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# The Computation of Prismatic Structures, Applied to Naval Architecture

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*ABSTRACT*

*In shipbuilding, most long vessels (barges, oil tankers, bulk carriers, etc.) are substantially prismatic in shape in the central region of their structure. These vessels have geometrical characteristics which make their calculation difficult, especially when the shell and plate panels which compose them are heavily stiffened.*

*One of the most challenging aspects of these structures is the computation of stresses and displacements (stress analysis). Consequently, the objective of this study was to develop a tool which allows the computation of such structures, reducing to the greatest extent the simplifications and approximations related to the representation of the structural geometry, the loading and the interdependence of the various elements (such as shell plating, stiffeners, deck beams, web frames and stringers).*

*To analyse the behaviour of each stiffened panel, the stiffened sheathing software L.B.R.-3 is presented, as well as some specific developments required for shipbuilding design. This is a new analytical method using Fourier series expansions to solve the differential equations of orthotropic prismatic shells. Using as an example the stress analysis of a tanker, the L.B.R.-3 software demonstrates its utility and efficiency in giving easily accurate stresses and displacements at every point of the structure.*

*Key words:* shell, plate, prismatic structure, orthotropic structure, stiffening, stress analysis, displacements, stresses, software, Fourier series.



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## 1 PRELIMINARY

The author does not aim to present the usual analytical developments of the stiffened sheathings method — the source of the L.B.R.-3 software. Only developments specifically related to the naval architecture field will be presented in this paper. They will be introduced by a short summary of the methodology. More details can be found in an earlier study by the author[1] or in other publications concerned with this method.[2, 3]

The reader must always keep in mind that the presented method is not able to compute all the structures included in the shipbuilding field. LBR-3 is a specific tool designed for a specific range of applications — the prismatic section of a ship which is at least 100 m long. In the future, this method should be able to help the designer because it is easier, faster and cheaper, and it provides a detailed stress analysis although it needs fewer data than much other commercial software used for this type of computation. LBR-3 will never be able to be used to study structures such as the bow, the stern, or pleasure craft, etc. It will only be an accurate tool complementary to conventional methods.

## 2 INTRODUCTION

Even with the computer science improvements of the last decade, to date there are still few ships computed with the help of a method which gives an accurate stress analysis and which takes into account interaction between all the different elements. Indeed, many structures in naval architecture are composed of several hundreds or thousands of elements (such as skin plating, stiffeners, deck beams, web frames, longitudinals and stringers). Ships and tankers are large structures (Figs 1 and 2) composed of several hundreds of elements which interact with each other[4]. Taking these interactions into account, their design is nowadays possible thanks to the emergence of powerful software. The stiffened sheathings software L.B.R.-3 is one of these programs.

The presented method lies between the conventional beam bending theory and finite element analysis (FEM). It has all the advantages of the traditional beam theory without its disadvantages. With the conventional theory, results of general bending are added to those arising from the local stiffeners bending and to those resulting from local unstiffened skin plate deformation. The behaviour of each element is analysed separately from the general behaviour of the complete structure.

In recent years,[5] digital simulation and FEM improvements have not changed this situation very much. One explanation could be that with

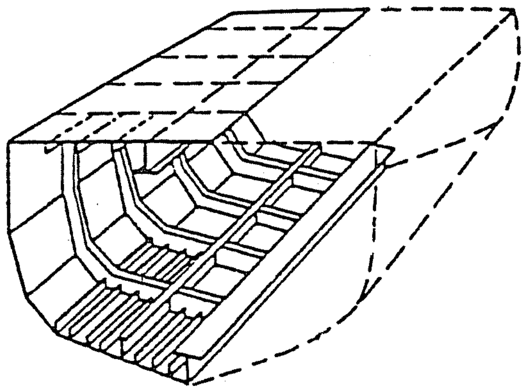


**Fig. 1.** Example of a prismatic structure which is the main application field of the LBR-3 method.

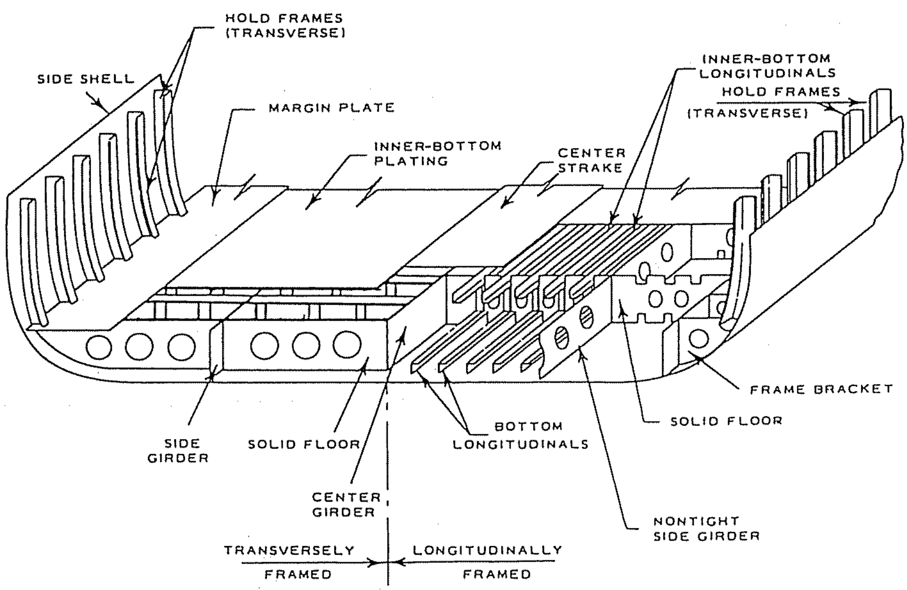



**Fig. 2.** Example of a prismatic structure of a ship which can be computed with LBR-3.[4]

conventional commercial software there is a conflict between cost and accuracy. It is always possible to discretize a system more finely, but the computer storage capacity, the time needed to discretize the system and the budget may become limiting factors. Keeping these limitations in mind, the author would like to propose a new computerized method which can be economically used for a specific range of ship structures, namely; the prismatic section of the ship.

