SENSITIVITY ANALYSIS
ON ULTIMATE HULL BENDING MOMENT

Ph. RIGO 1, C. TODERAN 2 and T. YAO 3

1 Department of Naval Architecture (ANAST), University of Liege
Chemin des Chevreuil n°1 (B52/7), B-4000 Liege, Belgium
2 National Fund for Scientific Research of Belgium – FNRS
3 Dept. of Naval Architecture and Ocean Engineering (NAOE), Osaka University
2-1 Yamada-Oka, Suita, Osaka 565-0871, Japan

ABSTRACT

The authors performed for the Committee VI.2 of ISSC’2000 an assessment of the sensitivity of the “average strain-average stress curves (σ-ε)” on the ultimate hull bending moment (M_0) and the moment-curvature relationship (M_0-φ). This assessment concerns the ultimate bending moments obtained through standard progressive collapse analysis (Smith Algorithm). Such progressive approach requires for each stiffened panel a “σ-ε” curve. The objective is to quantify the variation of the ultimate bending moment (M_0) induced by the uncertainties associated to the “σ-ε” curves. Extended and updated results are available in this paper.

KEYWORDS

Ultimate strength, Stiffened panels, Strain and stress curve, Ultimate bending moment, Hull girder, Progressive collapse analysis, Sensitivity analysis.

1 INTRODUCTION

Buckling, tripping, yielding... of plate, panels and longitudinal structural members affect the overall collapse behaviour of a ship hull cross-section under longitudinal bending moment. Figure 1 shows several typical “average stress-average strain (σ-ε)” relationships of a stiffened panel composed of a stiffener and attached plating under axial compression (beam-column). Even if all these curves consider the buckling effect they differ by the value of the “σ_ε”, compressive ultimate strength as well as by the shape of the post-collapse residual strength. The models used to get these curves are the following:

(M1) HULLST (Yao 1991, 1999): Rational numeric analysis of the “σ_ε” ult. strength and “σ_ε-ε” curve,
(M2) RAHMAN (Rahman and Chowdhury 1996) + HUGHES (Hughes 1988): Rational formulation for the ultimate strength “σ_ε” and simplified rational “σ_ε-ε” curve.
Figure 1: "α-ε" curves of the reference element of a box girder with 7 different "STR" models.

Figure 2: Moment-curvature relationships of a box girder subjected to longitudinal bending moment. Sensitivity of the "α-ε" element curve (STEP 2) on the ultimate bending moment.

Table 1: SENSITIVITY OF THE "α-ε" CURVES ON THE ULTIMATE BENDING MOMENT OF THEBOX GIRDER (Fig. 2) (Same mesh model [step 1] and same PCA model [step 3])

2 A GENERIC FRAMEWORK FOR PROGRESSIVE COLLAPSE ANALYSIS

Usually progressive collapse analysis to evaluate the ultimate bending moment of hull girder is achieved in 3 steps and each step is characterised by a numerical model associated with theoretical assumptions. For each step model, it is necessary to specify the assumptions and to assess their influence (sensitivity) on the result (ultimate bending moment of hull girder). This is particularly relevant as the paper’s aim is to assess the sensitivity of the "α-ε" curves on the ultimate bending moment (Mu).
At the beginning of a “Progressive Collapse Analysis” (PCA) the “raw data” are the same for each model/user and they concern:
- the scantling (plates, stiffeners, ...) of the midship section and the frame spacing,
- the initial imperfections (plate deflection, stiffener deflection and residual stress),
- the lateral loadings (if considered by the “a–e” element model)

The 3 main steps of a complete “Progressive Collapse Analysis” (PCA) are the following:

**STEP 1:** Modelling (to decompose the structure in elements and to build the mesh model).

**STEP 2:** Evaluate the “a–e” element curve. This requires a stress-strain model called “STR” model.

**STEP 2.1:** Assessment of the “a–e” compressive strength of each element.

This requires to survey all the failure modes (plate and stiffener induced buckling, tripping, yielding and local instabilities) and to model of the initial imperfections and boundary conditions.

**STEP 2.2:** Definition of the “shape” of the “a–e” curves.

NB: Steps 2.1 and 2.2 can be performed together in the same routine (e.g.: HULLST, Yao and Nikolov, 1991) or independently (e.g.: Rahman and Chowdhury, 1996).

**STEP 3:** Perform the Progressive Collapse Analysis (using a “PCA” model that includes an incremental procedure of the curvature).

