UNIVERSITY of Liège

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Sensitivity in shape optimization of complex 3D geometries using levels-sets and non-conforming finite elements

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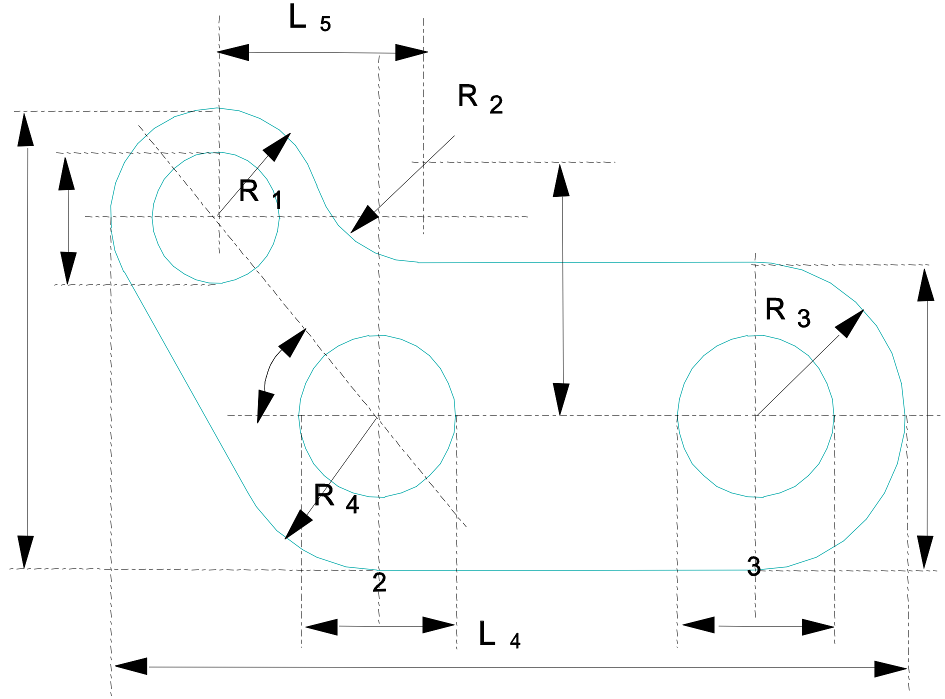
OUTLINE

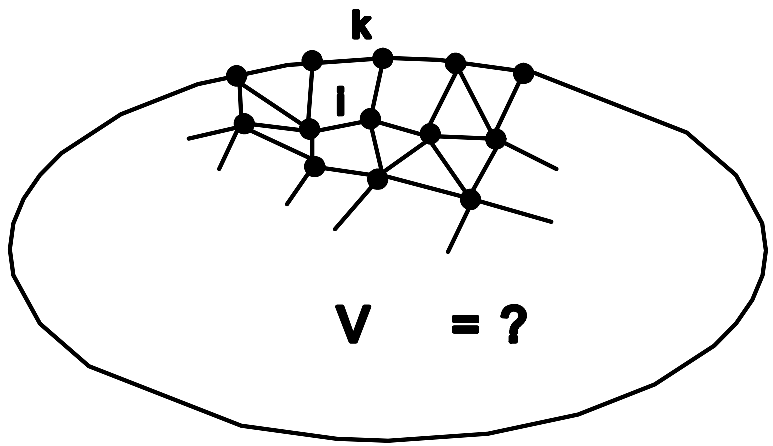
INTRODUCTION: Shape optimization

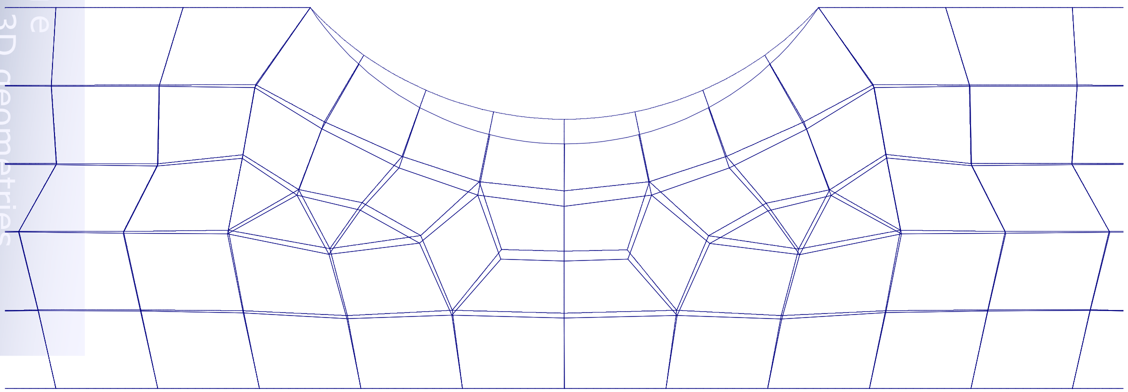
- Modification of boundaries of CAD model
- Design variables = CAD model parameters
- Restricted number of design variables
- Regular design including many geometrical constraints
- Key issue: Velocity field
- Mesh management problems
  - Mesh modification / mesh distortion
  - Error control



INTRODUCTION: Shape optimization

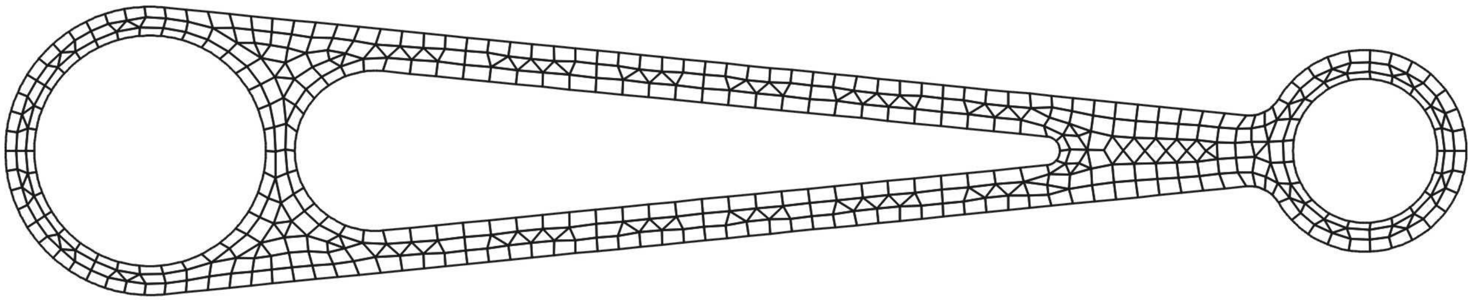
- Key issue: Velocity field
- Practical calculation of velocity field
  - Boundary velocity field: CAD model
  - Inner field:
    - Velocity law



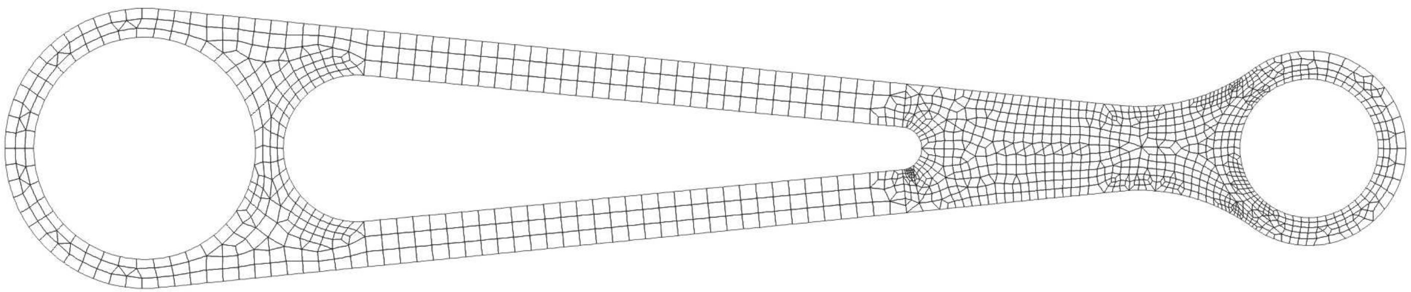


- Inner field:
  - Transfinite mapping
  - Natural / mechanical approach
  - Laplacian smoothing
  - Relocation schemes

