OF SWAY COMPOSITE FRAMES

R.MAQUOI, J.P. JASPART and J.F. DEMONCEAU
Department M&S, Liège University
Liège, Belgium
r.maquoi@ulg.ac.be
jean-pierre.jaspart@ulg.ac.be
ifdemonceau@ulg.ac.be

ABSTRACT

This paper presents numerical and analytical studies carried out at Liège University, as part of a European research project, with the objective to investigate the in-plane behaviour of composite sway frames under static loading.

1. INTRODUCTION

As most multi-storey structures, composite structures are often laterally restrained by efficient bracing systems. But moment resisting frames are an alternative that offers much flexibility for internal arrangement and exploitation of the construction. When sufficient stiffness and strength with regard to lateral forces are achieved, they are a structural solution which can resist lateral loads. In seismic regions, properly designed moment resisting frames constitute the best choice regarding the available ductility and the capacity of energy dissipation. This is stated in Eurocode 8 [Eurocode 8 2003] devoted to earthquake engineering in which high values of the behaviour factor are recommended. Such frames are however prone to second-order effects; the latter have to be predicted carefully because they may govern the design.

Eurocode 4 [Eurocode 4 1992], which deals with composite construction under static loading, contains mainly design procedures for non-sway composite buildings. That is why a research project on global instability of composite sway frames has been funded by the European Community for Steel and Coal (ECSC). The objective of this project was to provide background information on the behaviour of such frames under static and seismic loads and to provide simplified design rules.

In the present paper, numerical investigations conducted at Liège University on two composite frames under static loading are described. Also, the applicability, to one of the studied structures, of simplified analytical methods used for steel sway frames is investigated.

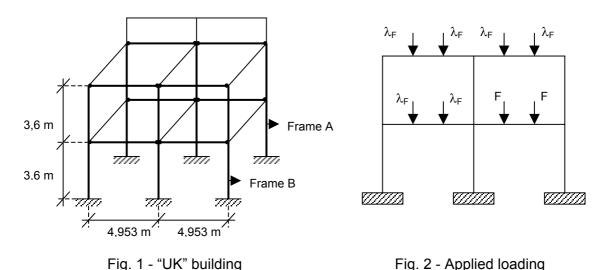
2. REFERENCE STRUCTURES

2.1 "UK" building

The "U.K." building has been tested at BRE (Building Research Establishment), U.K. The test report is well documented (yield strengths, dimensions, type of loading); in particular, the behavioural curves of the structural joints are given ([Li et al 1996 a], [Li et al 1996 b]). Even if the building is braced and therefore does not exhibit sway effects, its has been initially used as

a benchmark to demonstrate the ability of different FEM software (in particular the Liège one called FINELG) to simulate the behaviour of composite systems.

The structure (Fig. 1) is composed of two parallel two-storey two-bay main frames (namely "Frame A" and "Frame B") connected by secondary beams. The bare steel columns support composite floors made of steel beams connected to a reinforced concrete slab by shear studs welded on their upper flange.



In Frame A, all the columns are bent about their major axis, while those of Frame B are about their minor axis. Both frames are subjected to concentrated loads F applied at one third and two thirds of each beam span (Fig. 2). These forces are increased proportionally (λ load factor) until failure, except those applied to the lower right beam that are kept constant.

2.2 "Bochum" frame

The "Bochum" structure is a 2-D full-scale structure tested at Bochum University (Germany) under static loading [ECSC "Sway" project 2004]. Tests on joints in isolation have been performed so as to get their actual properties in terms of moment-rotation curves. The structure has been designed at Liège University in kind collaboration with Bochum University so as to fail by global in-plane instability.

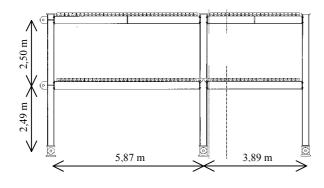


Fig. 3 - General layout of the "Bochum" frame

The "Bochum" building (Fig. 3) is a two-bay two-storey frame with a total height of 4,99 m and a total width of 9,76 m. Columns A and C are made of HEB260 profiles and column B of a HEB280 one. Columns bases are pinned.

