

# Economic growth and under-investment: a nonparametric approach\*

Sergio Perelman<sup>†</sup> and Barnabé Walheer<sup>‡</sup>

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## Abstract

This paper expounds a ratio to measure countries' under-investment which combines the advantages of being simple, intuitive, easy to compute, and non-parametric. Applying this to a panel of 81 countries over the period extending between 1965 and 2014 evidences a case of under-investment in all countries. Over this time period, OECD countries and Asian Tigers show less and less constrained by their available resources, while the contrary is true of Latin American and African countries. The distribution of our under-investment ratio also appears to become bi-modal over time, which implies that under-investment plays a role in economic divergence between countries. Our findings are supported by several statistical tests and a sensitivity analysis.

**Keywords:** Economic growth; under-investment; nonparametric; labor productivity.

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<sup>†</sup>HEC Liège, Université de Liège, Belgium. email: sergio.perelman@ulg.ac.be.

<sup>‡</sup>HEC Liège, Université de Liège, Belgium. email: barnabe.walheer@uliege.be.

# 1 Preliminaries

We posit that we observe a balanced panel of  $n$  countries where each country produces output using labor and capital at time  $t$ . In addition, we assume that the production process (i.e. the technology) is unobserved while the production function fulfills standard macroeconomic assumptions: this is quasi-concave, continuous, strictly increasing, and satisfies constant returns-to-scale. Last, we also posit that technological degradation is impossible over time (Henderson and Russell, 2005).

Our aim is to analyze how labor productivity (i.e. output divided by total worker force), denoted  $y_{it}$  at time  $t$  for country  $i$ , is impacted by capital per worker, denoted  $k_{it}$  at time  $t$  for country  $i$ . Labor productivity was established as a crucial indicator of welfare in the macroeconomic literature at least two decades ago. An advantage of using labor productivity rather than output per capita (i.e. output divided by total population) is that both the numerator and the denominator correspond to the market sector, which is not the case for the denominator of output per capita. This might be particularly problematic for countries with substantial non-market production activity (Jones, 1997).

Several theoretical and empirical works have demonstrated the crucial role of capital per worker for economic growth (e.g. Solow, 1956; Barro, 1991; Baumol, 1996; Temple, 1999; Kumar and Russell, 2002; Henderson and Russell, 2005). Our interest is different: we aim at quantifying potential under-investments for the countries and, if they exist, verify how they evolve over time.

To define under-investment, we use two measurements: efficiency and performance. Efficiency captures countries' ability to use their resources to generate economic growth; this then is related to the production process. As it is assumed to be unobserved, we reconstruct the technology using Farrell's (1957) deterministic production-frontier method. In short, a production possibility set is defined using observed data while imposing a minimal set of assumptions (we use those discussed above for the production function). Note that we adopt a sequential reconstruction of the production process to avoid technological degradation (Diewert, 1980). We

obtain the following set at time  $t$ :

$$T_t = \left( \begin{array}{l} (y, k) \mid y \leq \sum_{\tau=1}^t \sum_{j=1}^n \lambda_{j\tau} y_{j\tau}, \\ k \geq \sum_{\tau=1}^t \sum_{j=1}^n \lambda_{j\tau} k_{j\tau}, \\ 1 \geq \sum_{\tau=1}^t \sum_{j=1}^n \lambda_{j\tau}, \\ \lambda_{j\tau} \geq 0 \ \forall j, \forall \tau. \end{array} \right). \quad (1)$$

Efficiency is defined as the distance to the frontier of the reconstructed possibility set  $T_t$ . Formally, it is given for country  $i$  at time  $t$  as follows:

$$e_t(y_{it}, k_{it}) = \min \left\{ e \mid \left( \frac{y_{it}}{e}, k_{it} \right) \in T_t \right\}. \quad (2)$$

$e_t(y_{it}, k_{it})$  captures the maximal degree to which labor productivity can be expanded while keeping capital per worker constant.  $e_t(y_{it}, k_{it}) \leq 1$  and  $e_t(y_{it}, k_{it}) = 1$  means that the maximal level of output per worker is produced at time  $t$  for country  $i$ .

