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AUGUST 14 - 18

International colloquium in mechanics of solids, fluids, structures and interactions

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APPLICATION OF THE STIFFENED PLATE METHOD IN OPTIMIZING HYDRAULIC STRUCTURES

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Abstract

Use of stiffened plates is very common in a large range of naval and civil engineering structures. Such is the case for hydraulic structures with lock gate, movable weir, canal bridge,... and in shipbuilding where stiffened plates are the basic components of ships, vessels, barges and in many components of offshore structures. This paper introduces an effective method to facilitate and improve the design of these stiffened structures. With this model (called LBR-5), even at the preliminary design stage, we can perform a least cost structural optimization. LBR5 is a judicious combination between a strength analysis module and an optimization module and is particularly relevant to perform design assessment at the preliminary stage. To illustrate this concept, the method is applied to a 36-m span floating gate designed for the "Canal du Centre" in Belgium.

Keywords
Stiffened panel, optimization, floating gate.

1. INTRODUCTION

The guidelines and major orientations of a structural design are always defined during the earliest phases of a project, i.e. the preliminary design stage or the first draft that corresponds in most cases to the offer. It is thus not difficult to understand why an optimization tool is attractive, especially one designed for use at the preliminary stage. This is precisely the way the LBR-5 optimization software for stiffened hydraulic and naval structures was thought through, created and developed [Rigo 1998, 1999].

The target is to link standard design tools (steel structure CAD, hull form, hydrostatic curves, floating stability, weight estimation...) with a rational optimization design module that, as of the first draft (or preliminary design), allows for:

- a 3D analysis of the general behaviour of the structure, or at least of the basic transverse cross-section (midship section);
- to explicitly take into account all the relevant limit states of the structure (service limit states and ultimate limit states) thanks to a rational analysis of the structure based on the general rules of solid-mechanics and structure behaviour. By rational analysis, we mean a coherent and homogeneous group of analysis methods based on physics, solid mechanics, strength and stability treatises, etc. and that differ from empirical and parametric formulations;
- as of the first draft, an optimization of the sizing/scantling (profile sizes, dimensions and spacing) of the structure's constituent elements;
- integration of construction and manufacturing costs in the optimization process (through the cost objective function).

The advantages of this optimization module appear mainly at the preliminary stage. It is indeed during the first stages of the project that flexibility, modelling speed and method's easy use provide precious help to designers. At this moment, few parameters/dimensions have been definitively fixed and a coarse modelling by standard finite elements is often unusable for reasons of budget, modeling duration, available data,... and this, particularly for design offices and modest-sized yards (small and medium-sized enterprises).

This optimization module can also be used in final stage of the project to perform a general verification or to refine the scantling.

Application fields of LBR-5 include hydraulic structures and naval structures. For the former, the application domain is clearly the ship's central parts (cylindrical and prismatic zones of cargo ships, passenger vessels, etc.). This zone is the most important in length for the big floating units. For smaller units (sailboats, small craft, etc.), the cylindrical zone is smaller, or even non-existent. In this case, the LBR-5 model can be used to perform transverse cross-section optimization (midship section).

The LBR-5 optimization tool is based on important know-how in the stiffened structures domain that was materialised in 1988 by the development of the LBR-4 linear analysis software for stiffened structures analysis [Rigo 1992a and b]. The scientific environment in which the optimization part was developed mainly concerns naval architecture. This work was made possible by unifying analysis methods and by using rational approaches to assess structure limit states. LBR-5 definitively favors an unified optimisation approach.

2. LBR-5 AND THE CONCEPT OF OPTIMIZATION-ORIENTED MODULES

The general problems to be solved can be summarised as follows:

- X_i $i = 1, N$, the N design variables.
- $F(X_i)$ the objective function to minimize.
- $Cj(X_i) \leq CM_j$ $j = 1,M$ the M structural and geometrical constraints.
- $X_{i\,min} \leq X_i \leq X_{i\,max}$ upper and lower bounds of the X_i design variables: technological bounds (also called *side constraints*).

The objective is to create a user-oriented optimization technique, in permanent evolution, i.e. that evolves with the user and his individual needs. We define these as "Programming-Oriented Modules". The LBR-5 optimization model is based on this new concept and is composed of several modules. Neither the module number nor their type is imposed. At the start, the whole model is made up of 3 basic modules (Figure 1) and forms the framework of the tool (COST, CONSTRAINT and OPTI).

Figure 1 shows the basic configuration of the LBR-5 software with the 3 fundamental modules (COST, CONSTRAINT and OPTI) and the "DATABASES" in which the user can do his "shopping", i.e. choose the relevant constraints and cost data. Figure 3 succinctly shows the LBR-5 software chart.

With regard to structural constraints, the user must first choose the types of constraints (yielding, buckling, deflection, etc.) then, for each type of constraint, select the method, the code or the rules to use and finally the points/areas/panels where these constraints will be applied.

The structure is modelled with stiffened panels (plates and cylindrical shells) (Figure 2). For each panel one can associate, up to 9 design variables (XI). These 9 design variables are respectively:

- Plate thickness.

- For longitudinal members (stiffeners, crossbars, longitudinals, girders, etc.):
  - web height and thickness,
  - flange width,
  - spacing between 2 longitudinal members.
- For transverse members (frames, transverse stiffeners, etc.):
  - web height and thickness,
  - flange width,
  - spacing between 2 transverse members (frames).