The beams are IPE300 sections, the upper flange of which is connected to a composite slab by means of shear studs. According to the draft European standard prEN1994-1-1, still available for confidential use only, all the joints are classified as semi-rigid and partial-strength.

Because of the available experimental facilities, the applied "service" loads on the frame are as follows:

- a load of 400 kN applied at the top of each column (gravity loads from the upper storeys);
- uniform and concentrated gravity loads as indicated in Fig. 4;
- horizontal loads of 50 kN applied at both floor levels.

The actual loading sequence is the following: all the gravity loads are first increased up to their nominal values (1262 kN in total); they are then kept constant while the horizontal loads are progressively magnified by a load factor λ (see Fig. 4) until failure (λ_u).

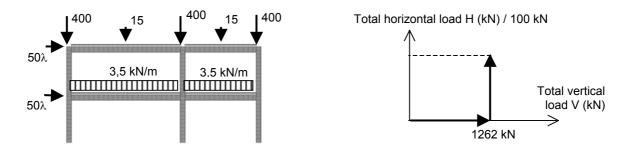


Fig. 4 - Loading conditions & loading sequence for the "Bochum" frame (kN)

3. VALIDATION OF THE FINELG SOFTWARE

3.1 "UK" building

In this section, a benchmark study aimed at validating several finite element software used for the numerical simulation of the non-linear behaviour of composite sway structures is described. The reference structure is the "U.K." building (§ 2.1) for which both detailed data and test results are available.

The European project partners involved in the numerical studies were: (i) RWTH Aachen (Germany) – DYNACS software, (ii) Pisa University (Italy) – ADINA 7.5 software and (iii) Liège University (Belgium) – FINELG software [FINELG 1999]. FINELG is a full non-linear finite element analysis program developed at Department M&S of Liège University.

Frames A and B (Fig. 1) have been modelled as planar frames and simulated separately up to failure so as to get the ultimate load factor, the corresponding failure mode as well as the load-deflection curves. A detailed comparison of the results may be found in [ECSC "Sway" project, 2004]; as an example, the load-deflection curves (mid-span) for the lower floor beam of Frame B are given in Fig. 5.

The simulations conducted with different software are found in a reasonably good agreement with test results. More especially, the validity of the FINELG software is demonstrated.

3.2 "Bochum" frame

Here, a comparison between the numerical prediction obtained through a "full" non-linear analysis (see § 4.2) performed with FINELG and the experimental results obtained through the "Bochum" test is presented.

The actual material properties have been integrated into the numerical computation; these properties have been obtained through coupon tests for the steel elements and through compression tests for the concrete ones [ECSC "Sway" project, 2004].

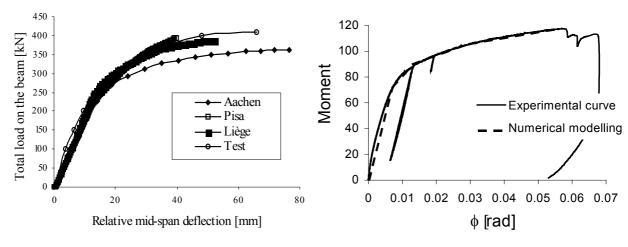


Fig. 5 - Frame B: lower beam load-deflection curves

Fig. 6 – Numerical modelling of the joint behaviour

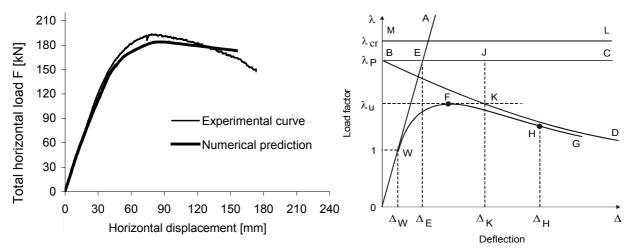


Fig. 7 - Comparison between numerical prediction and experimental result

Fig. 8 – Graphical representation of the results got from different structural analyses

As all the joint configurations of the "Bochum" frame have been tested in isolation [ECSC "Sway" project, 2004], the actual M- ϕ joint behavioural curves have been considered through a multi-linear approximation (see Fig. 6).

