DETERMINATION OF THE SPIN VECTOR OF A SYNTHETIC ASTEROID

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ABSTRACT

In order to test various methods for determining asteroid poles, a set of artificial lightcurves were computed using the program described in Karttunen (1989). The lightcurve and ephemeris data (but no information concerning the pole, shape or albedo distribution) were sent to those interested in applying their methods to determine the pole of this model asteroid.

THE DATA

The fictitious asteroid had the following orbital elements:

$$a = 2.80\mathrm{AU}, \quad e = 0, \quad i = 15°, \quad \Omega = 0°.$$

The object was a triaxial ellipsoid with axes 1.5, 1.0, and 0.8. The Lumme-Bowell scattering law was used with $\varpi_0 = 0.5$, except in one octant, where $\varpi_0 = 0.25$.

The data consisted of fifteen lightcurves, seven of which were "observed" during one apparition, phase angle ranging from 0° to 17.3°. The other eight lightcurves were from eight successive apparitions, giving a reasonably good coverage in longitude.

The following table shows the phase angle α, geocentric longitude λ and latitude β, and the three lowest Fourier amplitudes A_0, A_1 and A_2 of these lightcurves.

α	λ	β	A_0	A_1	A_2
0.0	0.0	0.0	20.7	0.31	0.99
1.8	-1.0	0.4	17.9	0.24	0.95
5.4	-2.8	1.0	14.9	0.18	0.88
10.2	-5.1	2.0	12.7	0.13	0.80
13.0	-6.2	2.6	11.9	0.12	0.76
15.4	-6.9	3.1	11.3	0.10	0.72
17.3	-7.2	3.6	10.9	0.09	0.68
8.0	98.1	22.8	10.9	0.47	1.71
2.2	-165.4	-6.1	13.8	1.52	0.40
7.4	-65.8	-21.2	8.4	1.62	2.09
4.2	29.4	11.8	17.9	0.02	0.12
6.4	130.0	18.0	8.9	1.26	2.37
5.9	-135.4	-16.6	11.9	1.07	0.18
4.8	-34.7	-13.6	10.7	0.83	2.25
7.1	60.2	20.2	15.1	0.08	0.54

THE POLE SOLUTIONS

The actual pole direction was $\lambda_0 = 30°$, $\beta_0 = 30°$.

1) Spherical harmonics method (Karttunen)

This method is described in Lumme *et al.* (1989). In the first iteration the pole was totally unknown, and no aspect correction was applied. Two symmetric solutions were found: $\lambda_0 = 30°$, $\beta_0 = 24°$, and $\lambda_0 = 210°$, $\beta_0 = -24°$. A second iteration was also carried out, using the first pole to correct for the aspect. This yielded the poles $\lambda_0 = 31°$, $\beta_0 = 26°$, and $\lambda_0 = 211°$, $\beta_0 = -26°$. With this method it is not possible to determine the sense of rotation. As expected with this method, the latitude was a little too small.

2) Free albedo method (Hainaut, Detal, Surdej)

The Free Albedo Map Method (FAMM) makes use of all the information contained in an asteroid lightcurve, i.e. the magnitude and the epoch of each data point.

The asteroid is modelled as a sphere covered with many surface elements (typically 32 or 128). For a given pole orientation, the albedos of the elements are adjusted until the best match is obtained between the model and the observed lightcurves. The final residual is defined as

$$R_{\lambda,\beta} = \sum_{k=1}^{N} (I_{k,obs} - I_{k,model})^2,$$

where $I = 10^{(M/2.5)}$ are the intensities and N the total number of data points contained in the lightcurves. This residual constitutes a good indicator of the quality of the trial pole.

