Photometry, Pole Orientation and Shape Parameters of the Minor Planets 624 Hektor and 43 Ariadne:

A. Detal[1], P. Collette[1], O. Hainaut[1,2], A. Ibrahim-Denis[1], A. Pospieszalska-Surdej[1], P. Schils[1], H.J. Schober[3] & J. Surdej[1].

[1] *Institut d'Astrophysique de Liège, Avenue de Cointe, 5, B-4000 Liège, Belgium*

[2] *European Southern Observatory, La Silla, Casilla 19001, Santiago, Chile*

[3] *Institut für Aeronomie, Universitätsplatz, 5, A-8010 Graz, Austria*

1. Introduction

Combining new measurements (described elsewhere[1]) with previously published photometric data for the minor planets 624 Hektor and 43 Ariadne, we have applied the Revisited Amplitude/Magnitude-Aspect relation (RAMA), the Free Albedo Map method (FAM), two independent variants of the Free Shape method (FS) and the Photometric Astrometry (PA) in order to derive their pole orientation (λ_0, β_0). These methods are shortly described and their results are compared.

2. The methods

The "Revisited Amplitude/Magnitude-Aspect relation"[2] (RAMA) is derived from the "historical" Amplitude-Aspect Relation. It assumes that the asteroid shape is a triaxial ellipsoid with no albedo variations. The shadowing effects are neglected, and the diffusion law adopted is that of Bowell and Lumme[3]. It is easy to show[2] that the expression of the lightcurve can be reduced to $F^2 = B.(\cos^2\Psi) + C$, where F is the total observed flux and Ψ the rotational phase. B and C are functions of the unknowns: λ_0, β_0 (pole coordinates), a, b, c (ellipsoid semi-axes), Q (multiple scattering coefficient[3]). A least square fit of each lightcurve allows one to determine the numerical values of B and C, providing two equations for the unknowns. In practice, the 6 unknowns are obtained by scanning the celestial sphere (with a typical 1-10 degrees step in both λ_0 and β_0), while the four others are fitted for each trial pole using a least squares method. The χ^2 of these fits are then plotted as a contour map (cf Fig. 1). The best pole corresponds to the lowest point. This kind of plot is *very* useful to show secondary minima, which would not appear in an automatic minimization.

Russell[4] showed in a very general way that the lightcurves of any convex body, with any albedo distribution, can be exactly reproduced by a sphere with the original axis orientation, covered with a suitable albedo distribution. This idea is used in the Free Albedo Map method[5],[6] (FAM): the asteroid is modeled by a sphere covered with many facets which albedoes are adjusted to fit at best the observations. As the Russell law is valid only if the sphere has the same orientation of the rotation axis as the asteroid, lower is the χ^2, better is the determination of the pole orientation. A χ^2 map is generated by scanning the celestial sphere and fitting an albedo distribution for each considered trial pole. The albedo distribution obtained for the best poles has to be considered very carefully: *i)* it is not unique (i.e. adding any combination of odd spherical harmonics will not change the resulting lightcurves, and the χ^2 will remain exactly the same[4]), *ii)* the real shape of the asteroid is probably not spherical at all. So, the pseudo-albedo of the model is related to the ratio between real albedo and the local curvature[4]. Additional parameters are simultaneously obtained: a normalization factor (corresponding to the radius of the sphere), and the multiple scattering factor.

In the "Free Shape method"[5] (FS), the asteroid is modeled by an irregular polyhedron with a constant albedo. The free parameters are the distances between the facet summits and the asteroid center, and the multiple scattering coefficient. A constraint has to be applied in order to ensure the convexity of the shape. Two completely independent programs have been developed, one using a minimization of the Surface/Volume ratio, the other maximizing the radii entropy. Again, the shape model corresponding to the best pole orientation is not unique but this does not affect the resulting pole orientation.

While the three previous methods use the photometric information of the lightcurves, the "Epoch Method", traditionally called "Photometric Astrometry" (PA)[7], only takes into account the chronometric information. The relative motion and positions between the asteroid and the Earth lead to slight variations in the observed rotational period of the asteroid. As these variations are involving the geometry of the rotation, it is possible to derive the pole position. This method has the advantage to make no assumption on the asteroid (shape nor albedo), but it is very demanding concerning the quality of the data.

3. The Asteroids

We have applied the above methods to previously published and newly recorded lightcurves of the asteroids 624 Hektor and 43 Ariadne. Table 1 lists the corresponding observational parameters.