In order to choose a method to evaluate the ultimate strength of a hull girder, a designer have in fact to select 2 models, one for STEP 2 (the “STR” element model) and one for STEP 3 (the “PCA” model). Sensitivity of both has to be evaluated.

### 2.1 Sensitivity of the PCA Model (STEP 1 & STEP 3) on the Ultimate Bending Moment

Most of the available PCA models are based on the model of Smith (1977). This means that the background of all the current PCA models is the same. So, only the quality of the numerical procedure can generate some differences between the different PCA models. On the other hand, the accuracy and reliability of a progressive collapse analysis are strongly influenced by the mesh model “quality” (STEP 1). In fact, the mesh model of the considered structure (STEP 1) depends on the considered PCA model and the available elements: plates, stiffeners, stiffened plates, plates, curved elements, etc. To short, we can say that a PCA model is “better” than another if the mesh model of the structure requires less simplifications/assumptions. For instance, the simplified Rahman progressive collapse model (Rahman et al. 1996) has the same background as the sophisticated HULLST model (Yao, 1999) but is less accurate as it requires a simplified modelling.

### 2.2 Sensitivity of the STR Model (STEP 2) on the Ultimate Bending Moment

To provide reliable information to select a relevant “STR” model (STEP 2), it is necessary to assess separately the sensitivity of STEP 2 on the ultimate bending moment. This job was initiated by Rigo and the ISST’2000 Committee V.l.2 through a series of analysis that quantify the sensitivity of the “a–e” curves on the ultimate bending moment (Yao et al. 2000). To achieve this goal, the ultimate bending moment of 3 ships studied by the ISST Committee were re-evaluated with different STR models (STEP 2) but with the same PCA model (STEP 1 and STEP 3) using the PROCOL software.

PROCOL links the HULLST’s PCA model with several STR models developed by different authors (Paik et al. 1997; Rahman et al. 1996; Dowling 1991; Yao and Nikolov 1991). For each ship an identical mesh model is used, including 105 elements for the VLCC, 99 elements for the Energy Concentration and 99 for the Container. Only the stress-strain curves differ. All the relevant data are available in Yao et al. (2000).

### Table 2

<table>
<thead>
<tr>
<th>Vlcc (105 elements)</th>
<th>Energy Concentration (99 elements)</th>
<th>Container (99 elements)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sensitivity of the</td>
<td>Sensitivity of the</td>
<td>Sensitivity of the</td>
</tr>
<tr>
<td>Ultimate Bending</td>
<td>Ultimate Bending</td>
<td>Ultimate Bending</td>
</tr>
<tr>
<td>Moment to the</td>
<td>Moment to the</td>
<td>Moment to the</td>
</tr>
<tr>
<td>Von Mises Stress</td>
<td>Von Mises Stress</td>
<td>Von Mises Stress</td>
</tr>
<tr>
<td>Model</td>
<td>Model</td>
<td>Model</td>
</tr>
<tr>
<td>S400/1600 Mpa</td>
<td>S400/1600 Mpa</td>
<td>S400/1600 Mpa</td>
</tr>
<tr>
<td><strong>Model 3</strong></td>
<td><strong>Model 3</strong></td>
<td><strong>Model 3</strong></td>
</tr>
<tr>
<td>0.63</td>
<td>0.63</td>
<td>0.63</td>
</tr>
<tr>
<td>0.63</td>
<td>0.63</td>
<td>0.63</td>
</tr>
<tr>
<td>0.63</td>
<td>0.63</td>
<td>0.63</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>S400/1600 Mpa</th>
<th>S400/1600 Mpa</th>
<th>S400/1600 Mpa</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Model 4</strong></td>
<td><strong>Model 4</strong></td>
<td><strong>Model 4</strong></td>
</tr>
<tr>
<td>0.63</td>
<td>0.63</td>
<td>0.63</td>
</tr>
<tr>
<td>0.63</td>
<td>0.63</td>
<td>0.63</td>
</tr>
<tr>
<td>0.63</td>
<td>0.63</td>
<td>0.63</td>
</tr>
</tbody>
</table>
In addition to these rationally-based \( \sigma - e \) curves (HULST, Rahman-Hughes, etc.), five simplified \( \sigma - e \) curves are considered (see Figure 3, Shape 1 to Shape 5). These simplified curves are built by PROCOL and are composed of a perfect elastic deflection, a plastic deflection (ultimate strength's plateau) and a linear post-collapse deflection. All consider a buckling effect and have the same \( \sigma_u \) compressive ultimate strength. They only differ by the shape of their post-collapse residual strength.

