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ULTIMATE STRENGTH OF STIFFENED PANELS ASSESMENT TAKING INTO ACCOUNT MODEL UNCERTAINTY

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ABSTRACT

This paper presents a methodology to take into account model uncertainties related to the load-end shortening curve of stiffened panels. This method is a part of a research activity carried out at University of Liege. His goal is to propose a reliability based model to assess hull girder ultimate strength using a progressive collapse algorithm.
The numerical results presented here concern the load-shortening model chosen for this research, which is based on Bureau Veritas rules. Model uncertainty is quantified as a parameter considered as a random variable. Four statistical moments (mean, standard deviation, skewness and kurtosis) of this parameter are calculated and analyzed using a data-base of stiffened panels test results published in recent years. In order to increase the number of "actual" values for the statistical assessment, the data-base is completed with non-linear finite element analysis results.
The paper contains also some recommendation for the implementation of the proposed method in ultimate strength reliability based analysis.

KEYWORDS

Ultimate strength; Reliability ; Load shortening model; Statistical moment; Model uncertainty; Progressive collapse analysis.

INTRODUCTION

Simulation of the collapse behavior is an essential issue in assessing the safety of marine structures. An accurate estimate of the maximum load-carrying capacity, also called ultimate strength, is required to determine the maximum load that the structure can support. Ultimate strength of plates and stiffened panels is a complex function of a large number of parameters (geometry, material properties, and imperfections) and a deterministic way of assessment is often not sufficient for practical applications.
The uncertainties related to hull girder ultimate strength are usually classified in two classes:

- parameter uncertainties – related to physical model (the geometrical properties, scantlings of hull components, material properties, imperfections,…)
- model uncertainties – related to mathematical model (hypothesis, analytical or numerical methods in use)

Parameter uncertainties could be quantified by measurement, but in practice the available data for a case study is often insufficient. Some general models were proposed in the literature for plates thickness, initial deflection, corrosion thickness, residual stress, etc. A sensitivity analysis of parameter uncertainties on hull girder ultimate strength was carried out by ISSC 2000 IV.2 Committee [1] and it was found that the most influential parameters are yielding stress of the material and plate and stiffener thicknesses.

Model uncertainties could be defined as the gap between the numerical results and the actual value of the ultimate strength obtained via full scale tests or model tests. A study on model uncertainties was recently proposed by *Moan and al.* [2] using several tests on box-girders models, simplified models and scale models. However, the number of available tests published until now seems to be still insufficient to perform an accurate statistical estimation of hull girder ultimate strength model uncertainty for different types of ships.

Different reliability-based methods to assess ultimate strength of ships were published in recent years. In all the cases, ultimate bending moment distribution is evaluated using two statistical moments (mean and standard deviation) or assumed to have a defined shape. *Teixeira and Guedes-Soares* [3] proposed a log-normal distribution with a 10 – 15% standard deviation for tankers and bulk carriers.

It is known that target probability related to ultimate strength limit state of ships is less than 10^{-7} in many cases, so it is crucial to model accurately ultimate bending moment distribution and particularly its tails. For this purpose, it is important to take into account the third and mainly the fourth statistical moment of this distribution (skewness and kurtosis). Some reliability analysis methods using four moments are successfully applied in different fields of activity as civil engineering or nuclear power.

A similar method based on independent perturbation propagation is developed by University of Liege in order to assess hull girder ultimate bending moment distribution using a progressive collapse analysis (Smith Method). This paper present a part of this research focusing on the assessment of load-end shortening curves of stiffened panel taking into account model uncertainties. The results presented here concern to a given model (BV rules) but the methodology could be generalized.