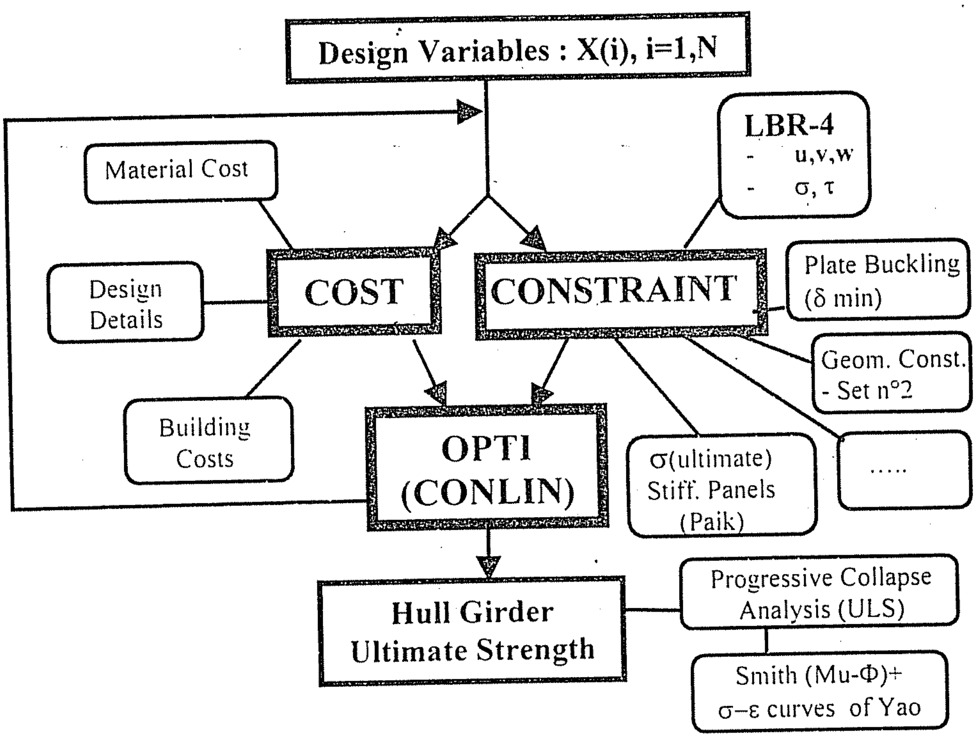

Figure 1: Basic configuration of the LBR-5 optimization model

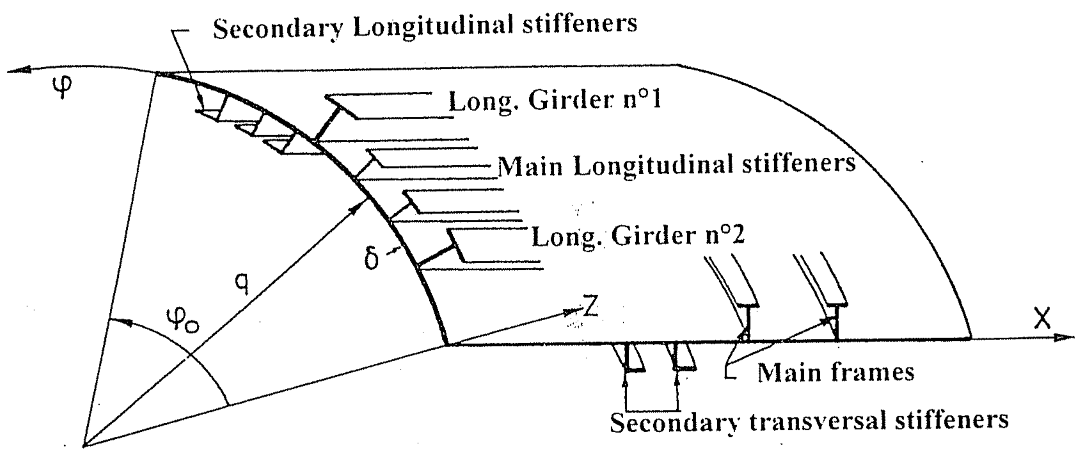

Figure 2: Basic stiffened panel (or basic element) used to model the structures.