The LBR-3 software is as easy to use as beam bending theory even if the theory used to establish the equations is very complex. Advantages of the LBR-3 methods are:

- Rapid, cheap and accurate determination of the stresses ($\sigma_x$, $\sigma_\phi$), the shear stresses ($\tau_{x\phi}$) and the displacements ($u$, $v$, $w$) can be obtained at every point of a structure (skin plating, stringers, deck beams, girders, longitudinals, web frames, in their webs and flanges, and at the web–flange and web–skin junctions). Stresses can be obtained at every point without the problems of interpretation that sometimes occur with the FEM analysis.
- The method easily allows the study of plate and shell panels (see Figs 3 and 4) which are heavily stiffened (including longitudinal stiffeners such as girders, stringers and longitudinals, and transverse stiffeners such as web frames). Calculations are not based on 'smeared-out' rib properties; rather, the torsional stiffness, the lateral (tangent to the shell) bending stiffness and the exact position of each stiffener are taken into account. Such parameters are usually not taken into account by available commercial methods.
- In shipbuilding, the number of longitudinal and transverse stiffeners (all types included) is so large that treating them by the usual stress analysis methods may often become very cumbersome. In this paper, using as an example the computation of a 103-m sea–river-going vessel, it will be shown that the most important advantage of the LBR-3 method is the ease of modelling of the structure and the short time needed to generate the required data

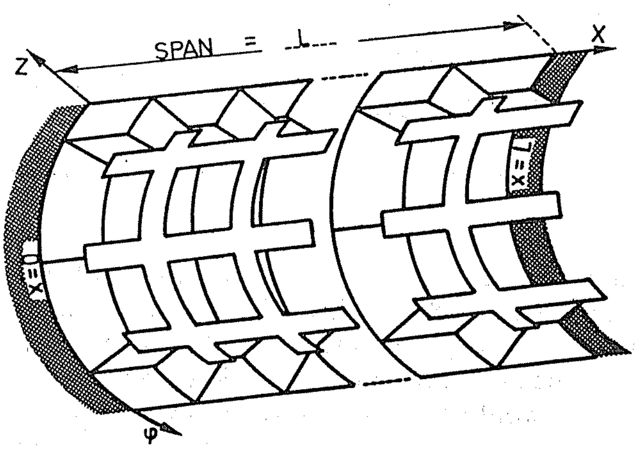



**Fig. 3.** Orthotropic prismatic shell panel.

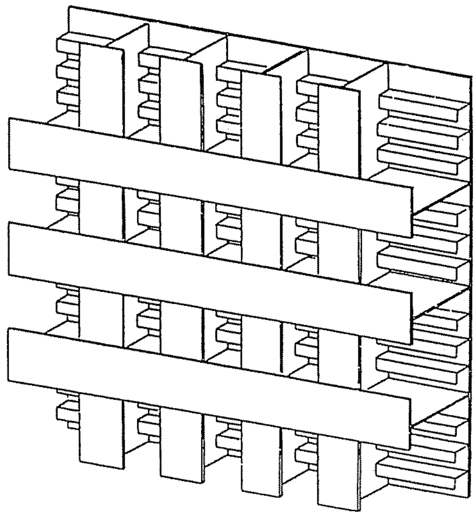


**Fig. 4.** Stiffened plate panel (longitudinals, girders, web frames, etc.).

(fewer than 120 data lines are enough for this ship). Moreover, the first results can be obtained within 12 h. Each stiffened shell or plate is considered as one 'panel' (Figs 3 and 4). A 'panel' has the meaning of a 'strip' in the finite strip method. However, a panel includes without difficulties all the stiffeners which are present. For each panel of a structure, analytical relationships of the solution stresses ($\sigma_x$, $\sigma_\phi$, shear stresses $\tau_{x\phi}$, and displacements $u$, $v$, $w$) are obtained. The method takes into account the compatibility conditions and the equilibrium equations which must be satisfied at the junction between two or more panels. Therefore, no problem occurs related to the transfer of shear stresses between two panels. The shear effects inside panels and at their junctions are fully and accurately modelled.

Nevertheless, the method has a major disadvantage. It is limited to the prismatic section of the ship, neglecting the bow and the stern. It is clear that this method will not allow the analysis of every kind of structure as is possible with the FEM. Therefore, it is necessary to repeat that the aim of the method is limited. The intent was only to develop an efficient tool to obtain a detailed and accurate stress analysis of the prismatic section of the ship.

Except for this restriction, no other limitations or approximations are made. There are no orthotropic plate assumptions. Heavily stiffened and double skin structures can be analysed without difficulties. Internal decks and longitudinal bulkheads can be modelled using plate panels, i.e. the No. 4 panel is a longitudinal bulkhead (see Figs. 15 and 16 later). Displacements and stresses (shear stresses included) are obtained at every point on the panels (skin plating, longitudinals, web frames, etc.). Loads resulting from deck-houses or other structures not extending the full length of the ship can be applied to the main structure.

## 3 METHOD

The method[1,3] is based on an analytical resolution of the differential equations governing orthotropic cylindrical shells, the shell thickness being constant (Fig. 5). These differential equations result from the DKJ method (Donnell, von Kármán and Jenkins). Within the linear DKJ method, no additional restrictions have been introduced on the orthotropic shell with 'smeared stiffener' theory, the torsional stiffness or the lateral bending stiffness.

