W C_u{}^a\left(\log_{10}\frac{gd_{60}^3}{\nu^2}\right)^{-1}+b\Delta n\right)\frac{g}{\nu}d_{60}^2 \tag{16}$$

where b is a fitting parameter. It should be noted that the linear relationship here may not be a rigorous assumption and here it quantitatively reflects that Δn could be an additional parameter to develop. Due to the limitation of the available data, we leave the Δn effect to future research.

3. Comparison With Equations in the Literature

To discuss the feasibility of the proposed equations in the previous section, comparisons with the previous models are essential. The hydraulic conductivity predicted by a variety of the literature models and the conductivity models of equations (13), (15), and (16) are compared to measured values from the database of 431 sediments [*Rosas et al.*, 2014] (Figure 3) in logarithm scales. The straight solid line is the equality line corresponding to a perfect prediction ($K_{predict}=K_{measure}$). Figure 3a shows the prediction of the typical Hazen model. It can be seen that the Hazen equation gives good prediction for relatively high hydraulic conductivity values, but it may underestimate most of the samples with $K<3\times10^{-4}$ m/s. The prediction of the Beyer model is presented in Figure 3b. Although the effect of C_u is considered in the equation, it only improves the slope of the regression line slightly and most of the predictions overestimate the hydraulic conductivity obviously by using the suggested coefficient. For the model of Kozeny-Carman (Figure 3c), in which the effect of porosity is embedded, the prediction is better than Beyer's model and the slope of the regression line is improved but it still underestimates most hydraulic conductivity values (when $K<4\times10^{-4}$ m/s).

After a dimensional analysis and a regression analysis on the database, the effect of C_u is formulated by equation (13). By approximating the parameter a as -2, the equation yields to the typical Hazen type equation. However, the coefficient is different from classic Hazen equation as it is from the regression analysis of C_u effect. Figure 3d depicts the prediction of equation (13). It can be seen that by using coefficient C_1 instead of C_H improves the prediction of Hazen type equation as it leads more data fall on the equality line. The prediction of equation (15), with the effect of $\frac{gd_{60}^3}{\nu^2}$ considered, is presented in Figure 3e. It can be seen that by introducing the $\frac{gd_{60}^3}{\nu^2}$ term, the prediction is obviously improved especially for high hydraulic conductivity values comparing to the prediction of equation (13). Furthermore, if the correction from the porosity is accounted, the prediction of equation (16) is demonstrated in Figure 3f with the fitting parameter $b=0.5$. It can be seen that the variation of the data set against the equality line is slightly reduced. The slope of the regression line is also closer to that of the equality line. For reason of practicability, as the value of porosity and the intrinsic porosity are not always easily accessible, equation (15) could be a more practical equation. It should be noted that although we use suggested coefficient values in the literature for the three classic

