Enhanced CUSUM control charts for monitoring Coefficient of Variation: A case study in Textile industry ⋆

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Abstract: The recent blooming developments of Artificial Intelligence (AI), Internet of Things (IoT), and Data Science (DS) have put Smart Manufacturing (SM) into a new context. This leads to more attractions on control charts as one of the useful tools that contribute to the success in SM by anomaly detection (AD) approach. Coefficient of variation (CV) is a recent popular statistic that is used in the quality control of SM. In this paper, we propose investigating the performance of Cumulative sum (CUSUM) control charts monitoring CV with a fast initial response (FIR) strategy. The chart parameters are also optimized according to the random shift size in a given interval with the proposed Nelder-Mead optimization algorithm. The numerical results show that the performance of FIR CUSUM-γ^2 charts are greater than the initial CUSUM-γ^2 ones. An example in monitoring yarn quality at the spinning mill with the design of FIR CUSUM-γ^2 charts is also proposed. These findings are useful for practitioners as well as managers and researchers. The proposed design of FIR CUSUM-γ^2 charts could be applied in other processes of various domains such as finance, business, industrial processes, etc.

Keywords: Smart Manufacturing, SPM, CUSUM control charts, Coefficient of Variation, FIR, Textile Industry, AI, IoT, DS, quality control.

1. INTRODUCTION

Together with the rapid development of Artificial Intelligence (AI), Internet of Things (IoT), and Data Science (DS), manufacturing industry has been faced with various opportunities and challenges. Manufacturing process takes advantages of these advancements that lead to boosting the productivity and efficiency. Meanwhile, manufacturing industry is also under the pressure adaptation to the new trend namely Smart Manufacturing (SM). SM is a complicated term that describes the manufacturing of the future with both objects and process integration including current and tomorrow manufacturing assets with sensors, computing platforms, communication technology, data intensive modelling, control, simulation, and predictive engineering, see Kusiak (2018). Generally, SM relates to terms such as the cyber-physical systems, IoT, cloud computing, service oriented computing, AI and, DS. Therefore, the recent blooming developments of AI, IoT, and DS have put SM into a new context, see more in Phuyal et al. (2020); Khan et al. (2020).An efficiently SM implementation is an object that have received a great attention from the science community as well as government.

Recently, Tran et al. (2022) suggested control charts as one of the useful tools that contribute to the success in SM by anomaly detection (AD) approach. The idea of this AD approach is to find the instance that does not conform to previously defined normal one, see Chandola et al. (2009). Control charts therefore can be used by detecting early the out-of-control state to guarantee the product quality in manufacturing process, see Tran (2022). Control chart is a tool of Statistical Process Monitoring (SPM) that is inspired by Shewhart (1924) from the 20th century. The practitioners monitor the process throughout a set of activities such as sampling, calculating the quality of interest, and plotting this value on the chart. A Shewhart control chart includes a central line and the control limits. The process is in-control if the sample point falls

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into the interval between the control limits, if not, it can be said out-of-control. The role of the practitioners then is to determine special causes and find the way to eliminate them. The Coefficient of Variation (CV) is a common statistical measure of dispersion that has several applications to detect changes in the manufacturing process. It is defined as a ratio of the standard deviation σ to the mean μ of a normal distribution. In textile industry, the CV represents for the variation in thickness of the yarn, see Spencer-Smith and Todd (1941). The CV of mass is calculated by the ratio of the standard deviation of mass variation to average of one. Besides, another notation is also used to determine the thickness of the yarn or variation in weight per unit length of the yarn is percentage of mean deviation (PMD). It means the average value for all the deviations from the mean, see Hasanuzzaman et al. (2015). PMD is also denoted by Unevenness% or U% that found by the Uster Company, see Hossain and Samanta (2019). Both CV and U% are used to express the irregularity of yarns in which CV has received more attention than U% from the science community, see Gauri (2005); Jambur et al. (2018). The higher CV value, the more irregular the yarn. In a spinning mill, yarn is the final product that is produced by steps as follows. According to Gauri (2005), a mass of raw cotton is gradual cleaned and drafted to a long thin strand of cotton. This procedure then twists this strand of cotton to create a yarn. It is noted that the yarn production involves the processing of raw cotton in different machines at different stages. The existence of any difference between machines could be the cause that leads to high variability in any output stage. He also mentioned that because of the series of the operations, the variation induced at any stage still effects on yarn resulting in high variability of count at the final output. The more increase in variability of textile strands, the more increase in linear density. The linear density is defined as the count of a yarn. This provides attribute information like the fineness, and governs the appearance and behavior of the various types of yarns and fabrics. When a yarn has higher variability of linear density, a larger number of thin and thick places along the length of the yarn are found. As a result, the yarn easily is breakage during the spinning and subsequent weaving/knitting operations. This leads to lower productivity as well as poorer appearance quality of the woven/knitted fabric. Moreover, uneven shades are found when this woven/knitted fabric is dyed .Thus, Gauri (2005) considered CV as a good measure of the variability of linear density because of the possibility of the comparison of different yarns and intermediate output stage without weight reference. As we can see, CV plays an important role in the control issues of yarn quality at the spinning mill; however, there is a lack of study about monitoring CV with control charts in textile industry in the literature until now.