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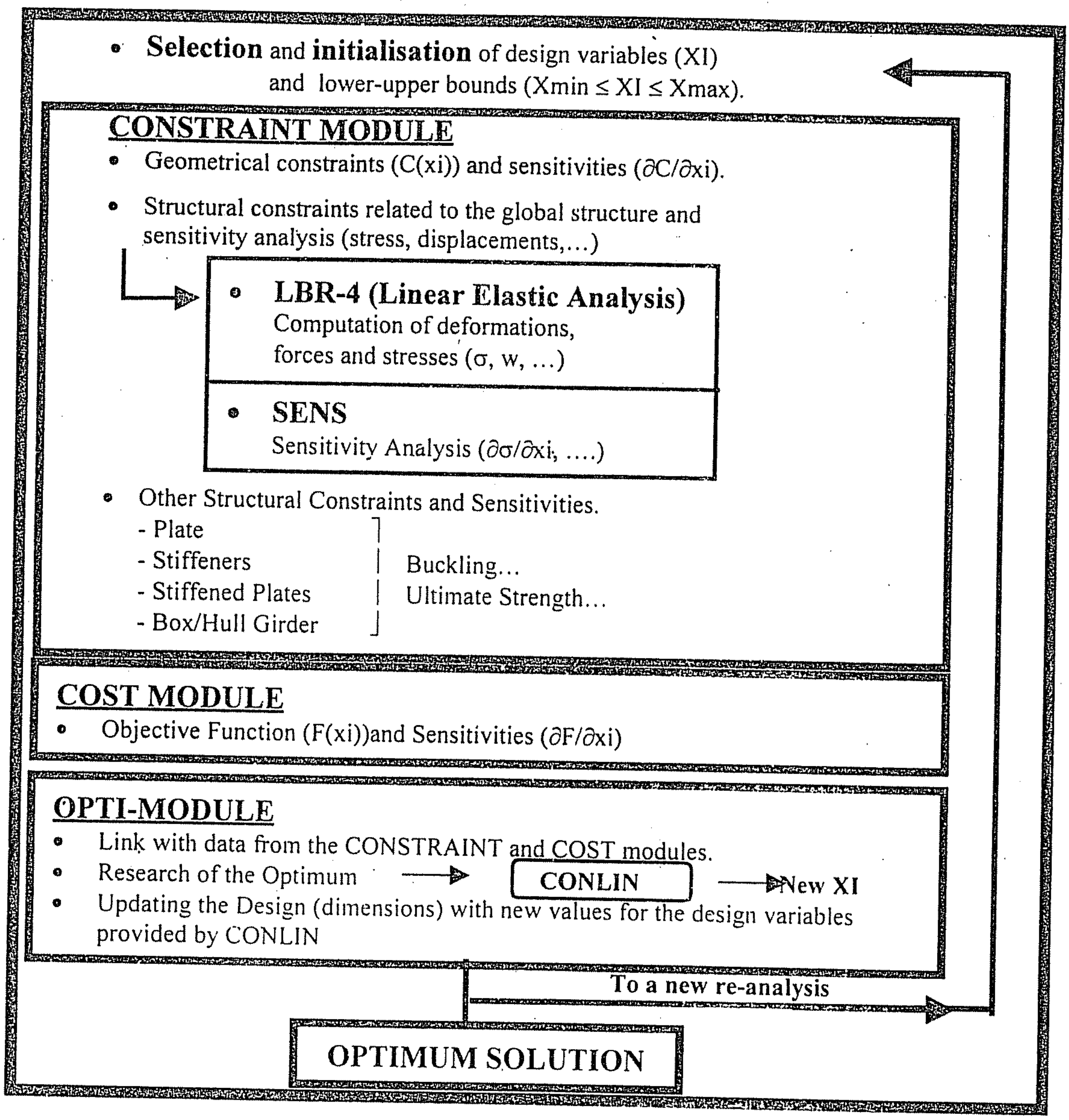

Figure 3: Chart of the LBR-5 model with CONSTRAINT, COST and OPTI modules

3. DESCRIPTION OF THE 3 BASIC MODULES: OPTI, CONTRAINT AND COST

The **OPTI module** contains the mathematical optimization algorithm (CONLIN) that allows solving non-linear constrained optimization problems. It is especially effective because it only requires a reduced number of iterations. In general, fewer than 15 iterations (including a structure re-analysis) are necessary, even in presence of several hundred design variables (XI) (Figure 4). CONLIN is based on a convex linearization of the non-linear functions (constraints and objective functions) and on a dual approach [C. Fleury, 1989 and 93]. This module uses as inputs the results/outputs of the two other basic modules, i.e. CONTRAINT for the C(XI) constraints and COST for the F(XI) objective function.

The main difficulty in solving a dual problem is dealing with the non-linear and implicit constraints. In order to avoid a large number of time-consuming re-assessments of these non-linear and implicit functions, Fleury suggests applying convex approximations. At each iteration, all the functions (objective function and constraints) are replaced by an approximation called 'convex'. In a word, the complex initial optimization problem is decomposed in a sequence of more simple convex optimization problems (obtained through a convex linearization) that can be easily solved using a dual approach (Figure 4).

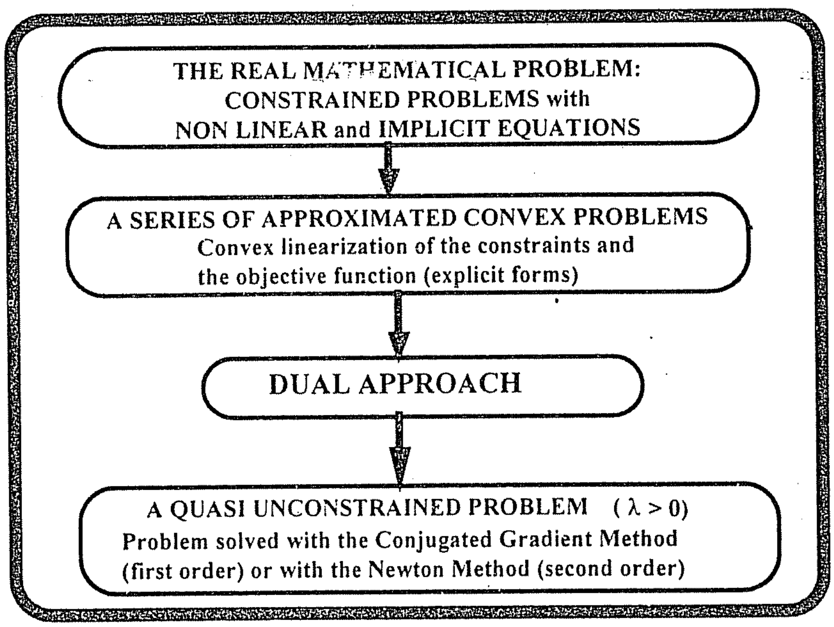

Figure 4:
The CONLIN model:
convex approximations
and
dual approach.

The **CONSTRAINT module** helps the user to select relevant constraints within constraint groups at his disposal in a databank (Figure 1). In fact, the user remains responsible for his choice. However, in order to facilitate this selection, several coherent constraint sets are proposed to the user. These sets are based on national and international rules/codes (Eurocodes, ECCS Recommendations, Classification Societies, etc.).