The basic element is a cylindrical orthotropic shell of length $L$ and radius $q$. The coordinate system is presented in Fig. 5, where the $x$-axis is along the prismatic generator, the $\phi$-axis is along the circumference and the $z$-axis is perpendicular to the shell. Displacements ($u$, $v$ and $w$) are associated with the $x$-, $\phi$- and $z$-axes respectively. The reference shell is situated between the external and the internal surface of the plating.

The main loads are (Fig. 6):

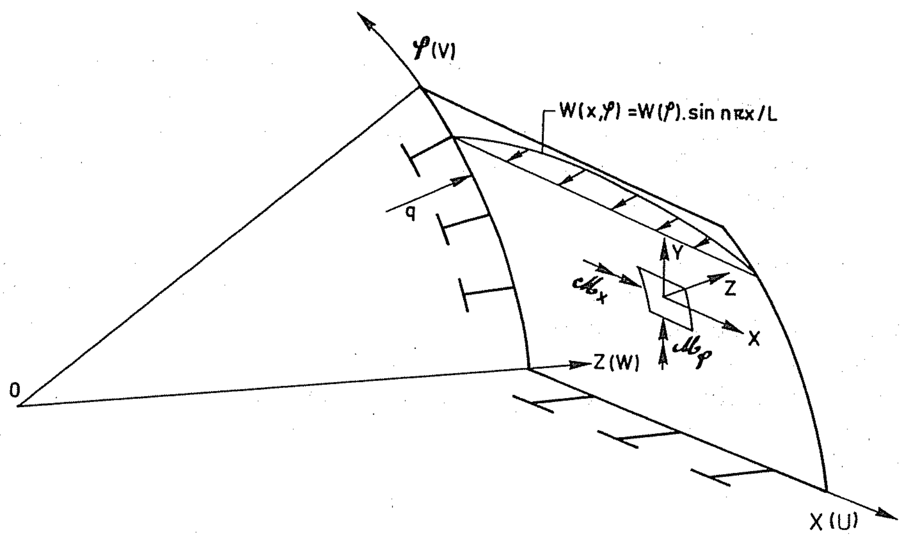



**Fig. 5.** Cylindrical shell panel and its coordinate system.

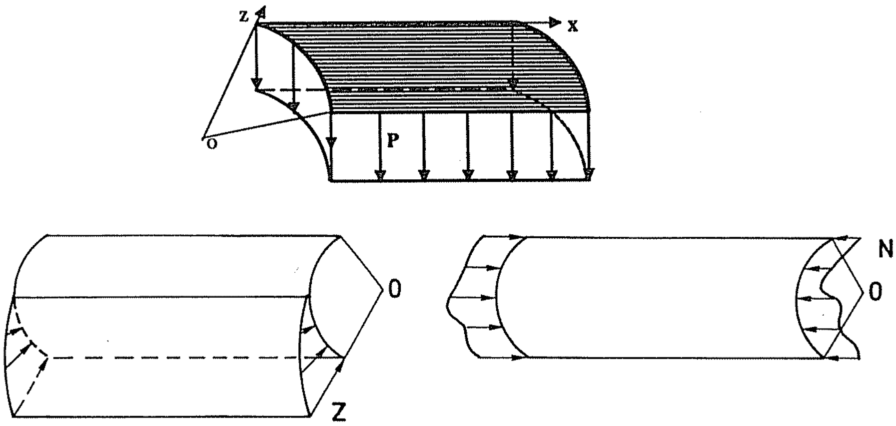



**Fig. 6.** Loads $Z$, $P$ and $N$.

— the pressure loads $Z$, such as hydrostatic pressure, which act perpendicular to the shell;
— the vertical loads $P$, such as dead load, live load, deck-house, crane, etc.;
— the longitudinal forces (end forces $N$) which are applied at the ends along the $x$-axis. These forces allow for the introduction of a bending moment at the ends of the prismatic structure.

To solve analytically the differential equations, displacements $(u, v, w)$ and loads $(Z, P, N)$ are decomposed along the $x$-axis, using a Fourier series (Fig. 5); thus, the two edges ($x = 0$ and $x = L$), between which the Fourier series expansions are applied, are simply supported ends (Fig. 7). Indeed, for $x = 0$ and $x = L$, $v$, $w$, $N_x$ and $M_x$ are zero (sine series) but $u$, $N_{x\phi}$, $M_{x\phi}$ and $Q_x$ are not zero (cosine series).

These descriptions, presented for shell panels (Fig. 3), are also valid for orthotropic plate panels (Fig. 4). Indeed, the relationships for stiffened plate panels can be established using a simplified form of the stiffened shell equations. The radius $q$ must be taken as equal to infinity ($q \gg 10 \times 10^{10}$ m) to find an accurate form of the orthotropic plate equations.

## 4 ASSUMPTIONS AND COMPUTATION OF PRISMATIC STRUCTURES RELATED TO NAVAL ARCHITECTURE

The following developments were undertaken to extend the L.B.R.-3 application range to the design and the computation of large ships which

