FAS: A NEW METHOD FOR DERIVING THE POLE ORIENTATION AND SHAPE PARAMETERS OF ASTEROIDS

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1. Introduction.

Several photometric methods are being used to determine the pole orientation, the shape and the albedo distribution of asteroids. In general, these methods make use of the dependence existing between the photometric information (magnitudes and amplitudes of lightcurves) and the relative orientations and aspects of a modeled asteroid, at different positions along its trajectory.

The main restriction on these methods is, of course, the implicit choice of an asteroid model. In Liège, we have studied successively the classical ellipsoidal model (Surdej & Surdej, 1985[1], Surdej et al., 1986[2]) and the FAM model (Free Albedo Map method : a spheroid with local adjustable albedo; Hainaut et al., 1989[3]). But, are these shapes and albedo distributions sufficiently general to model adequately any asteroid ?

In order to push back that limitation, we have developed a new tool, directly derived from the FAM method, and based on a deformable polyhedron covered with facets of variable albedo. We have named it FAS, for Free Albedo and Shape method.

2. The Free Albedo Map method (Hainaut et al., 1989[3]).

As shown by H.N. Russell in 1906[4], the lightcurves of any convex asteroid, with any albedo distribution, can be exactly reproduced by a sphere covered with a suitable albedo distribution, provided that the latter has the same axis orientation as that of the asteroid.

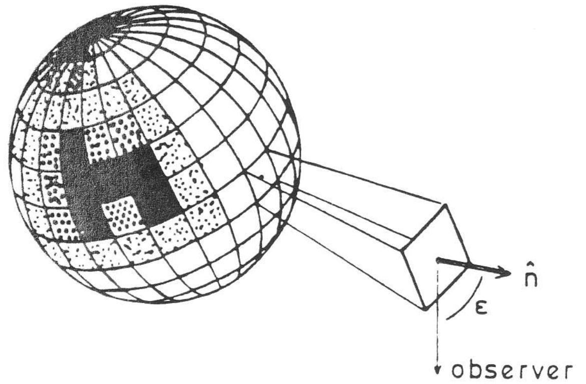


Figure 1 : The FAM model.

So, we model the asteroid by a sphere approximated by a large set of planar facets, each of them having an adjustable albedo (Figure 1).

Assuming that we are near the opposition, and that the geometrical approximation is valid, the relative flux of the reflected sunlight is given for every facet i= 1,...,N by the expression :

$$F_i = k * f(\alpha,Q) * A_i * S_i * \cos^{+}\varepsilon_i ,$$

with : k : a normalizing constant,

$f(\alpha,Q)$: a phase (α) dependence function, using for instance the multiple scattering parameter Q of the Bowell and Lumme theory[5],

A_i : the geometrical albedo of facet i,

S_i : the area of facet i,

and $\cos^{+}\varepsilon_i$: the cosine of the angle between the normal of the facet and the line-of-sight, if the facet is visible :

$$\cos^{+}\varepsilon = \cos\varepsilon \quad \text{if } \varepsilon \le 90^\circ$$
$$= 0 \quad \text{if } \varepsilon > 90^\circ .$$

As the sphere is a convex solid, every facet is either completely visible or completely hidden, and we have not to bother about shadowing effects.

The total flux of the reflected sunlight is equal to the sum of all the facet contributions :

$$F(\lambda_{pole},\beta_{pole},A_i) = \sum_{\text{all the facets}} F_i .$$

The search for the pole and for an albedo distribution is then achieved by minimizing, in terms of both the polar coordinates and the albedos, the sum of the squares of the residuals between all the observed and computed fluxes :

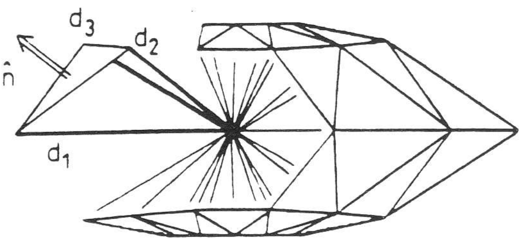
$$K^2(\lambda_{pole},\beta_{pole},A_i) = \sum_{\text{all the data}} \left(F_{observed} - F(\lambda_{pole},\beta_{pole},A_i)\right)^2 .$$

Whereas the lowest K^2 generally provides us with a reliable pole, we have to mention that the derived albedo distribution is only one among an infinity of equivalent possibilities, as the addition of any combination of odd spherical harmonics does not change the resulting lightcurves.

3. <u>The Free Albedo and Shape method</u> (Detal[6]; Schils, 1991[7]).

