Bottom friction formulations for free surface flow modeling

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Free surface flow modeling

Shallow Water equations :

$$\begin{cases} \dfrac{\partial h}{\partial t} + \dfrac{\partial u_i h}{\partial x_i} = (r - i) \\ \dfrac{\partial (u_i h)}{\partial t} + \dfrac{\partial}{\partial x_j}(u_i u_j h) = -gh\dfrac{\partial h_s}{\partial x_i} + t_i \end{cases}$$

Bottom friction representation :

$$t_i = -\rho g \omega J_i$$

2 “schools” :

Empirical school
(Chézy, Manning)

$$U = \alpha J^{1/2} R_h^{\chi}$$

Modern school
(Prandtl)

$$J = \frac{\lambda}{4R_h}\frac{U^2}{2g}$$

$$U = \alpha J^{1/2} R_h^{\chi}$$

Author	χ
Chézy	0.5
Manning	0.667
Gauckler	0.4
Forchheimer	0.7
Christen	0.625
Hagen	0.714
Tillman	0.7

Modern school

$$J = \frac{\lambda}{4R_h}\frac{U^2}{2g}$$

Flow turbulence regime	Author	Law	Validity field
Laminar	Poiseuille	$\lambda = \frac{64}{Re}$	Re < 5000
Smooth turbulent	Prandtl	$\sqrt{\frac{1}{\lambda}} = -2\,log\,\frac{2.51}{Re\sqrt{\lambda}}$	k/h = 0
Transitional	Colebrook	$\sqrt{\frac{1}{\lambda}} = -2\,log\left(\frac{k}{14.8h} + \frac{2.51}{Re\sqrt{\lambda}}\right)$	k/h < 2240/Re
Rough turbulent	Nikuradse	$\sqrt{\frac{1}{\lambda}} = -2\,log\,\frac{k}{14.8h}$	k/h > 2240/Re
Macro-roughness	Bathurst	$\sqrt{\frac{1}{\lambda}} = -1.987\,\log\,\frac{k}{5.15h}$	k/h > 0.25

Colebrook (implicit form) :

$$\sqrt{\frac{1}{\lambda}} = -2\,log\left(\frac{k}{14.8h} + \frac{2.51}{Re\,\sqrt{\lambda}}\right)$$

Barr (explicit form) :

$$\sqrt{\frac{1}{\lambda}} = -2\,log\left(\frac{k}{14.8h} + \frac{4.518\,log\left(\frac{Re}{7}\right)}{Re\left(1 + \frac{Re^{0.52}\left(\frac{k}{R_h}\right)^{0.7}}{76.53}\right)}\right)$$

Rough turbulent regime (Nikuradse) :

$$\sqrt{\frac{1}{\lambda}} = -2\,log\,\frac{k}{14.8h} \qquad J = \frac{\lambda}{4R_h}\frac{U^2}{2g}$$

$$U = (8g)^{1/2}(Ak)^{-1/2M}\,J^{1/2}R_h^{1/2+1/2M}$$

$$\text{with}\begin{cases} A = \dfrac{R_h}{k}\left[-2\,log\left(\dfrac{k}{14.8R_h}\right)\right]^{-ln(10)log\left(\frac{14.8R_h}{k}\right)} \\ M = \dfrac{ln(10)}{2}\,log\left(\dfrac{14.8R_h}{k}\right)\end{cases}$$

$$U = \alpha J^{1/2}R_h^{\chi}$$

$$\begin{cases}\chi = \dfrac{1}{2} + \dfrac{1}{2M} \\ \alpha = \sqrt{\dfrac{8g}{(Ak)^{1/M}}}\end{cases}$$

Empirical laws validity fields

$$\begin{cases} \chi = \dfrac{1}{2} + \dfrac{1}{2M} \\ \alpha = \sqrt{\dfrac{8g}{(Ak)^{1/M}}} \end{cases}$$

Author	**χ**	**k/R_h**	**α**
Chézy	0.5	0	∞
Christen	0.625	0.005	$31.71/k^{0.125}$
Manning	0.667	0.037	$26.61/k^{0.167}$
Tillman	0.7	0.1	$24.26/k^{0.2}$
Forchheimer	0.7	0.1	$24.26/k^{0.2}$
Hagen	0.714	0.138	$23.51/k^{0.214}$

Validity fields extension :

$$\frac{\Delta h}{h} = \frac{h_{\text{modern}} - h_{\text{empirical}}}{h_{\text{modern}}}$$

Author	Validity field (*k/h*)
Chézy	0
Christen	[0 ; 0.032]
Manning	[0.007 ; 0.1]
Tillman	[0.023 ; 0.29]
Hagen	[0.034 ; 0.38]
Gauckler	No validity
Poiseuille	Only laminar
Prandtl	0
Colebrook	[0 ; 0.1]
Barr	[0 ; 0.1]
Nikuradse	[0 ; 0.1]
Bathurst	[0.1 ; 5.15]

