THERMO-PLASTICITY OF SOILS AT VARIOUS SATURATION STATES: A CONSTITUTIVE MODEL

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ABSTRACT. This paper presents a highly-coupled thermo-plasticity model for unsaturated soils. The effect of temperature on the mechanics of unsaturated soils is briefly addressed. Then the equations of the developed model, so-called ACMEG-TS, are detailed. Finally, the model is validated by the means of comparisons with experiments. This constitutive model constitutes an effective tool for modelling the thermo-hydro-mechanical (THM) behaviour of geomaterials involved in the confinement of nuclear waste disposal.

1. Introduction

The geomaterials that will be involved in the confinement of radioactive waste in deep geological formations will be submitted to strong thermal, hydraulic, and mechanical modifications. Those modifications may produce a significant change of the characteristics of the confinement barrier, partially due to thermo-plasticity effects in the confining soil (Laloui et al., 2008). Following the need for understanding and quantifying such effects, a constitutive model that deals with the thermo-mechanical modelling of unsaturated soils is proposed (François and Laloui, 2008). In light of elasto-plasticity, this model is based on the relevant temperature and suction effects on the mechanical behaviour of fine-grained soils, as observed in experiments (Salager et al., 2008).

2. Thermo-plasticity in soils

The thermal effects on the mechanical response of soils must be considered not only in terms of reversible phenomena, but also in term of thermo-plasticity. The predominant effect of temperature on the behaviour of fine-grained soils is the causation of successively lower void ratios with temperature increasing for a given stress level. The normally consolidated lines at different temperatures are parallel and shifted to the left with increasing temperature. As a consequence, in a normally consolidated state, the soil undergoes thermal hardening (i.e. densification) upon heating in order to reach the normally consolidated line corresponding to the current temperature. Under undrained conditions, the generation of pore water pressure upon heating is a consequence of a higher thermal expansion coefficient of water than of the mineral phase. Also, thermo-plastic processes may induce additional pore water pressure. Moreover, the deviatoric behaviour of soils may also be affected by temperature variations (Hueckel et al., 2008).

In addition to the effect of temperature on the saturated soils, the unsaturated conditions bring additional thermo-hydro-mechanical couplings in the materials. In particular, the water retention capacity of soils decreases with increasing temperature. The retention curves, expressed in the degree of saturation vs suction axis, are shifted to the left with increasing temperature, mainly because the interfacial tension between the water and the grains decreases as temperature increases. This thermal effect indirectly influences the mechanical response of the soils by changing the suction value for the same degree of saturation.

From an experimental study of the combined effects of suction s and temperature T on the preconsolidation pressure, Salager et al. (2008) deduced logarithmic functions to describe the evolution of p'_c with temperature and suction:

$$p'_c(s,T)=\begin{cases} p'_{c0}\left\{1-\gamma_T \log\left[T/T_0\right]\right\} & \text{if } s\le s_e \\ p'_{c0}\left\{1-\gamma_T \log\left[T/T_0\right]\right\}\left\{1+\gamma_s \log\left[s/s_e\right]\right\} & \text{if } s\ge s_e \end{cases} \tag{1}$$

where p'_{c0} is the preconsolidation pressure at ambient temperature T_0 and for suction lower than the air-entry value s_e. γ_T and γ_s are material parameters.

3. The constitutive equations

The developed model uses the generalized effective stress approach, which aims to use a single stress to describe the mechanical behaviour of unsaturated soils through combinations of mechanical stresses and fluid pressures. This averaged stress variable converts a multi–phase porous media into a mechanically equivalent, single-phase, single-stress state continuum according to the following expression (Bishop, 1959):

$$\sigma'_{ij} = \left(\sigma_{ij} - p_a \delta_{ij}\right) + S_r \left(p_a - p_w\right)\delta_{ij} \tag{2}$$

where σ_{ij} is the total external stress tensor, p_a and p_w the air and water pore pressures, respectively, δ_{ij} Kroenecker's symbol and S_r the degree of saturation.

Mechanical scheme

The model, called ACMEG-TS, is based on an elasto-plastic-framework, the total strain increment $d\varepsilon$ being decomposed into non-linear, thermo-elastic, $d\varepsilon^e$, and thermo-plastic, $d\varepsilon^p$, components. The elastic part of the deformation is expressed as follows:

$$d\varepsilon^e_{ij} = E^{-1}_{ijkl} d\sigma'_{kl} - \beta_{\mathrm{T},ij} dT \tag{3}$$

The first term of Equation (3) is the contribution of the effective stress increment $d\sigma'_{kl}$ to the total elastic strain increment, through the elastic tensor E_{ijkl}. According to Equation (2), this part may follow from total stress or fluid pressure variations. The second term of Equation (3) is related to the thermo-elastic strain of the material, through the thermal expansion coefficient vector, $\beta_{\mathrm{T},ij} = \left(1/3\right)\beta'_s\ \delta_{ij}$.