**Shape 1:** no residual strength after ultimate strength.
**Shape 2:** a sharp reduction of the residual strength and no ultimate strength’s plateau.
**Shape 3:** a smooth reduction of the residual strength and no ultimate strength’s plateau.
**Shape 4:** a smooth reduction of the residual strength and a long ultimate strength’s plateau.
**Shape 5:** no reduction of the strength after the ultimate strength.

HULST provides the reference \( \sigma_{u0} \) compressive ultimate strength and reference \( \sigma - e \) curve. To compare the different \( \sigma - e \) curves of the same element, we classify the \( \sigma - e \) curves based on:
- the element ultimate compressive average-stress and associated average-strain (noted \( \sigma_u \) and \( \varepsilon_u \)),
- the “shape” of the \( \sigma - e \) curve.

### 2.3 Assessment of the Sensitivity of the \( \sigma_u \) Ultimate Strength (Same Shape)

The \( \sigma_u \) compressive ultimate strength of each element is evaluated with 4 different models. Then, 4 different \( \sigma - e \) curves are obtained for each element. They have the same shape, i.e. same elastic regime, same “ultimate strength’s plateau” (plateau) and post-collapse strength (e.g. curves 3 and 7 of Figure 1). They only differ one to the other by the level of \( \sigma_u \). Then, using the same PCA algorithm (model), these 4 sets of \( \sigma - e \) curves are used to compute the (Mu) ultimate bending moment. Table 2 gives the sensitivity of the \( \sigma_u \) element ultimate strength on the ultimate bending moment (Mu). It is observed that a variation of 1% on the ultimate strength of the elements induces a variation of the ultimate bending moment of 1% and 2% for, respectively, hogging and sagging. This means that the uncertainty on the Mu sagging ultimate bending moment is double the uncertainty on the \( \sigma_u \) element ultimate strength.

### 2.4 Assessment of the Sensitivity of the Shape of the \( \sigma - e \) Curve (Same \( \sigma_u \))

To assess the impact of the “shape” of the stress-strain curves on the ultimate bending moment (Mu), a second group of \( \sigma - e \) curves is defined. They are characterised by the same \( \sigma_u \): element ultimate strength but different shapes. As 5 different standard shapes are considered (see Figure 3), 5 sets of \( \sigma - e \) curves are defined. Then, the sensitivity of the “shape” of the \( \sigma - e \) curves on the (Mu) ultimate bending moment (see Table 3) are evaluated by using the 5 sets of \( \sigma - e \) curves in the PROCOL model.

Table 3 shows that the shape sensitivity on (Mu) is higher for sagging than hogging (double). Between shape 3 (no post-collapse strength) and shape 5 (perfect elastoplastic strength) the average variation is about 25% in sagging and 12% in hogging. Comparison between shape 3 and shape 4 shows that the length of the “ultimate strength’s plateau” is the more sensitive parameter.

An overestimation of the length of the plateau (shape 4) increases the ultimate bending moment of 10 to 20%. On the other hand, shape 2, which has no plateau, provides more conventional results (i.e. close to the reference value). Mu is similar for the VLCC, is underestimated by 3% for Energy Concentration and about 7% for the container. However, it should be noticed that the actual length of the plateau depends on the scantlings of the panel and the stiffeners.

Table 3: Sensitivity of the Shape of the \( \sigma - e \) Curves (Step 2.2) on the Ultimate Bending Moment of Hull Girder (VLCC: Energy Concentration and Container Ship)

<table>
<thead>
<tr>
<th>Shape</th>
<th>Sagging</th>
<th>Mu (1000 MN.m)</th>
<th>Energy Concentration</th>
<th>Container</th>
<th>Mean Variation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mu</td>
<td>Mu/Mu(ref)</td>
<td>Mu</td>
<td>Mu/Mu(ref)</td>
<td>Mu</td>
</tr>
<tr>
<td>Shape 1</td>
<td>20.42</td>
<td>16.84</td>
<td>6.72</td>
<td>0.859</td>
<td></td>
</tr>
<tr>
<td>Shape 2</td>
<td>19.64</td>
<td>16.27</td>
<td>3.77</td>
<td>0.859</td>
<td></td>
</tr>
<tr>
<td>Shape 3</td>
<td>20.28</td>
<td>16.28</td>
<td>5.89</td>
<td>0.876</td>
<td></td>
</tr>
<tr>
<td>Shape 4</td>
<td>20.94</td>
<td>16.43</td>
<td>6.08</td>
<td>0.905</td>
<td></td>
</tr>
<tr>
<td>Shape 5</td>
<td>21.65</td>
<td>18.91</td>
<td>7.15</td>
<td>1.004</td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>22.91</td>
<td>19.39</td>
<td>7.35</td>
<td>1.094</td>
<td></td>
</tr>
</tbody>
</table>

Mean Variation = 7.32% for Sagging and 10.35% for Energy Concentration.