INTRODUCTION: Shape optimization



Without error control



With error control

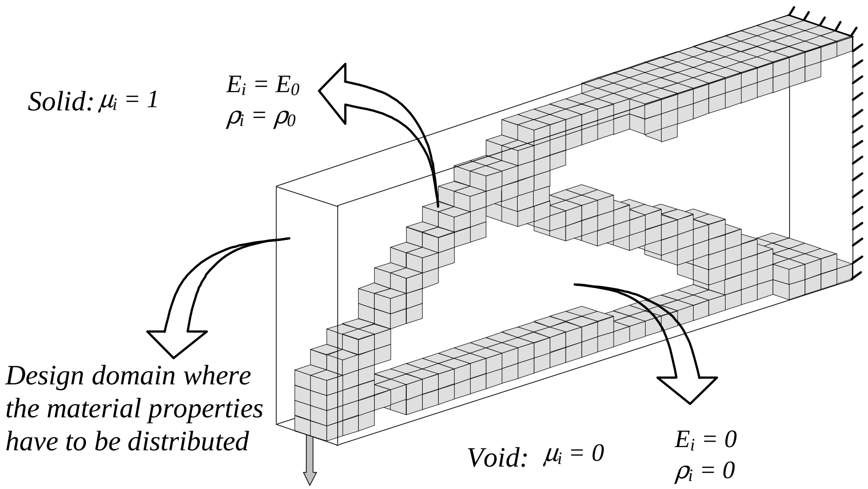
- Mesh management problems
  - Mesh modification / mesh distortion
  - Error control

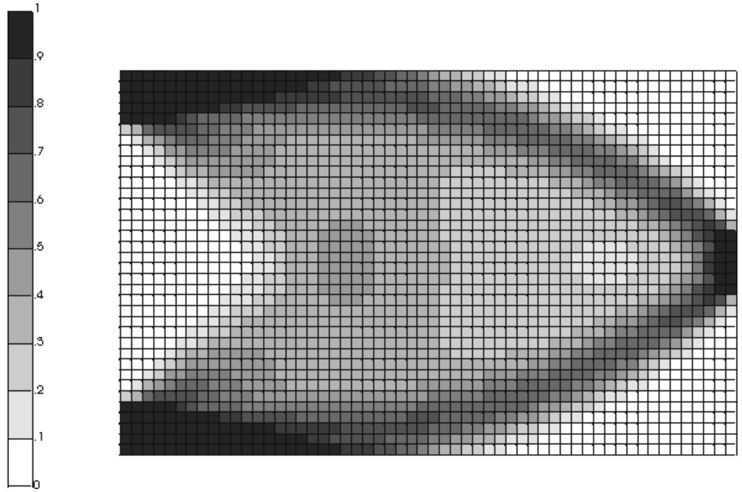
INTRODUCTION: Topology optimization

- TOPOLOGY OPTIMIZATION (Bendsøe & Kikuchi, 1988)
  - Formulated as an optimal material distribution
  - Optimal topology without any a priori
  - Fixed mesh
  - Design variables = Local density parameters
  - Homogenization law for continuous interpolation of effective properties (e.g. SIMP / power law)

$$E = \mu^3 E_0$$

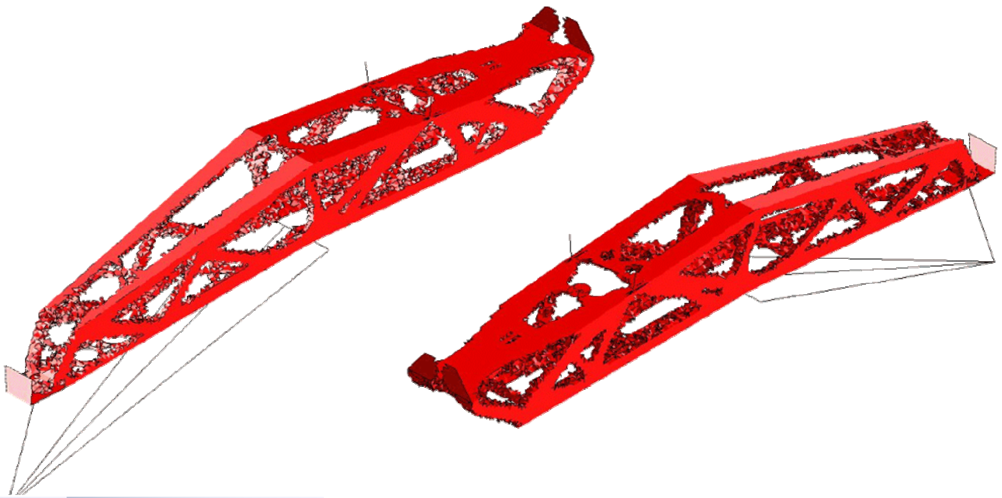
$$\rho = \mu \rho_0$$





INTRODUCTION: Topology optimization

- TOPOLOGY OPTIMIZATION (Bendsøe & Kikuchi, 1988)
  - Simple design problem:
    - Minimum compliance s.t. volume constraint
    - Local constraints are difficult to handle
    - Geometrical constraints (often manufacturing constraints) are difficult to define and to control
  - Preliminary design: interpretation phase necessary to come to a CAD model
  - Great industrial applications



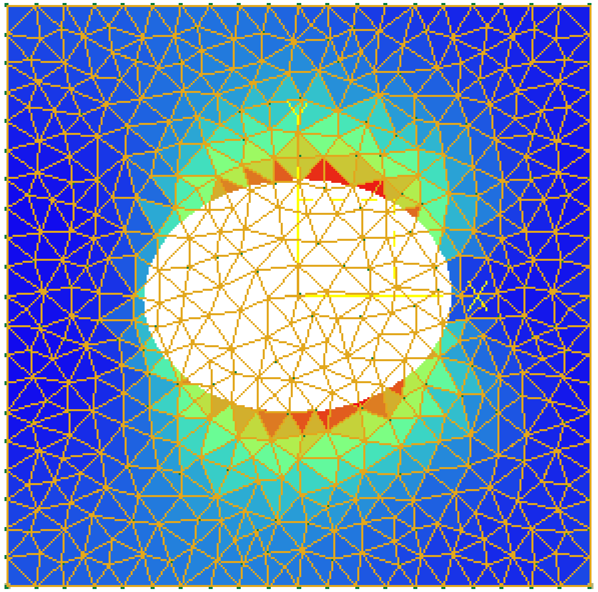
With courtesy by Samtech and Airbus Industries

INTRODUCTION: Level Set and XFEM

- LEVEL SET METHOD
  - Alternative description to parametric description of curves
- EXTENDED FINITE ELEMENT METHOD (XFEM)
  - Alternative to remeshing methods
  - Alternative to homogenization: void is void!
- XFEM + LEVEL SET METHODS
  - Efficient treatment of problem involving discontinuities and propagations
  - First applications to crack problems. Moës et al. (1999)
  - Early applications to topology optimisation Belytschko et al. (2003), Wang et al. (2003), Allaire et al. (2004)

INTRODUCTION: Level Set and XFEM

- XFEM + Level Set methods = alternative (intermediate) approach to shape and topology optimisation
- Level Set
  - Constructive geometry using parametric level sets
- XFEM
  - Void / solid approach
- Problem formulation:
  - Global and local constraints
  - Limited number of design variables
- Sensitivity analysis
  - Material derivative approach
  - FE implementation

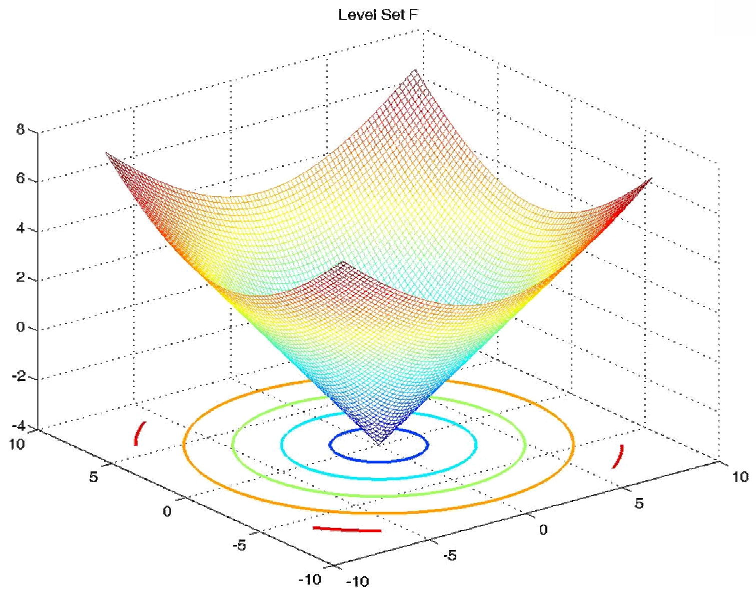


GEOMETRICAL DESCRIPTION USING LEVEL SETS

- Principle (Sethian, 1999)
  - Implicit representation by using a higher dimension surface

$$\Phi(\mathbf{x}) = 0$$

$$\begin{aligned} \Phi(\mathbf{x}, s) &< 0 \quad \text{if material} \\ \Phi(\mathbf{x}, s) &> 0 \quad \text{if void} \\ \Phi(\mathbf{x}, s) &= 0 \quad \text{on boundary} \end{aligned}$$