Continuous friction formulation

- Colebrook and Barr valid for $k/h < 0.1$
- Bathurst valid for $k/h > 0.1$
- Bathurst ≠ Colebrook or Barr for $k/h = 0.1$

Continuous link close to $k/h = 0.1$

For $\frac{k}{h} \leq 0.05$:

$$\frac{1}{\sqrt{\lambda}} = -2 \log \left(\frac{4.518 \log\left(\frac{Re}{7}\right)}{Re\left(1 + \frac{Re^{0.52}(k/h)^{0.7}}{76.531}\right)} + \frac{k}{14.8\,h} \right)$$

For $0.05 \leq \frac{k}{h} \leq 0.15$:

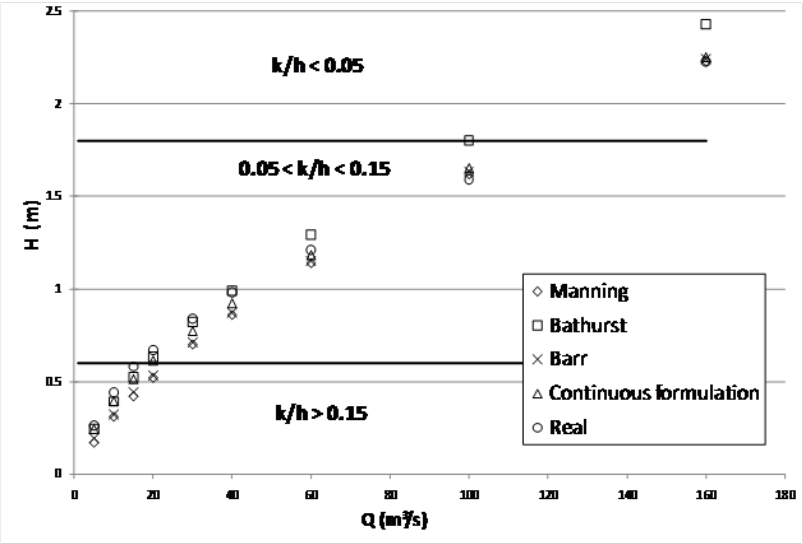
$$\frac{1}{\sqrt{\lambda}} = 1469.76\left(\frac{k}{h}\right)^3 - 382.83\left(\frac{k}{h}\right)^2 + 9.89\left(\frac{k}{h}\right) + 5.22$$

For $\frac{k}{h} \geq 0.15$:

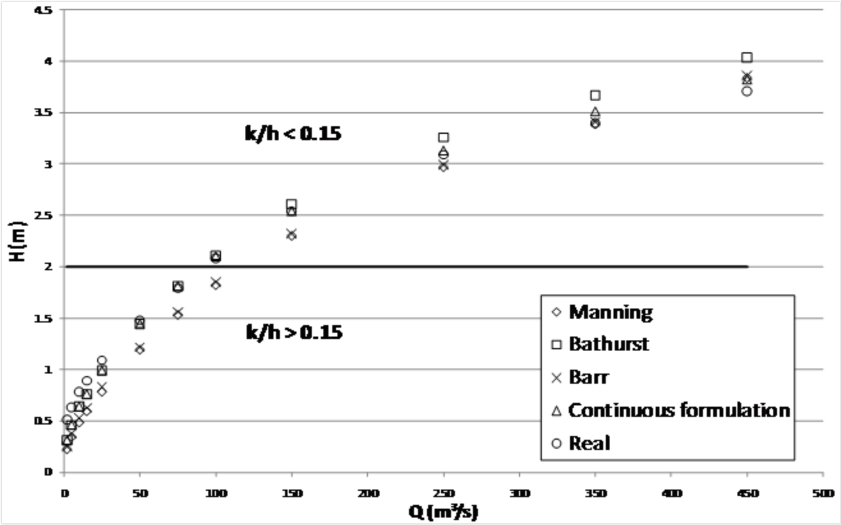
$$\frac{1}{\sqrt{\lambda}} = -1.987 \log\left(\frac{k}{5.15\,h}\right)$$

Validation

Ourthe River near Hamoir



Semois River near Membre



Situation	Modeling law	k/h <0.05	0.05 < k/h < 0.15	k/h > 0.15
Hamoir – Ourthe	Manning	0.9	14.3	30.6
	Barr	1.5	13.1	26.1
	Bathurst	11.1	4.0	9.8
	Continuous formulation	2.3	6.4	10.3
Membre - Semois	Manning	-	5.9	33.9
	Barr	-	5.5	29.5
	Bathurst	-	5.3	15.9
	Continuous formulation	-	1.7	16.0