### Figure 4: Moment-Curvature Relationships for the VLCC with 5 Different Sets of \( \sigma - e \) Curves

Figure 4 shows the (M-4) moment-curvature relationships obtained for the VLCC with 5 different sets of \( \sigma - e \) curves (simplified shapes) and the HULST’s set of curves (same mesh model, same \( \sigma_u \) element ultimate strength and same PCA algorithm). An example of these \( \sigma - e \) curves for a typical element are presented at Figure 3. This element corresponds to the ‘element type #2’ of the mesh model presented by Yao et al., 2000.

Sensitivity analysis on the ultimate bending moment with respect to various factors (plate and stiffener thickness, yield stress, residual stress, initial deflections and frame spacing) is included in Yao et al. (2000).

### 3 CONCLUSIONS

Different \( \sigma - e \) curves have been used with the same (PCA) progressive collapse analysis model (STEP 3) and using the same structure mesh model (STEP 1). The comparison of the results assesses the sensitivity of the \( \sigma - e \) curves on the (Mu) ultimate bending moment. Sensitivity of the \( \sigma_u \) element ultimate strength and the sensitivity of the \( \sigma - e \) curves have also been performed. The results show that the main significant factor in the complete procedure of evaluation of the ultimate bending moment is the STEP 2.1: Evaluation of the \( \sigma - e \) ultimate compressive strength of
the elements/components (S). Moreover the results show that the “shape” (Step 2.2) is not negligible and particularly the length of the ultimate strength's plateau. Such a conclusion is not surprising as the main uncertainties and numerical assumptions are linked with STEP 2.1 (for instance: modelling of the initial imperfections and the boundary conditions, local buckling, tripping, bi-axial compression, lateral loading.…).

![Diagram showing shape curves](image)

Figure 3: Shapes of the simplified “σ-ε” element curves (with same σ,) for the sensitivity analysis of the shape of “σ-ε” curve (STEP 2.2).

![Diagram showing M-ε curves](image)

Figure 4: Moment-curvature relationships (VLCC) with different “σ-ε” curves (same mesh model, same “σ,” element ultimate strength and same PCA model). Sensitivity analysis of the shape of “σ-ε” curve (STEP 2.3)

References

Practical Design of Ships and Other Floating Structures

Proceedings of the Eighth International Symposium on Practical Design of Ships and Other Floating Structures

16 - 21 September 2001
Shanghai, China

Edited by
You-Sheng Wu
China Ship Scientific Research Center, Wuxi, Jiangsu, China
Wei-Cheng Cui
School of Naval Architecture & Ocean Engineering, Shanghai Jiaotong University, Shanghai, China

and
Guo-Jun Zhou
China Ship Scientific Research Center, Wuxi, Jiangsu, China

Volume II

2001
ELSEVIER
AMSTERDAM - LONDON - NEW YORK - OXFORD - PARIS - SHANNON - TOKYO
CONTENTS

VOLUME I

Preface

PLENARY LECTURES

Maritime Safety Culture and Development of Ship and Offshore Installations Design Standards in the 21st Century
Ke-Jun Li

Structural Safety of Ships
D. Liu

Shipping Industry in the 21st Century
Jia-Fu Wei

1. DESIGN SYNTHESIS FOR SHIPS AND FLOATING SYSTEMS

LIFE CYCLE COST AND SHIPPING SYSTEM

A Consideration of Life Cycle Cost of a Ship
Bamahli Komakara and Hiroshi Sanctino

The Experiment of River-Sea-Going Ore Barge Fleet and Renovation of Existing Integrated Barge
Shan-Hua Chen, Wei Zhang, Jun-Ming Li and Chang-Feng Wang

DESIGN OPTIMISATION

Optimization of a Wave Cancellation Multihull Ship Using CFD Tools
C. Teng, R. Lohner and O. Soto

A Module-Oriented Optimization Tool
P.H. Rigo

The Fine Optimization of Ship Hull Lines in Resistance Performance by Using CFD Approach
L. Xu and F.T. Wong

HULL FORM DESIGN

Parametric Hull Form Design - A Step Towards One Week Ship Design
C. Ali, S.D. Bade, L. Birk and S. Harries