To date, only a limited number of modules are available (in general 1 or 2 for each constraint type). It is up to the user to complete, adapt and add new modules according to his specific requirements (type of structure, codes and regulations to be followed, technical and scientific level, available hardware, etc.). The objective is to enable the user himself to build the tool he needs.

Constraints are linear or non-linear functions, either explicit or implicit of the design variables (XI). These constraints are analytical "translations" of the limitations that the user wants to impose on the design variables themselves or to *parameters* like displacements, stresses, ultimate strength, etc.

So, one can distinguish:

- Technological constraints (or side constraints) that provide the upper and lower bounds of the design variables.
  For example: Xi min = 4mm ≤ Xi ≤ Xi max = 40 mm,
  with: Xi min a thickness limit dues to corrosion, etc;
  Xi max a technological limit of manufacturing or assembly.

- Geometrical constraints impose relationships between design variables in order to guarantee a functional, feasible, reliable structure. They are generally based on "good practice" rules to avoid local strength failures (web or flange buckling, stiffener tripping, etc.), or to guarantee welding quality and easy access to the welds. For instance, welding a plate of 30 mm thick with one that is 5 mm thick is not recommended.

  Example: $0.5 \leq X2 / X1 \leq 2$

  with X1, the web thickness of a stiffener and X2, the flange thickness.
- Structural constraints represent limit states in order to avoid yielding, buckling, cracks etc. and to limit deflection, stress, etc. These constraints are based on solid-mechanics phenomena and modeled with rational equations.

  Thus, these constraints can limit:
  - Deflection level (absolute or relative) in a point of the structure,
  - Stress level in an element (σ_x, σ_y and $\sigma_c = \sigma_{von\ Mises}$),
  - Safety level related to buckling, ultimate resistance, tripping, etc. (example: $\sigma / \sigma_{ult} \leq 0.5$).

For each constraint (or solid-mechanics phenomenon), the selected behaviour model is especially important since this model fixes the quality of the constraint modeling.

The list of constraints included in the LBR-5 model is intimately bound to the types of structures targeted by this research. Let us recall that these are mainly metallic, prismatic (box girders) and stiffened (orthotropic) structures used for hydraulic and marine structures. These structures are composed of stiffened panels that are either cylindrical or plane. The panels are joined one to another by generating lines (edges of the prismatic structure) and are stiffened longitudinally and transversely (Figure 5).

When going from the "local" to the "general" components (Figure 5), one differentiates three types of constraints:

- constraints on panels and components,
- constraints on frames and transversal stiffening,
- constraints on the general structure.

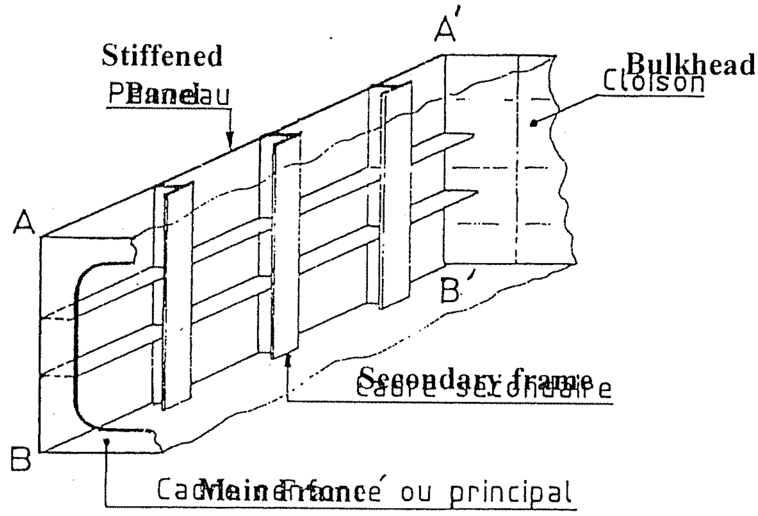

Figure 5: A stiffened panel

- Constraints on stiffened panels (Figure 5).

  Panels are limited by their lateral edges (junctions with other panels, AA' and BB') either by watertight bulkheads or transverse frames. These panels are orthotropic plates and shells supported on their four sides, laterally loaded (bending) and submitted, at their extremities, to in-plane loads (compression/tensile and shearing).

  The global buckling of panels (including the local transverse frames) must also be considered.

  Supports of panels and, in particular those corresponding to the reinforced frames, are assumed infinitely rigid. This means that they can distort themselves significantly only after the stiffened panel collapse.

- Constraints on the transverse frames (Figure 5)

The frames take the lateral loads (pressure, dead-weight, etc.) and are therefore submitted to combined loads (large bending and compression). The rigidity of these frames must be assured in order to respect the hypotheses on panel boundary conditions (undeformable supports).

- Constraints on the global structure (box girder/hull girder).
  The ultimate strength of the global structure or a section (block) located between two rigid frames (or bulkheads) must be considered as well as the elastic bending moment of the hull girder (against yielding).

The limit states that will be considered are:
-A *service limit state* that corresponds to a situation where the structure can no longer assure the service for which it was conceived (examples: excessive deflection cracks).
-An *ultimate limit state* that corresponds to collapse/failure.

The **COST module:** In 2000, even for a first draft, a least weight optimization process can no longer be justified and should be replaced by a least construction cost or, even better, by a minimum global cost (including operational costs).