The "total horizontal load – horizontal displacement" curves obtained numerically and in laboratory are compared in Fig. 7. It can be observed that the numerical curve fits very well with the experimental one; the numerically predicted ultimate load is lower than the experimental one but with an acceptable difference of 4.8%. This justifies the further use of the FINELG software in the present paper to investigate the behaviour of composite sway frames.

4. NUMERICAL INVESTIGATIONS

4.1 Introduction

The present paragraph introduces the numerical investigations carried out by means of FINELG on the "Bochum" frame. Reference is made in these calculations to the nominal material and connection properties and not to the actual measured ones as in section 3.2 (see [Demonceau

2004]). Moment-rotation curves for joints are defined according to Eurocode 3 and Eurocode 4 provisions.

4.2 Types of analysis

Different types of analysis have successively been performed; there are briefly presented below:

- Elastic critical analysis: the aim is to evaluate the elastic critical load factor λ_{cr} that corresponds to the first mode of global instability. According to Eurocode 3 for steel frames [Eurocode 3, 1992], the $\lambda_{Sd}/\lambda_{cr}$ ratio λ_{Sd} being the load factor associated to the design loads allows to determine whether a frame is laterally rigid or, in contrast, prone to sway. When this ratio is lower than 0,1, the frame is said "rigid", otherwise it is "sway".
- First-order rigid-plastic analysis: this calculation results in the first-order rigid-plastic load factor λ_p . It can be obtained easily by hand-calculation, or by using appropriate software.
- Second-order rigid-plastic analysis: this analysis differs from the previous one by the fact
 that equilibrium equations are now expressed with reference to the deformed frame
 configuration. It gives an indication on how second-order effects develop once the first-order
 rigid-plastic mechanism is formed and on how much they affect the post-limit resistance.
 The second-order effects are without significant influence on the plastic beam mechanisms
 but strongly affect the post-limit response for panel and combined beam-panel ones.
- Non-linear analysis: in this type of analysis, all the geometrical and material non-linearities are accounted for: actual material stress-strain curves, semi-rigid response of the joints and second-order effects induced by frame and element geometrical imperfections. The initial deformation of the buildings is evaluated in accordance with Eurocode 4. Such an analysis enables an accurate estimation of the actual ultimate load factor λ_u .

In Fig. 8, the results of the different analyses described in this paragraph are qualitatively illustrated. This figure shows how the total sway displacement Δ influences the value of the load factor λ got from the several types of analysis.

Line "OA" corresponds to a purely elastic first-order analysis. The result of the elastic critical analysis is given by the horizontal line "ML" (elastic critical load factor λ_{cr}). The first-order rigid-plastic analysis is represented by the curve "OBC". The "OBD" curve represents the second-order rigid-plastic analysis: when the rigid-plastic load factor λ_p is reached (in "B"), its value decreases with an increasing transverse displacement ("BD" curve). The "OFG" curve results from the non-linear analysis; it reflects the "actual" frame behaviour. The ultimate load factor λ_u corresponds to the peak ordinate of the load-displacement curve (in "F"). If, at λ_u , the failure of the structure is due to the formation of a full plastic mechanism, the "actual" behavioural curve "OFG" and the second-order rigid-plastic analysis line "OBD" join at point "F" (which should then correspond to point "K"). If a global frame instability occurs before the development of a plastic mechanism, the "actual" curve remains below the second-order rigid-plastic one "OBD" and point "F" differs from point "K". This situation is the one illustrated in Fig. 8.

4.3 Numerical results

From the elastic critical analysis of the "Bochum" frame, $V_{sd}/V_{cr}=0.11$. The latter one is just slightly higher than 0.1 that corresponds to the sway/non-sway classification boundary introduced in § 4.2.

A first-order rigid-plastic analysis has also been performed. Amongst the possible plastic mechanisms, the beam ones may be disregarded as the vertical loads, once applied, are kept

constant (Fig. 4). As far as panel and combined beam/panel mechanisms are concerned, the following minimum values of λ_p are found:

• panel mechanism: $\lambda_p = 1.82$ • combined mechanism: $\lambda_p = 9.42$

So, the first-order rigid-plastic failure mode is associated to the development of a global panel mechanism.