Besides, Kang et al. (2007) has been as the pioneer in the SPM community that put attention in monitoring the CV based Shewhart control chart. Recent time have been a flourish period with numerous studies about CV based on control charts such as the exponentially weighted moving average (EWMA) (Tran et al. (2019)), the synthetic (Tran et al. (2018)), the Run Rules (Tran et al. (2021)), the variable sampling interval(Nguyen et al. (2019)), and the cumulative sum (CUSUM)(Tran and Tran (2016), Tran and Heuchenne (2021), Tran et al. (2019)).Moreover, in comparison with various numerous control charts, it is worth to highlight that the CUSUM control chart monitoring CV give a better statistical performance in detecting small shifts of the process, see in Tran and Tran (2016). Furthermore, an important assumption in these designing CUSUM charts is that the process stars from an in-control state. People, however, find it difficult in some real cases that process is not the same as proposed assumption. In order to deal this issue, an approach suggested by Lucas and Crosier (2000) namely fast initial response (FIR) strategy is used. The value of specified positive *headstart* $C_0^+ = C_0^- = c$ in the design of CUSUM with FIR strategy is set between 0 and control limit H instead $c = 0$ like in tradition CUSUM control charts. Although this approach has competitive advantages in performance, there is still lack of understand about design of CUSUM control charts with FIR strategy monitoring CV.

The purpose of this study therefore is to investigate the design of CUSUM control charts monitoring the CV with FIR strategy in order to fill these current gaps in the literature. A context of textile industry implementation is also considered. Moreover, the chart parameters are also optimized. The research outcomes could be applied not only in textile industry but also in diverse fields such as finance, business, industry processes, etc.

The rest of the paper is organized as follows. A brief introduction about the distribution of the sample CV as well as the CUSUM-γ^2 control charts is presented in Section 2 and 3, respectively. The performance evaluation and comparison between initial CUSUM-γ^2 and CUSUM-γ^2 with FIR strategy control charts are proposed in Section 4. Section 5 provides an illustrative example in a spinning mill of the use of the proposed charts. Some suggestions and remarks are given in Section 6.

2. THE DISTRIBUTION OF THE SAMPLE CV SQUARED

The distribution of the sample CV squared is briefly described in this section.

Let us consider X as a positive random variable with the mean μ and the standard deviation σ are defined as follows, respectively: $\mu = \mathrm{E(X)}$, $\sigma = \mathrm{V(X)}$. The CV of the random variable X is defined as the ratio of the σ to the μ

$$\gamma = \frac{\sigma}{\mu}.$$

Suppose that observed subgroups $\{X_1, \ldots, X_n\}$ are n independently and identically distributed normal (i.i.d.) random variables with the mean μ and standard deviation σ. Let $\bar{X}$ and S be the sample mean and the sample standard deviation of $X_1, \ldots, X_n$. They are defined as follows

$$\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i,$$

and

$$S = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2}.$$

The sample CV $\hat{\gamma}$ then is

$$\hat{\gamma} = \frac{S}{\bar{X}}.$$

The exact following $c.d.f$ (cumulative distribution function) of the sample CV squared $\hat{\gamma}^2$ is:

$$F_{\hat{\gamma}^2}(x|n,\hat{\gamma}) = 1 - F_F\left(\frac{n}{x}\middle| 1, n-1, \frac{n}{\gamma^2}\right), \tag{1}$$

where $F_F\left(.\middle|1, n-1, \frac{n}{\gamma^2}\right)$ is the $c.d.f.$ of the noncentral F distribution. The $p.d.f$ (probability density function) of the sample CV squared $\hat{\gamma}^2$ is

$$f_{\hat{\gamma}^2}(x|n,\hat{\gamma}) = \frac{n}{x^2} f_F\left(\frac{n}{x}\middle| 1, n-1, \frac{n}{\gamma^2}\right), \tag{2}$$

where $f_F\left(.\middle|1, n-1, \frac{n}{\gamma^2}\right)$ is the $p.d.f$ of the noncentral F distribution, see more in the studies of Tran et al. (2019) and Tran et al. (2021).