Up to now, the objective function of the LBR-5 software has considered both construction costs (COST module) and weight (example: 60% of the cost and 40% of the weight). In order to link the objective function (Euro) to the design variables (Xi), the unit costs of raw materials (Euro/Kg), the productivity rates for welding, cutting, assembling, ... (man-hours/unit of work = m-h/unit) and labour costs (Euro/m-h) must be specified by the user.

These unit costs vary according to the type and the size of the structure, the manufacturing technology (manual welding, robots,...), the experience and facilities of the construction site, the country, etc. It is therefore obvious that the result of this optimization process (sizing optimization) will be valid only for the specific economic and production data under consideration. Sensitivity analysis of the economic data on the optimum scantling can also be performed, providing the manager with valuable information for improving the yard.

4. OPTIMIZATION OF A FLOATING GATE

The present optimization with the LBR-5 model concerns a 36-m span floating gate designed for the "Canal du Centre" in Belgium.

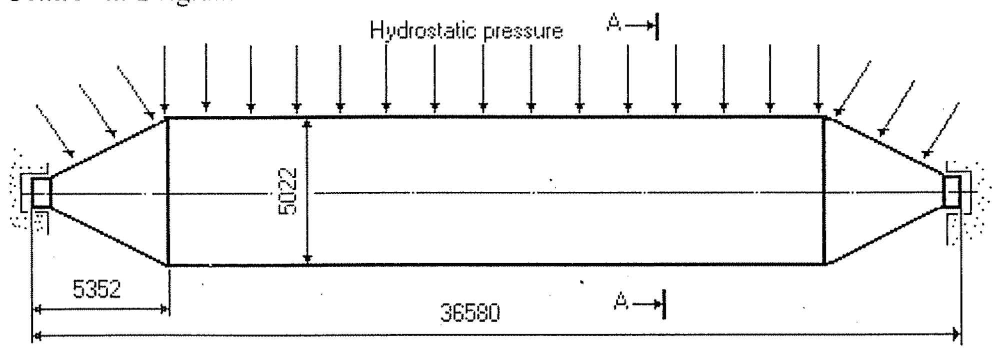

Figure 6: The Plan view and pressure in working state.

This structure can be used for maintenance of the canal or in the emergency case, when there is an accidental, such as breaking of the banks of the canal. It will be placed transversally to interrupt the water spilling to the neighbor area.
We consider responses of the floating gate with three load cases which correspond to the sunk state, the floating state, and the working sate. Figure 10 shows the pressures acting on the structure at working state.

To model exactly the response of the floating gate, we estimate the loads applied on the whole structure, (see Figure 8). The associated shearing force and vertical bending moment are 9.366 t and 16.708 t.m, respectively. These forces will be inserted in the data file as boundary conditions.

The structure is modeled as in Figure 9 into 12 panels. Following are the main parameters of the problem:

- 96 design variables:
  + Plate thickness.
  + For frame :
    * web height and thickness.
    * flange width,
    * frame spacing.
  + For stiffener : idem.
- 61 equality constraints to guarantee standardized sizes.
- 84 geometrical constraints to avoid local web and flange buckling and to consider welding and manufacturing feasibility.
- 113 structural constraints are used to consider the response of the structure members, such as:
  + von-Mises combined stresses, $1.5\sigma_c \leq \sigma_y$.
  + ultimate strength of stiffened plate, $\sigma/\sigma_{ult} \leq$. 0.7
  + deflections smaller than 7cm.
  + the minimum plate thickness should always be larger than the value at which buckling or yielding happens.

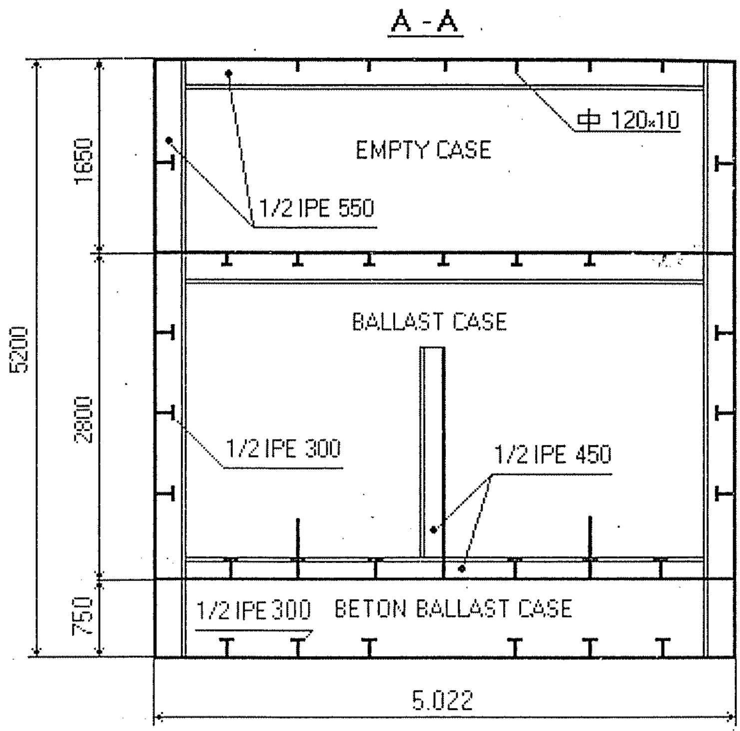

Figure 7: The cross section.