- Possible practical implementation: Approximated on a fixed mesh by the signed distance function to curve Γ:

$$\Phi(\mathbf{x}) = \pm \min_{\mathbf{x}_\Gamma \in \Gamma} ||\mathbf{x} - \mathbf{x}_\Gamma||$$

THE LEVEL SET METHOD

- In XFEM framework,
  - Each node has a Level Set dof
  - Interpolation using classical shape functions

$$\Phi(x,s) = \sum_{i=1}^{n} N_i(\mathbf{x})\,\Phi_i$$

  - Material assigned to a part of the Level Set (positive or negative)

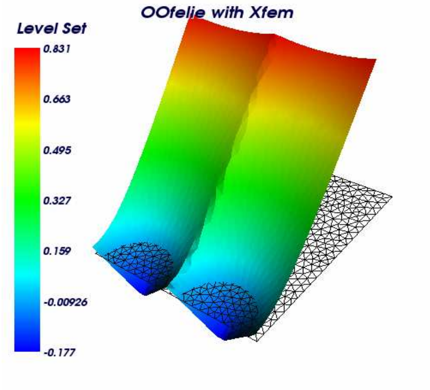
GEOMETRICAL DESCRIPTION USING LEVEL SETS

- Constructive geometry approach
  - Elaborate complex geometries using Level Sets:
    - Primitive shapes with dimension parameters

$$\Phi = \Phi(\mathbf{x}, s)$$

    - Linear combinations of basic functions

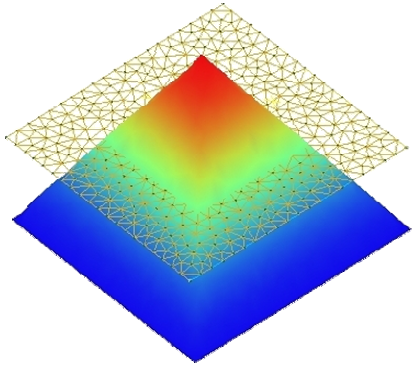
$$\Phi(x, s) = \sum_{i=1}^{n} s_i \, \chi_i(\mathbf{x})$$

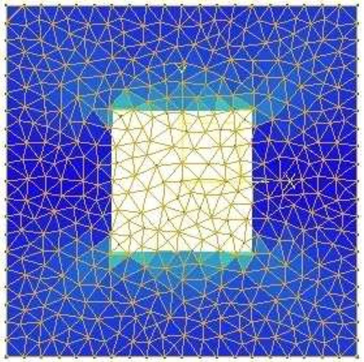


  - Library of graphic primitives and features
    - Lines, circles, ellipses, rectangles, triangles
    - NURBS
- Combine the basic levels sets using logic and Boolean operations

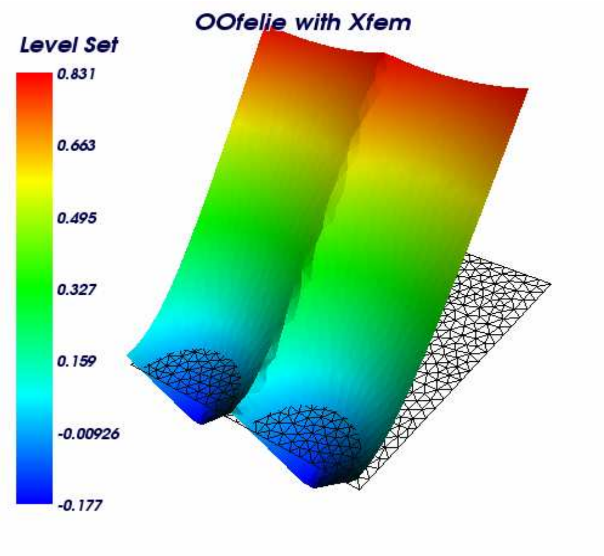
GEOMETRICAL DESCRIPTION USING LEVEL SETS