3. CUSUM-γ^2 CONTROL CHARTS

Let us assume that $\{X_{i,1}, X_{i,2}, \ldots, X_{i,n}\}$ at time $i = 1, 2, \ldots$ is an observed subgroup of n i.i.d. random variables, in which each random variable $X_{i,j}$ follows a normal distribution with parameters (μ_i, σ_i). The CV of in-control process is denoted by γ_0 with the constraint relation $\gamma_i = \frac{\sigma_i}{\mu_i} = \gamma_0$. This leads an important implication that each subgroup might has diverse μ_i and σ_i values, but it's CV $\gamma_i = \frac{\sigma_i}{\mu_i}$ remains constant, i.e. equal to some predefined in-control values γ_0, see Tran et al. (2019). Althought there are no closed-form of the in-control mean μ_0 and standard deviation σ_0 of the sample CV squared $\hat{\gamma}^2$, the accurate approximations of these parameters are provided by Breunig (2001).

According to Tran and Tran (2016), the following two separate one-sided CUSUM charts are used for monitoring CV squared:

- An increase in the CV is detected by an upward CUSUM chart as follows:

  $$C_i^+ = \max(0, C_{i-1}^+ + (\hat{\gamma}_i^2 - \mu_0(\hat{\gamma}^2)) - K^+) \tag{3}$$

  with the initial value $C_0^+ = 0$ and the corresponding upper control limit $UCL^+ = H_U \times \mu_0(\hat{\gamma}^2) > 0$,

- A decrease in the CV is detected by a downward CUSUM as follows:

  $$C_i^- = \max(0, C_{i-1}^- - (\hat{\gamma}_i^2 - \mu_0(\hat{\gamma}^2)) - K^-) \tag{4}$$

  with the initial value $C_0^- = 0$ and the corresponding lower control limit $UCL^- = H_D \times \mu_0(\hat{\gamma}^2) > 0$.

where $K^+ = K_U \times \sigma_0(\hat{\gamma}^2)$ and $K^- = K_D \times \sigma_0(\hat{\gamma}^2)$ are the reference charts'parameters, in which K_U and K_D are the charts parameters needed to be found.

The performance of the CUSUM-γ^2 charts is evaluated through average run length (ARL), see in study of Tran and Tran (2016). This statistic is considered as the average number of samples before the first out-of-control point is plotted in the control chart. In order to calculate the ARL, a method suggested by Tran et al. (2019) is to use the Markov chain. It is noted that the first sub-interval is $\delta = \frac{H^+}{2p-1}(\delta = \frac{H^-}{2p-1})$ in width while the others are 2δ in width. Figure 1 shows these subdivisions for the upward chart.

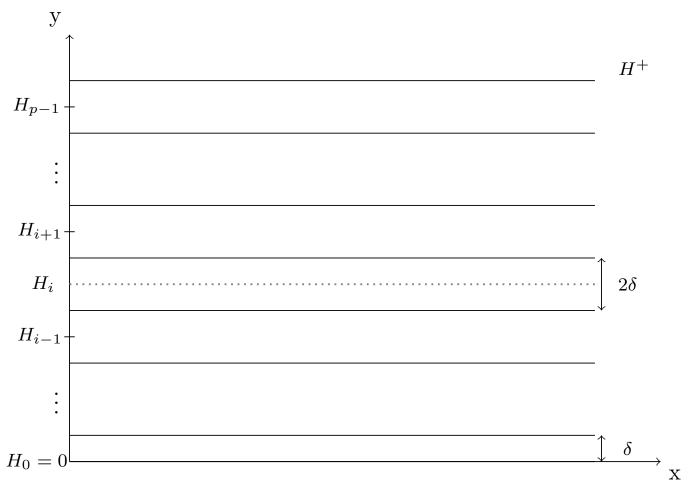

Fig. 1. Division of control limit interval for upward chart in the Markov chain approach

Each transient state of the Markov chain is figured by each sub-interval $(H_j - \delta, H_j + \delta]$, where the midpoint of the sub-interval j is denoted by H_j with j=0,..,p-1, see more Tran and Tran (2016).

Second, the discrete-time Markov chain has the transition probability matrix as follows $\mathbf{P}$

$$\mathbf{P} = \begin{pmatrix} \mathbf{Q} & \mathbf{r} \\ \mathbf{0}^T & 1 \end{pmatrix} = \begin{pmatrix} Q_{0,0} & Q_{0,1} & \ldots & Q_{0,p-1} & r_0 \\ Q_{1,0} & Q_{1,1} & \cdots & Q_{1,p-1} & r_1 \\ \vdots & \vdots & & & \vdots \\ Q_{p-1,0} & Q_{p-1,1} & \ldots & Q_{p-1,p-1} & r_{p-1} \\ 0 & 0 & \cdots & 0 & 1 \end{pmatrix},$$

where $\mathbf{Q}$ is the (p,p) matrix of transient probabilities, $\mathbf{0} = (0, 0, \ldots, 0)^T$ and $\mathbf{r}$ is p-vector satisfying $\mathbf{r} = (1 - \mathbf{Q1})$ with $\mathbf{1} = (\mathbf{1}, 1, \ldots, 1)^T$.

