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Generalized shape optimization with X-FEM and Level Set description applied to stress constrained structures

L. Van Miegroet, T. Jacobs E. Lemaire, P. Duysinx & C. Fleury

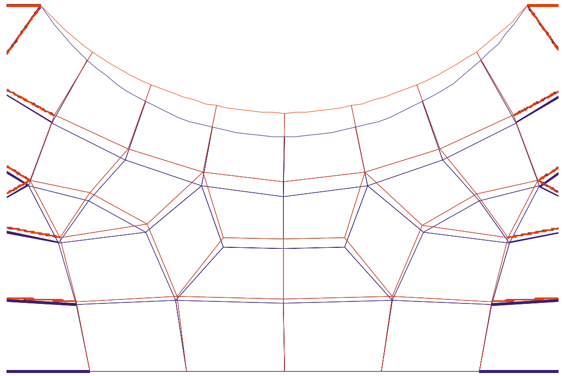
Automotive Engineering / Multidisciplinary Optimization
Aerospace and Mechanics Department
University of Liège

Outline

- Introduction
- eXtended Finite Element Method (XFEM)
- Level Set Method
- Sensitivity Analysis
- Applications
- Conclusion

Introduction – *Optimization of mechanical structures*

- Shape optimization :
  - Variables : geometric parameters
  - Limitation on shape modification
  - Mesh perturbation – Velocity field
  - CAD model of structure
  - Various formulations available



- Topology optimization :
  - Variables : distribution of material element densities
  - Strong performance – deep change allowed
  - Fixed Mesh
  - « Image » of the structure
  - Limitation of objective function and constraints

Introduction – *X-FEM and Level Set optimization*

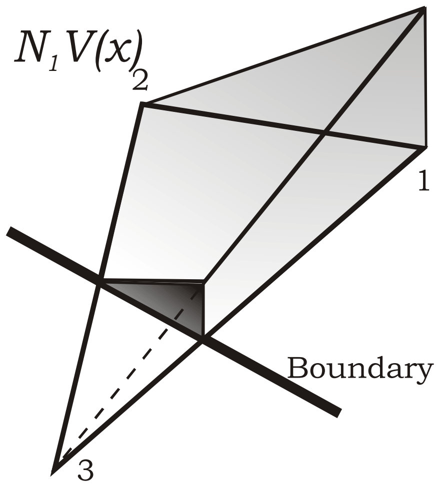
- XFEM + Level Set description = Intermediate approach between shape and topology optimisation
  - eXtended Finite Element Method → Alternative to remeshing
    - work on fixed mesh
    - no velocity field and no mesh perturbation required
  - Level Set Method → Alternative description to parametric CAD
    - Smooth curve description
    - Topology can be altered as entities can be merged or separated → generalized shape
  - Problem formulation:
    - Global and local constraints
    - Small number of design variables
- Introduction of new holes requires a topological derivatives

eXtended Finite Element Method

- Early motivation :
  - Study of propagating crack in mechanical structures → avoid the remeshing procedure
- Principle :
  - Allow the model to handle discontinuities that are non conforming with the mesh
- Representing holes or material – void interfaces
  - Use special shape functions $V(x)N_i(x)$ (discontinuous)

$$u = \sum_{i \in I} N_i(x)\, V(x)\, u_i \Rightarrow \qquad K_{uu} u = f$$

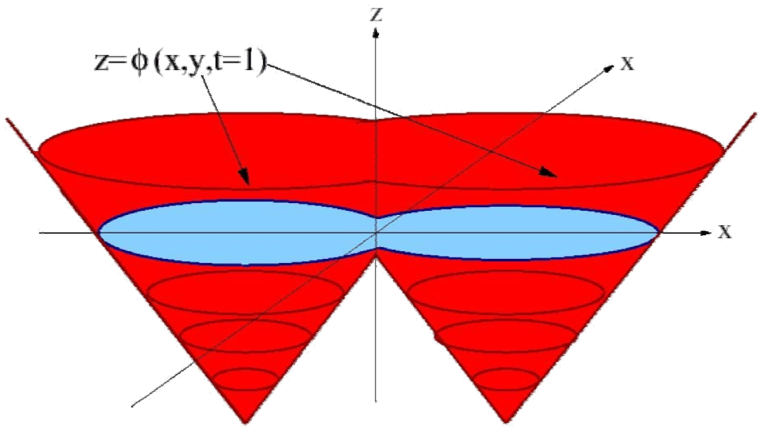
$$V(x) = \begin{cases} 1 & \text{if node} \in \text{solid} \\ 0 & \text{if node} \in \text{void} \end{cases}$$