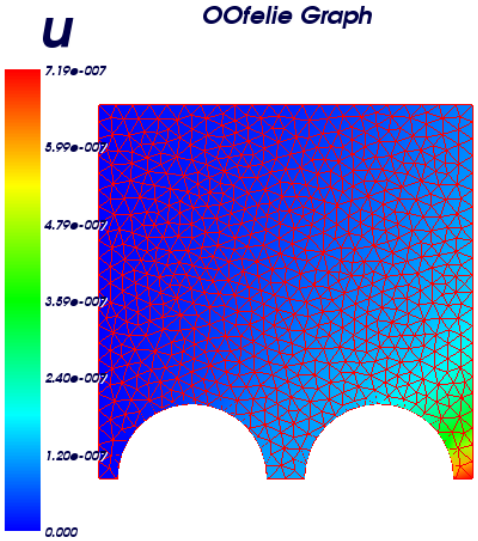
- Level Set of a square hole





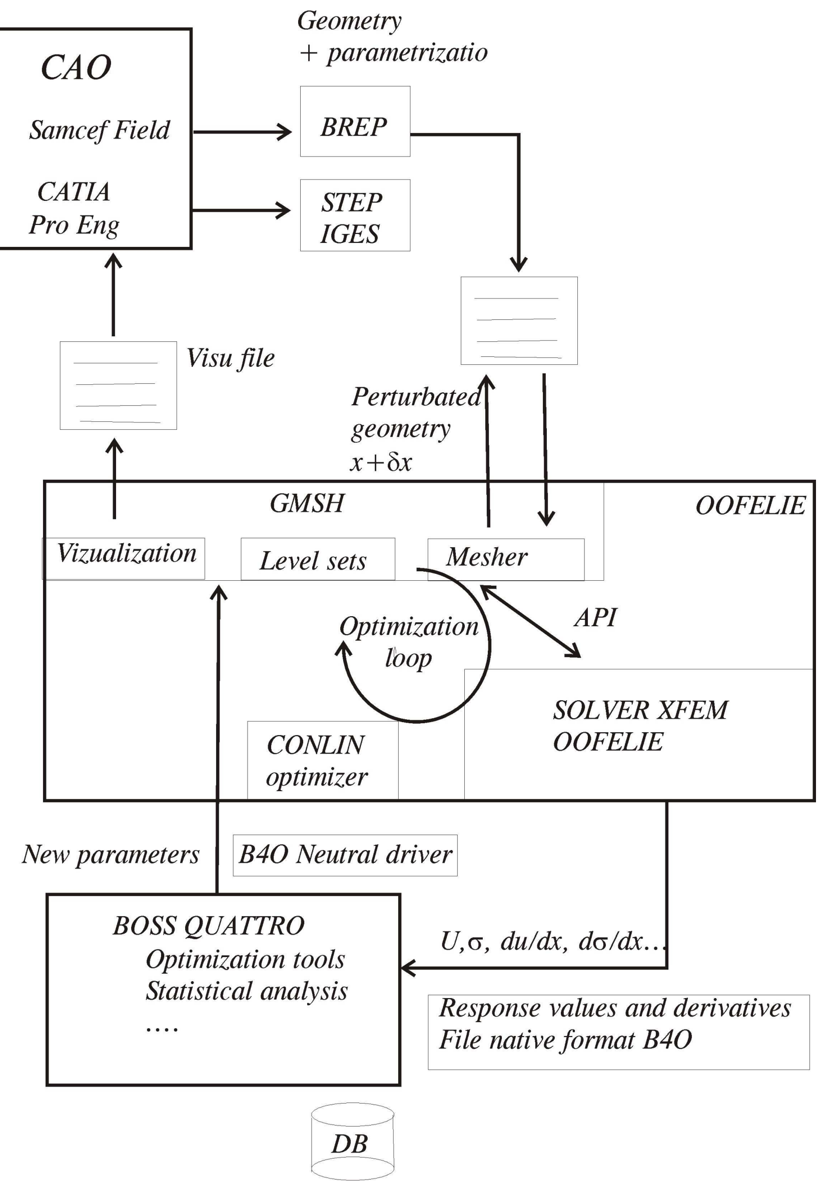
- Combination of two holes



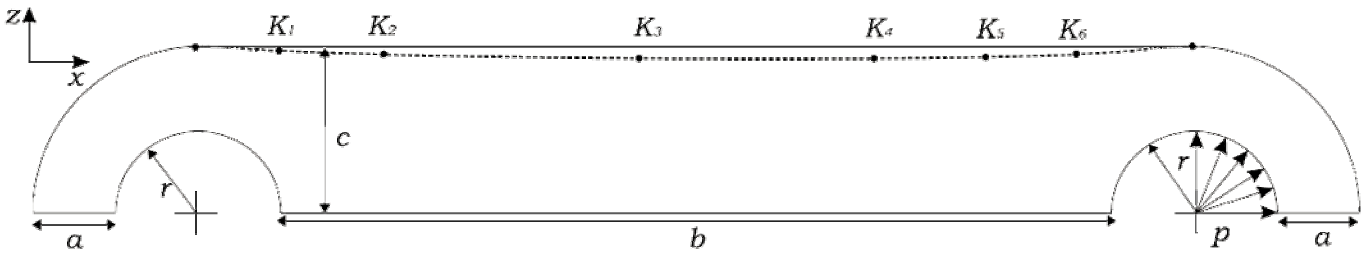


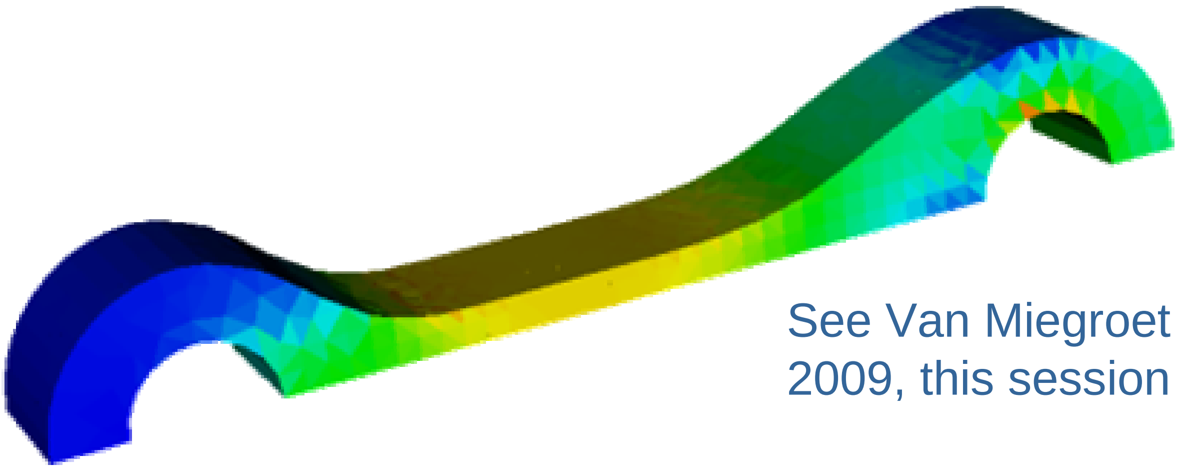
GEOMETRICAL DESCRIPTION USING LEVEL SETS

- Under development EFCONIVO project sponsored by Walloon Region of Belgium:
  - Level Set geometrical modeling (GMSH)
  - Meshing (GMSH)
  - XFEM (OOFELIE) and non conforming numerical methods
  - Optimization (Boss Quattro)



GEOMETRICAL DESCRIPTION USING LEVEL SETS
(Van Miegroet et al., 2007)





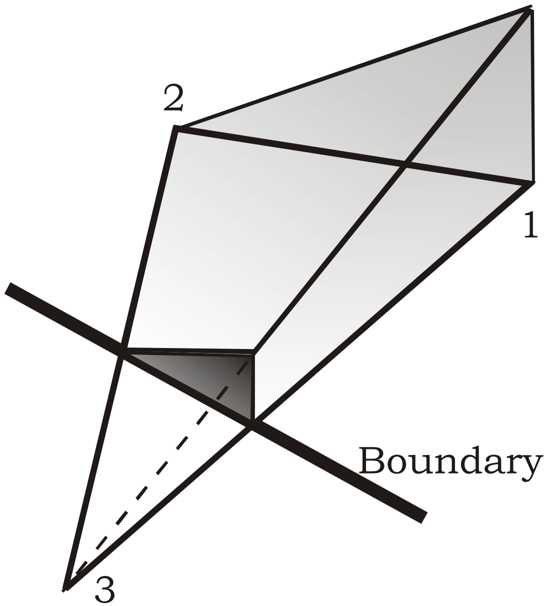
See Van Miegroet (XFEM 2009, this session)

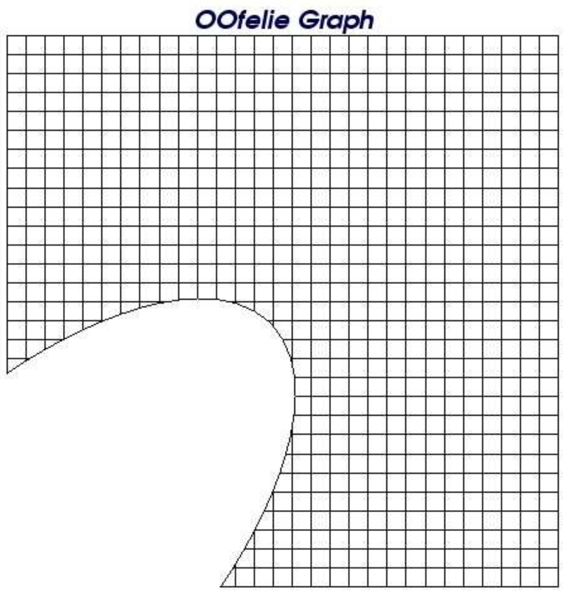
- External boundary : 3D Level Set surface defined by a Nurbs curve
- Parameters : Ki control points
- Design variable: position of control nodes in z direction

EXTENDED FINITE ELEMENT METHOD

- Modelling void-solid boundaries using XFEM
  - No grey or artificial material
  - Using non conforming fixed mesh: avoid remeshing, mesh deformation, etc.

$$u = \sum_{i \in I} N_i(x)\, V(x)\, u_i \qquad V(x) = \begin{cases} 1 & \text{if node} \in \text{solid} \\ 0 & \text{if node} \in \text{void} \end{cases}$$





PROBLEM FORMULATION

- Design Problem
  - Find the best shape to minimize a given objective functions while satisfying design constraints
- Design variables:
  - Parameters of Level Sets
- Objective and constraints
  - Mechanical responses: global (compliance) or local (displacement, stress), eigenfrequencies
  - Geometrical characteristics: volume, distance
- Problem formulation similar to shape optimization but simplified thanks to XFEM and Level Set!

PROBLEM FORMULATION

- Design problem is cast into a mathematical programming problem

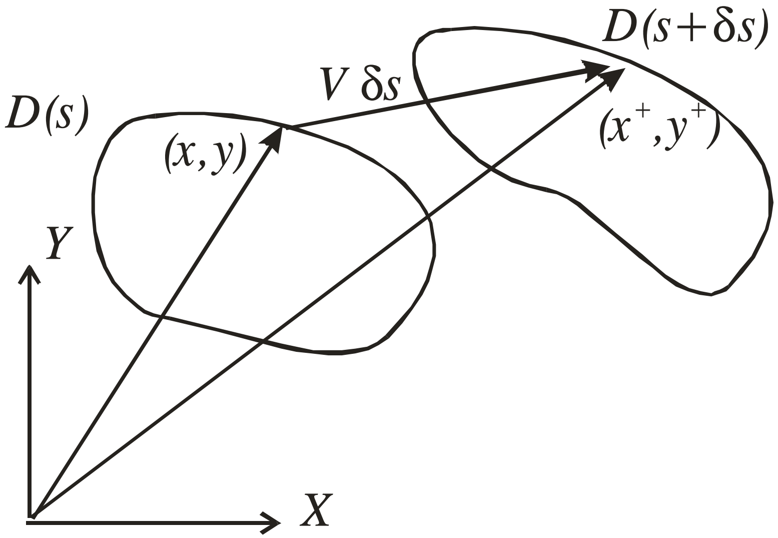
$$\begin{array}{lll} \min\limits_{\mathbf{x}} & g_0(\mathbf{x}) & \\ \text{s.t.:} & g_j(\mathbf{x}) \leq \bar{g}_j & j = 1 \ldots m \\ & \underline{x}_i \leq x_i \leq \overline{x}_i & i = 1 \ldots n \end{array}$$

- Take benefit of the available efficient solvers :
  - CONLIN (Fleury, 1989); MMA (Svanberg, 1987)
  - Solution of large scale problems:
    - 100.000 design variables (topology)
    - 5.000 constraints (shape)
    - 5.000 constraints and 5.000 design variables (composite)
- Requires sensitivity (derivatives) of functions