The matrix $\mathbf{Q}$ of the upward and the downward charts has the elements $Q_{i,j}$ which are calculated by the formulas respectively as follows:

- The upward chart has:

$$Q_{i,0} = F_{\hat{\gamma}^2}\left(\mu_0(\hat{\gamma}^2) - H_i + K^+ + \delta\middle| n, \gamma_1\right) \text{ and} \tag{5}$$

$$\begin{aligned} Q_{i,j} = {} & F_{\hat{\gamma}^2}\left(\mu_0(\hat{\gamma}^2) + H_j - H_i + \delta + K^+\middle| n, \gamma_1\right) \\ & - F_{\hat{\gamma}^2}\left(\mu_0(\hat{\gamma}^2) + H_j - H_i - \delta + K^+\middle| n, \gamma_1\right); \end{aligned} \tag{6}$$

- The downward chart has:

$$Q_{i,0} = 1 - F_{\hat{\gamma}^2}\left(\mu_0(\hat{\gamma}^2) + H_i - K^- - \delta\middle| n, \gamma_1\right) \text{ and} \tag{7}$$

$$\begin{aligned} Q_{i,j} = {} & F_{\hat{\gamma}^2}\left(\mu_0(\hat{\gamma}^2) + H_j - H_i + \delta + K^-\middle| n, \gamma_1\right) \\ & - F_{\hat{\gamma}^2}\left(\mu_0(\hat{\gamma}^2) + H_j - H_i - \delta + K^-\middle| n, \gamma_1\right), \end{aligned} \tag{8}$$

where $F_{\hat{\gamma}^2}(.)$ is the $c.d.f.$ of $\hat{\gamma}^2$ in (1).

Next, let $\mathbf{q}$ be the $(p-1, 1)$ vector of initial probabilities associated with the p transient states, $\mathbf{q} = (q_0, q_1, \ldots, q_{p-1})^T$. Regarding the zero-state ARL of performance, it can be seen the "restart state" corresponds to the initial state with $\mathbf{q} = (1, 0, \ldots, 0)$.

The ARL value of the CUSUM-γ^2 control chart is computed by the formula as follows

$$ARL = \mathbf{q}^T(\mathbf{I} - \mathbf{Q})^{-1}\mathbf{1}. \tag{9}$$

Finally, a CUSUM-γ^2 control chart is defined by the upward and downward CUSUM-γ^2 chart coefficients, denoted by the parameters (K^+, H^+) and (K^-, H^-), respectively. In order to evaluate performance of the charts, the ARL measure, can be numerically calculated for a particular shift size τ is used. If the values $\tau \in (0, 1)$, it can be said that is a decrease of the nominal CV while $\tau > 1$, that is an increase of the nominal CV.

In general, the design of the CUSUM-γ^2 charts is implemented by finding out the optimal couples (K^+, H^+) or (K^-, H^-) that minimize the out-of-control ARL_1 for a given in-control value ARL_0 with Nelder Mead method where the ARL value is calculated based on the formula (9) with the *c.d.f.* of $\hat{\gamma}^2$ is defined in (1). This procedure includes two main steps:

(i) The potential combinations (K^+, H^+) or (K^-, H^-) would to be defined in case $ARL = ARL_0$, where ARL_0 is a predefined in-control ARL value.
(ii) An optimal combination chosen among these ones which is said (K^{*+}, H^{*+}) or (K^{*-}, H^{*-}), that gives the best performance, i.e. the smallest out-of-control ARL value for a particular shift τ, from an in-control value γ_0 to an out-of-control value $\gamma_1 = \tau\gamma_0$.

It is noted in the study of Tran and Tran (2016) that the process starts from an in-control state. Thus, the value is set by $C_0^+ = C_0^- = 0$ for the design of the CUSUM-γ^2 control charts. In fact, however, this assumption is not appropriate in some practice cases. The out-of-control state may be appear when the process starts or is restarted after an adjustment. Therefore, Lucas and Crosier (2000) suggested a fast initial response (FIR) strategy that can be used in such special situations. Following this method, the design of the FIR CUSUM chart is still the same as the design of the standard CUSUM chart but the process is modified by initially setting a CUSUM to a specified positive *headstart* $C_0^+ = C_0^- = c$ with the value of $c \in (0, H)$ in stead of $c = 0$. A significant advantage of the FIR strategy is that the CUSUM chart performance is improved when the process starts from an out-of-control state but costs only a small penalty when it starts in control. See more discussion about FIR features in the studies of Sanusi et al. (2017) and Hawkins and Olwell (1998).