Here under, Figures 1 to 4 illustrate for each asteroid the χ^2 maps obtained by each method. Note that the χ^2 scaling is not normalized for the different methods. From these maps, one can find the best pole orientations by locating the lowest χ^2 points. The best and secondary solutions, as well as additional parameters are given for each method.

The shape models can be visualized using a PC program, and the resulting lightcurves compared with the observed ones. Such simulations were shown during the conference.

4. Discussion

One should compare the morphology of the χ^2 maps obtained with the PA on one side, and with the 3 other methods on the other side. The main valleys are nearly perpendicular. This corresponds to the fact that completely independent information is being used: the chronometric information for the PA and the photometric one for the others. Usually, for asteroids which orbit inclination are low, the precision of the PA is good in latitude and poor in longitude, while it is the opposite for the photometric methods. This stresses how important it is to use both types of methods in order to derive reliable pole orientations. We plan to implement a new FS-like method using directly both types of complementary information.

One has to remember that the shape or albedo models obtained by the FAM and FS methods are only *one* among the infinity of possible models reproducing exactly the same lightcurves[4]. The use of moderating procedures like the maximization of the radii entropy or the minimization of the surface/volume ratio ensures that the calculated model is the simplest one compatible with the data. Consequently, it is one of the most probable. The only way to determine the actual shape and albedo distribution of an asteroid consists in also making use of some thermal IR lightcurves. This will also be included in our next improvement of the FS method.

5. References

1. Detal, A., Hainaut, O., Schils, P., Surdej, J.: submitted to A&A; 2. Pospieszalska-Surdej, A. & Surdej, J. (1985) A&A 149 p186; 3. Bowell, E. & Lumme, K. (1979) in "Asteroids", ed. Gehrels, Univ. of Az Press, p132; 4. Russell, H.N. (1906) ApJ 24 p1; 5. Schils, P. (1991) Master thesis, University of Liège; 6. Hainaut, O., Detal, A., Ibrahim-Denis, A., Pospieszalska-Surdej, A. & Surdej, J. (1989) in the proceedings of "Asteroids Comets Meteors III", Uppsalà; 7. Taylor, R. (1979) in "Asteroids", ed. Gehrels, Univ. of Az Press, p480; 8. Dunlap & Gehrels (1969) Astron.J. 74 p796; 9. Van Houten Groeneveld et al. (1979) AASS 35 p223; 10. Burchi & Milano (1974) AASS 15 p173; 11. Di Martino & Cacciatori (1984) Icarus 60 p75; 12. Di Martino et al. (1987) Icarus 69 p338 ; 13. Weidenschilling et al. (1987) Icarus 70 p191.

Table 1: Observational parameters: λ and β are the geocentric ecliptic coordinates of the asteroids; Δ and R are their geocentric and heliocentric distances; α is the solar phase angle.

624 Hektor							43 Ariadne						
Date	λ	β	Δ	R	α	Ref.	Date	λ	β	Δ	R	α	Ref.
1957 05 04	249.5°	-22.1°	4.212	5.088	6.2°	[8]	1965 05 01	221.8°	-4.9°	0.961	1.0079	2.5°	[9]
1957 05 30	246.0°	-22.6°	4.142	5.093	4.4°	[8]	1965 05 03	221.3°	-4.8°	0.957	1.0084	2.6°	[9]
1965 02 04	119.4°	14.6°	4.114	5.028	4.1°	[8]	1972 08 09	344.6°	6.3°	0.982	1.0137	14.3°	[10]
1967 03 07	192.9°	-9.9°	4.114	5.007	5.5°	[8]	1972 08 13	343.9°	6.4°	0.972	1.0130	12.1°	[10]
1968 05 01	225.8°	-20.3°	4.109	5.063	4.1°	[8]	1982 10 15	13.8°	5.8°	1.200	0.9969	4.5°	[11]
1984 10 03	7.2°	10.1°	4.288	5.276	2.0°	[1]	1984 02 01	128.6°	-4.1°	1.524	0.9854	2.0°	[12]
							1984 02 20	123.8°	-4.2°	1.564	0.9986	10.3°	[13]
							1985 08 16	319.8°	6.2°	0.891	1.0126	3.7°	[1]

Table 2: **Results of the different methods.** λ_0 and β_o are the ecliptic coordinates of the pole; a, b, c are the ellipsoid semi-axes; Q is the multiple scattering parameter; T is the sidereal Period.