SENSITIVITY ANALYSIS

- Position of a point after a perturbation of the design variable s_i

$$x_i^+ = x_i + s\,V_i(x_i)$$



With the velocity field V,
i.e. the first order derivative
of position field x:

$$V_i = \frac{\partial x_i}{\partial s}$$

- Material derivative of the displacement u in a given point:

$$\frac{Du(x)}{Ds} = \lim_{s \to 0} \frac{\tilde{u}_j(x) - u_j(x)}{s}$$

$$\frac{Du(x)}{Ds} = \frac{\partial u}{\partial s} + V_i\,\partial_i u_j$$

SENSITIVITY ANALYSIS

- The sensitivity of integral function

$$F = \int_\Omega f(\mathbf{x})\, H(\Phi)\, d\Omega$$

- is

$$\frac{DF}{Ds} = \int_\Omega [\frac{\partial f}{\partial s} + \nabla f.\mathbf{V}]\, H(\Phi)\, d\Omega + \int_\Omega f(x)\, [\delta(\Phi)\, \nabla\Phi.\mathbf{V} + H(\Phi)\, div(\mathbf{V})]\, d\Omega$$

$$\begin{aligned} \frac{DF}{Ds} &= \int_\Omega \frac{Df}{Ds} H(\Phi)\, d\Omega + \int_\Omega f(x)\, div(H(\Phi)\, \mathbf{V})\, d\Omega \\ &= \int_\Omega \frac{Df}{Ds} H(\Phi)\, d\Omega + \int_{\partial\Omega} f(x)\, H(\Phi)\, \mathbf{V}.\mathbf{n}\, d\Gamma \end{aligned}$$

SENSITIVITY ANALYSIS

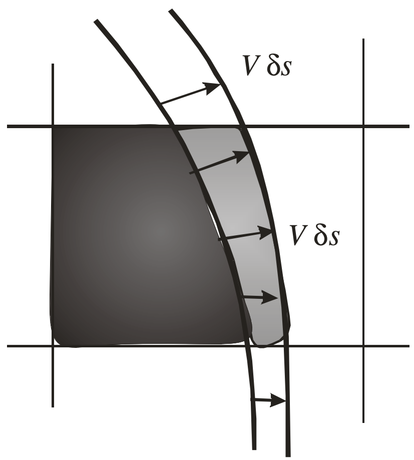
- Proof

$$\frac{DF}{Ds} = \lim_{\delta s \to 0} \frac{1}{\delta s}\{\int_\Omega f^{s+\delta s}(x) H(\Phi^{s+\delta s})\, d\Omega^{s+\delta s} - \int_\Omega f^s(x) H(\Phi^s)\, d\Omega^s\}$$

$$\frac{DF}{Ds} = \lim_{\delta s \to 0} \frac{1}{\delta s} \int_\Omega (f^s(x) + \frac{\partial f}{\partial s}\delta s + \nabla f . \mathbf{V} \delta s)$$
$$(H(\Phi^s) + \delta(\Phi^s) \nabla \Phi . \mathbf{V} \delta s)\, |J|\, d\Omega$$
$$- \int_\Omega f^s(x) H(\Phi^s)\, d\Omega\}$$

$$|J| \simeq 1 + div(\mathbf{V})\, \delta s$$

$$\frac{DF}{Ds} = \int_\Omega [\frac{\partial f}{\partial s} + \nabla f . \mathbf{V}]\, H(\Phi)\, d\Omega + \int_\Omega f(x)\, [\delta(\Phi)\, \nabla \Phi . \mathbf{V} + H(\Phi)\, div(\mathbf{V})]\, d\Omega$$



SENSITIVITY ANALYSIS: DISPLACEMENT FIELD

- The sensitivity of the displacement field comes from the derivative of the state equation (virtual work principle)

$$a(u,v) \;=\; l(v) \quad \forall\, v \in\; H_1^0(\omega)$$

$$a(u,v) \;=\; \int_\Omega H_{ijkl}\varepsilon_{ij}(u)\varepsilon_{kl}(v)\; H(\Phi)\; d\Omega \qquad l(v) = \int_\Omega f_i\, v_i + div(t_i v_i \mathbf{n})\; d\Omega$$

- Applying the previous result gives:

$$\frac{D}{Ds}a(u,v) = \int_\Omega H_{ijkl}\frac{\partial \varepsilon_{ij}(u)}{\partial s}\varepsilon_{kl}(v)\; H(\Phi)\; d\Omega + \int_\Omega H_{ijkl}\varepsilon_{ij}(u)\varepsilon_{kl}(v)\; div(H(\Phi)\,\mathbf{V})\; d\Omega$$

$$\frac{D}{Ds}l(v) \;=\; \int_\Omega (f_i\, v_i \;+\; div\;(t_i\; v_i\; \mathbf{n})\; div(H(\Phi)\,\mathbf{V})\; d\Omega \;=\; \int_{\partial\Omega} \{f_i\, v_i \;+\; div\;(t_i\; v_i\; \mathbf{n})\}\; \mathbf{V}.\mathbf{n}\; d\Gamma$$

SENSITIVITY ANALYSIS: DISPLACEMENT FIELD

- F.E. discretization:

$$\mathbf{u}_h = \mathbf{N}\,\mathbf{q} \qquad \varepsilon(\mathbf{u}_h) = \mathbf{B}\,\mathbf{q}$$

- The equilibrium

$$\mathbf{K}\,\mathbf{q} = \mathbf{g}$$

- The stiffness matrix and the load vector

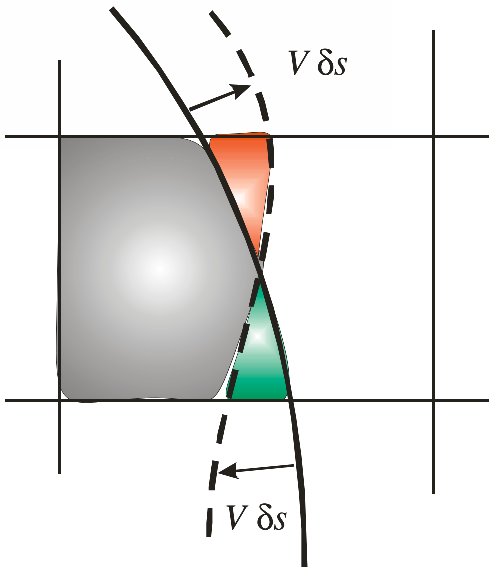
$$\mathbf{K} = \int_\Omega \mathbf{B}^T\,\mathbf{H}\,\mathbf{B}\,d\Omega \qquad \mathbf{g} = \int_\Omega \mathbf{N}^T\,\mathbf{f}\,d\Omega$$

SENSITIVITY ANALYSIS: DISPLACEMENT FIELD

- F.E. discretization of the sensitivity of the virtual work principle

$$\delta \mathbf{q}^T \left[\int_\Omega \mathbf{B}^T \,\mathbf{H}\,\mathbf{B}\, d\Omega\right] \frac{\partial \mathbf{q}}{\partial s} + \delta \mathbf{q}^T \left[\int_\Omega \mathbf{B}^T \,\mathbf{H}\,\mathbf{B}\, div(H(\Phi)\,\mathbf{V})\, d\Omega\right] \mathbf{q}$$

$$= \delta \mathbf{q}^T \left[\int_\Omega \mathbf{N}^T \,\mathbf{f}\, div(H(\Phi)\,\mathbf{V})\, d\Omega\right]$$



- With

$$\frac{\partial \mathbf{K}}{\partial s} = \int_\Omega \mathbf{B}^T \,\mathbf{H}\,\mathbf{B}\, div(H(\Phi)\,\mathbf{V})\, d\Omega$$

$$\frac{\partial \mathbf{g}}{\partial s} = \int_\Omega \mathbf{N}^T \,\mathbf{f}\, div(H(\Phi)\,\mathbf{V})\, d\Omega$$

- Gives the sensitivity of the displacements

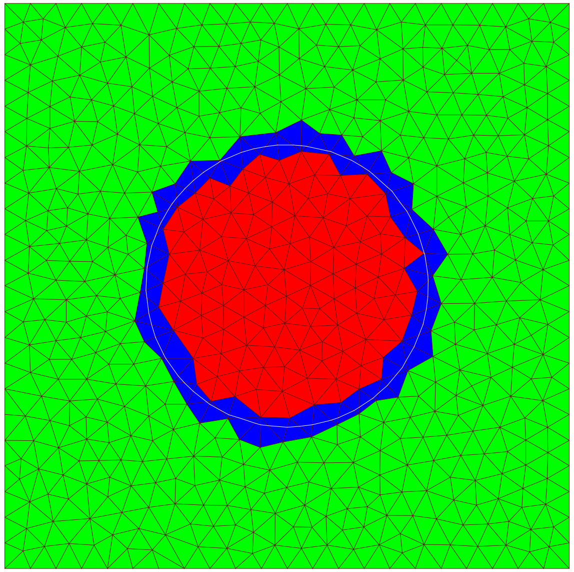
$$\mathbf{K}\frac{\partial \mathbf{q}}{\partial s} = \left\{\frac{\partial \mathbf{g}}{\partial s} - \frac{\partial \mathbf{K}}{\partial s}\,\mathbf{q}\right\}$$

SENSITIVITY ANALYSIS

- Generally, the sensitivity analysis is carried out using the semi-analytical approach

$$\frac{\partial \mathbf{K}}{\partial s} \simeq \frac{\mathbf{K}(s+\delta s)-\mathbf{K}(s)}{\delta s} \qquad \frac{\partial \mathbf{g}}{\partial s} \simeq \frac{\mathbf{g}(s+\delta s)-\mathbf{g}(s)}{\delta s}$$

- The derivatives are non trivial and only evaluated in the boundary layer



- The approach requires:
  - To keep the same mesh and the same FE discretization
  - To keep the same number of degrees of freedom including the number of extra dof.