The Level Set Description

- Principle (Sethian & Osher, 1999)
  - Introduce a higher dimension
  - Implicit representation
  - Interface = the zero level of a function : $\psi(x,t) = 0$

- Possible practical implementation:
  - Approximated on a fixed mesh by the signed distance function to curve Γ:

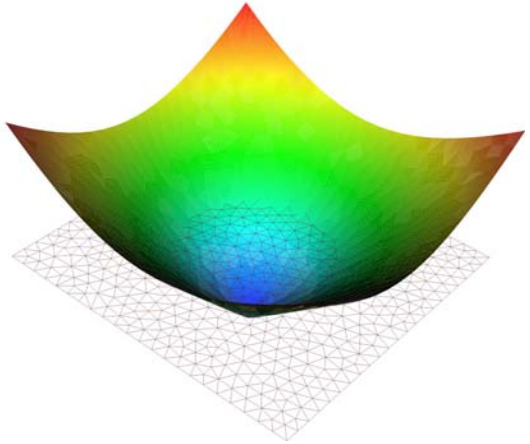


$$\psi(x,t) = \pm \min_{x_\Gamma \in \Gamma(t)} \left\| x - x_\Gamma \right\|$$

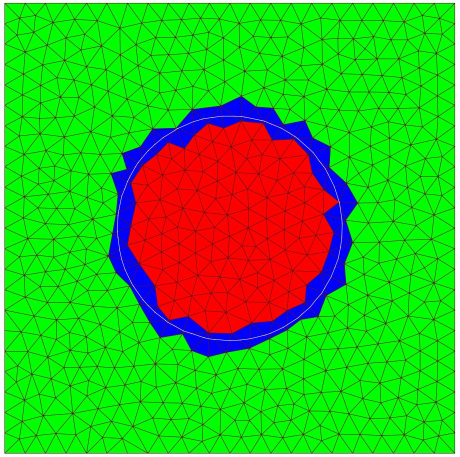
- Advantages:
  - 2D / 3D
  - Combination of entities: e.g. min / max

The Level Set and the X-FEM

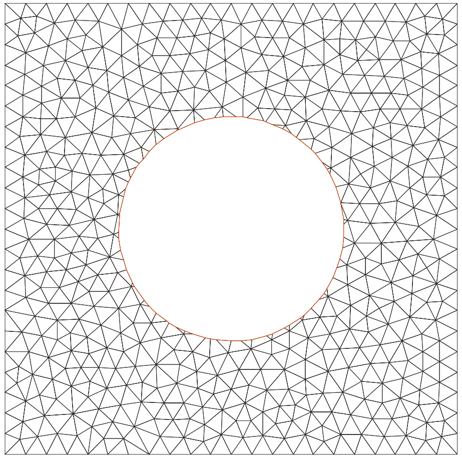
- 1. Build Level Set



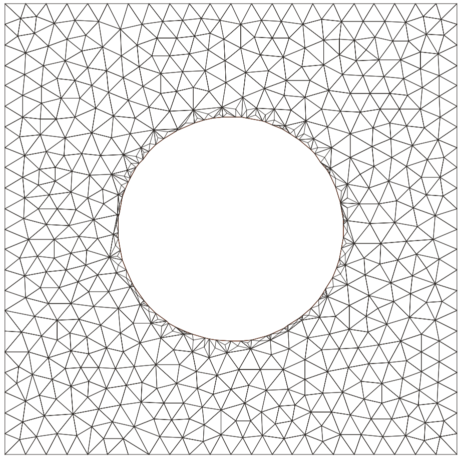
- 2. Detect type of element



- 3. Cut the mesh



- 4. Working mesh for integration



Optimization Problem Formulation

- Geometry description and material layout:
  - Using Basic Level Sets description : ellipses, rectangles, NURBS,...
- Design Problem:
  - Find the best shape to minimize a given objective functions while satisfying design constraints
- Design variables:
  - Parameters of Level Sets
- Objective and constraints:
  - Mechanical responses: compliance, displacement, stress,...
  - Geometrical characteristics: volume, distance,...
- Problem formulation similar to shape optimization but simplified thanks to XFEM and Level Set!