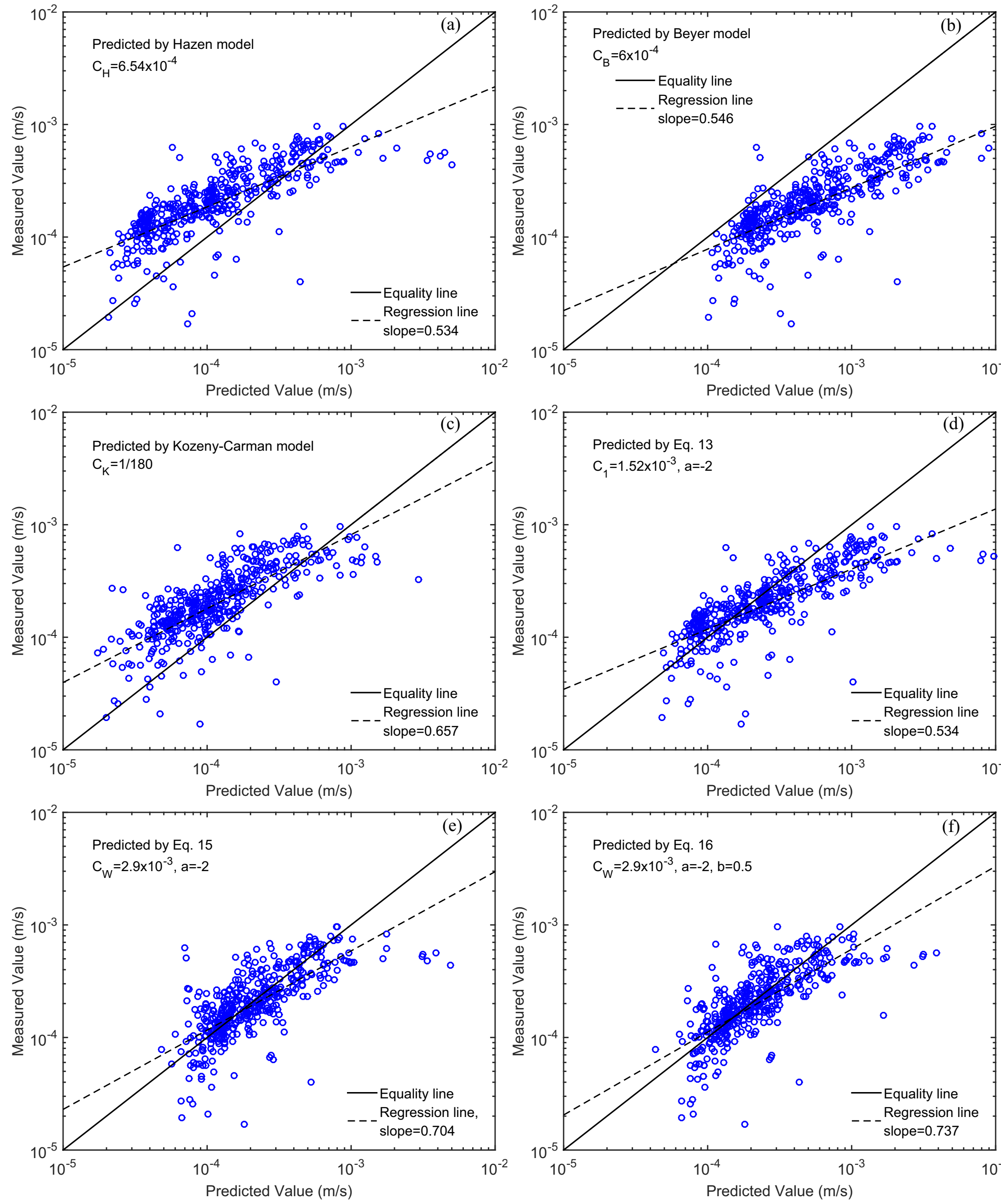

Figure 3. Prediction of different models on the data set of *Rosas et al.* [2014]. (a) Hazen model; (b) Beyer model; (c) Kozeny-Carman model; (d) Prediction of equation (13); (e) Prediction of equation (15); (f) Prediction of equation (16).

equations, change the coefficient value will only shift the data set without changing the slope of regression line. This means that the predictions of equations (15) and (16) are always better than other models (for this set of data).

As the data set of *Rosas et al.* [2014] contains hydraulic conductivity values for materials in various depositional environments and the samples are collected on global sites, the equations developed based on dimensional analysis and this data set can be widely applicable to different sandy soils beyond this data set. Validation of the new model can be carried out on other sandy soils in the literature (tested by the same constant head method). *Alyamani and Sen* [1993] tested the hydraulic conductivity of 22 quaternary deposit samples from Saudi Arabia. *Wiebenga et al.* [1970] tested 13 samples of quartz sands and silts from Australia. *Cabalar and Akbulut* [2016] measured hydraulic conductivity values of 16 Narli sands and 16 crushed stone sands from Turkey. Figure 4 compares the prediction values and the measured values of these three data sets as a validation of the equation developed from dimensionless terms and regression analysis.