As most of the observed magnitude variations are caused by shape effects[8], the FAM "exclusive albedo" model has no real significance for the majority of asteroids. So, we have tried the complementary approach : to keep the individual albedos constant and to modify the shape of the polyhedron.
This is the Free Albedo and Shape method (FAS, or sometimes FS as the albedo does not play a prominent rôle).

The FAM model is slightly adapted for that purpose. Let us consider plane triangular facets, which summits lie on axes emerging from the center of the asteroid. Keeping the orientation of these axes fixed, the surface and orientation of a facet simply depend on the length of its supporting axes (Fig. 2).



$$\vec{N} = d_2 d_3 \, (\hat{d}_2 x \hat{d}_3) + d_3 d_1 \, (\hat{d}_3 x \hat{d}_1) + d_1 d_2 \, (\hat{d}_1 x \hat{d}_2)$$

$$S = 1/2 \, |\vec{N}| \qquad \hat{n} = \vec{N}/|\vec{N}|$$

Figure 2 : The FAM model.

So, the flux recorded by a distant observer from facet i may now be expressed as :

$$F_i = k * f(\alpha,Q) * A_i * S_i(d_{j(i)}) * COS^{+}\varepsilon_i(d_{j(i)}),$$

where $d_{j=1,2,3(i)}$ are the lengths of the 3 supports of facet i.

We may then restart the search for the pole and, this time for the shape, by minimizing the $K^2(\lambda_{pole}, \beta_{pole}, d_{j(i)}, [A_i])$ in terms of the polar coordinates and of the lengths of the axes. The advantage of this faceted model is the wider range of shapes that are allowed in comparison with the classical ellipsoidal or spherical models. Of course, the computational cost is also more important, although not too excessive.

This method has been tested on a synthetical asteroid of ellipsoidal type, developed as a tool for the verification of our methods[8]. Two numerical programs (FAS1 and FAS2), based on the same approach but completely different in their designs (Fig. 3) give us quite good correlated results, as it is shown on figure 4. The longitude of the pole is determined with an excellent precision ($\delta\lambda$ < 1°) but there is a difference of nearly 20° between the real and the calculated latitudes; this probably results because of both the low inclination orbit (i < 3°) and the low pole latitude, around 17°, of the asteroid. The relative dimensions of the ellipsoid are reproduced with a quite good accuracy (see Table 1).

Application of this method to real asteroids is presented elsewhere in the Proceedings (see "Photometry, Shape Parameters and Pole Orientation of the Minor Planets 624 Hektor and 43 Ariadne" by A.Detal et al.).

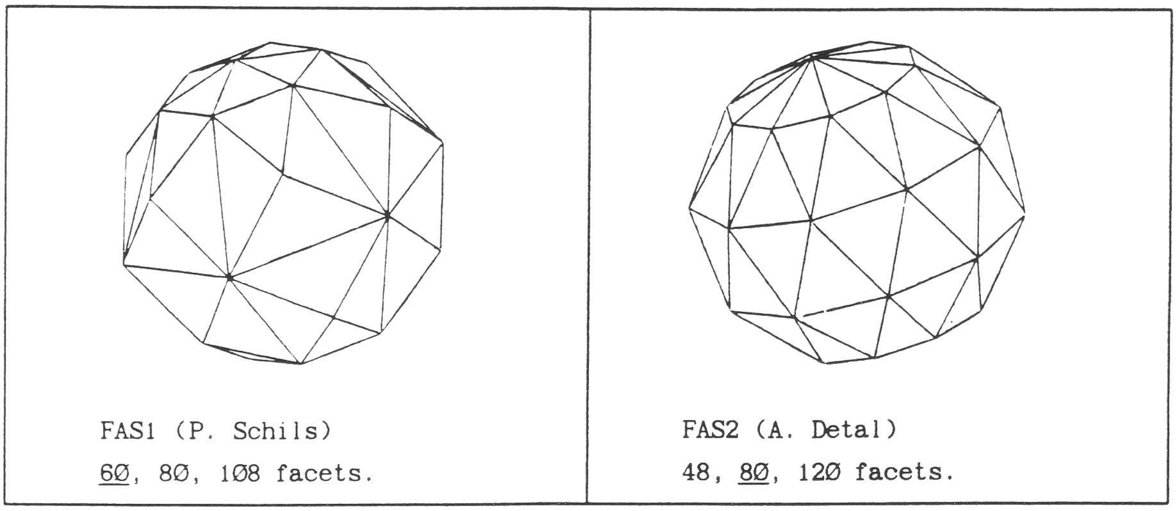


Figure 3 : Examples of FAS1 and FAS2 asteroid models.