A full non-linear analysis gives an ultimate load factor λ_u = 1,41 to which corresponds a top sway displacement of 85 mm (initial out-of-plumb not included). The horizontal load-displacement curve (Fig. 9) starts at a non-zero abscissa which represents the initial out-of-plumb of the frame. The general shape of this curve, especially the descending branch in the post-limit regime, indicates that failure results from an instability and, in this specific case, from a global frame instability. Fig. 10 shows indeed that the number of plastic hinges at failure is smaller than the one required to form a full plastic mechanism (7 hinges instead of 8). The elastic load factor λ_e , corresponding to the formation of the very first plastic hinge is equal to 1,26.

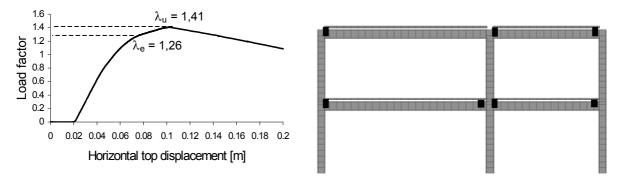


Fig. 9 - Total horizontal load - top displacement curve for the "Bochum" structure

Fig. 10 - Location of the plastic hinges at failure for the "Bochum" structure

As yet said above, panel mechanisms are significantly influenced by second-order effects. A second-order rigid-plastic analysis is therefore conducted in order to evaluate the influence of the geometrical non-linearities on the value of the first-order rigid-plastic load factor λ_p . The relevant results are given in Fig. 9. The descending branch of the frame response obtained from a non-linear analysis is below but quite close to the one deduced from the second-order rigid-plastic analysis even if, as shown in Fig. 10, the corresponding failure load is not, strictly speaking, associated to a full plastic mechanism with eight plastic hinges.

That is due to the fact that (Fig. 10): i) only one plastic hinge at one beam end (right hand-side of the left upper beam) is missing before a global panel mechanism is formed, and ii) the bending moment in this cross-section when the last hinge (the seventh) forms is only 10% lower than the plastic moment resistance of the cross section.

Finally, an elastic critical analysis is performed on the frame in which seven perfect plastic hinges are introduced at the beam ends and located as shown in Fig. 10. The elastic critical load of this structure is seen to amount 95 percent of the total applied vertical loads (see Fig. 4). This last analysis confirms the above prediction that failure is due to a global frame instability subsequently to the development of a seventh plastic hinge in the frame. Also this failure mode is the one observed during the test carried out in Bochum.

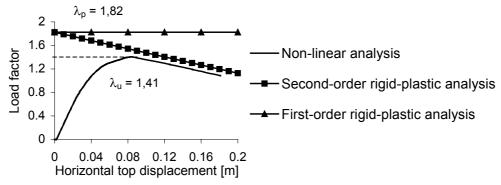


Fig. 11 - Second-order rigid-plastic analysis for the "Bochum" structure

5. APPLICABILITY OF SIMPLIFIED ANALYTICAL METHODS

5.1 Introduction

Several simplified analytical methods for frame analysis and design do exist and some of them are referred to in Eurocode 3. Investigate whether and how these design procedures could be generalised to composite sway frames is the aim of the present section. Two of these methods are examined in the following paragraphs: the amplified sway moment method and the Merchant-Rankine approach.

Investigations on the applicability of both methods to sway composite structures have been conducted with satisfying results [Demonceau et al 2004]. In the following paragraphs, the Bochum structure is similarly examined.

5.2 Amplified sway moment method

In this method, a cracked first-order linear elastic analysis is first carried out; then, the resulting internal forces are amplified by a "sway factor" so as to ascertain for second-order sway effects. Finally, the design load resistance of the frame is obtained by computing the load at which a first plastic hinge develops in the frame. This method permits a direct comparison with the elastic load factor λ_e got from numerical simulation.