The plastic mechanism of the material is induced by two coupled hardening processes: an isotropic and a deviatoric one. Using the concept of multi-mechanism plasticity, both mechanisms may induce volumetric plastic strain. Therefore the total volumetric plastic strain rate $d\varepsilon^p_v$ is the coupling variable linking the two hardening processes. The yield functions of the two mechanical, thermo-plastic mechanisms have the following expressions (Figure 1):

$$f_{iso} = p' - p'\ r_{iso} \quad ; \quad f_{dev} = q - Mp'\left(1 - b\ Log\frac{d\ p'}{p'_c}\right) r_{dev} = 0 \tag{4}$$

where p'_c is the preconsolidation pressure. b, d and M are material parameters. p'_c depends on the volumetric plastic strain, ε^p_v, in addition to temperature and suction:

$$p'_c = p'_c\left(s, T\right)\exp\left(\beta\ \varepsilon^p_v\right) \tag{5}$$

where β is the plastic compressibility modulus and $p'_c\left(s,T\right)$ is expressed in Equation (1). r_{iso} and r_{dev} are the degree of mobilization of the isotropic and the deviatoric mechanisms and are hyperbolic functions of the plastic volumetric strain induced by the isotropic and the deviatoric mechanisms, respectively (Hujeux, 1979; Laloui and François, 2008).

The flow rule of the isotropic mechanism is associated, while the deviatoric one is not, and they are assumed to take the following forms, respectively:

$$d\varepsilon^{p,iso}_{ii} = \frac{\lambda^p_{iso}}{3} \quad ; \quad d\varepsilon^{p,dev}_{ij} = \lambda^p_{dev}\frac{1}{Mp'}\left[\frac{\partial q}{\partial \sigma'_{ij}} + \alpha\left(M - \frac{q}{p'}\right)\frac{1}{3}\delta_{ij}\right] \tag{6}$$

where α is a material parameter. The plastic multipliers, λ^p_{iso} and λ^p_{dev}, are determined using the consistency equation.

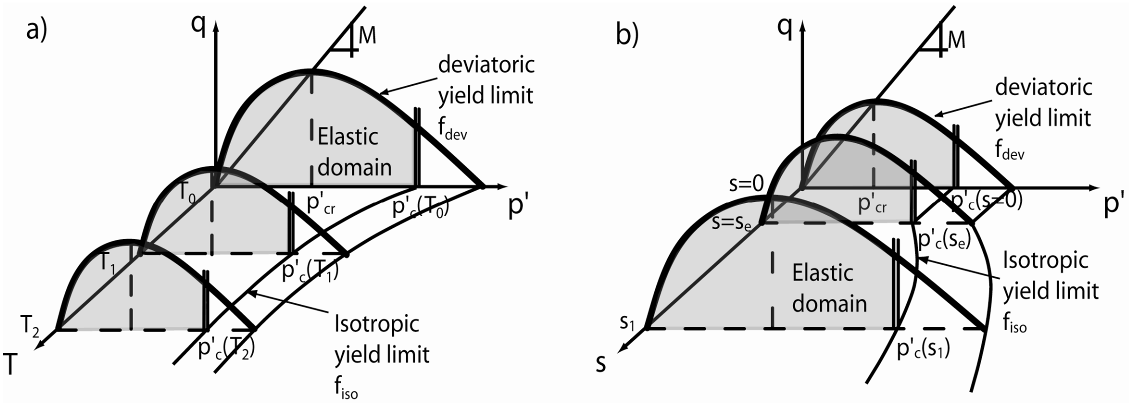

Figure 1: Effect of (a) temperature and (b) suction on the shape of coupled mechanical yield limits.

Water retention scheme

In terms of water retention response, desaturation is also a yielding phenomenon. Hysteresis in water retention behaviour is modelled as a plastic process. As long as the soil is drying, suction increases, and the degree of saturation, S_r, tends to decrease mainly when the air-entry suction s_e is reached. Under re-wetting, a hysteretic phenomenon occurs, also represented by a yielding process (Figure 2). A wetting-drying cycle activates two successive yield limits in the ($S_r - s$) plane (f_{dry} and f_{wet}, along the drying and wetting paths, respectively):

$$f_{dry} = s - s_d = 0 \quad ; \quad f_{wet} = s_d s_{hys} - s = 0 \tag{7}$$

where s_d is the drying yield limit and s_{hys} a material parameter considering the size of the water retention hysteresis. Because air-entry suction of the materials depends on temperature and dry density, s_d is a function of temperature and volumetric strain (François and Laloui, 2008):