4. PERFORMANCE EVALUATION AND COMPARISON

In the current study, the FIR CUSUM-γ^2 control chart is applied with the values of k^{*+} and k^{*-} following the study of Tran and Tran (2016). Table 1 and Table 2 present the value of reference coefficients $h^{*+}(h^{*-})$ and the corresponding ARL_1 of the FIR CUSUM-γ^2, respectively. A various of situations of the headstart as $c \in \{0, 0.25H, 0.5H, 0.75H\}$ is considered. The performance of the CUSUM-γ^2 control chart is improved when using FIR

Table 1. The chart parameters h^{*+} when $\tau = \{0.5, 0.7, 0.8\}$, h^{*-} when $\tau = \{1.3, 1.5, 2.0\}$ of the CUSUM-γ^2 charts for $n = \{5, 7; 10, 15\}$, $\gamma_0 = \{0.05, 0.1, 0.2\}$, and different values of the headstart c.

	$\gamma_0 = 0.05$					$\gamma_0 = 0.1$					$\gamma_0 = 0.2$				
τ	$C_0 = 0$	$C_0 = 0.1H$	$C_0 = 0.25H$	$C_0 = 0.5H$	$C_0 = 0.75H$	$C_0 = 0$	$C_0 = 0.1H$	$C_0 = 0.25H$	$C_0 = 0.5H$	$C_0 = 0.75H$	$C_0 = 0$	$C_0 = 0.1H$	$C_0 = 0.25H$	$C_0 = 0.5H$	$C_0 = 0.75H$
								n = 5							
0.5	0.8105	0.8108	0.8119	0.8168	0.8351	0.7915	0.7918	0.7929	0.7975	0.8149	0.8127	0.8130	0.8141	0.8188	0.8369
0.7	1.6283	1.6291	1.6323	1.6474	1.6994	1.6444	1.6452	1.6484	1.6635	1.7161	1.7126	1.7135	1.7168	1.7325	1.7874
0.8	3.3713	3.3745	3.3867	3.4426	3.6154	3.3938	3.3970	3.4092	3.4648	3.6381	3.5586	3.5619	3.5747	3.6330	3.8153
1.3	5.9685	5.9728	5.9924	6.0875	6.3666	4.9781	4.9781	4.9781	4.9781	4.9781	5.6251	5.6251	5.6251	5.6251	5.6251
1.5	4.2236	4.2254	4.2321	4.2644	4.3623	3.6291	3.6291	3.6291	3.6291	3.6291	4.1954	4.1954	4.1954	4.1954	4.1954
2.0	3.0593	3.0602	3.0632	3.0750	3.1082	2.6300	2.6300	2.6300	2.6300	2.6300	3.1351	3.1351	3.1351	3.1351	3.1351
								n = 7							
0.5	0.5307	0.5309	0.5316	0.5351	0.5429	0.5274	0.5275	0.5282	0.5315	0.5392	0.5421	0.5423	0.5429	0.5463	0.5542
0.7	1.1324	1.1329	1.1348	1.1434	1.1740	1.1400	1.1406	1.1424	1.1509	1.1819	1.1631	1.1636	1.1655	1.1740	1.2056
0.8	2.3581	2.3599	2.3667	2.3984	2.4982	2.3851	2.3868	2.3937	2.4255	2.5256	2.5021	2.5040	2.5112	2.5449	2.6516
1.3	4.0796	4.0821	4.0925	4.1432	4.2962	3.5308	3.5308	3.5308	3.5308	3.5308	3.9375	3.9375	3.9375	3.9375	3.9375
1.5	2.8258	2.8269	2.8306	2.8478	2.8992	2.4893	2.4893	2.4893	2.4893	2.4893	2.8450	2.8450	2.8450	2.8450	2.8450
2.0	1.9521	1.9526	1.9542	1.9602	1.9754	1.7075	1.7075	1.7075	1.7075	1.7075	2.0195	2.0195	2.0195	2.0195	2.0195
								n = 10							
0.5	0.5557	0.5562	0.5568	0.5606	0.5687	0.3581	0.3582	0.3586	0.3602	0.3633	0.3554	0.3555	0.3559	0.3573	0.3600
0.7	0.7932	0.7935	0.7949	0.8002	0.8157	0.7884	0.7888	0.7899	0.7950	0.8108	0.7917	0.7921	0.7931	0.7982	0.8141
0.8	1.7024	1.7035	1.7077	1.7273	1.7884	1.6708	1.6719	1.6758	1.6942	1.7541	1.7415	1.7426	1.7467	1.7657	1.8295
1.3	2.7817	2.7831	2.7888	2.8161	2.9000	2.4735	2.4735	2.4735	2.4735	2.4735	2.7378	2.7378	2.7378	2.7378	2.7378
1.5	1.8720	1.8726	1.8747	1.8837	1.9098	1.6786	1.6786	1.6786	1.6786	1.6786	1.9056	1.9056	1.9056	1.9056	1.9056
2.0	1.1979	1.1982	1.1991	1.2019	1.2082	1.0606	1.0606	1.0606	1.0606	1.0606	1.2547	1.2547	1.2547	1.2547	1.2547
								n = 15							
0.5	0.6833	0.6840	0.6849	0.6900	0.7029	0.3121	0.3121	0.3124	0.3136	0.3159	0.2973	0.2974	0.2977	0.2986	0.3003
0.7	0.6833	0.6840	0.6849	0.6900	0.7029	0.6981	0.6985	0.6994	0.7042	0.7179	0.5143	0.5145	0.5151	0.5183	0.5265
0.8	1.1938	1.1945	1.1971	1.2091	1.2442	1.1810	1.1817	1.1841	1.1952	1.2311	1.1855	1.1861	1.1884	1.1992	1.2354
1.3	1.8224	1.8232	1.8262	1.8402	1.8832	1.6566	1.6566	1.6566	1.6566	1.6566	1.8239	1.8239	1.8239	1.8239	1.8239
1.5	1.1747	1.1750	1.1761	1.1805	1.1921	1.0689	1.0689	1.0689	1.0689	1.0689	1.2060	1.2060	1.2060	1.2060	1.2060
2.0	0.6405	0.6406	0.6410	0.6421	0.6441	0.5695	0.5695	0.5695	0.5695	0.5695	0.6898	0.6898	0.6898	0.6898	0.6898