Here, we study the complete floating gate structure but the analysis is concentrated on sizing (scantling) optimization. The other problems such as stability, propulsion, ... are ignored. Nevertheless, solution of the optimization problem is changing with these parameters. Therefore we gradually alter the side constraints, compare the objective function between configurations to pick out the most attractive one. Because the software allows to choose the objective function either least cost or least weight, cost parameters should also be considered..
For cost evaluating, we use the following parameters:

- Reference plate thickness	:	14 mm
- Unitary labor cost/ Unitary material cost	:	0.02
- Unitary price (for plate)	:	C1 = 0.644 Euro for 1 kg of steel AE355(HTS)
	:	C1 = 0.570 Euro for 1 kg of steel AE235(MS)
- The unitary price of 1kg of stiffener	:	1.25*C1
- The unitary price of 1kg of frame	:	1.30*C1
- Working load for welding 1m of both longitudinal and transverse members	:	P4 = P5 = 1.1m-h/m
- Welding cost per meter	:	1 Euro /m ....

Table 2 shows the results of 32 scenarios corresponding to different parameters:

- For least cost objective function, when the side constraints ($\delta \leq \delta_{max}$) are released, the weight of the structure increased and the total cost goes down.
  The solution with a least cost objective function usually goes with a more attractive price. Specially, when there is no equality constraint, the least cost problem get us a cost is

(6.9 – 19.2)% lower than the corresponding least weight.

- With the same objective function, when the problems have a very large feasible design space, the optimized solutions are not very different from one to the others. There is not any obstacle in searching optimized results, see the columns with δ_{max} = 25mm and δ_{max} = 30 mm in table 2 .

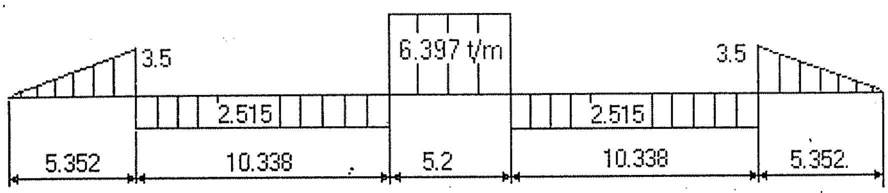

Figure 8: Load on the floating state.

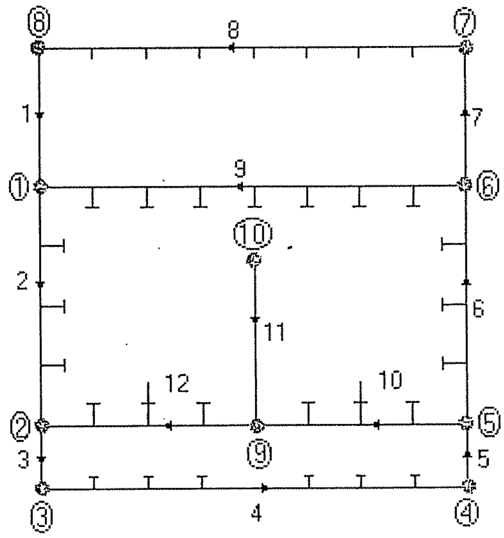

Figure 9: Mesh model.

n : plate number
ⓝ : node number

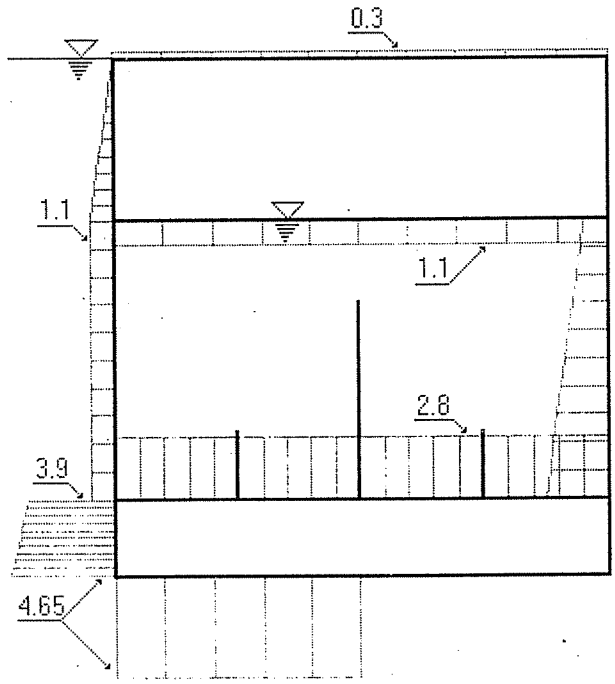

Figure 10: Hydrostatic pressure (N/m²) acting on the working state.

- With the least cost objective function, the frame spacing increases as the plate thickness becomes thicker to maintain the strength of the structure. The result is a smaller cost as the manpower requirements change with the frame quantity. The same process does not happen for least weight problem. We see that for the least weight problem, the frame spacing is reduced, while the other parameters of panels are changed. This phenomenon reduces the total weight of the structure but doesn't decrease the total cost, in some cases, the cost is even increased.
- The stiffener spacing in least weight cases is usually smaller than for least cost. Generally, the stiffeners dimension for least weight problem are also smaller. In addition, their height almost takes the lower bound of the side constraint. These effects with the frame spacing adjustment induce a reduced total weight of the structure.