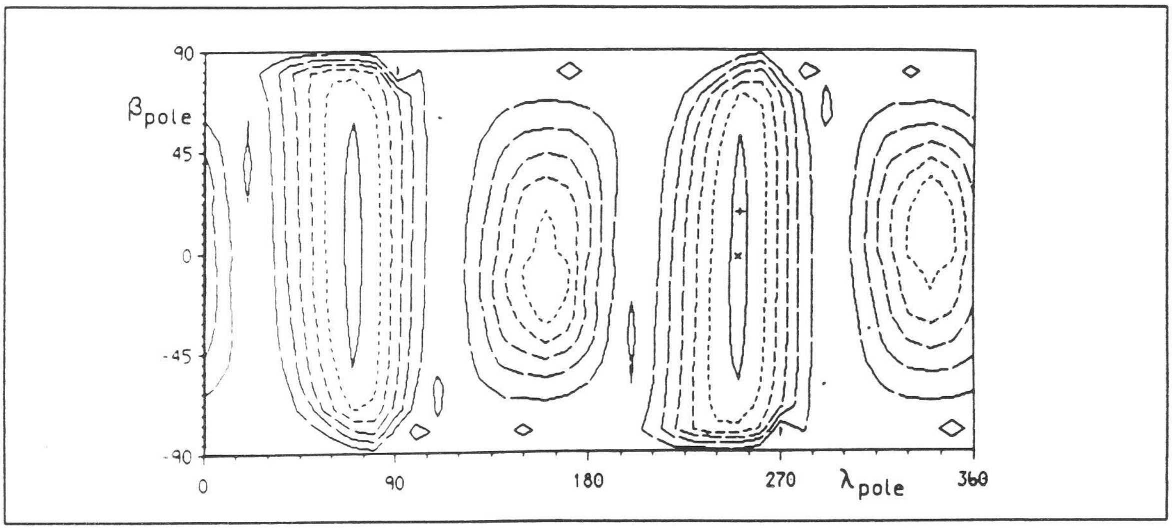


Figure 4 : Map of the residual $K^2(\lambda_{pole}, \beta_{pole}, d_{j(i)}, [A_i])$ as a function of the coordinates of the trial pole for the synthetical asteroid. The real pole is marked with a "+" symbol and the FAS ones, with a "x".

Table 1

	pole longitude	pole latitude	Q	a/b	b/c
Original	25Ø.9°	17.6°	Ø.1Ø5	1.45	1.97
FAS1	25Ø°	-3°	Ø.Ø93	1.47	1.79
FAS2	25Ø°	-3°	Ø.Ø89	1.49	1.8Ø

Figure 5 illustrates the synthetical asteroid at different aspect angles (A) for (retrograde) rotational phases (φ) of Ø°, 3Ø°, 6Ø° and 9Ø°.