Sensitivity analysis - *principles*

- Classical approach for sensitivity analysis in industrial codes: semi analytical approach based on *K* derivative

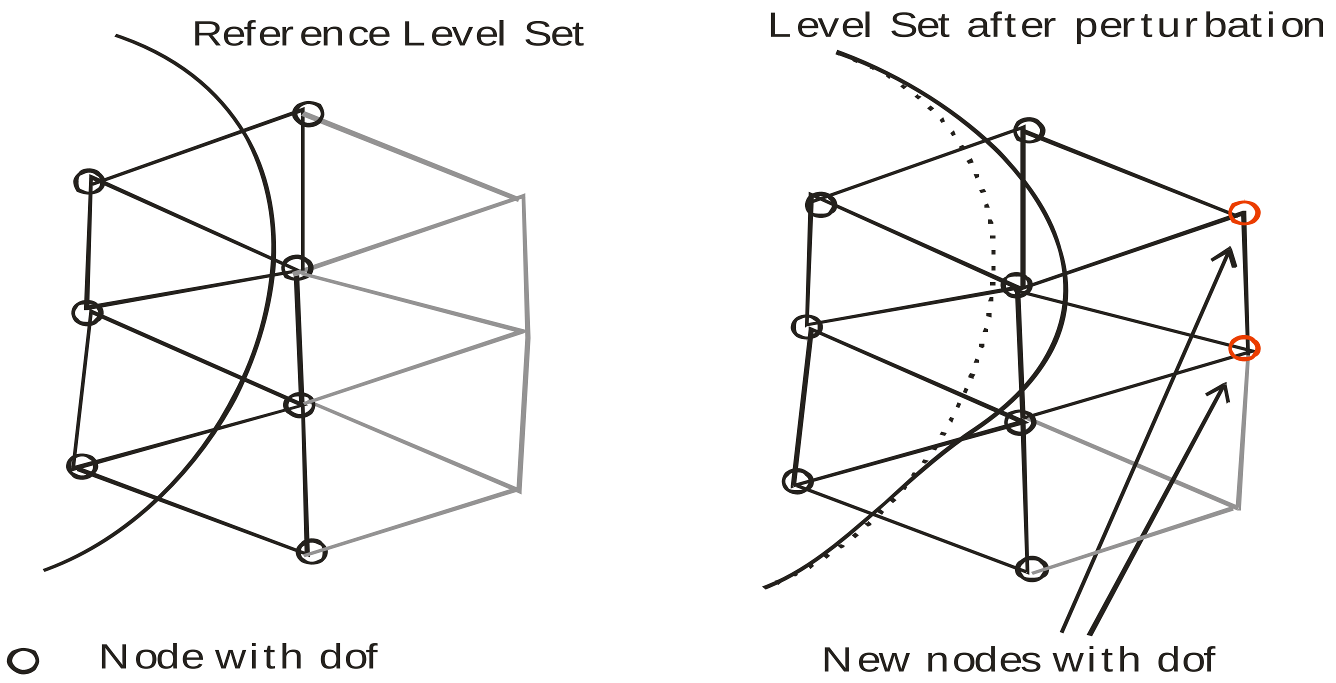
- Discretized equilibrium: $$\mathrm{K}\,\mathrm{u} = \mathrm{f}$$

- Derivatives of displacement: $$\mathrm{K}\frac{\partial \mathrm{u}}{\partial \mathrm{x}} = \left(\frac{\partial \mathrm{f}}{\partial \mathrm{x}} - \frac{\partial \mathrm{K}}{\partial \mathrm{x}}\mathrm{u}\right)$$