STATISTICAL MOMENTS OF A DISTRIBUTION

Generally, a random variable X is associated with a function called probability distribution – f(X). It is usual to model this distribution by several statistical moments, as follows :

$$E[X_1]=\int_{-\infty}^{\infty} X^1 \, f(X)dX = m_1(X) \quad (1)$$

$$E[X_2]=\int_{-\infty}^{\infty} (X-m_1(X))^2 \, f(X)dX = \mu_2(X) \quad (2)$$

$$E[X_3]=\int_{-\infty}^{\infty} (X-m_1(X))^3 \, f(X)dX = \mu_3(X) \quad (3)$$

$$E[X_4] = \int_{-\infty}^{\infty} (X - m_1(X))^4 \, f(X) dX = \mu_4(X) \quad (4)$$

The first moment is the mean value of X; the moments greater than 1 are expressed as centered moments, relatives to the mean value. Using equations (2) – (4), it

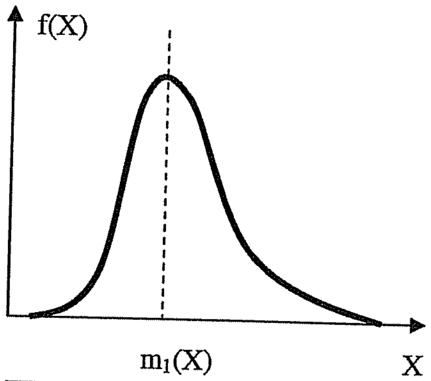

Fig.1 : Distribution of random variable X

is easy to define the following parameters :

$$\sigma_X = \sqrt{\mu_2(X)} \quad \text{- standard deviation} \quad (5)$$

$$\beta_1(X) = \frac{\mu_3(X)}{\sigma_X^3} \quad \text{- skewness coefficient} \quad (6)$$

$$\beta_2(X) = \frac{\mu_4(X)}{\sigma_X^4} \quad \text{- kurtosis coefficient} \quad (7)$$

In this convention, skewness coefficient is negative for a left dissymmetry distribution and zero for a symmetric distribution. The kurtosis coefficient has a value greater than 1.80 for usual distribution function and less than 1.80 for multi-modal distributions (β_2=3 for Gauss distribution).

LOAD-END SHORTENING MODEL

The method chosen in this paper to simulate stiffened panel behavior in compression is based on the recommendation of Bureau Veritas rules [8]. This method was also proposed in 2005 for Joint Bulker Project Rules (JBP). The axial compressed stiffened panel is supposed to collapse through the 4 following failure modes:

- beam-column buckling (mode I)
- tripping of stiffener (mode II)
- web local buckling of flanged stiffener (mode III)
- web buckling of flat-bar stiffener (mode IV)

In tension, only the elasto-plastic collapse is considered. For this reason, the load-end lengthening model is not the object of the present investigation. The critical (ultimate) stress is computed for each failure mode and the smallest one gives the equation to be used for the load-end shortening curve. These equations are given below:

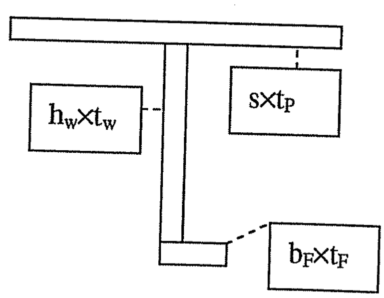

Fig. 2 : Beam-column scantlings

$$\sigma_{\text{mode}\,I} = \Phi \sigma_{c1} \frac{As + b_E t_P}{As + s t_P} \quad (8),$$

$$\sigma_{\text{mode}\,II} = \Phi \frac{As\,\sigma_{c2} + s t_P \sigma_{CP}}{As + s t_P} \quad (9),$$

$$\sigma_{\text{mode}\,III} = \Phi \sigma_y \frac{b_E t_P + h_{WE} t_W + b_F t_F}{s t_P + h_W t_W + b_F t_F} \quad (10),$$

$$\sigma_{\text{mode } IV} = \Phi \frac{As\sigma_{c2} + st_P\sigma_{CP}}{As + st_P} \quad (11),$$

where:

As – net area of stiffener

σ_{c1}- critical stress for beam-column buckling

σ_{c2}- critical stress for tripping

σ_{c4}- critical stress for web buckling of flat-bar

σ_y – material yield stress

E – material Young modulus

ε – relative strain of beam-column

Φ –$\Phi(\varepsilon)$ edge function

$\beta_E = \frac{s}{t_p}\sqrt{\frac{\varepsilon\sigma_Y}{E}}$, *plate slenderness*

$b_E = \left(\frac{2.25}{\beta_E} - \frac{1.25}{\beta_E^2}\right)s$ *for* $\beta_E > 1.25$

$b_E = s$ *for* $\beta_E \leq 1.25$

The initial imperfections (initial distortion, residual stress) are not explicitly taken into account by this model. However, model uncertainty estimation will be carried out for several cases corresponding to three levels of imperfections.

METHODOLOGY PROPOSED TO ASSESS MODEL UNCERTAINTIES

As mentioned above, model uncertainty of hull girder ultimate strength could be defined as a gap between the calculated value of bending moment and the actual one, obtained by tests. In this paper, model uncertainty parameter respects the following equation :

$$M_A = M_C + \chi \quad (12),$$

M_A – actual value of ultimate moment,

M_C – calculated value of ultimate moment,

χ – global model uncertainty parameter

If a progressive collapse analysis based on Smith method is used to compute M_C, it is known that the model uncertainty relate to two important aspects of the method :

- Smith method incremental algorithm
- Load-end shortening model.

In this case, the global model uncertainty could be expressed as :

$$\chi = \chi_S + \chi_{SE} \quad (13).$$

χ_S – Smith method uncertainty parameter

χ_{SE} – load-end shortening model uncertainty parameter

Smith method uncertainty parameter is difficult to quantify independently. If its take into account is required, it is better to perform directly an estimation of global model uncertainty, based on the available results of full scale tests, box girder and scale models tests. As mentioned above, the number of available data for this purpose is still insufficient to assure an accurate assessment of the uncertainty parameter distributions for all types of ships. Some tests, particularly box girder tests and simplified model tests, introduce a large scattering of data and χ distributions obtained present sometimes a kurtosis coefficient $\beta_2 < 1.80$ (multi-modal distribution).

Smith method uncertainty parameter is not considered in the methodology proposed here. Several sensitivity analysis on available progressive collapse analysis methods have shown that the main influential aspect is the quality of load-end shortening curves. *Rigo and al.* [4] found that the most important parameter of load-end shortening models is the ultimate stress of individual elements composing the hull girder, but the shape of the curve could not be neglected, mainly the post-collapse behavior.

In this paper, a direct assessment of uncertainty parameter related to load-end shortening model is presented. This methodology must be considered as a partial assessment of ultimate moment model uncertainty, particularly when the simplified beam theory considered by the Smith method is not valid (ex : multi-deck passengers vessels).

The uncertainty parameter to estimate is defined as follows :

$$\sigma_A = \sigma_M + \chi_P \qquad (14)$$

σ_A – actual value of beam-columns compressive stress during Smith method curvature increment,
σ_M – beam-columns compressive stress as computed using the model presented here (based on BV rules)
χ_P – partial uncertainty parameter

The parameter χ_P is considered as a random variable. Its distribution is estimated by four statistical moments calculated using several control points positioned along the load-end shortening curve. The curve is divided in 7 specified control regions as indicated in Fig 3 and χ_P statistical moments are evaluated for each region using 5 control points. The positions of those regions and their control points are defined by the yield strain (ε_Y) of the calculated curve (σ_M) – see Fig3.

In a first step, the width of the region was taken to 10% of the strain inducing yield stress (ε_Y). It was found the number of control points was too large and the values of the actual curves were often obtained by linear interpolation of measured values. It was decided to fix the width of regions to 33.33% of ε_Y(Fig 3 and Fig 4).