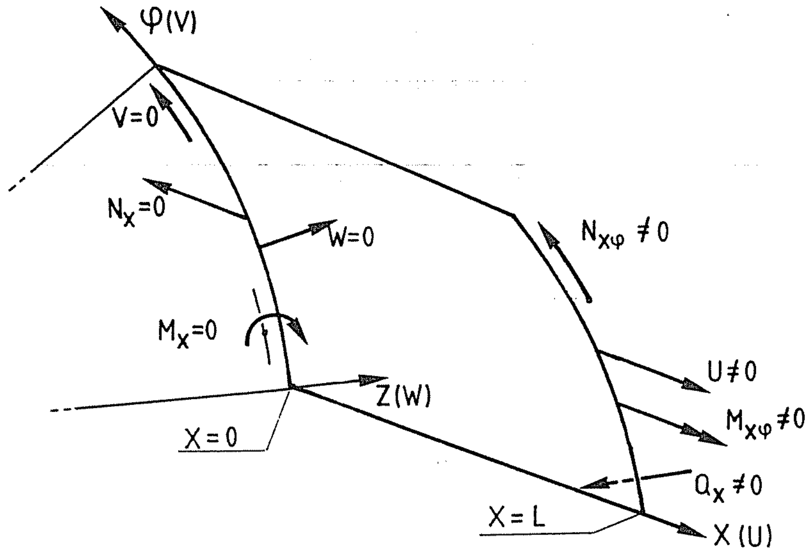



**Fig. 7.** End conditions of a panel which is simply supported at the two edges ($x = 0$ and $x = L$).

have a long prismatic section (70–85% of the length). As indicated, the aim of the method is not to study the bow and the stern. This application is mainly for barges for fluvial navigation, and tankers, bulk carriers, etc. for maritime navigation. For such structures, the maximum longitudinal bending moment usually occurs inside the prismatic section and the maximum shear forces near an end of this section. Effects of the bow and stern on the prismatic section are taken into account but the stress analysis cannot be carried out in these elements.

For each ship investigation, the first step (Fig. 8) is to determine the prismatic section of the ship, composed of shell and plate panels,

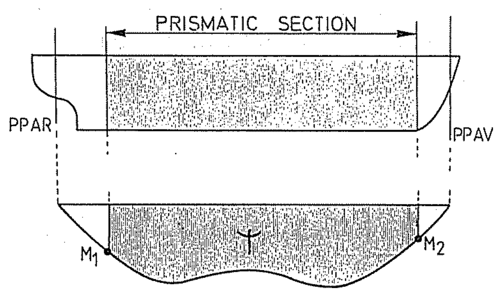



**Fig. 8.** Longitudinal view of the ship to determine the length of the prismatic section and the end moments ($M_1$ and $M_2$).

stiffened by stringers, longitudinals, deck beams, web frames, etc. These shell and plate panels are bound together by common generators along the $x$-axis. Therefore, together they accurately model the ship cross-section. For instance, Fig. 9 shows the modelling of the structure presented in Fig. 1. It is composed of two shell panels and five plate panels, each with its own stiffening.

The $M_1$ and $M_2$ end moments can be obtained using the longitudinal bending moment diagram on the ship (Fig. 8). These moments are applied (Fig. 10) at the ends of the prismatic section of the ship by way of a suitable distribution of $N$ longitudinal forces so that the effects of the bow and the stern on the prismatic section are taken into account.

Therefore, as far as the computation is concerned, it is as if a simply supported box-girder (the prismatic section) loaded with end moments ($M_1$ and $M_2$ applied at the ends) and loaded by the real loads ($P$ and $Z$) acting along the span of the structure (hydrostatic pressure, tanker hydrostatic pressure, dead load, live load, freight, loads resulting from deck-houses and other structures not extending the full length of the ship, etc.) is studied. Moments ($M_1$ and $M_2$) and loads ($P$ and $Z$) are different for every load situation.

The simply supported ends of the considered prismatic section behave like vertical partition walls which are infinitely rigid in their planes

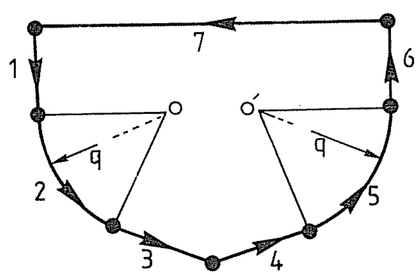



**Fig. 9.** Ship cross-section and its modelling.

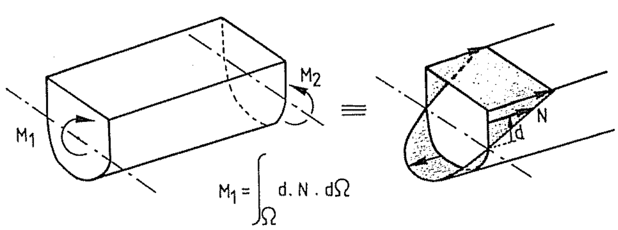



**Fig. 10.** Schematic representation of the end moments and their method of application to the structure.

($v = w = 0$). The longitudinal displacements remain free ($u$ being free and warping allowed for). Therefore Poisson effects and tranverse bending are inhibited at these ends. This assumption is fully valid when rigid frames or transverse bulkheads are located at the ends, which is usually the case.

### 4.1 Fourier series expansions related to the naval architecture field

When the stiffened sheeting method was developed,[3] its application range was mainly hydraulic structures such as lock gates or navigation dams. Loads (dead load $P$ and pressure $Z$) were constant along the longitudinal $x$-axis. For instance, the hydrostatic pressure acting on a lock gate varies only with depth, the load distribution being the same for every cross-section.

Other types of load exist in ship structure studies. Indeed, the loadings vary longitudinally and there are also many confined loads (deck-houses, cranes, etc.) acting on short parts of the ship. Therefore, it was necessary to study Fourier series expansions related to loads varying gradually along the longitudinal axis. Figure 11 shows the theoretical longitudinal distribution of a ship's loading and the load diagram obtained by use of a Fourier series expansion.

Another Fourier series expansion specific for ship design is necessary because of the inequality of the longitudinal forces $N$ acting at the end sections ($M_1$ and $M_2$, Fig. 8). For the study of a radial gate of a navigation dam,[1] the forces acting in the arms of the gate were symmetrical, so the end resulting bending moments in the box-girder of the radial gate were also symmetrical. On the other hand, ship loads are non-symmetrical

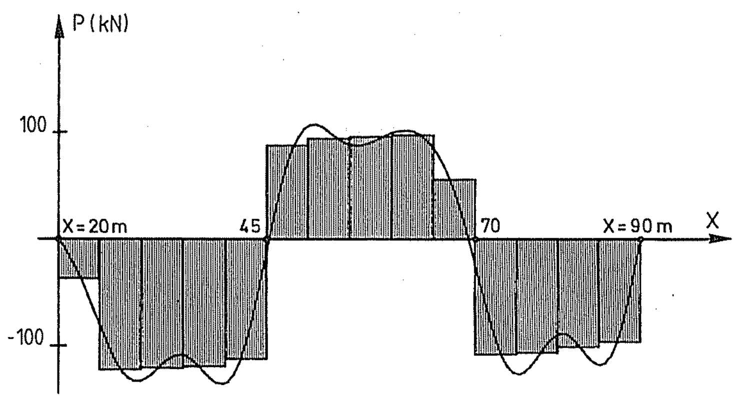



**Fig. 11.** Theoretical longitudinal distribution of the vertical loads and those resulting from the Fourier series expansion (13 terms).