- Semi analytical approach:
  - Stiffness matrix derivative: $$\frac{\partial \mathrm{K}}{\partial \mathrm{x}} \approx \frac{\mathrm{K}(\mathrm{x}+\delta\mathrm{x})-\mathrm{K}(\mathrm{x})}{\delta \mathrm{x}}$$
  - Compliance derivative: $$\frac{\partial C}{\partial x} = -u^T \frac{\partial K}{\partial x} u$$
  - Stress derivative: $$\frac{\partial \sigma}{\partial x} = \frac{\sigma(u(x+\delta x)) - \sigma(u(x))}{\delta x}$$

Sensitivity analysis – technical difficulty

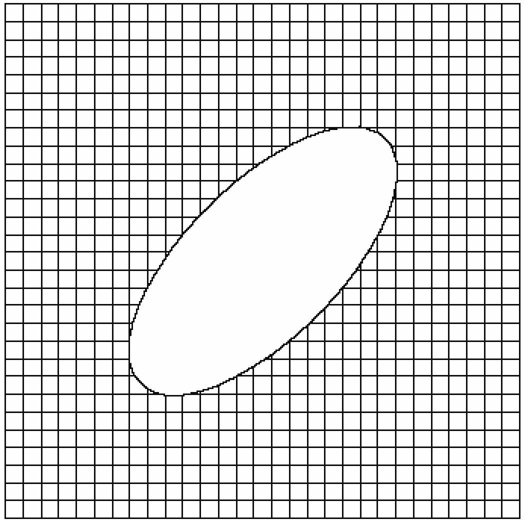
- Introduction of new dofs → K dimension changes → not possible



- Ignore the new elements that become solid or partly solid
  - Small errors, but minor contributions
  - Practically, no problem observed
  - Efficiency and simplicity
  - Validated on benchmarks

Sensitivity analysis – *validation*

- Validation of semi-analytic sensitivity:
  - Elliptical hole
  - Parameters: major axis *a* and Orientation angle θ w.r.t. to horizontal axis
  - Perturbation: $\delta=10^{-4}$



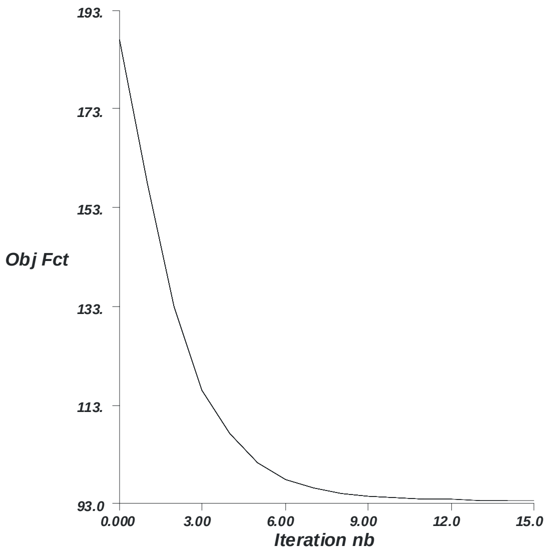
Design variables	*Finite differences*	*Semi-analytical approach*	*Relative error (%)*
$a = 0.6$	$3698,0000$	$3691,3344$	$0,1802$
$\theta = \pi/4$	$478,0000$	$477,0641$	$0,1957$
$a = 0.6$	$2712,000$	$2707,328$	$0,1722$
$\theta = \pi/6$	$523,70000$	$523,4099$	$0,0553$
$a = 0.6$	$783,8000$	$781,3920$	$0,3072$
$\theta = 0$	$11,6239$	$11,6235$	$0,0029$

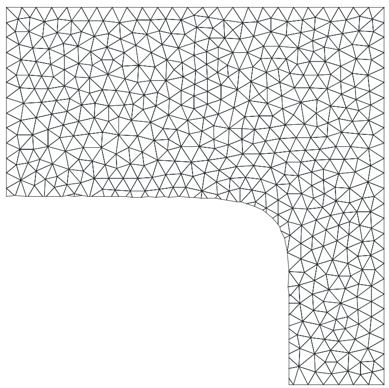
Applications - 2D plate with a hole

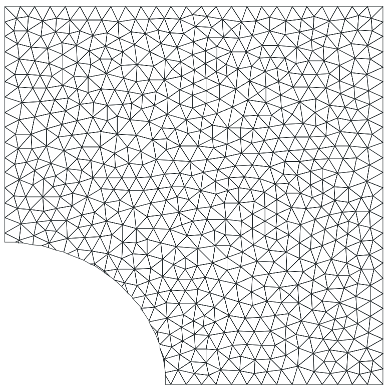
- Plate with generalized super elliptical hole :