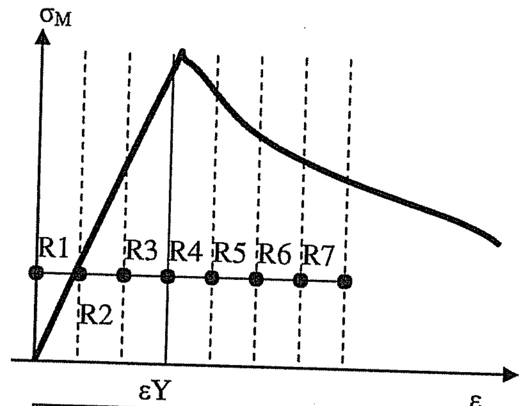

Fig 3 : definition of control regions on the calculated load-end shortening curve

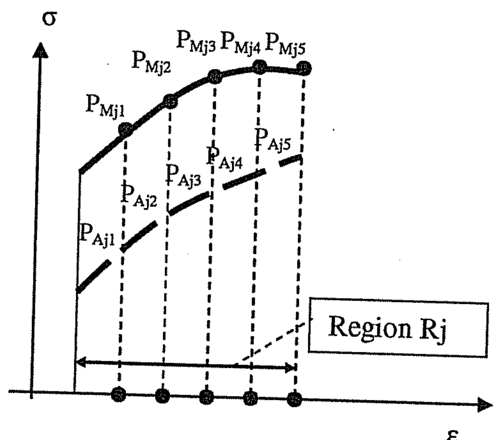

Fig 4 : Definition of control points for region j

P_{Mj1} – stress at the first control point of region j on the calculated curve
P_{Aj1} – stress at the first control point of region j on the actual curve

Once the values of beam-column compressive stress are known at all control points, statistical moments are computed for each region using the following equations :

$$\chi_{jk} = P_{Ajk} - P_{Mjk} \quad (15)$$

$$m_{1j\chi} = \frac{1}{k}\sum_k \chi_{ik} \quad (16)$$

$$\sigma_{j\chi} = \sqrt{\frac{1}{k-1}\sum_k (\chi_{jk} - m_{1jk})} \quad (17)$$

$$\beta_{1j\chi} = \frac{1}{\sigma_{jk}^3}\frac{k}{(k-1)(k-2)}\sum_k (\chi_{jk} - m_{1jk})^3 \quad (18)$$

$$\beta_{2j\chi} = \frac{1}{\sigma_{j\chi}^4}\frac{(k+1)\sum_k (\chi_{jk} - m_{1j\chi})^4}{\frac{(k-1)(k-2)(k-3)}{k}} \frac{-3\frac{k-1}{k}\left[\sum_k (\chi_{jk} - m_{1j\chi})^2\right]^2}{\frac{(k-1)(k-2)(k-3)}{k}} + 3 \quad (19)$$

As mentioned above, the load-end shortening model proposed here does not consider explicitly initial imperfections. The level of initial imperfections will be included implicitly in the estimation of model uncertainty. To this purpose, the database containing the actual load-end shortening curves selected for this analysis was divided in 3 classes, following the level of plate initial deflection used during testing (or finite element analysis) – see Table 1.

Only plate initial deflection are taken into account in the definition of levels :

$W_{O\,max}/t_P$ $< 0.05\beta^2$ – *slight level*
$0.05\beta^2$ *to* $0.2\beta^2$ – *moderate*
$> 0.2\beta^2$ – *serious level*

$W_{O\,max}$ – *amplitude of initial deflection of plate,*
β – *plate slenderness.*

ACTUAL CURVES SELECTION FOR THE PRESENT ANALYSIS

Actual curves selected for the present model uncertainty analysis are results of tests and finite element analysis published in the literature. These tests, their specifications and references are presented in the Table 1.a,b and c.

In a first step, 45 tests on stiffened panels were investigated, but only a few of them were selected. The reasons of this selection are different testing conditions (mainly the presence of lateral pressure on different sides and different boundary conditions and imperfections) inducing a large scattering of data and a non-homogeneous population for the present statistical estimation.