Sensitivity in Shape Optimization of complex 3D geometries using Level Sets and non conforming finite elemnts

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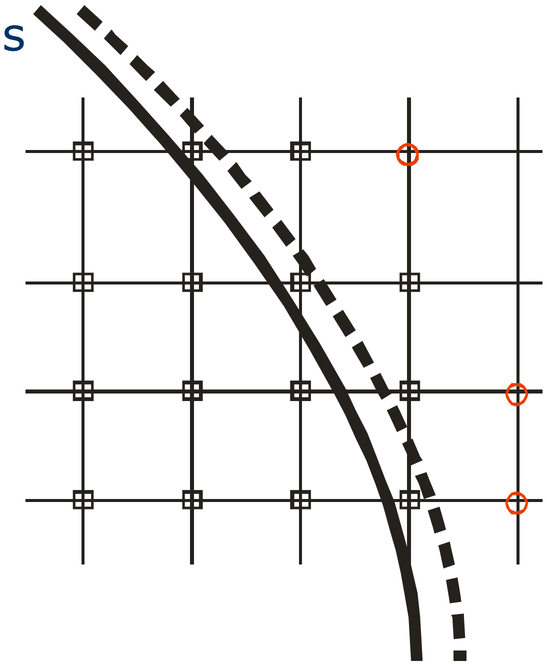
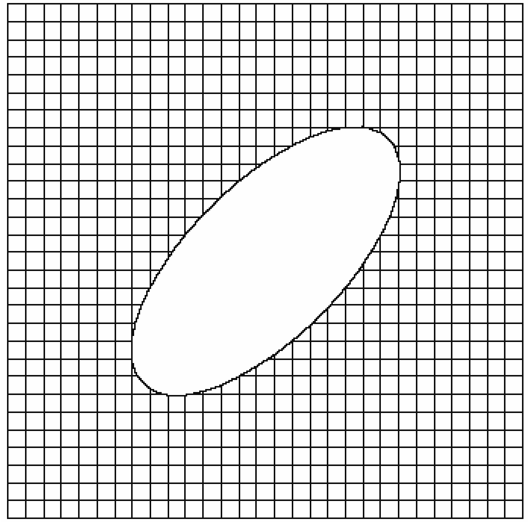
SENSITIVITY ANALYSIS

- Strategies to freeze the number of dof
  - What happens if perturbed level sets comes into new FE?
  - Ignore the new elements that become solid or partly solid
    - small errors, but minor contributions
    - practically, no problem observed
    - efficiency and simplicity
    - validated on benchmarks



SENSITIVITY ANALYSIS– validation

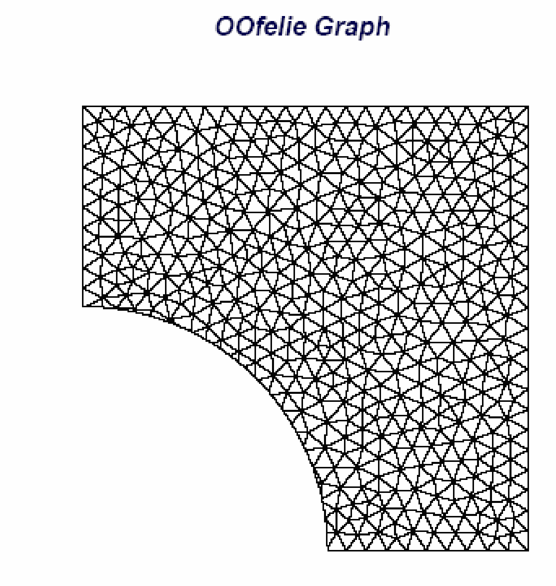
- Validation of semi-analytic sensitivity:
  - Elliptical hole
  - Parameters: major axis *a* and Orientation angle θ w.r.t. to horizontal axis
  - Perturbation: $\delta = 10^{-4}$
  - Sensitivity of compliance



Design variables	*Finite differences*	*Semi-analytical approach*	*Relative error (%)*
$a = 0.6$	$3698,0000$	$3691,3344$	$0,1802$
$\theta = \pi/4$	$478,0000$	$477,0641$	$0,1957$
$a = 0.6$	$2712,000$	$2707,328$	$0,1722$
$\theta = \pi/6$	$523,70000$	$523,4099$	$0,0553$
$a = 0.6$	$783,8000$	$781,3920$	$0,3072$
$\theta = 0$	$11,6239$	$11,6235$	$0,0029$

SENSITIVITY ANALYSIS– *validation*

- Validation of semi-analytic sensitivity:
  - Elliptical hole
  - Parameters: axes *a* and b
  - Perturbation: $\delta=10^{-4}$
  - Sensitivity of maximum stress

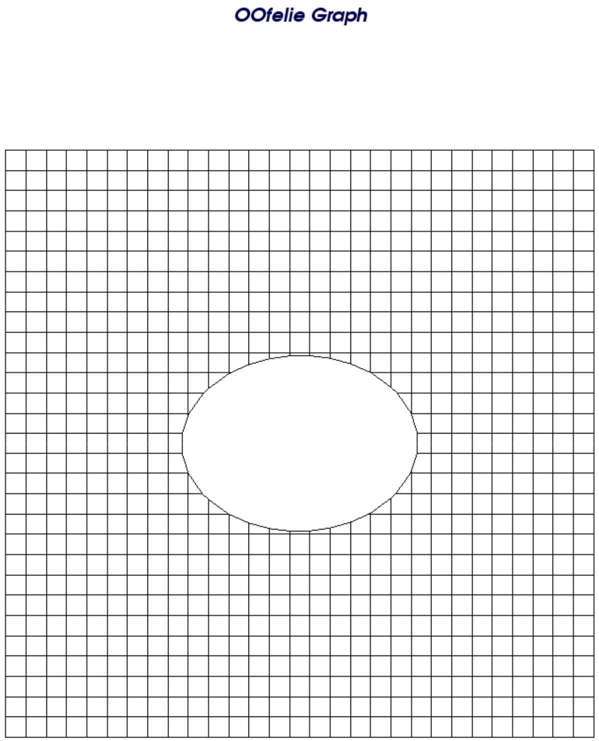


Variable	Finite difference	Semi-analytical	Relative error
$a = 0.41$	-213.2172401	-213.2279160	0.005%
$b = 0.41$	2075.6012932	2074.0930198	0.073%
$a = 0.55$	-51.9288917	-51.888882944695	0.077%
$b = 0.55$	5019.8392594	5003.5389826985	0.32%