Conclusion : Obviously, we do not recommend to use the unequality constraint models with inherent complications in manufacturing because these models require scantling with various plate thickness'. Moreover it is no aesthetic, (see Figures 11 and 13). Our concerns for a floating structure is not only the least cost but also stability of the floating process. Finally, we choose a least cost

Table 1 : Side constraints in plates

	δ	H_f	T_f	W_f	Δ_f	H_s	T_s	W_s	Δ_s
Min	0.00500	0.10000	0.00800	0.05000	0.25000	0.05000	0.00500	0.02000	0.25000
Max	δmax	0.50000	0.04000	0.30000	3.50000	0.30000	0.02000	0.20000	1.00000

Notations:

δ : plate thickness, W : Flange width, Δ : Frame spacing,
H: Web height, T: Web thickness, f, s : frame and stiffener.

configuration: 129475 Euro, 161.875 t, δ_{max} = 30 mm, steel AE235, (see shadowed square in table 2). The weight 161.875 t of this model is almost heavier than weight of the other ones. Hence, we have a more stable structure and we will save time for ballasting water when placing the structure in working state.

By comparison between the chosen optimum solution and the initial design, we will save 8.2% in total cost, but an improved safety is the main benefit. For the initial design,the maximum stresses occur in the middle cross section A-A and are:

- δ_c = 263 MPa in SEM of frame at node 5 in panel 10 - Figure 9.
- δ_c = 202 MPa in JAB of frame at node 5 in panel 10.
- δ_c = 289 MPa in JAS of frame at node 9 in panel 10.

and the maximum displacements are:

- U_{max} = 0.0135 m,
- V_{max} = 0.0038 m,
- W_{max} = 0.0135 m.

While in the optimum, they are :

- δ_c = 120 MPa in SEM of frame at node 2 in panel 12.
- δ_c = 140 MPa in JAB of frame at node 4 in panel 4.
- δ_c = 140 MPa in JAS of frame at node in panel 12.

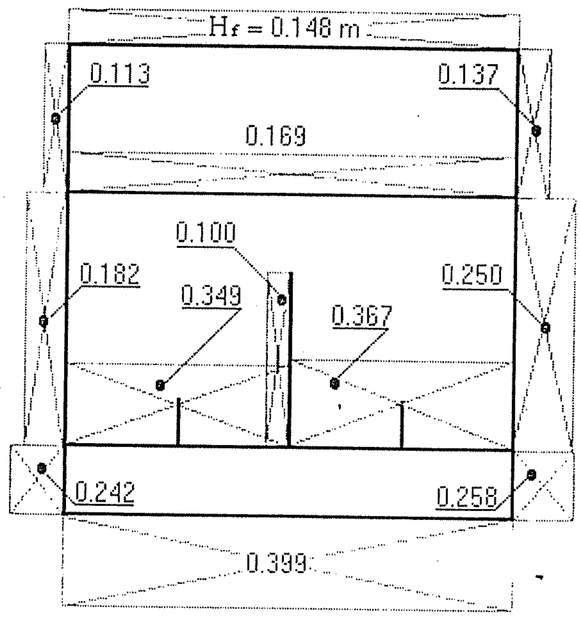

Figure 11: The optimum frame web height (m) without equality constraint ($\delta_{max} \le 30$, AE 235).

without equality constraint .
and the maximum displacements are:

- U_{max} = 0.0124 m,
- V_{max} = 0.0038 m,
- W_{max} = 0.0120 m.

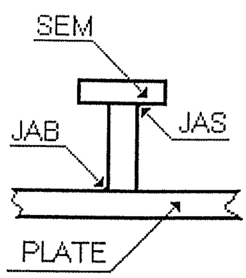

Figure 12: Locations of the computing stresses.

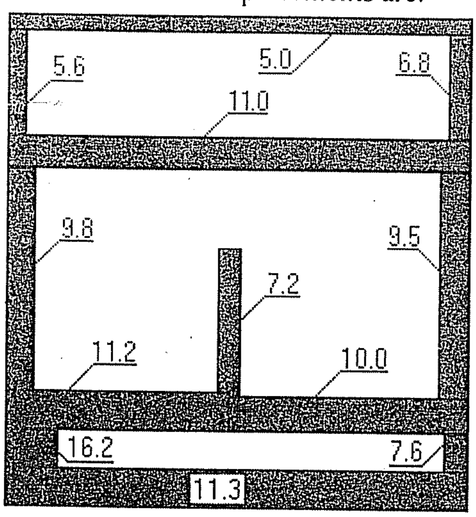

Figure 13: The optimum plate thickness (mm) without equality constraint ($\delta_{max} \le 30$, AE 235)

Notations in table 2:
C : least cost.
W : least weight.
U : without equality constraints.