Table 2. The ARL_1 values of the CUSUM-γ^2 control charts for $n = \{5, 7; 10, 15\}$, $\gamma_0 = \{0.05, 0.1, 0.2\}$, and different values of the headstart c.

	$\gamma_0 = 0.05$					$\gamma_0 = 0.1$					$\gamma_0 = 0.2$				
τ	$C_0 = 0$	$C_0 = 0.1H$	$C_0 = 0.25H$	$C_0 = 0.5H$	$C_0 = 0.75H$	$C_0 = 0$	$C_0 = 0.1H$	$C_0 = 0.25H$	$C_0 = 0.5H$	$C_0 = 0.75H$	$C_0 = 0$	$C_0 = 0.1H$	$C_0 = 0.25H$	$C_0 = 0.5H$	$C_0 = 0.75H$
								n = 5							
0.5	4.3079	4.0416	3.5052	2.6804	1.6680	4.3003	4.0345	3.5043	2.6776	1.6772	4.3456	4.0784	3.5421	2.7091	1.6926
0.7	8.1989	7.7462	6.7763	4.9751	3.0666	8.2106	7.7572	6.7855	4.9817	3.0706	8.3359	7.8771	6.8912	5.0604	3.1204
0.8	19.962	19.085	16.928	12.498	7.5627	20.025	19.149	16.992	12.555	7.6025	20.397	19.510	17.326	12.821	7.7683
1.3	18.091	17.552	15.973	12.336	8.1670	14.795	14.362	13.149	10.267	6.8895	15.351	14.909	13.641	10.637	7.1267
1.5	6.4474	6.2173	5.6783	4.5463	3.3161	5.6136	5.4229	4.9876	4.0679	3.0505	5.8872	5.6906	5.2320	4.2526	3.1651
2.0	2.5488	2.4682	2.3062	2.0105	1.7044	2.3424	2.2760	2.1435	1.9036	1.6523	2.4857	2.4145	2.2698	2.0000	1.7122
								n = 7							
0.5	3.1006	2.9333	2.6218	1.9139	1.3787	3.0740	2.9104	2.6007	1.8967	1.3736	3.1024	2.9353	2.6262	1.9143	1.3806
0.7	5.9432	5.6091	4.9077	3.6746	2.2885	5.9321	5.5986	4.8980	3.6680	2.2830	6.0297	5.6933	4.9851	3.7350	2.3244
0.8	14.800	14.118	12.500	9.2497	5.6957	14.834	14.149	12.527	9.2676	5.7046	15.155	14.457	12.803	9.4744	5.8323
1.3	12.956	12.519	11.359	8.7826	5.8859	11.088	10.720	9.7721	7.6264	5.1789	11.576	11.198	10.204	7.9513	5.3834
1.5	4.6381	4.4592	4.0692	3.3018	2.4934	4.1667	4.0136	3.6849	3.0400	2.3533	4.3996	4.2397	3.8906	3.1942	2.4474
2.0	1.9006	1.8462	1.7418	1.5653	1.3949	1.7959	1.7503	1.6631	1.5161	1.3734	1.9117	1.8608	1.7618	1.5907	1.4206
								n = 10							
0.5	2.3814	2.2766	2.0160	1.4281	1.1129	2.3258	2.1787	1.8805	1.4515	1.1937	2.2976	2.1498	1.8563	1.4419	1.1935
0.7	4.3608	4.1065	3.6087	2.7167	1.7882	4.2995	4.0487	3.5620	2.6781	1.7704	4.3602	4.1081	3.6183	2.7220	1.8019
0.8	10.941	10.403	9.1791	6.7932	4.2074	10.919	10.388	9.1791	6.8088	4.2264	11.171	10.630	9.3950	6.9681	4.3246
1.3	9.3038	8.9547	8.1020	6.2910	4.2987	8.2461	7.9417	7.2120	5.6479	3.9130	8.6491	8.3337	7.5660	5.9111	4.0739
1.5	3.3568	3.2225	2.9461	2.4387	1.9285	3.0931	2.9758	2.7365	2.2998	1.8590	3.2794	3.1550	2.8977	2.4194	1.9299
2.0	1.4621	1.4305	1.3725	1.2809	1.1987	1.4156	1.3889	1.3399	1.2623	1.1919	1.4995	1.4678	1.4087	1.3133	1.2251
								n = 15							
0.5	2.0939	2.0444	1.8546	1.1956	1.0153	1.6885	1.5623	1.3555	1.1366	1.0434	1.6301	1.5138	1.3278	1.1318	1.0451
0.7	3.2237	3.0295	2.6799	1.9900	1.3743	3.1449	2.9562	2.6169	1.9369	1.3432	3.1235	2.9569	2.6047	1.9594	1.4511
0.8	7.9487	7.5346	6.6324	4.9397	3.0974	7.8287	7.4208	6.5330	4.8698	3.0579	7.9730	7.5646	6.6716	4.9862	3.1368
1.3	6.5050	6.2372	5.6301	4.4103	3.1105	5.9320	5.6919	5.1539	4.0703	2.9129	6.2438	5.9932	5.4248	4.2705	3.0318
1.5	2.3839	2.2914	2.1113	1.8054	1.5195	2.2475	2.1658	2.0075	1.7401	1.4897	2.3860	2.2975	2.1240	1.8251	1.5403
2.0	1.1719	1.1596	1.1379	1.1053	1.0776	1.1583	1.1479	1.1293	1.1013	1.0771	1.2069	1.1928	1.1677	1.1292	1.0959