APPLICATIONS: Elliptical hole

CLASSICAL PROBLEM OF PLATE WITH A HOLE REVISITED

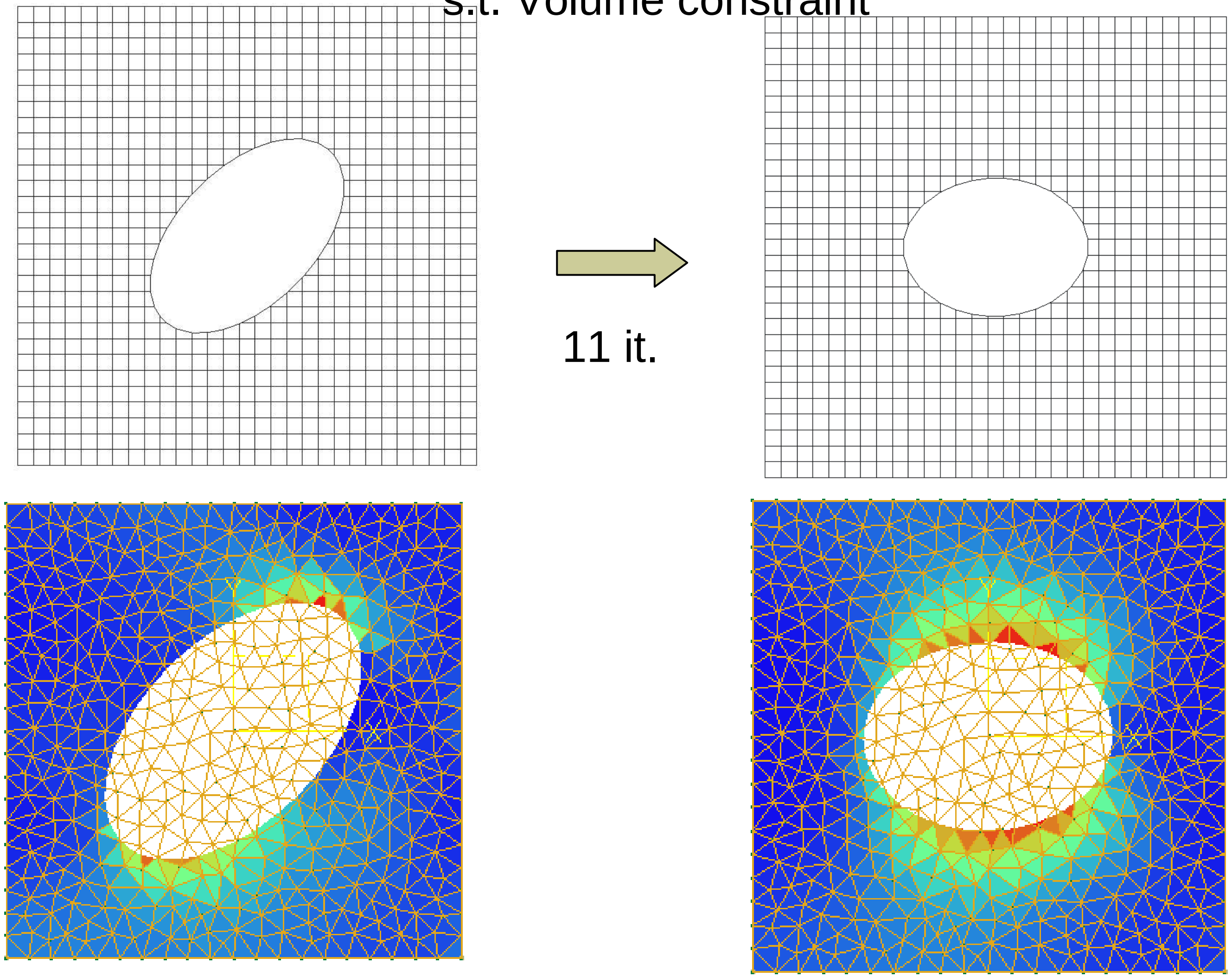
- Square plate with a hole
- Bidirectional stress field
- $\sigma_x = 2\ \sigma_0 \qquad \sigma_y = \sigma_0$
- E= 1 N/m², ν=0.3
- Minimize compliance
  - st volume constraint
- Design variables: major axis a and orientation θ
- Mesh 30 x 30 nodes