Method	Solution	Pole	Additional parameters
624	**Hektor**		
RAMA	Best Solution	$\lambda_0 = 315°$ $\beta_0 = -16°$	a/b = 2.27 ; b/c = 1.41
	Secondary Solution	$\lambda_0 = 152°$ $\beta_0 = +27°$	a/b = 2.26 ; b/c = 1.32
FAM	Best Solution (72 facets)	$\lambda_0 = 145°$ $\beta_0 = +3°$	$Q = 0.717$
FS	**Minimization of S/V**	(80 facets)	
	Best Solution	$\lambda_0 = 149°$ $\beta_0 = +22°$	$Q = 0.38$
	Maximisation of Entropy	(80 facets)	
	Best Solution	$\lambda_0 = 144°$ $\beta_0 = +11°$	$Q = 0.35$
PA	Best Solution	$\lambda_0 = 156°$ $\beta_0 = +33°$	$T = 6.9205094$ h $\pm 1.0 \times 10^{-6}$ (Retrograde)
	Secondary Solution	$\lambda_0 = 313°$ $\beta_0 = +17°$	$T = 6.9205109$ h $\pm 1.3 \times 10^{-6}$ (Retrograde)
43	**Ariadne**		
RAMA	Best Solution	$\lambda_0 = 72°$ $\beta_0 = +13°$	a/b = 1.84 ; b/c = 1.53
	Secondary Solution	$\lambda_0 = 250°$ $\beta_0 = +8°$	a/b = 1.84 ; b/c = 1.50
FAM	Best Solution (72 facets)	$\lambda_0 = 250°$ $\beta_0 = +1°$	$Q = 0.192$
FS	**Minimization of S/V**	(80 facets)	
	Best Solution	$\lambda_0 = 248°$ $\beta_0 = +20°$	$Q = 0.098$
	Secondary Solution	$\lambda_0 = 73°$ $\beta_0 = +25°$	$Q = 0.092$
	Maximisation of Entropy	(60 facets)	
	Best Solution	$\lambda_0 = 70°$ $\beta_0 = +5°$	$Q = 0.19$
PA	Best Solution	$\lambda_0 = 250°$ $\beta_0 = +22°$	$T = 5.7619820$ h $\pm 0.6 \times 10^{-6}$ (Retrograde)
	Secondary Solution	$\lambda_0 = 74°$ $\beta_0 = +24°$	$T = 5.7619819$ h $\pm 0.9 \times 10^{-6}$ (Retrograde)

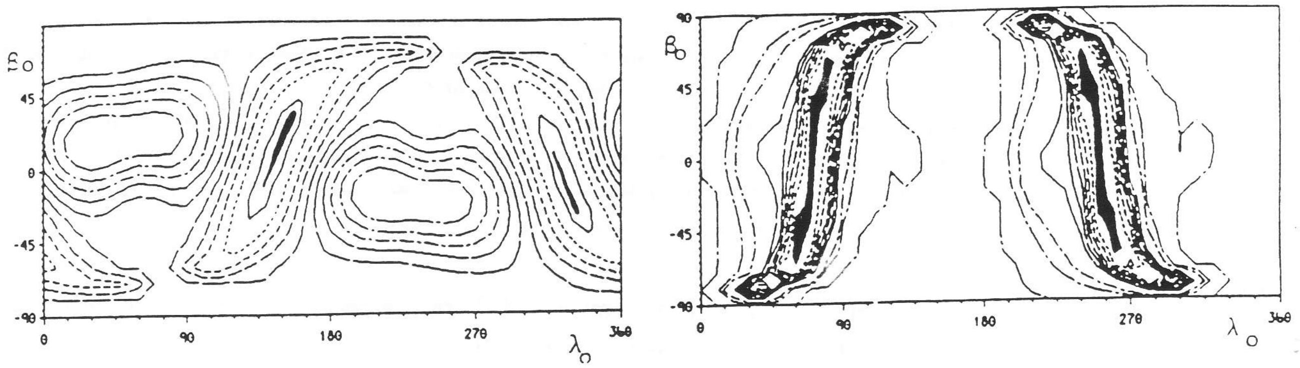

Figure 1: The RAMA χ^2 maps.
For 624 Hektor (left), 43 Ariadne (right). The best pole solutions lie in the shaded regions