The steps to be crossed when applying this elastic design procedure are as follows:

- A cracked first-order elastic analysis is performed on the frame fitted with horizontal supports at each floor level; it results in a distribution of bending moments in the frame and reactions at the horizontal supports. In this analysis, the location and the extension of the cracked zones are defined according to Eurocode 4.
- Then, another cracked first-order elastic analysis is conducted on the initial frame subjected to the sole horizontal reactions obtained in the first step; the resulting bending moments are the so-called "sway moments". For this particular loading situation, cracked zones extend to half of the beam spans so as to be in accordance with the general provisions of Eurocode 4.
- Approximate values of the "actual" second order moments are got by summing up the moments obtained respectively in the two frame analyses with amplification of the sole sway moments by the sway factor:

$$\frac{1}{1 - \frac{V_{sd}}{V_{cr}}} \tag{1}$$

where V_{sd} and V_{cr} are respectively the resultant of all the gravity design loads and the lowest elastic critical load associated to a global sway instability mode. According to the latest still unpublished draft of Eurocode 4, an estimation of the "cracked" value of V_{cr} is required. But

it is shown in [Demonceau 2004] that the influence of this parameter on the accuracy of the amplified sway moment method is not significant; as a result, an "uncracked" value of V_{cr} is considered for sake of simplification.

• The maximum elastic resistance of the frame is reached when the first plastic hinge forms.

Above design procedure is rather simple, as it only requires first-order elastic analyses. Also the principle of superposition remains applicable (in case of numerous loading cases). According to Eurocode 3, the amplified sway moment method is restricted to structures characterized by 0,1 < $V_{sd}/V_{cr} \le 0,25$; this condition is met for the "Bochum" frame.

The application of this method leads here to an elastic load multiplier λ_e = 1,2; this value has to be compared with λ_e = 1,26 drawn from a full non-linear analysis (see § 4.3); the difference amounts 4,9 % (on the safe side). It may therefore be concluded to a pretty good agreement between the two results; this confirms previous investigations (Demonceau et al 2004).

5.3 Merchant-Rankine approach

The Merchant-Rankine method is a second-order elasto-plastic approach, which was developed for bare steel frames; it aims at assessing the ultimate load factor through a formula that reflects the interaction between plasticity (λ_p) and instability (λ_{cr}) in a simplified and empirical way. A direct comparison with the ultimate load factor λ_u got from the numerical investigations may be achieved. The Merchant-Rankine basic formula (MR) writes:

$$\frac{1}{\lambda_u} = \frac{1}{\lambda_{cr}} + \frac{1}{\lambda_p} \tag{2}$$

or:

$$\lambda_u = \frac{\lambda_p}{1 + (\lambda_p / \lambda_{cr})} > \lambda_p \tag{3}$$

Should the frame be very stiff against sway displacements, then λ_{cr} is much larger than λ_p with the result of a low λ_p/λ_{cr} ratio: a minor influence of the geometrical second-order effects is expectable and the ultimate load is therefore close to the first-order rigid-plastic load. In contrast, a flexible sway frame is characterised by a large value of the λ_p/λ_{cr} ratio. It shall collapse according to a nearly elastic buckling mode at a loading magnitude, which approaches the elastic critical bifurcation load.

To allow, in a general treatment, for the minimum beneficial effects to be expected from both strain hardening and cladding, Wood suggested a slightly modified Merchant-Rankine formula (MMR):

$$\lambda_u = \frac{\lambda_p}{0.9 + (\lambda_p / \lambda_{cr})} > \lambda_p \tag{4}$$

in the range $\lambda_{cr}/\lambda_p \ge 4$. He recommended not to use it in practice when $\lambda_{cr}/\lambda_p < 4$ and suggested a second-order elastic-plastic analysis.

When $\lambda_p/\lambda_{cr} \leq 0.1$, λ_u is limited to λ_p , what means that the frame can be designed according to the simple first-order plastic hinge theory. A clear and direct relationship may be established between this criterion and the one, which, according to Eurocode 3, enables to classify steel frames as sway (V_{Sd} / V_{cr} > 0,1) or rigid (V_{Sd} / V_{cr} \leq 0,1). Similarly, the limitation of the range of application of the amplified sway moment method to V_{sd}/V_{cr} values lower than 0,25 is seen to be strongly related to the above $\lambda_{cr}/\lambda_p \geq 4$ range of the modified Merchant-Rankine formula.