Duysinx et al. 2006

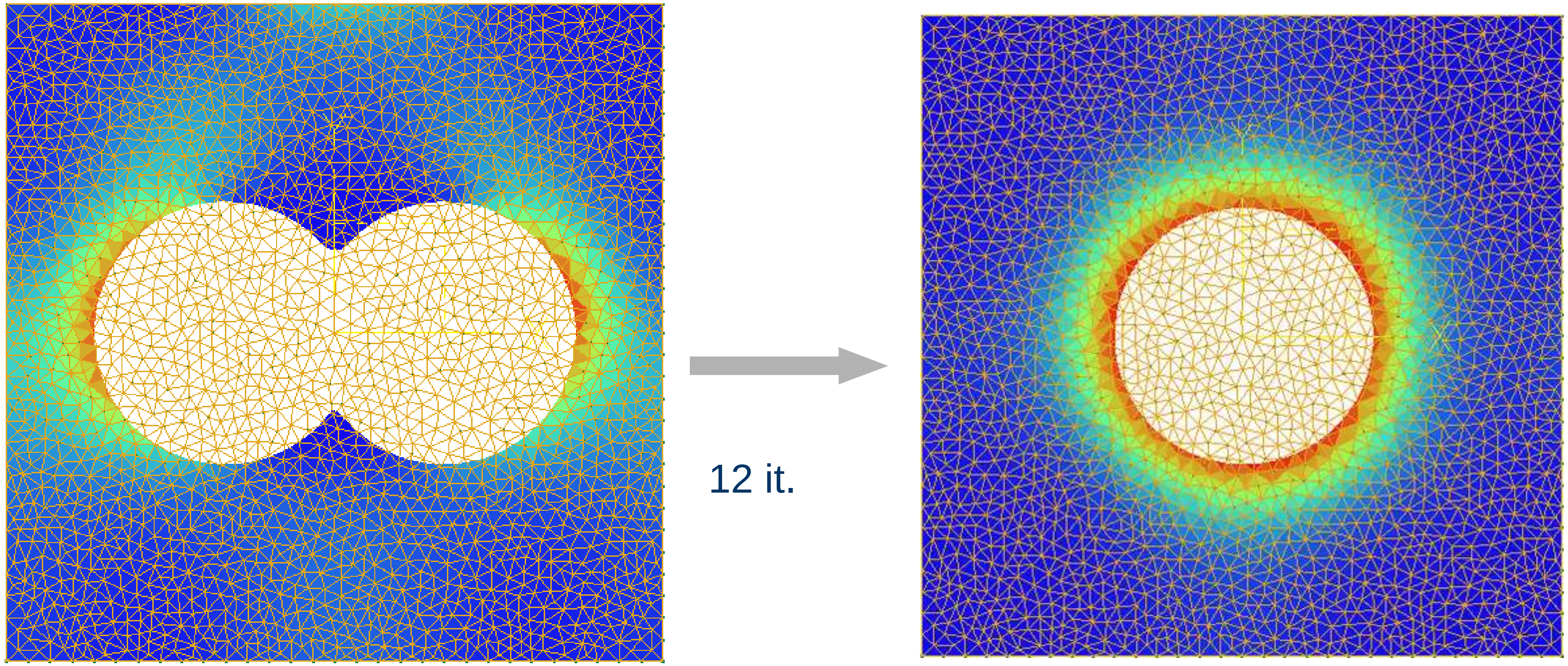
APPLICATIONS

Min Compliance
s.t. Volume constraint



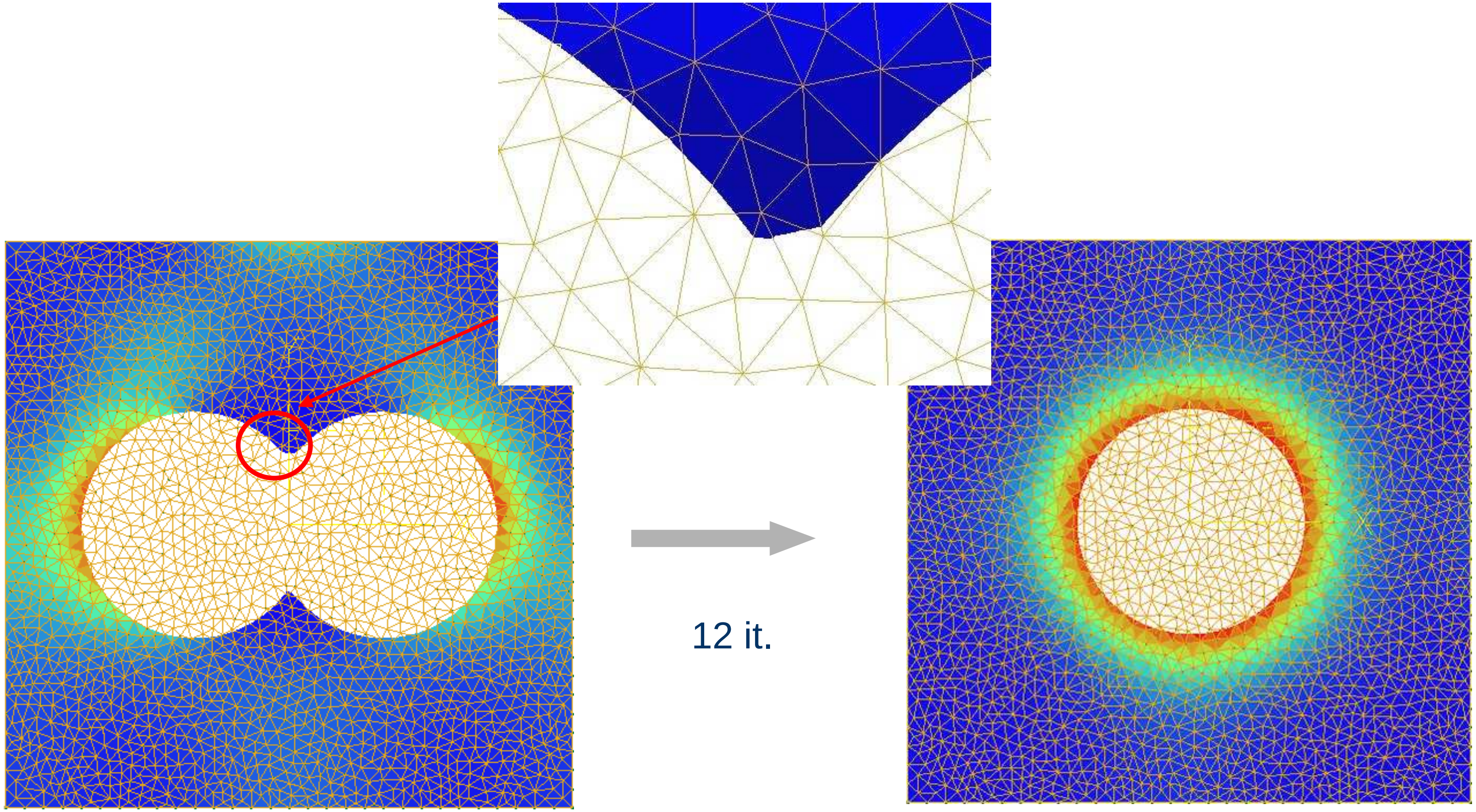
APPLICATIONS

- Topology modification during optimization
  - Two variables : *center* x_1, *center* x_2
  - Min. potential energy under a surface constraint
  - Uniform Biaxial loading : $\sigma_x = \sigma_0$, $\sigma_y = \sigma_0$



APPLICATIONS

- Mesh refinement for the Level Set representation of sharp parts
- Accuracy of stresses

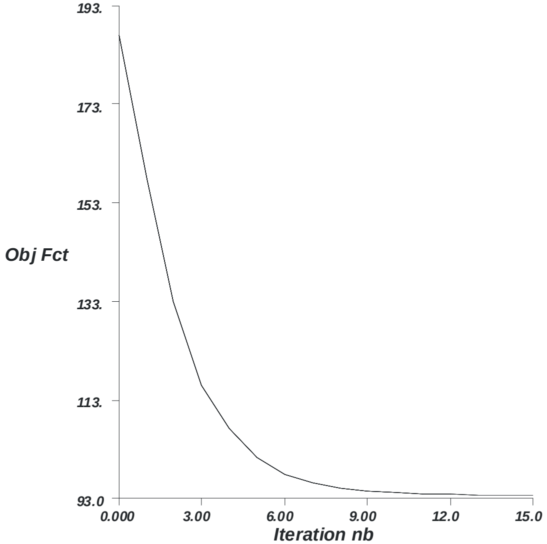


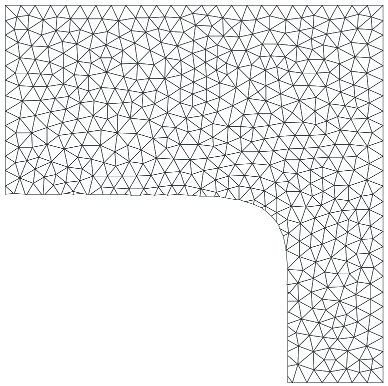
Applications - 2D plate with a hole

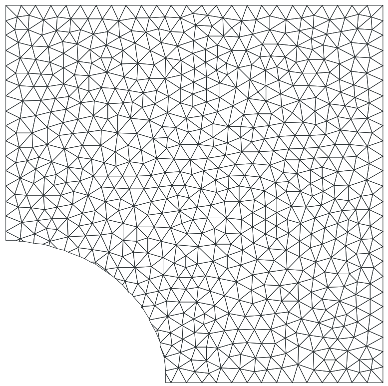
- Plate with generalized super elliptical hole :

$$\frac{|x|^{\alpha}}{|a|} + \frac{|y|^{\eta}}{|b|} = r$$

  - Parameters : $2 < a, b, \eta, \alpha < 8$
  - Objective: min Compliance.
  - Constraint: upper bound on the Volume.
  - Bi-axial Load: $\sigma_x = \sigma_y = \sigma_0$
  - Solution: perfect circle: $a = b = 2, \eta = \alpha = 2$



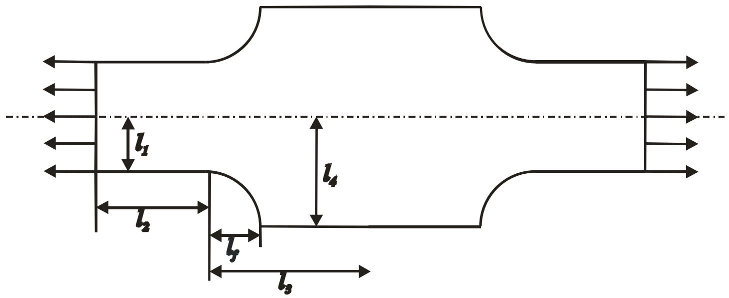


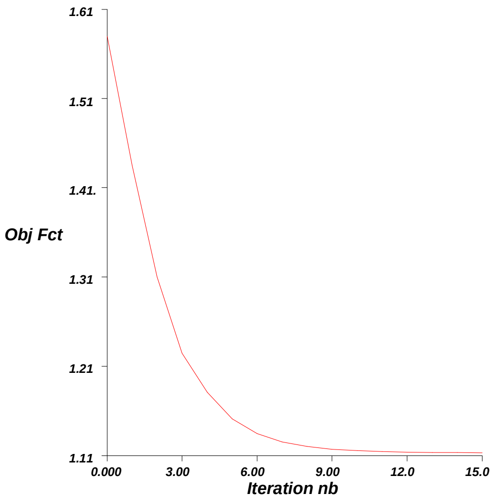


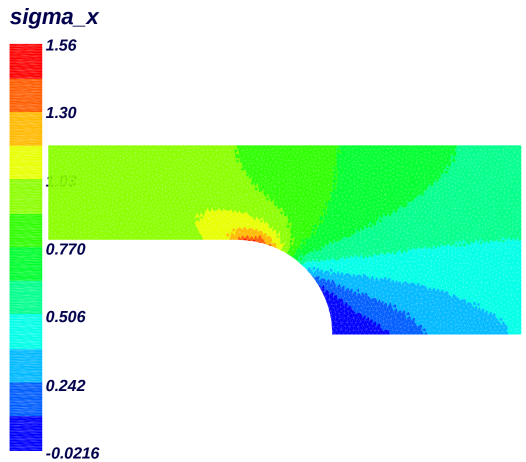
Van Miegroet & Duysinx, SMO, 2007

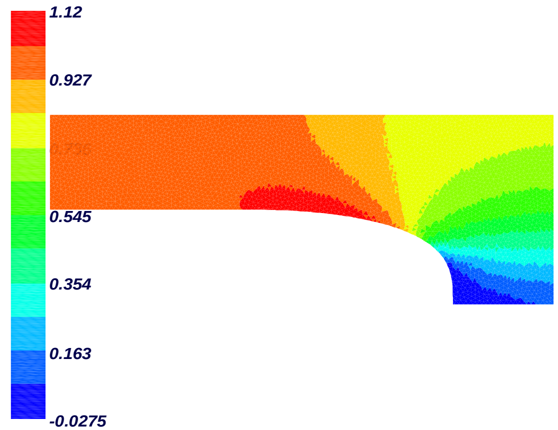
Applications – *2D fillet in tension*

- Shape of the fillet : generalized super ellipse $\frac{|x|^{\alpha}}{|a|} + \frac{|y|^{\eta}}{|b|} = r$
  - Parameters : a, η, α
  - Objective: min (max Stress)
  - No Constraint
  - Uni-axial Load: $\sigma_x = \sigma_0$
  - Solution: stress reduction of 30%









Van Miegroet & Duysinx, SMO, 2007

CONCLUSION

- XFEM and Level Set gives rise to a generalized shape optimisation technique
  - Topology can be modified:
  - Smooth curves description
  - Void-solid description
  - Small number of design variables
  - Global or local response constraints
  - Reduce velocity field and mesh perturbation problems

- Sensitivity analysis
  - Compliance : OK (Allaire et al., Wang et al., Belytchko)
  - Extension to displacement and local responses i.e. stresses
    - Better understanding using material derivative concept
    - Efficient numerical implementation (semi-analytical) for applications

PERSPECTIVES

- Work in progress:
  - Construction of an integrated design environment using constructive geometry and XFEM
    - EFCONIVO project
  - Local stress constraint estimation and error estimation in XFEM
  - Adaptive (but non conforming) meshing
  - Boundary conditions along non-conforming curves
  - Coupled electromechanical simulation and optimization using XFEM

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