$$s_d\left(T, \varepsilon_v\right) = s_{d0}\left\{1 - \theta_T \log\left[T/T_0\right] - \theta_e \log\left[1 - \varepsilon_v\right]\right\} \tag{8}$$

where θ_T and θ_e are material parameters describing the evolution of air-entry suction with respect to temperature and volumetric strain, respectively. If the initial state is saturated, the initial drying limit s_{d0} is equal to air-entry suction s_e and increases when suction overtakes s_e as follows:

$$s_d = s_d\left(T, \varepsilon_v\right)\exp\left(-\beta_h \ \Delta S_r\right) \tag{9}$$

where β_h is the slope of the desaturation curve in the $\left(S_r - \ln s\right)$ plane (Figure 2). $s_d\left(T, \varepsilon_v\right)$ is described by Equation (8).

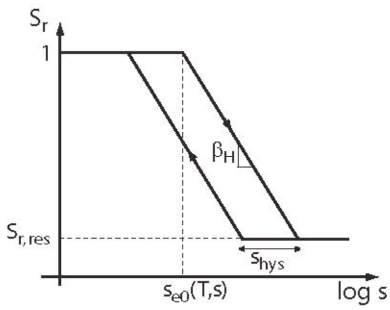

Figure 2: Schematic representation of water retention curve modelling.

4. Numerical simulations

The proposed model has been extensively validated with the results of different non-isothermal experiments under saturated and unsaturated conditions (François and Laloui, 2008; François, 2008). In this section, comparison between numerical simulations and experimental results on compacted FEBEX bentonite is briefly proposed. Figure 3a compares the numerical simulations with oedometric compression tests at different suctions (T= 22°C). The initial strain observed at 0.1 MPa of net stress is due to the suction path from 127 MPa to the suction applied during compression. The subsequent compression paths clearly shows the enhancement of elastic domain when suction increases. Figure 3b reproduces the numerical simulation of oedometric compression tests at two temperatures under 127 MPa of suction. The initial strain observed for the path at 50°C is due to the temperature increase.

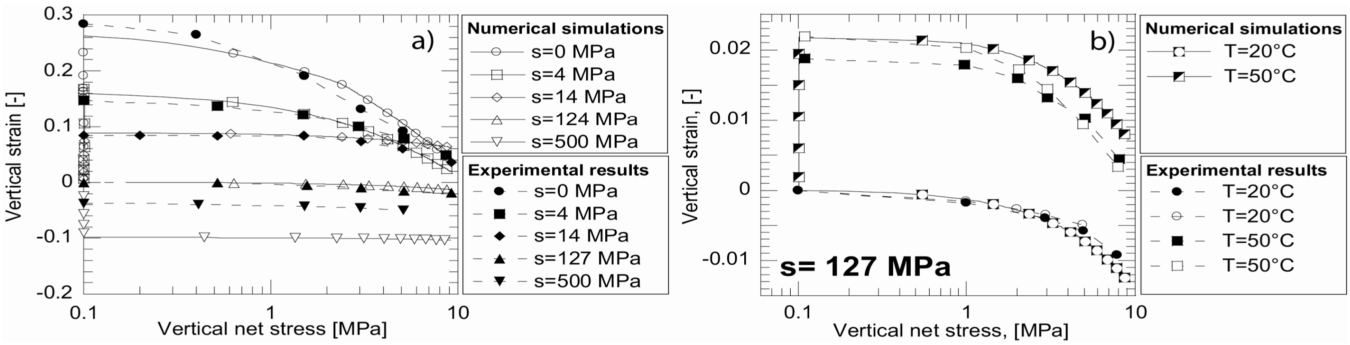

Figure 3: Numerical simulations of oedometric compression tests of FEBEX bentonite at (a) different suctions and (b) different temperatures. Comparisons with experiments.

5. Conclusions

When a soil is simultaneously submitted to mechanical, hydraulic and thermal variations, several coupling effects are involved in its global THM response. Those interactions have been introduced in a unified constitutive framework, so-called ACMEG-TS, including two interconnected aspects (a mechanical and a water retention framework) linked through a generalized effective stress expression. This constitutive approach has been confronted with experimental results through numerical predictions which tend to proof the accuracy of the developed model. ACMEG-TS constitutes an effective constitutive tool for modelling the THM behaviour of geomaterials. In addition, the model has been properly implemented in a finite element code in order to study the behaviour of the soils that confine the nuclear waste (François, 2008; François et al., 2008).

6. References

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