Phase II

k	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}	S_i	$\bar{X}_i$	$\hat{\gamma}_i^2$	C_i^+
1	9.59	10.21	10.23	9.58	9.76	9.69	10.46	9.21	10.26	10.08	0.3969	9.9071	0.0016	0.008985
2	9.48	10.71	10.04	9.99	10.54	10.73	10.00	11.16	9.76	10.52	0.5191	10.2918	0.0025	0.008902
3	20.49	18.92	20.11	20.01	20.86	20.71	18.94	19.28	20.72	20.86	0.7827	20.0901	0.0015	0.007799
4	20.48	20.92	19.60	21.20	20.77	21.83	20.70	19.33	20.78	20.17	0.7336	20.5774	0.0013	0.006443
5	18.83	19.91	18.87	20.90	21.03	20.19	19.59	20.38	20.97	19.24	0.8427	19.9919	0.0018	0.005601
6	19.62	19.34	21.17	17.87	20.66	19.77	18.81	19.21	18.92	20.57	0.9929	19.5939	0.0026	0.005548
7	21.54	21.21	20.55	20.26	20.49	19.74	20.69	18.63	19.36	22.05	1.0235	20.4530	0.0025	0.005425
8	10.66	9.58	10.06	8.87	10.44	10.12	9.53	9.20	9.92	9.68	0.5477	9.8067	0.0031	0.005915
9	21.04	18.78	19.62	20.95	19.49	19.42	20.65	21.02	17.97	18.45	1.1313	19.7405	0.0033	0.006575
10	11.43	9.92	10.65	8.71	9.31	9.72	10.02	9.89	10.75	9.59	0.7777	9.9995	0.0061	0.010005
11	10.27	9.93	11.06	9.78	9.21	10.42	9.46	10.77	8.42	9.86	0.7710	9.9198	0.0060	0.013419
12	9.38	9.43	9.04	10.30	8.21	10.37	10.18	9.96	9.83	10.18	0.6834	9.6886	0.0050	0.015766
13	10.41	10.71	9.84	10.15	10.33	8.64	10.44	10.58	9.80	10.92	0.6457	10.1819	0.0040	0.017162
14	19.89	20.20	18.01	19.19	21.04	18.66	19.93	20.27	18.79	18.17	0.9987	19.4151	0.0026	0.017181
15	22.51	19.60	21.26	19.50	18.52	18.54	18.30	20.60	21.52	21.07	1.4562	20.1424	0.0052	0.019785
16	22.04	18.73	18.94	18.67	20.76	20.55	20.15	20.20	20.21	21.68	1.1578	20.1939	0.0033	0.020445
17	9.55	9.65	11.64	9.92	10.53	10.42	7.91	11.50	10.22	9.86	1.0550	10.1210	0.0109	0.028679
18	10.33	9.14	9.66	11.50	11.03	8.31	10.13	12.04	9.66	9.09	1.1648	10.0898	0.0133	0.039373
19	19.54	17.58	19.84	22.34	19.89	20.78	22.59	22.12	20.30	17.13	1.8635	20.2101	0.0085	0.045251
20	20.54	21.20	19.48	19.74	20.83	20.55	21.47	17.47	20.26	18.54	1.2397	20.0065	0.0038	0.046471