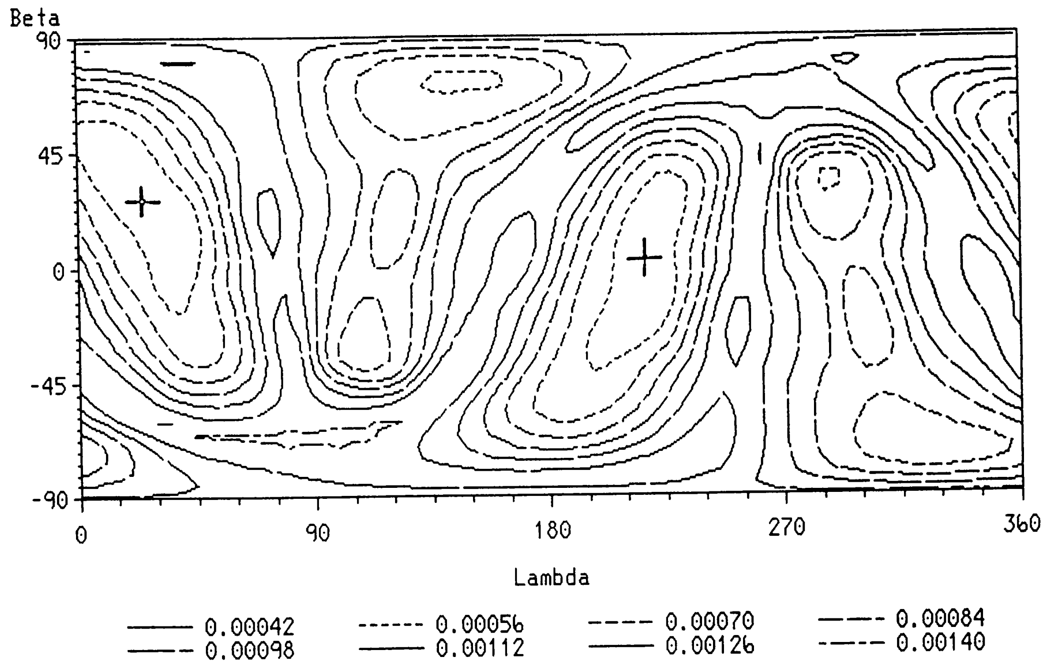
To find the pole, the entire celestial sphere is scanned, and for each trial pole, an albedo map is derived. The residuals are then plotted as a level-contour map, and the best poles are associated with the lowest points.

It is important to realise that the obtained albedo map has abolutely no physical meaning, not only because most of the magnitude variations are caused by shape effects (Barucci *et al.*, 1989), but also because there is an infinity of possible albedo distributions which reproduce exactly the same lightcurves. However, the pole solution is unique (Russell, 1906).

Since all the lightcurves produced by any convex asteroid, without any restriction on the albedo features, can be reproduced by a sphere covered with an adequate albedo distribution, FAMM can be used on a large variety of asteroids, without any model dependence problem.

The most probable pole is $\lambda_0 = 25°$, $\beta_0 = 27°$, and the secondary pole $\lambda_0 = 216°$, $\beta_0 = 4°$. The estimated errors on the pole coordinates are of the order of 10°. Even if it has no meaning for this particular lightcurve set, both poles seemed to have retrograde spins.

The following figure represents the total residual between observed and model lightcurves as a function of the coordinates of the trial pole. The solutions (+) are associated with the lowest points in this graph.



3) Amplitude–Magnitude method (Zappalà, Cellino)

The table below gives the solutions obtained by the following different methods:
(1) Maximum amplitude from primary minima.
(2) Maximum amplitude from secondary minima.
(3) Secondary amplitudes corrected for albedo effects.
(4) Determination of the shape and albedo variegation; computation of the aspect angles comparing observed and computed maximum amplitudes; and determination of the pole.

method	λ_0	β_0	λ_0	β_0
(1)	220	17	16	40
(2)	226	18	20	43
(3)	220	9	24	33
(4)	220	12	19	40

4) SAM, SPA and WAA methods (Drummond)

These methods have been described in Drummond *et al.* (1988). Without selecting subsets of the data the SAM method indicates one axis, but with no clear indication from SPA whether the prograde end or the retrograde end is the correct pole. The average of the SPA, WAA, and SAM methods gives the following possible poles: $\lambda_0 = 18°$, $\beta_0 = 60°$ with a 19° radius error, or $\lambda_0 = 172°$, $\beta_0 = -50°$ with a 17° error. Thus it is not possible to distinguish between the prograde and retrograde poles. Furthermore, the phase and theta (sub-latitude) plots are so ugly with these poles that one must revert to the orginal SAM axis for further modeling.

For the SAM pole at $\lambda_0 = 197°$, $\beta_0 = -42°$ one finds a triaxial ellipsoid shape with $a/b = 1.83(\pm 0.11)$ and $b/c = 1.66 \pm 0.04)$, with $H(X) = -3.02(\pm 0.01)$, $G(X) = 0.36(\pm 0.02)$, and $G(N) = 0.35(\pm 0.06)$. In this case there seems to be two 'spots', one at asterocentric longitude 180° (corresponding to one minimum in the lightcurve) and latitude −15°, with a radius of about 25° and an albedo ratio of two with respect to the unspotted region. The other spot is a dark south polar cap with a radius of 60° and an albedo ratio of 0, or completely dark. For the other end of this axis, at $\lambda_0 = 17°$, $\beta_0 = +42°$ the signs of the latitudes of the spots would be reversed. This spot model is not unique, of course, and is in fact only moderately succesful in replicating the major features of the lightcurves. The locations, size, and albedo ratios are therefore only approximate.

SUMMARY

The model had a large albedo feature, and the resulting high first harmonic caused problems in applying some methods. We intend to make some further tests with homogeneous objects.

Although there is a lot of scatter in the results, several methods seem to give reasonable good estimates for the pole. One obvious problem is how to select the correct pole from the numerous candidates.

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