$$\frac{|x|^{\alpha}}{|a|} + \frac{|y|^{\eta}}{|b|} = r$$

  - Parameters : $2 < a, b, \eta, \alpha < 8$
  - Objective: min Compliance.
  - Constraint: upper bound on the Volume.
  - Bi-axial Load: $\sigma_x = \sigma_y = \sigma_0$
  - Solution: perfect circle: $a = b = 2, \eta = \alpha = 2$

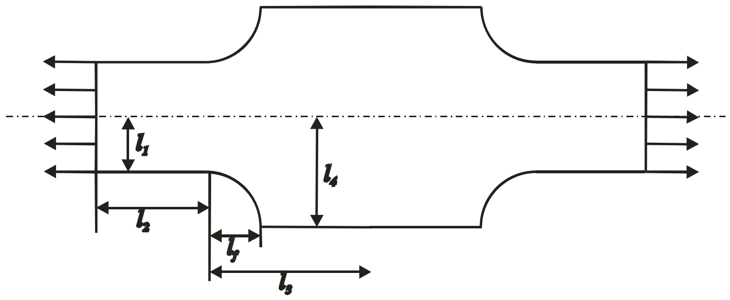


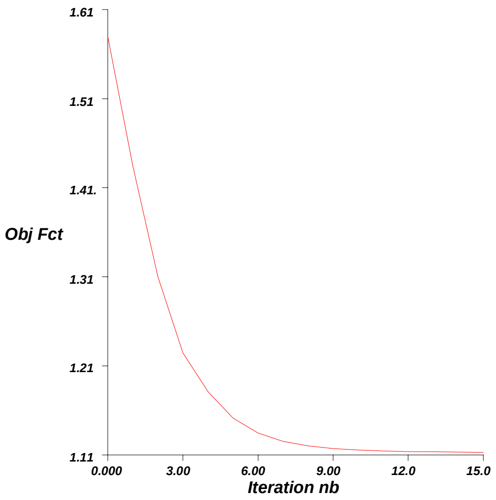


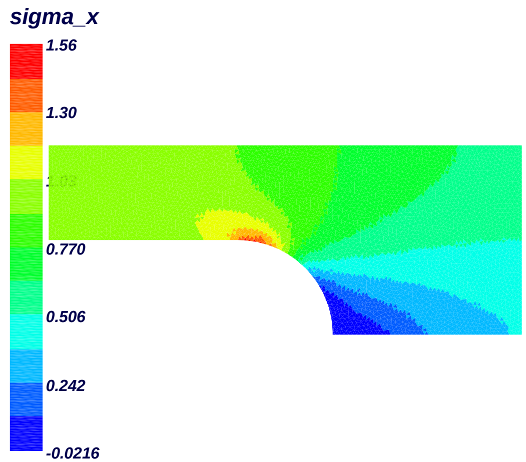


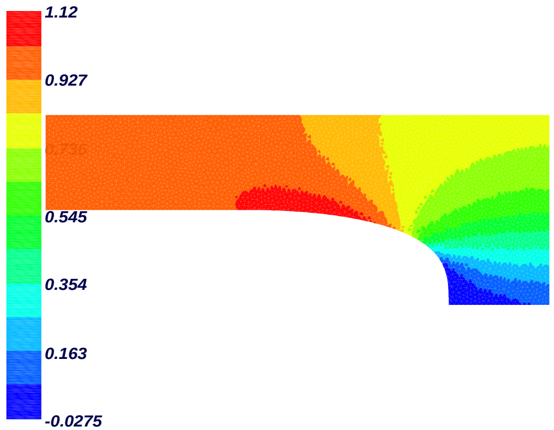
Applications – *2D fillet in tension*

- Shape of the fillet : generalized super ellipse $\frac{|x|^{\alpha}}{|a|}+\frac{|y|^{\eta}}{|b|}=r$
  - Parameters : a,η,α
  - Objective: min (max Stress)
  - No Constraint
  - Uni-axial Load: $\sigma_x=\sigma_0$
  - Solution: stress reduction of 30%