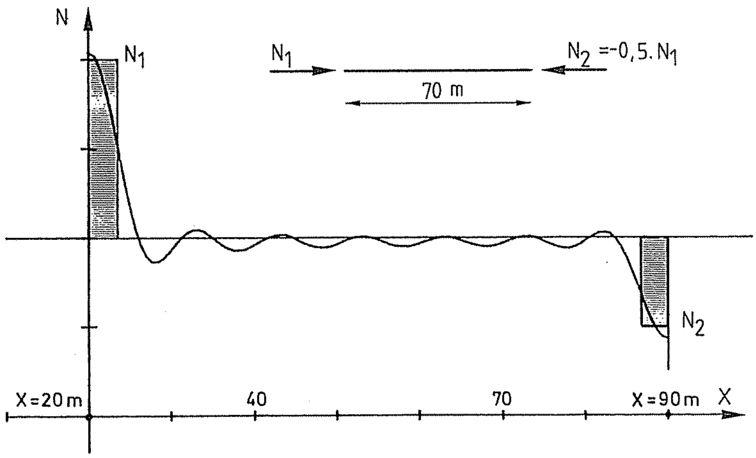



**Fig. 12.** Fourier series expansion (13 terms) of the longitudinal forces $N$ applied at a ship's ends. Case of two different forces ($N_2 = -0{\cdot}5N_1$).

although the structure of the prismatic section remains symmetrical. Thus, all the Fourier series expansions used for ship structure applications are different. Figure 12 shows the Fourier series expansions of non-symmetrical forces (for instance, $N_2 = 0{\cdot}5N_1$) acting at the ends of a structure.

The presented design is for a sea–river-going vessel of 103-m length, 12-m width and 5·2-m depth.[6] Its maximum capacity is 4340 $m^3$ (10 × 370 $m^3$) + (2 × 320 $m^3$). The length of the prismatic section of the ship is almost 70 m ($x$ = 20–90 m) and relates to the 12 cargo holds (Fig. 13).

The transverse structure of the prismatic section of the tanker is presented in Figs 14 and 15; Fig. 16 shows the modelling adopted for the system. The structure was discretized by seven plate panels, each of them being stiffened with several types of stiffeners.

At the time of the study, the designer analysed 30 load cases. As it is not possible to present every case here and as the aim in this application is to illustrate the computation method, only the load case of Fig. 17 will be studied. Its main features are 3·10 m draught, −0·008-m trim and 24 300-kN freight. Taking into account this load case, Fig. 11 presents the theoretical distribution of the vertical loading and that resulting from the Fourier series expansion.

According to the position of the prismatic section of the tanker (see Fig. 13), the end bending moments ($M_1$ and $M_2$) are obtained from the longitudinal bending moment diagram of the ship (Fig. 18) computed for the corresponding load case (Fig. 17).