Table 3. Illustrative example for dataset used for Phase II.

strategy. The higher $C_0 = c$, the lower ARL_1 is showed through the numerical results as follows. According to table 2, when $n = 5, \gamma_0 = 0.1, \tau = 0.8$, $ARL_1 = 20.025$ with $C_0^- = 0$ and $ARL_1 = 7.6025$ with $C_0^- = 0.75H^-$. Furthermore, the value of $h^{*+}(h^{*-})$ corresponding to the control limit $H^{*+}(H^{*-})$ increases when c increases. Table 1 presents the optimal values of reference coefficient are $h^{*+} = 4.0796$ and $h^{*+} = 4.2962$ when $C_0 = 0$ and $C_0 = 0.75H^+$ respectively with $n = 7, \gamma_0 = 0.05, \tau = 1.3$.As a result, a head start routinely is highly recommend in order to obtain better performance in designing a CUSUM control chart monitoring the CV.

5. APPLICATION IN REAL INDUSTRIAL SCENARIO

In this section, we discuss the implementation of an upward CUSUM-γ^2 control chart with FIR strategy in textile industry. The context of this study is on the quality monitoring of yarns at the spinning mill. Quality control is carried out on a production line with outputs of two yarns of different diameters. The quality practitioner decided to monitor irregularities in textile yarn by using the CV. According to Gauri (2005), an increase in CV would lead to a high possibility of yarn breakage during weaving. This can say that it is worth to detect early an increase in CV with the upward FIR CUSUM-γ^2 control chart. The data of this case study is simulated with the real context discussed above. Assume that, from phase I data, $\hat{\gamma_0}$ is estimated, and is equal to 0.05. Let us consider an experiment for Phase II. We have sample size $n = 10$ yarns with length 25 cm every 30 minutes. The yarns are then weighed (in mg) and recorded in the data sheet. Table 3 shows a set of simulated samples collected from the process, the corresponding 2 different sizes of yarns and the $\hat{\gamma}_i^2$, C_i^+ statistics. In fact, according to the process engineers, a shift of 10% in the CV should be interpreted as a signal that something is going wrong in the production of yarns. For the upward FIR CUSUM-γ^2 chart the parameters ($UCL^+ = 0.012921$, $K = 0.1058298$, $C_0 = 0.01$) can be obtained numerically. We assume that the process is to run in-control up to sample 10th. Next, we simulated the occurrence of an assignable cause shifting γ_0 to $\gamma_1 = 1.1 \times \gamma_0$ between samples 10th and 11th, i.e. a CV percentage increase equal to 10%. The Figure 2 showed that the upward CUSUM-γ^2 control chart signals the occurrence of the out-of-control signal at sample 11th. This confirms the greater effectiveness of the proposed CUSUM -γ^2 FIR charts in comparison with tradition CUSUM-γ^2 control charts in detecting process anomalies.

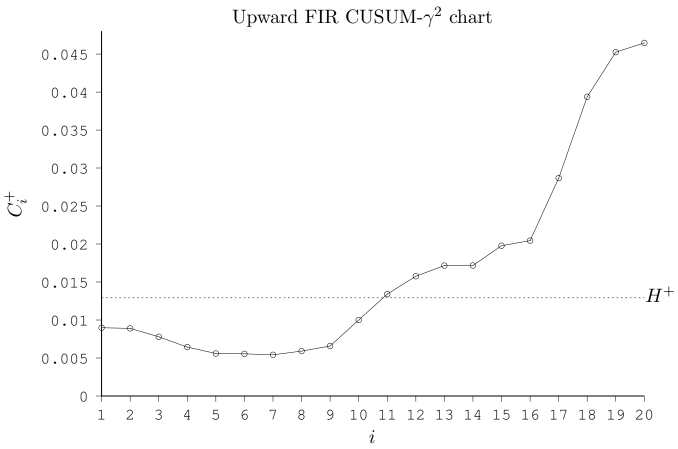

Fig. 2. FIR CUSUM-γ^2 charts applied to the textile process (Phase II)

6. CONCLUDING REMARKS

Designs of new CUSUM control charts with FIR strategy for monitoring the CV are developed in this paper. Moreover, the chart parameters are also optimized according to the random shift size in a given interval with the proposed optimization algorithm. The performance of FIR CUSUM-γ^2 charts are greater than initial CUSUM-γ^2 ones. These findings are useful for practitioners, as well as managers and researchers. An experiment in monitoring yarn quality at the spinning mill with design of FIR CUSUM-γ^2 charts also proposed that are the first contributions in the literature. The research outcomes could be applied in different areas such as finance, business, industry processes, etc.

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