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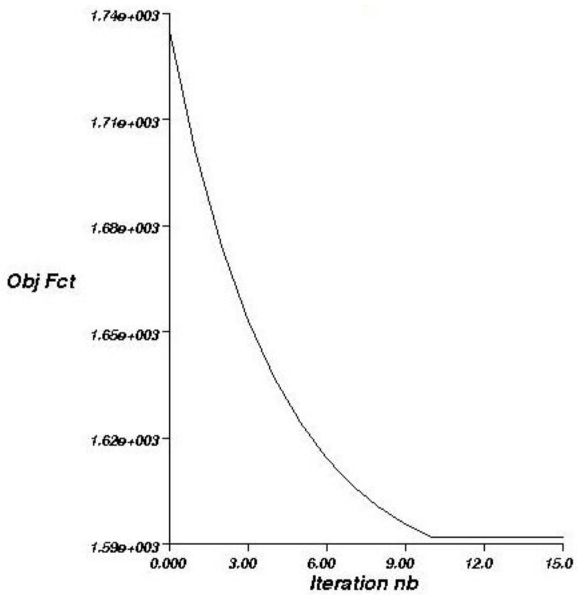
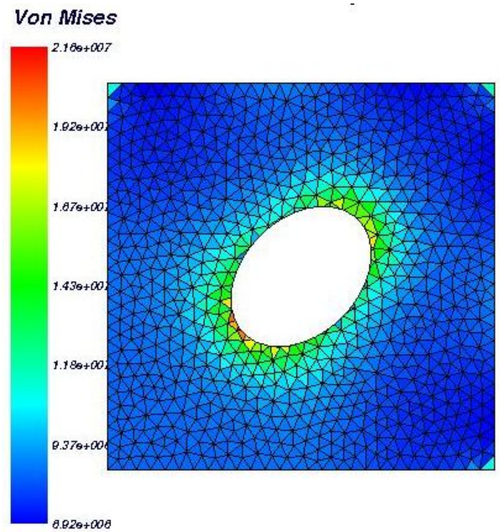
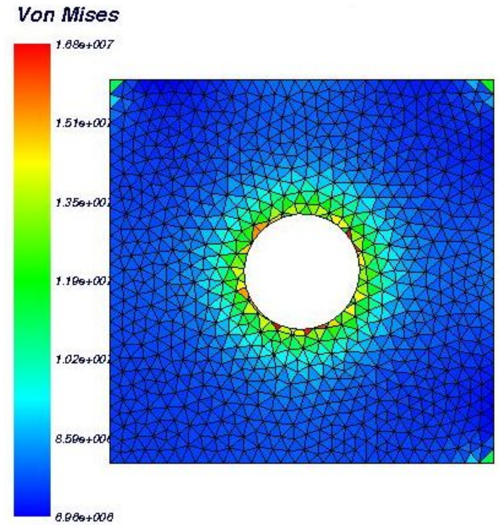
Applications – 2D plate with elliptical hole

- Plate with elliptical hole :
  - Parameters : a, θ
  - Objective: min Compliance.
  - Constraint: $\sigma_{VM} <= 0.8\ \sigma_{VM0}$
  - Bi-axial Load: $\sigma_x = \sigma_y = \sigma_0$
  - Solution: perfect circle. σ_{VM0} : 2.16

    $\sigma_{VM\ final}$: 1.68







Conclusion

- XFEM and Level Set gives rise to a generalized shape optimisation technique

- Intermediate to shape and topology optimisation
  - Work on a fixed mesh
  - Topology can be modified:
    - Holes can merge and disappear
    - New holes cannot be introduced without topological derivatives
  - Smooth curves description
  - Void-solid description
  - Small number of design variables
  - Global or local response constraints
  - No velocity field and mesh perturbation problems

Conclusion

- Contribution of this work
  - New perspectives of XFEM and Level Set
  - Investigation of semi-analytical approach for sensitivity analysis
  - Implementation in a general C++ multiphysics FE code

- Perspectives:
  - 3D problems
  - Dynamic problems
  - Multiphysic simulation problems with free interfaces