Table 2 : The results of different configurations

δ_{max} (mm)		STEEL AE235 15	20	25	30	STEEL AE355 15	20	25	30
Total weight (N).	C	1515152	1560090	1618757	1612761	1312782	1345548	1390464	1411726
	W	1364170	1327258	1326954	1326954	1057894	1054783	1054783	1054802
	UC	1107903	1099640	1094537	1098416	1041189	1029386	1027937	1027839
	UW	831626	830666	829489	829070	717584	717513	717634	717634
Total cost (Euro).	C	140261	130570	130369	129475	138161	133786	132382	131952
	W	141328	132453	131287	131287	139440	134953	134953	134971
	UC	107477.	105818	105731	105676	113022	113594	113574	113174
	UW	130579	129061	128335	128070	140015	140009	140065	140065
δ	C	0.01500	0.01606	0.01565	0.01581	0.01201	0.01203	0.01200	0.01200
Δ_r	C	1.32566	2.39531	3.31192	3.34426	1.58504	2.20987	3.00731	3.50000
H_r	C	0.21478	0.44714	0.50000	0.50000	0.19146	0.22744	0.22298	0.20657
Δ_s	C	1.00000	1.00000	1.00000	1.00000	1.00000	0.96952	0.94829	0.93500
Hs on panel 2 (m).	C	0.05000	0.06293	0.08368	0.08430	0.05826	0.07256	0.08954	0.09957
δ	W	0.01500	0.01873	0.01875	0.01875	0.01500	0.01573	0.01573	0.01573
Δ_r	W	3.26740	3.20577	3.20529	3.20529	3.50000	3.50000	3.50000	3.50000
H_r	W	0.50000	0.50000	0.50000	0.50000	0.40621	0.40721	0.40721	0.40697
Δ_s	W	1.00000	0.80661	0.80692	0.80692	0.65336	0.66708	0.66708	0.66659
Hs on panel 2 (m).	W	0.05813	0.05000	0.05000	0.05000	0.05000	0.05000	0.05000	0.05000
δ	UC	0.01082	0.00987	0.00983	0.00983	0.01382	0.00975	0.00975	0.00976
Δ_r	UC	3.50000	3.50000	3.50000	3.50000	3.50000	3.50000	3.50000	3.50000
H_r	UC	0.34371	0.22638	0.22717	0.22780	0.17649	0.18346	0.18272	0.18283
Δ_s	UC	0.83041	0.86072	0.85930	0.85968	0.95254	0.87530	0.87537	0.87535
Hs on panel 2 (m).	UC	0.11910	0.12233	0.12188	0.12143	0.12124	0.11700	0.11719	0.11701
δ	UW	0.00500	0.00500	0.00500	0.00500	0.00500	0.00500	0.00500	0.00500
Δ_r	UW	1.69512	1.67385	1.65821	1.64999	1.56759	1.56268	1.56455	1.56455
H_r	UW	0.22363	0.22572	0.22393	0.22233	0.10000	0.10000	0.10000	0.10000
Δ_s	UW	0.27196	0.27515	0.27926	0.28176	0.31384	0.31428	0.31405	0.31405
Hs on panel 2 (m).	UW	0.05572	0.05564	0.05503	0.05503	0.05000	0.05000	0.05000	0.05000

Initial design: Weight = 1549076 (N) ; Cost = 141159 Euro (AE235) ; Δ_r = 2.6 (m) ; H_r = 0.2578 (m) ; Δ_s = 0.7 (m) ; H_r = 0.1393 (m).

5. CONCLUSIONS

LBR-5 is a structural optimization tool for structures composed of stiffened plates and stiffened cylindrical shells. It is an integrated model to analyze and optimize naval and hydraulic structures at their earliest design stages: tendering and preliminary design.
Initial scantling is not mandatory to star optimization analysis. Designers can directly start by an automatic search for optimum sizing (scantling). Design variables (plate thickness, stiffener dimensions and their spacing) are freely selected by the user.
LBR-5 is composed of 3 basic modules (OPTI, CONSTRAINT and COST). The user selects the relevant constraints (geometrical and structural constraints) in external databases. Standard constraint

sets are proposed to users. Since the present optimization deals with least construction costs, unitary material costs, welding, cutting and labor costs must be specified by the user to

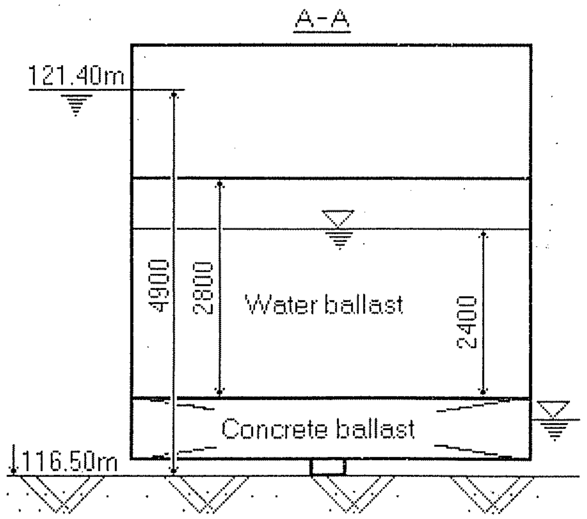

Figure 14: Structure in service.

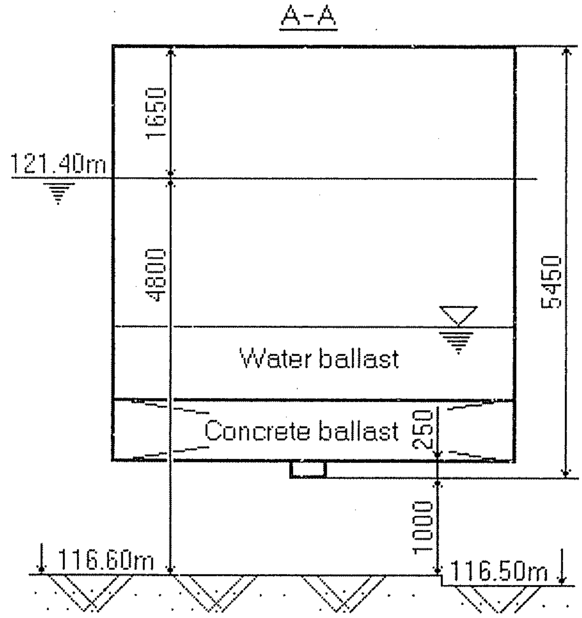

Figure 15: Structure in floating condition.

define an explicit objective function. Using all these data (constraints, objective function and sensitivity analysis), an optimum solution is found using an optimization technique based on convex linearizations and a dual approach. Independently of the number of design variables and constraints, the number of iterations requiring a complete structural re-analysis is limited to10 or 15.
Optimum analysis of a floating gate of 36m long is presented as an application of the LBR-5 optimization process.

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