Mission Based Hydrodynamic Design of a Hydrographic Survey Vessel
S.L. Tzouvegas, P.F. van Tervegot and C.H. Thill

Hull Form Design of a Passenger Catamaran for Operation in the Yellow Sea Region
Shuang-Hee Lee, Rong-Gui Lee and Jae Wook Lee

29
37
43
51
59
67
75
83
4. STRUCTURES AND MATERIALS

WAVE INDUCED LOADS AND RESPONSES

Prediction of Wave-Induced Rolling Responses by a Time-domain Strip Theory
Zhao-Hui Wang, J.J. Slesers and Jin-Zhu Xia

Methods to Reduce the Effects of Irregular Frequencies in Hydrodynamic Analyses of Vessels with Forward Speed

The Effects of Forward Speed on Hydrodynamic Pressure and Structural Response of Ships in Waves
Chih-Chung Fang, Hsu-Teng Wu, Hsiao-Chen Chou and Chiang-Yang Lu

Ship Motions and Sea Loads by a 3D Rankin Panel Method
D. Li, Wei-Xing Zhang, Chen-Si Zou, Bo-Yun Xu and Hwe-Fang Chou

EXTREME WAVE LOADS

Experiment on Extreme Wave Loads of a Flexible Ship Model
Rui-Zhang Chen, Shuang-Xing Du, Hsu-Sheng Wu, Ji-Ru Lin, Xin-Jun He and Yee-Lin Yue

Estimation of Nonlinear Long-Term Extremes of the Vertical Bending Moments in Ships
T. Baurhorn and T. Moan

A Direct Calculation Approach of Determining Extreme Combined Bending Moments for Fast Fine Form Ships
Xue-Kang Gu and Jun-Wei Shen

HYDROELASTICITY

Flutter of Hydrofoil in Viscous Field
Cao Sheng, Xin-CI Zhang and You-Sheng Wu

Symmetric and Antisymmetric Hydrodynamic Analysis of a Bulker in Waves
S.E. Hinders, W.G. Price and P. Tamarel

Hydroelastic Model for Bottom Slaming
A. Beremetski and Y. Pestov

Hydrodynamic Impulsive Loads Acting on Ship-Hull Plates
Gang Huang

RELIABILITY

Risk Analysis Applied to Occurrence of Maximum Wave Bending Moment
E.A. Dahle, D. Myhre and H.T. Wes

Fuzzy Reliability Analysis of a Ship Longitudinal Strength
J.M. Yang and J.Y. Huang

Reliability-Based Requalification of Existing Offshore Platforms
T. Maa and G.T. Vandal

Deterministic and Probabilistic Assessment of FFS Hull Fender Strength
A. Jusvik and Y. Ts

Consistent Code Formulation for Ship Structural Design
A.E. Moosavi, J.E. Spencer, P.H. Wenzel, J.E. McGruvish and D.D. Tarman

Reliability of Stiffened Ship Decks
K. Rajagopalan

ULTIMATE STRENGTH - SENSITIVITY

Total Analysis System for Ship Structural Strength
T. Yawaya, H. Kusakabe, M. Akegoodani, T. Saito and M. Iirisawa

Uncertainty and Sensitivity Analyses in the Predicted Critical Buckling Strength of a Longitudinally Stiffened Sub-Panel
Wei-Chung Cui, Li-Jun Shi and Jia-Fei Zhang

Sensitivity Analysis on Ultimate Hull Bending Moment
Ph. Rigo, C. Todesca and T. Tsu

ULTIMATE STRENGTH - HULL GIRDER

Assessment of Ultimate Longitudinal Strength of Aged Tankers
J. Roda, T. Ito, O. Kitamura, T. Yamamoto, M. Yorita and H. Oehlabe

Ultimate Strength and Reliability Assessment for the Ship Hull Girder Used in ISCC-2000

Benchmark Study
Hai-Hong Su and Yong Bai

An Assessment of the Ultimate Plastic Strength of the Ship's Aged Hulls
G.S. Egorov and V.V. Kostyukov

ULTIMATE STRENGTH - STIFFENED PLATES AND SHELLS

A New Design Model for Ultimate and Buckling Strength Assessment of Stiffened Plates
E. Steen, T.R. Elavard and S. Valberg

Ultimate Strength of Longitudinally Stiffened Panels: Multi-Criteria Comparative Analysis
J.Y. Peddiluv, T. Quasad, C. Todesca and Ph. Rigo