The target was to consider 8 to10 actual curves for each class, which means 40-50 control values for each region (5 control points per region). It was found that this number is sufficient to perform an accurate estimation of model uncertainty.

The number of tests selected was insufficient to reach the target, and the database was completed with results of non-linear finite element analysis. Estimation of uncertainty for moderate and serious level of imperfection is based on FEM (only one test result included) – See Tables 1.a, b and c.

TABLE 1.a : Actual curves for slight level of imperfection				
code	description	method	β	λ
SP1.2	T- stiffener, SSC399 [5]	test	2.32	0.61
SSP1	T- stiffener, SSC433 [6]	test	-	-
SSP2	T- stiffener, SSC433 [6]	test	-	-
SSP3	T- stiffener, SSC433 [6]	test	-	-
SSP4	T- stiffener, SSC433 [6]	test	-	-
P2S-1	Angle-bar, [6]	FEM ABAQUS	1.38	0.20
P5S-1	Angle-bar, [6]	FEM ABAQUS	0.93	0.34
A5 20 35 n	Bulb stiffener [6],[7]	FEM ABAQUS	1.34	0.32
P6S-1	Bulb-stiffener [6]	FEM ABAQUS	2.10	0.37
F31525n	Flat-bar, ISSC2000 [7]	FEM ABAQUS	2.09	0.41

TABLE 1.b : Actual curves for moderate level of imperfection				
code	description	method	β	λ
SP2.1	T- stiffener, SSC399 [5]	test	2.32	0.61
P2S-2	Angle-bar, [6]	FEM ABAQUS	1.38	0.20
P5S-2	Angle-bar, [6]	FEM ABAQUS	0.93	0.34
F151525-2	Flat-bar [6]	FEM ABAQUS	1.78	0.6
A51525-2	Angle-bar, [6]	FEM ABAQUS	1.79	0.51
A52035-2	Angle-bar, [6]	FEM ABAQUS	1.33	0.31
P6S-2	Bulb-stiffener [6]	FEM ABAQUS	2.10	0.37
P7s-2	T-bar [6]	FEM ABAQUS	1.73	0.45

TABLE 1.c : Actual curves for serious level of imperfection				
code	description	method	β	λ
SP2.2	T- stiffener, SSC399 [5]	test	2.32	0.61
P2S-5	Angle-bar, [6]	FEM ABAQUS	1.38	0.20
P5S-5	Angle-bar, [6]	FEM ABAQUS	0.93	0.34
F151525-5	Flat-bar [6]	FEM ABAQUS	1.78	0.6
A51525-5	Angle-bar, [6]	FEM ABAQUS	1.79	0.51
A52035-5	Angle-bar, [6]	FEM ABAQUS	1.33	0.31
P6S-5	Bulb-stiffener [6]	FEM ABAQUS	2.10	0.37
P7s-5	T-bar [6]	FEM ABAQUS	1.73	0.45

β - *plate slenderness*
λ - *stiffener slenderness*

TABLE 2.a : Model uncertainty estimators

Region	mean χ	st. deviation χ	$\beta 1\ \chi$ skewness	$\beta 2\ \chi$ kurtosis
	Slight level of imperfection			
1	0.094	0.227	2.625	10.194
2	3.760	4.795	1.697	2.704
3	11.278	16.622	1.264	1.946
4	-7.349	25.703	-0.895	1.864
5	-32.054	40.994	-1.606	2.136
6	-39.104	49.921	-1.578	1.956
7	-41.196	53.969	-1.604	2.044

TABLE 2.b : Model uncertainty estimators

Region	mean χ	st. deviation χ	$\beta 1\ \chi$ skewness	$\beta 2\ \chi$ kurtosis
Moderate level of imperfection				
1	0.141	0.253	1.667	4.864
2	5.029	6.543	1.881	3.850
3	14.507	17.450	1.552	2.016
4	6.802	20.659	1.269	1.822
5	14.952	33.235	1.386	1.711
6	17.147	39.998	1.412	1.799
7	23.790	46.560	1.477	1.814