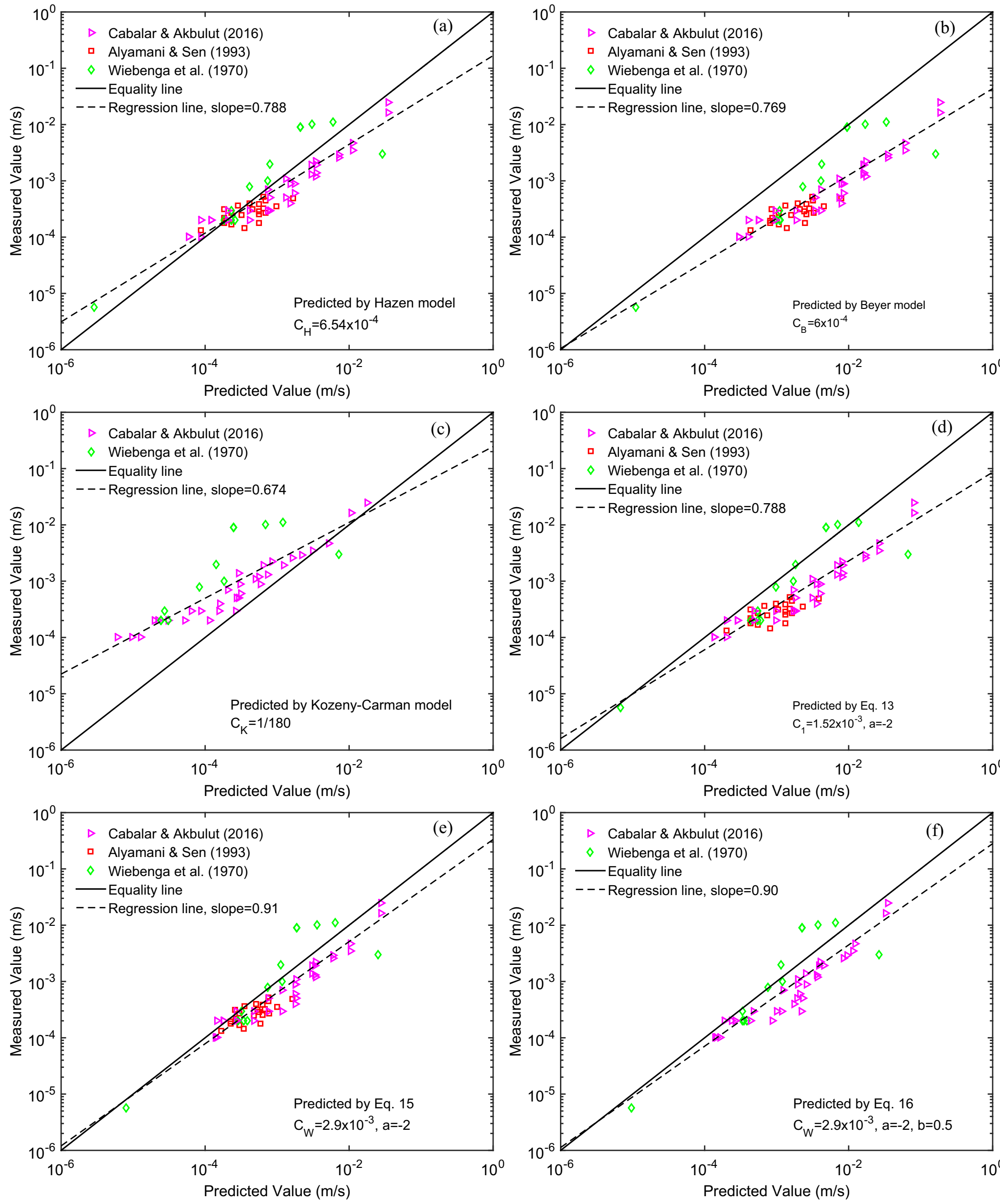

Figure 4. Prediction on experiment results of *Wiebenga et al.* [1970], *Alyamani and Sen* [1993], and *Cabalar and Akbulut* [2016]. (a) Hazen model; (b) Beyer model; (c) Kozeny-Carman model; (d) Prediction of equation (13); (e) Prediction of equation (15); (f) Prediction of equation (16).

In Figure 4a, the prediction of Hazen equation is presented. The Hazen model has a good prediction for these three data sets, and the slope of the regression line is slightly smaller than the equality line. The Beyer model in Figure 4b shows that this model overestimates the hydraulic conductivity values and the slope of the regression line is not improved. Estimation results of the Kozeny-Carman model is compared in Figure 4c. The porosity property is not available in the test results of *Alyamani and Sen* [1993] thus only the experiments of *Wiebenga et al*. [1970] and *Cabalar and Akbulut* [2016] are compared. It can be seen that the Kozeny-Carman model underestimates most of the results. The predictions by equations (13) and (15) are displayed in Figures 4d and 4e, respectively. Equation (13) is a similar type of Hazen equation with a different coefficient, and it also has a fair estimation on this two data sets. Furthermore, equation (15), in which both of the effects of particle size uniformity and gravity (proportional to mean particle volume) are considered, has the best prediction among the models. This proves the good applicability of equation (15) in different deposit materials. Figure 4f presents the prediction of equation (16) in which the effect of porosity variation around the so-called intrinsic porosity is considered. But the prediction accuracy is not obviously improved from equation (15). This means in practice equation (15) could be the best choice.

4. Conclusions

We applied dimensional analysis in the relationship between PSD and hydraulic conductivity and used a database containing 431 samples to carry out a regression analysis. The coefficient of uniformity C_u quantifying the grain size polydispersity and the dimensionless term of $\frac{gd_{60}^3}{\nu^2}$ are found to be the main controlling parameters of the estimation. After regression analysis, new equations for estimating hydraulic conductivity are developed. The classic Hazen type equation (K is propotional to d_{10}^2) is found to be in consistence with a simplified form of the regressed equation mainly considering C_u effect (equation (13)). By comparing with the prediction performances of the three classic equations in the literature based on the database of Rosas, the new equation with the effect of $\frac{gd_{60}^3}{\nu^2}$ (equation (15)) improved the prediction significantly. Moreover, by introducing the parameter of Δn, which is the difference between the measured porosity and the theoretical correlation with PSD, equation (16) further improved the prediction accuracy and reduced the variance slightly. To validate the applicability of the new model beyond the database, the new equations are employed to estimate hydraulic conductivity values of two set of experimental results in the literature. We find that equation (15), with the term $\frac{gd_{60}^3}{\nu^2}$ embedded, which is representing the dimensionless gravity effect (proportional to mean grain volume), has the best estimation accuracy. This implies the wide application prospect of the new equations proposed in this report.

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