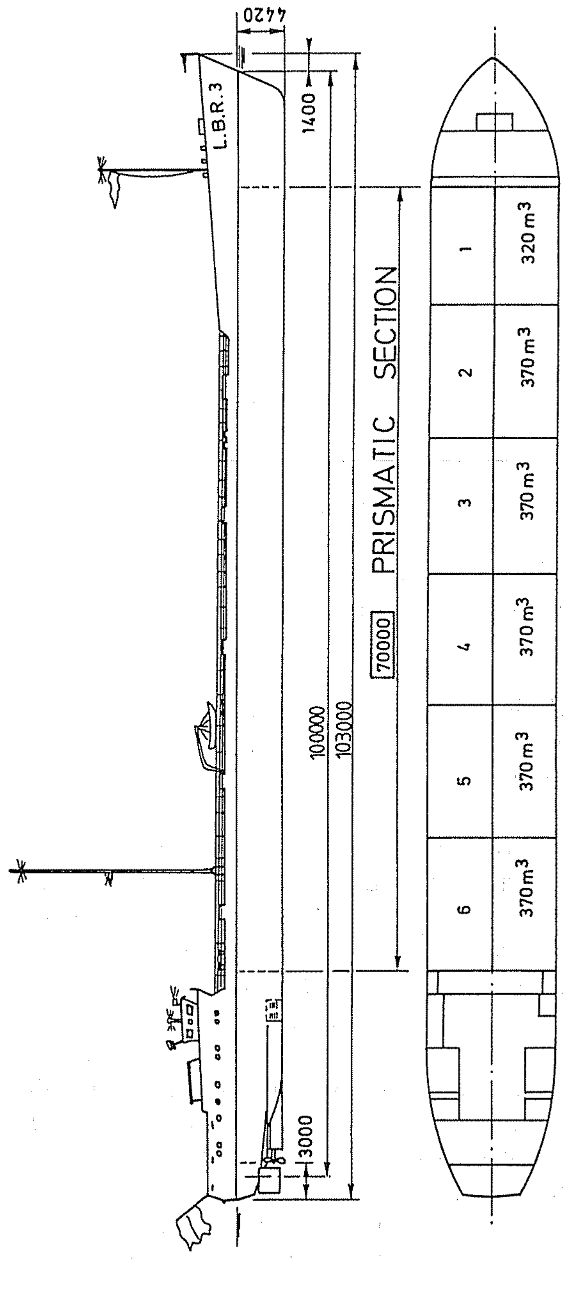



**Fig. 13.** Plan and side view of the tanker.

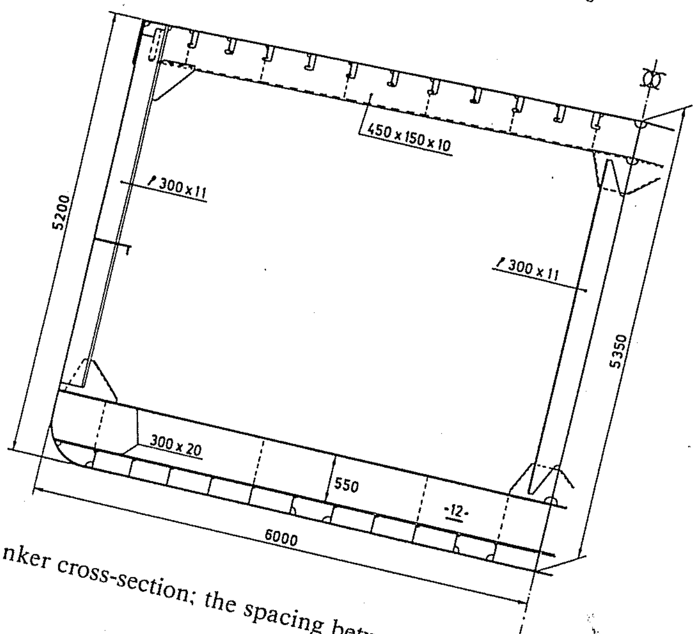
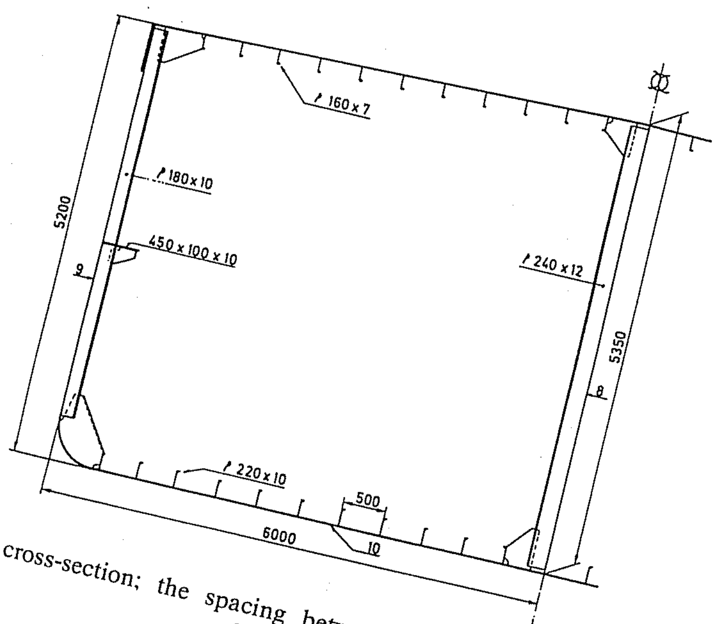



**Fig. 14.** Tanker cross-section; the spacing between web frames is 3·00 m.




anker cross-section; the spacing between small transverse stiffeners is 0·50 m.

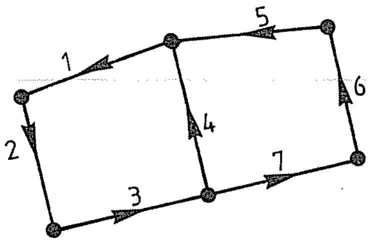



**Fig. 16.** Modelling of the prismatic section of the tanker using seven orthotropic plate panels.

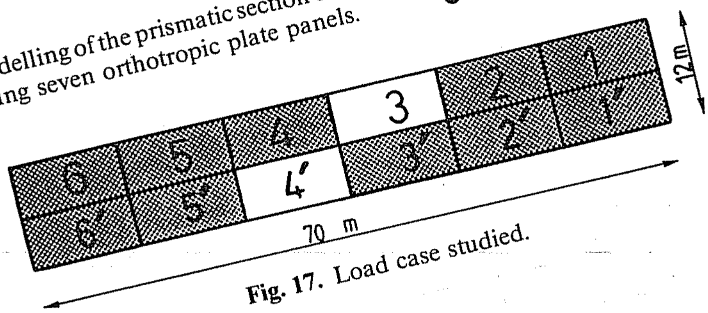



**Fig. 17.** Load case studied.

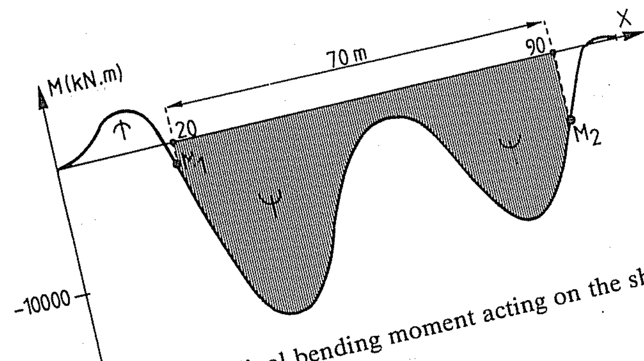



**Fig. 18.** Longitudinal bending moment acting on the ship.

After discretization, use of the Fourier series expansion is necessary to solve the differential equations and thus to obtain the analytical relationships of each displacement. Substituting these displacement into the stress–displacement relationships, the analytical stress relation ships including the shear stresses are obtained.