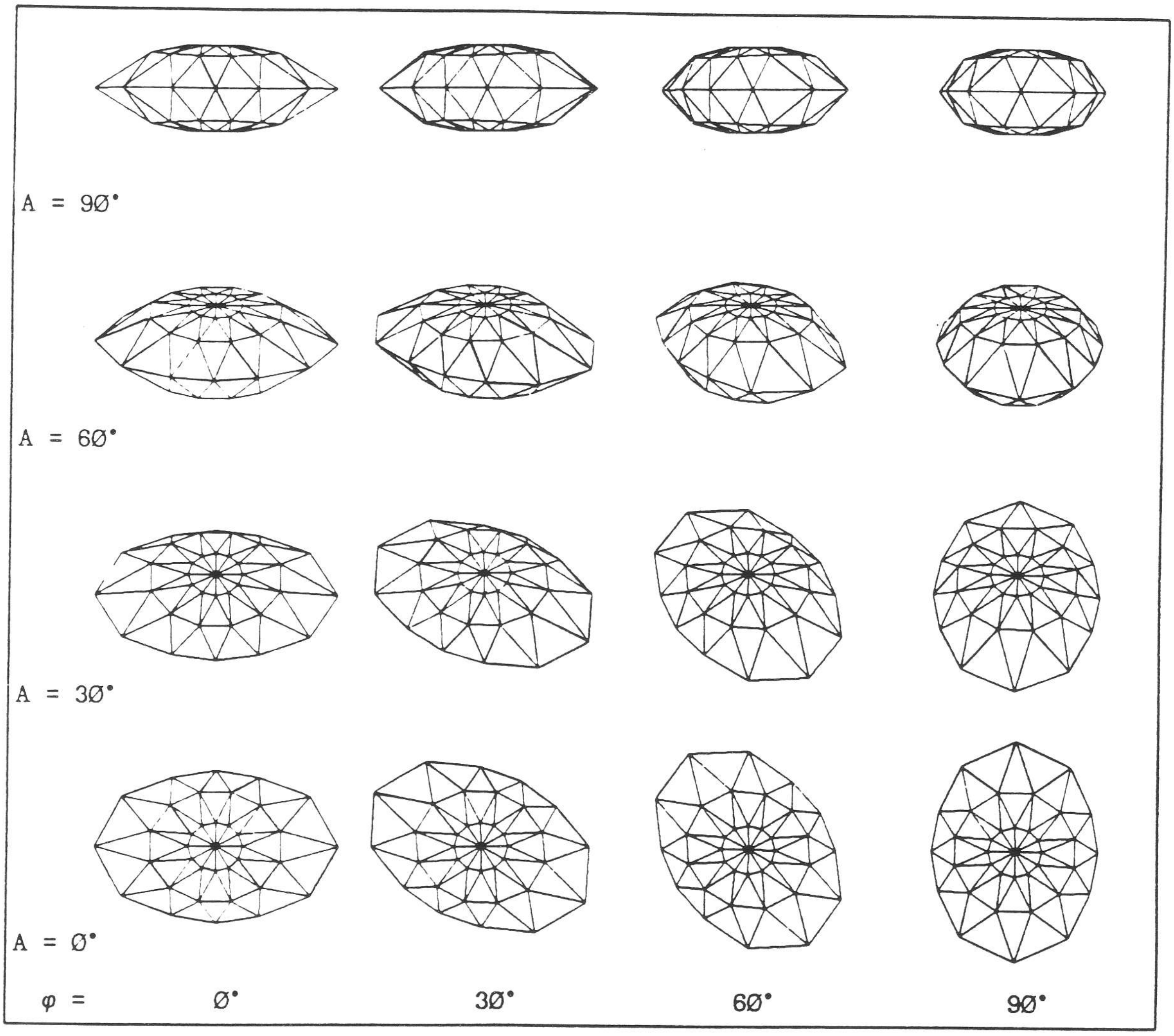


Figure 5 : different geometrical views of the synthetical asteroid.

The main difficulty of the FAS method consists in preserving the convexity of the surface of the model. We have not implemented a treatment of the shadowing effects that appear in the non-convex case : the subtraction of the contribution of normally hidden facets costs too much calculation time. Instead, we have used two types of constraints that turn out to be quite effective : a maximization of the entropy of the axis lengths (these ones being normalized to the best sphere or ellipsoid; FAS1) or a minimization of the surface/volume ratio of the asteroid model (FAS2). Even if those constraints bring us back to the classical shapes, their fine tuning has enabled us to obtain reasonable shapes and lightcurve fittings.

Two major improvements of FAS will be made in the near future :

- The inclusion of a treatment for infrared lightcurves.
  It is not possible now to perform a simultaneous determination of shape and albedo with the only information contained in visible lightcurves. What we can always do is adjusting the albedo when the shape is stabilized or vice-versa in

order to fit at best the observed lightcurves. But, such a modeling can not give us any usable shape or albedo distribution, even if the determination of the pole position is quite reliable. Using the complementary information of thermal infrared lightcurves, in order to separate the shape effects from the albedo ones, should improve future attempts of shape and albedo determination of asteroids.

- The combination of FAS with a chronometric method, namely the photometric astrometry. The latter method is completely independent of FAS : it does not require a model, does not use the photometric information, excepted for the determination of epochs corresponding to particular phases of rotation, and gives a complementary information on the pole position - (i.e. a better precision in latitude than in longitude, just the reverse of FAS) - plus the sidereal period and the sense of rotation.
  This combination of a photometric and a chronometric method should result in optimal determinations of the pole orientation of asteroids.

4. Conclusions.

FAS consists in a new photometric method for the determination of the shape and the orientation of the pole of asteroids. The poles derived with FAS turn out to be quite reliable, better in longitude than in latitude, as expected with that kind of methods. The reproduced shapes are also acceptable as long as the observed asteroid does not depart too much from a convex solid.

As the computational cost of using FAS is important, this method should be applied preferentially to asteroids showing regular, non-ellipsoidal type, lightcurves.

5. References.

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DISCUSSION

H. DEBEHOGNE: The problem upon the latitude determination seems to be the same as the graphs presented by Per Magnusson. Have you discussed this altogether ?

A. DETAL: Yes.