TABLE 2.c : Model uncertainty estimators

Region	mean χ	st. deviation χ	$\beta 1\ \chi$ skewness	$\beta 2\ \chi$ kurtosis
Serious level of imperfection				
1	0.141	0.253	1.667	4.864
2	5.029	6.543	1.881	3.850
3	8.154	12.662	1.351	1.939
4	-1.943	15.082	-0.013	2.132
5	1.107	16.692	0.688	1.781
6	-5.358	9.418	-1.702	2.749
7	-2.086	4.756	-1.446	1.818

ANALYSIS RESULTS

The results obtained using the proposed methodology are given in Table 2.a, b and c. Some examples of load-end shortening curves are presented in Fig. 5 to 7. As expected, the most significant values of parameter mean and standard deviation are obtained for post-collapse regions (4 to 7). The absolute value of mean and standard deviation increases from regions 1 to 7, except for the serious level of imperfection, where smaller values are obtained for region 6 and 7.

Generally, the selected load-end shortening model underestimates the post-collapse curve (regions 4 to 7) for moderate level of imperfection (positive mean values) and overestimate it for slight and serious level of imperfection (negatives mean values). Kurtosis parameter $\beta 2$ takes important values for the regions 1 to 3. Some kurtosis coefficients are smaller than 1.80, which means a multi-modal distribution, but the values are very close to the limit (the smallest is 1.71). In this case a uniform distribution should be used to model uncertainty parameter.

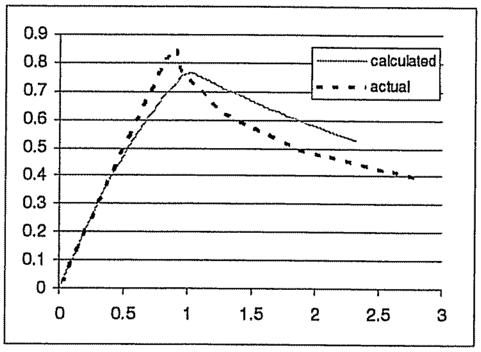

Fig 5 : Actual and calculated curve for test P6s-1 (slight level of imperfection)

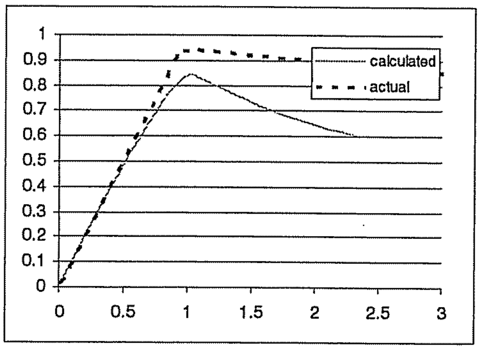

Fig 6 : Actual and calculated curve for test P2s-2 (moderate level of imperfection)

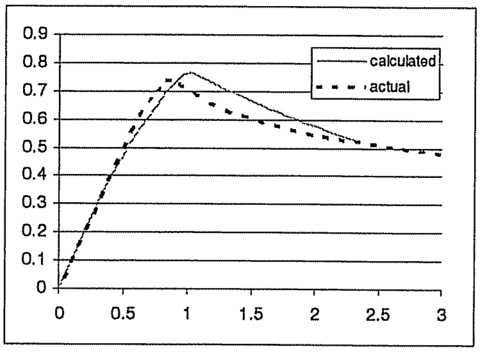

Fig 7 : Actual and calculated curve for test P6s-5 (serious level of imperfection)

CONCLUSIONS

A methodology to take into account model uncertainty of a load-end shortening model for beam-columns is presented in this paper. The method could be generalized, but the results presented here are only valid for the present curve model based on Bureau Veritas rules.

The results given in table 2.a to c shows that the range of kurtosis obtained for the uncertainty parameter corresponds to usual probability distribution functions (uniform, Gauss, Beta-function,...). The values of the four estimators presented in this paper can be directly implemented in a reliability model based on statistical moments.

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