The use of Formula (4) is commonly restricted to frames in buildings, in which:

- the frame is braced perpendicular to its own plane;
- the average bay width in the plane of the frame is not less than the greatest storey height;
- the frame does not exceed 10 storeys in height;
- the sway at each storey, due to non-factored wind loading, does not exceed 1/300 of the storey height;
- $\lambda_{cr}/\lambda_{p} \geq 4$.

From complementary studies carried out at Liège University [Maquoi & Jaspart 2001], the MMR approach is seen to exhibit a different degree of accuracy according to the type of first-order rigid-plastic failure mode which characterises the frame under consideration: (i) safe for beam plastic mechanisms, (ii) adequate for combined plastic mechanisms and (iii) unsafe for panel plastic mechanisms. As a result, the application of the MMR approach to structures exhibiting a first-order panel plastic mechanism should therefore be prohibited.

In [Maquoi & Jaspart 2001], the scope of the MMR formula has been extended to structures with semi-rigid and/or partial-strength joints and its applicability to composite structures is contemplated.

The MMR approach cannot be applied to composite construction in a straightforward way. It has been developed for sway steel building frames where the loss of stability is related to the onset of plastic hinges. Another source of deformability exists in composite structures: concrete cracking, which develops well before the formation of the first plastic hinge. This effect, which is specific to composite construction, tends to increase the lateral deflection of the frame, amplifies consequently the second-order effects and reduces the ultimate resistance. In order to incorporate this detrimental effect into the MMR approach, it is suggested [Demonceau et al 2004] to substitute the critical "uncracked" instability load factor $\lambda_{\text{cr,uncracked}}$ by a "cracked" one, noted $\lambda_{\text{cr,cracked}}$. This one is obtained – in accordance with Eurocode 4 - by using an effective stiffness for the columns and by defining cracked zones in the beams. The validity of the so-obtained MMR approach for composite structures (CMMR approach) has been investigated and satisfactory results have been obtained.

The MR approach is based on the concept of "proportional loading", what is not the case for the "Bochum" structure (Fig. 4). So an alternative method is presented herein; it enables an assessment of the ultimate load factor by means of the modified Merchant-Rankine approach applied to composite structures (CMMR approach).

This alternative method consists (Fig. 12):

- in deriving a CMMR interaction curve in a "normalised V H" diagram, V and H (Fig . 4) being respectively the total vertical and horizontal applied loads at failure;
- in plotting in this diagram the actual loading path followed during the test (Fig. 4);
- in defining the failure load at the intersection of the CMMR interaction curve with the one representing the actual loading path.

This approach is based on the assumption that the ultimate load factor of a structure is independent of the loading history; this is not rigorous but it usually appears as acceptable.

When low vertical loads are applied to the structure, $\lambda_u = \lambda_p = 1,82$ corresponds to the development of a panel plastic mechanism (line "AB"). This indicates that second-order effects are quite negligible in this loading range and that no instability phenomena occurs. Beyond point "B", second-order effects can no more be neglected and the CMMR computed values λ_u reduce when the importance of the vertical loads in the load combinations increases. At point "B", the ratio $\lambda_{cr,cracked}/\lambda_p$ is equal to 10. When low horizontal loads are applied to the structure, the first-order rigid-plastic load factor corresponds to the development of a beam mechanism (λ_p =

12,88) and the CMMR ultimate load factor λ_u - which is equal to 5,68 (line "CD") - takes into account the interaction between the plasticity and instability phenomena under vertical loads.

The actual loading path is represented in Fig. 12 by the arrow "OEF". At its intersection with the interaction curve, a CMMR estimated failure load multiplier λ_u = 1,80 is derived (which may now be compared with the FINELG numerical result λ_u = 1,41). The difference between the two approaches is equal to 22% and the analytically predicted value is seen to be quite unconservative. Such a conclusion has already been drawn (see § 5.3) from previous studies on steel structures characterised by the development of a first-order rigid-plastic panel mechanism. Furthermore, the fact that "F" is very close to "B" in Fig. 12 confirms the low importance of the sway effects in the "Bochum" frame (in accordance with the numerical results where it is observed that only one plastic hinge is missing to form a full panel plastic mechanism at failure, see Fig. 10).