From practical considerations, 13 terms of the Fourier ser expansion are necessary to obtain diagrams shown in Figs 19–21. In skin plating these figures respectively show the longitudinal stresse the transverse stresses $\sigma_\phi$ and the shear stresses $\tau_{x\phi}$; the diagram presented respectively for cross-sections $x = 35$ m, $x = 60$ m and $x =$ The $x = 35$ m abscissa corresponds to the maximum bending m (see Fig. 18), $x = 60$ m to the No. 3 compartment position and $x =$ to one end of the prismatic section.

Stress discontinuities occur at the corners in the diagram longitudinal stresses (Fig. 19) because the stresses $\sigma_x$ are influe the value of the stresses $\sigma_\phi$. These transverse stresses $\sigma_\phi$ ch

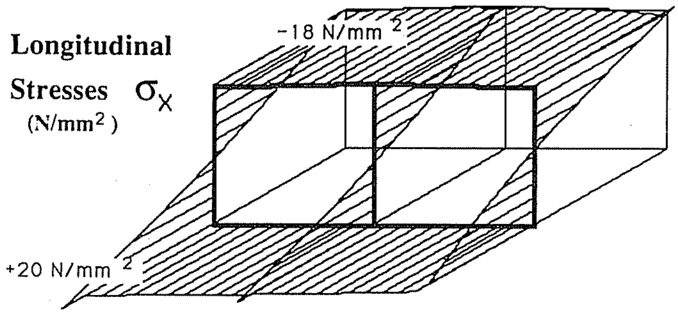



**Fig. 19.** Longitudinal stresses ($\sigma_x$) in the skin plating at cross-section $x = 35$ m.

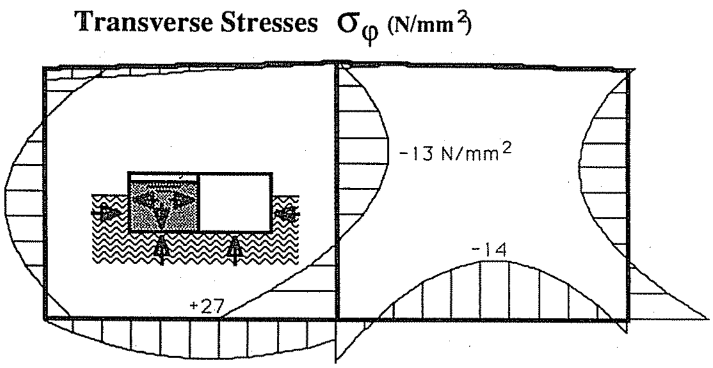



**Fig. 20.** Transverse stresses $\sigma_\phi$ in the skin plating at cross-section $x = 60$ m.

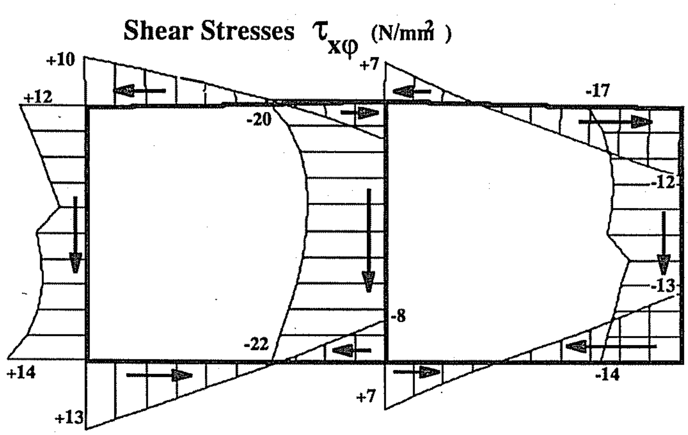



**Fig. 21.** Shear stresses ($\tau_{x\phi}$) in the skin plating at cross-section $x = 20$ m.

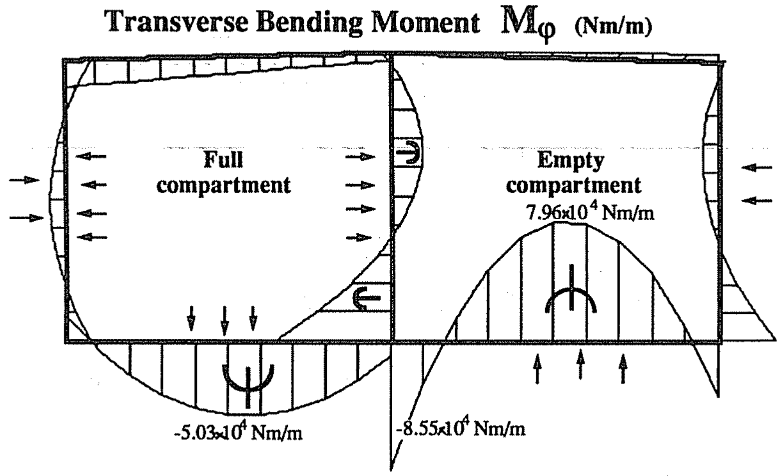



**Fig. 22.** Transverse bending moment ($M_\phi$) in the panels at cross-section $x = 60$ m.

rigidity variation occurs between two adjoining panels (see Fig. 20), for instance, in the thickness of the skin plating and above all the size of the stiffeners.

On the other hand, force and bending moment equilibrium are verified, as shown in Fig. 22. This diagram shows the load effects induced by a full compartment located next to an empty one. This diagram is similar to the transverse bending moment distribution obtained with the Hardy Cross method.

The highest stresses were noted in the longitudinal bulkhead frame flanges (plate No. 4, Fig. 16) which are located between the No. 3 and the No. 3′ compartments (Fig. 17). Indeed, near this abscissa, there are two compartments, one of which is empty and the other full, so the longitudinal bulkhead between these two compartments is heavily loaded. The maximum combined stresses were found in the bulkhead frame flanges. This frame has a behaviour intermediate between that of a simply supported beam ($Pl^2/8$ at mid-span) and a fixed beam ($-Pl^2/12$ at the ends and $Pl^2/24$ at mid-span). As it is the most heavily loaded, this bulkhead transmits some load to the main deck and some to the bottom, which are less heavily loaded. This phenomenon explains the shape of the $M_\phi$ transverse bending moments diagram (Fig. 22).