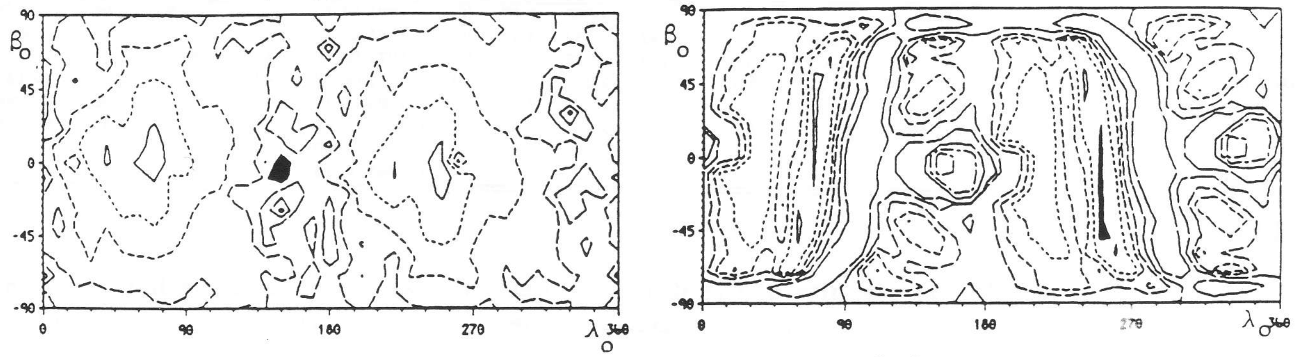

Figure 2: Same as Fig. 1 for the FAM method.

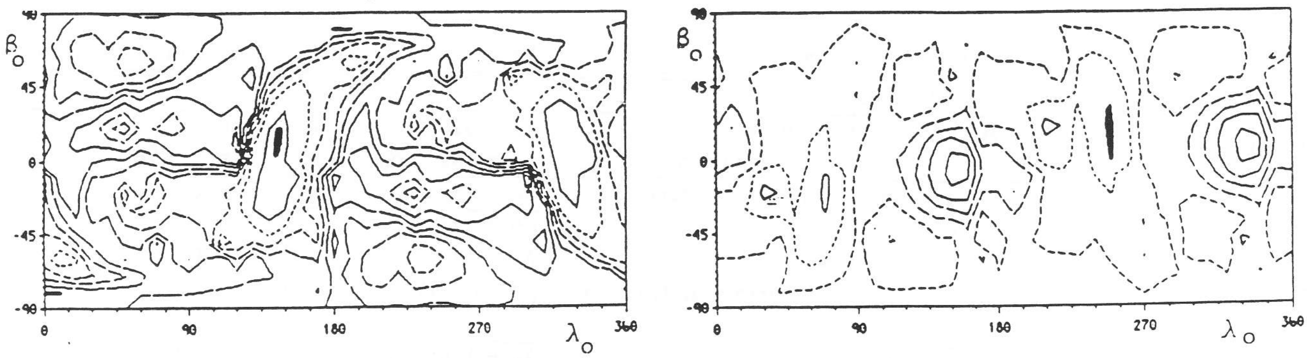

Figure 3: Same as Fig. 1 for the FS method.

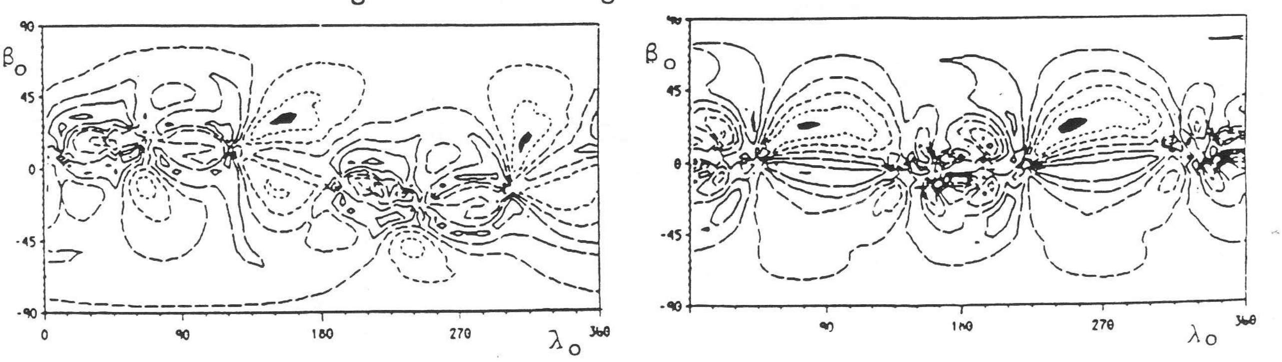

Figure 4: Same as Fig. 1 for the PA method.