Obviously, as said before, the application of this design method to structures where λ_p is associated to a panel mechanism could be simply prohibited. For steel structures, this limitation of the field of application is not very restrictive as steel frames exhibiting a first-order rigid-plastic mechanism are usually not fulfilling the drift requirements under service loads. But for composite structures, in which the stiffness and resistance properties in bending of the composite beams are rather high in comparison to those of the steel or even composite columns, a satisfactory lateral response of the structure under service loads may be combined with the development of a panel plastic mechanism in a first-order rigid-plastic analysis.

As a consequence, further studies are therefore carried out with the objective to derive an amended CMMR approach in which the nature of the first-order rigid-plastic mechanism would be explicitly taken into consideration.

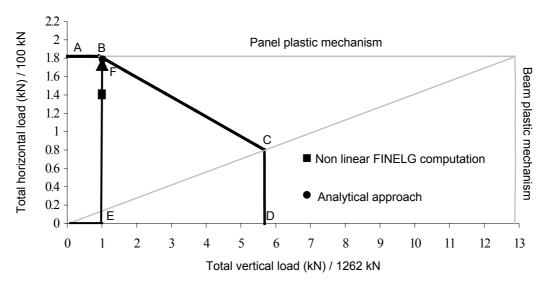


Fig. 12 - CMMR interaction curve for the "Bochum" structure

6. CONCLUSIONS

In the paper, numerical and analytical investigations performed on composite sway frames at Liège University, as part of a European project, are presented. The validity of the homemade FEM software FINELG for the numerical simulation until failure of composite structures has

been shown; then, as an example, a detailed numerical study of a composite sway frame tested in Bochum (Germany) has been achieved, and interesting phenomena have been identified.

Finally, the applicability to the "Bochum" frame of two simplified design analytical methods known as the "Amplified sway moment method" and the, respectively based on elastic and plastic design philosophies, has been investigated.

From this example, the accuracy of the "Amplified sway moment method" is demonstrated and the need for further improvements of the "Merchant-Rankine" approach is pointed out. In [Demonceau 2004] it is shown that these conclusions may be generalised to any sway composite building under static loading.

Further works on the "Merchant-Rankine approach" are presently in progress at Liège University.

7. REFERENCES

Demonceau J.F. Applicability of composite structures to sway frames. Master thesis, department M&S, Liège University, June 2004.

Demonceau J.F., Jaspart J.P. & Maquoi R., 2004. Design of composite sway building frames for global instability. ASCE journal of engineering mechanics, Special issue on advances in the stability of framed structures, to be published.

ECSC "Sway" project, 2004. ECCS project 7210-PR-250 "Applicability of composite structures to sway frames", Final Report, European Commission for Steel and Coal, now renamed Research Fund for Coal and Steel, Brussels, Belgium, March 2004.

Eurocode 3, 1992. Design of Steel Structures. Part 1.1: General Rules and Rules for Buildings. European Prestandard, ENV 1993-1-1, CEN, Brussels, February 1992.

Eurocode 4, 1992. Design of Composite Steel and Concrete Structures. Part 1.1: General Rules and Rules for Buildings. European Prestandard, ENV 1994-1-1, CEN, Brussels, 1992.

Eurocode 8, 2003. Design of structures for earthquake resistance. Part 1: General rules, seismic actions and rules for buildings. European Prestandard, ENV 1998-1, CEN, Brussels, 2003.

FINELG, 1999. FINELG User's Manual. Non linear finite element analysis program. 7yh update, Liège University and BEG Design Office, July 1999.

- Li T. Q., Moore D. B., Nethercot D. A. & Choo B. S., 1996 a. *The experimental behavior of a full-scale, semi-rigidly connected composite frame: overall considerations. Journal of Constructional Steel Research, Vol.* 39, 1996, pp. 167-191.
- Li T. Q., Moore D. B., Nethercot D. A. & Choo B. S., 1006 b. The experimental behavior of a full-scale, semi-rigidly connected composite frame: detailed appraisal. Journal of Constructional Steel Research, Vol. 39, 1996, pp. 193-220.

Maquoi R. & Jaspart J.P., 2001. A simple approach for the design of steel and composite frames accounting for effective overall stability. Festschrift Prof. Richard Greiner, Graz University, Austria, October 2001.