The maximum longitudinal bending moment computed according to the 'Bureau Veritas' rules[7] was 58 000 kN m. The maximum stresses corresponding to this bending moment were determined. The combined stresses in the skin plating are shown in Fig. 23.

With the L.B.R.-3 software, relatively few data are required to study

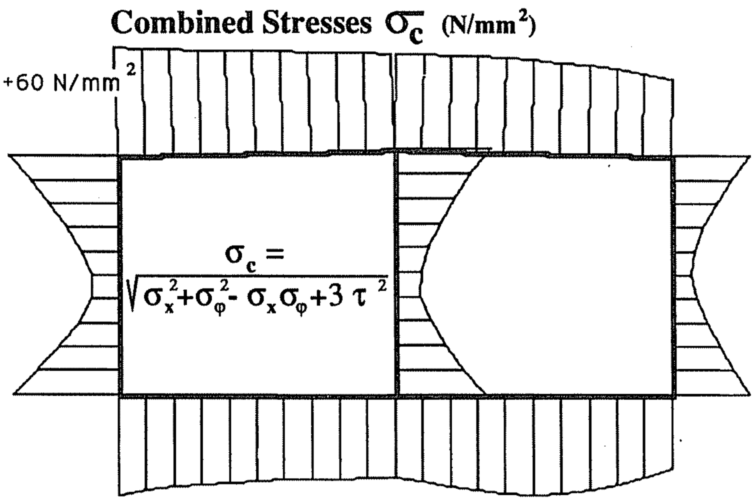



**Fig. 23.** Combined stresses in the skin plating at the cross-section of maximum bending moment (58 000 kN m).

such a tanker; fewer than 120 data lines are enough. An example of the required data is shown below. The discretization of the system and the data generation take less than 2 h. Moreover, the computer time, including the printing time, is less than 1 h. Therefore, the first results can be obtained within a day.

It is obvious that the complete study of a long prismatic ship with CAD software is possible, can be quickly executed and, moreover, does not require long training.

### 5.1 Example of data required by the LBR-3 software

| | |
|---|---|
| TANKER, *M*max = 58 000 kN m | Title |
| 0 −2 0 0 1 7 0 7 | Output parameters, number of terms (Fourier series) and number of panels |
| 2·1E + 11 0·3 70· | Young's modulus, Poisson's coefficient, span |
| 20· 35· 50· 70· 90· | Results will be obtained at $x$ = 20 m, . . . and $x$ = 90 m |
| 0 | Dead load (1 yes, 0 no) |
| PLATE 1 | Panel No. 1 |
| 6·002 | Height of the panel |
| 3·0 0·5 0·012 | Web frame spacing, longitudinal spacing, skin plating thickness |

| | |
|---|---|
| 0·44 0·01 0·15 0·01 | Web frame dimensions |
| 0·1446 0·007 0·029 0·0154 | Longitudinal dimensions |
| 0 1 1 1 0 0 1 | Web frame and longitudinal parameters, load parameters |
| 178·568 0· 0· | Slope of the panel related to a horizontal line, loads |
| 0 | Key in 0 or 1 (0 without overloads, 1 with overloads) |
| 2 0 0 0 0 0 0 0 0 0 | Panels (No. 2) which follow this panel (No. 1) |
| PLATE 2 | Panel No. 2 |
| . . . | |
| etc. | Panels Nos 3, 4, 5 and 6 |
| . . . | |
| PLATE 7 | Panel No. 7 |
| 6· | Height of the panel |
| 3· 0·5 0·01 | Web frame spacing, longitudinal spacing, skin plating thickness |
| 0·74 0·012 0·3 0·02 | Web frame dimensions |
| 0·1975 0·01 0·041 0·0225 | Longitudinal dimensions |
| 0 1 1 1 0 0 1 | Web frame and longitudinal parameters, load parameters |
| 0· 0· 0· | Slope of the panel related to a horizontal line, loads |
| 1 | Key in 0 or 1 (0 without overloads, 1 with overloads) |
| 14 | Number of spacings for the overloads |
| −18·66 | Load (kN/m between $x$ = 20 and 25 m) |
| −61·475 | Load (kN/m between $x$ = 25 and 30 m) |
| etc. | |
| −50·845 | Load (kN/m between $x$ = 80 and 85 m) |
| −48·375 | Load (kN/m between $x$ = 85 and 90 m) |
| 6 0 0 0 0 0 0 0 0 0 | Panels (No. 6) which follow this panel (No. 7) |
| Extreme bending moments | Title |
| 4458·52E + 04 4763·43E + 04 | Bending moment at the ends of the prismatic structure |
| 1 2 3 5 6 etc. 19 20 | Graphic output parameters |

## 6 CONCLUSION

The advantages of the stiffened sheathings software are numerous: simple discretization (few data are needed), speed, simplicity, and above all, ease of use. Taking into account the analytical resolution possible in the elastic regime and the accuracy of the Fourier series expansions, results are always clear, accurate and complete. Stresses and displacements can be obtained at every point of the structure (skin plating, stringers, girders, longitudinals, web frames, etc.).

Although the aim in this study was not to comment upon the design quality of the tanker considered (every loading case has not been checked), according to this limited study, the maximum stresses obtained look acceptable.

This stress analysis example of a tanker shows a new application for the L.B.R.-3 software, to the design of long ships (such as tankers or barges). This approach is also synonymous with the economical utilization of steel. Indeed, structural optimization produced with a tool such as L.B.R.-3 should help to reduce the structural weight and the cost in a non-negligible way.

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