Next, performance also captures countries' ability to generate output per worker, while ignoring capital per worker variability. To define this dimension, we first introduce the production possibility set when ignoring capital per worker variations across countries.<sup>1</sup> It is obtained as an adapted version of (1) for each period  $t$ :

$$\mathcal{T}_t = \left( \begin{array}{l} (y, 1) \mid y \leq \sum_{\tau=1}^t \sum_{j=1}^n \lambda_{j\tau} y_{j\tau}, \\ 1 \geq \sum_{\tau=1}^t \sum_{j=1}^n \lambda_{j\tau}, \\ \lambda_{j\tau} \geq 0 \ \forall j, \forall \tau. \end{array} \right). \quad (3)$$

We again define performance as the distance to the frontier of  $\mathcal{T}_t$ . It is given for country  $i$  at time  $t$  as follows:

$$p_t(y_{it}) = \min \left\{ p \mid \left( \frac{y_{it}}{p}, 1 \right) \in \mathcal{T}_t \right\}. \quad (4)$$

$p_t(y_{it})$  has to be interpreted in an analogous fashion that  $e_t(y_{it}, k_{it})$ , but when ignoring the capital per worker constraint. That is,  $p_t(y_{it})$  captures the maximal possible expansion of labor productivity. When it is one, it shows that the maximal value has

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<sup>1</sup>We refer to Cherchye et al. (2019) for detailed explanations about (1) and (3) and their interconnection. At this point, we highlight that these sets are well-known in the nonparametric efficiency and performance literature. The twist, here, is to compute ratios of distance to the frontier of these sets, i.e. (2) and (4), to measure under-investment (see(5)).

been reached at time  $t$  for country  $i$ .

Finally, we point out that, generally, the nonparametric reconstructions in (1) and (3) are significantly impacted by the presence of outliers and biased. Using the well-established method described in Daraio and Simar (2007), we compute the bias-corrected efficiency and performance scores. Also, we have verified that no outliers are present in our sample using Simar’s (2003) procedure.<sup>2</sup>

## 2 Under-investment

Recently, Cherchye et al. (2019) have suggested using the ratio of the performance and efficiency measurements to capture how resources impact outcomes. Adapting their initial definition to our specific case enables us to propose a new way of looking at countries’ under-investment. In particular, we define under-investment as the ratio between maximal labor productivity improvements without and with capital per worker constraints. It is given for country  $i$  at time  $t$  by:

$$r_{it}(y_{it}, k_{it}) = \frac{p_t(y_{it})}{e_t(y_{it}, k_{it})}, \quad (5)$$

$r_{it}(y_{it}, k_{it})$  is labeled the resource constraint ratio. When  $r_{it}(y_{it}, k_{it}) < 1$ , it reveals that country  $i$  is limited in terms of economic growth by its resources, i.e. there is under-investment. In that case,  $p_t(y_{it}) < e_t(y_{it}, k_{it})$ . That is, ignoring the capital per worker variations across countries reveals more potential output per worker improvement. In other words, country  $i$  fully explodes its capital per worker at time  $t$  and therefore more capital per worker is requested to further improve labor productivity.

As a final remark, we highlight that  $r_{it}(y_{it}, k_{it}) > 1$  is possible. This case reveals that country  $i$  does not fully use its resources at time  $t$ :  $p_t(y_{it}) > e_t(y_{it}, k_{it})$  implies that output per worker performance is better when ignoring capital per worker constraints. That is, there is unexploited capacity, which seems hardly plausible in our economic growth context, as is confirmed by our empirical analysis in the next Section.<sup>3</sup>

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<sup>2</sup>Roughly speaking, these procedures use sub-samples of the observations when computing efficiency or performance. As a result, the estimators are less sensitive to potential issues (e.g. outliers, measurement errors), i.e. more robust.

<sup>3</sup>We may relate our findings to the concept of steady state in macroeconomic theory (e.g. in Solow’s famous model). Broadly speaking, if countries are below (above) the steady state, it shows

### 3 Empirical results

Output and capital per worker are constructed according to the common practice in the literature. Data are taken from the most recent Penn World Table.<sup>4</sup> We obtain a sample of 81 countries for the time span 1965–2014. We start by showing the world labor productivity (kernel) distributions and the regression line between growth and initial level of labor productivity in Figure 1.

Figure 1: Labor productivity

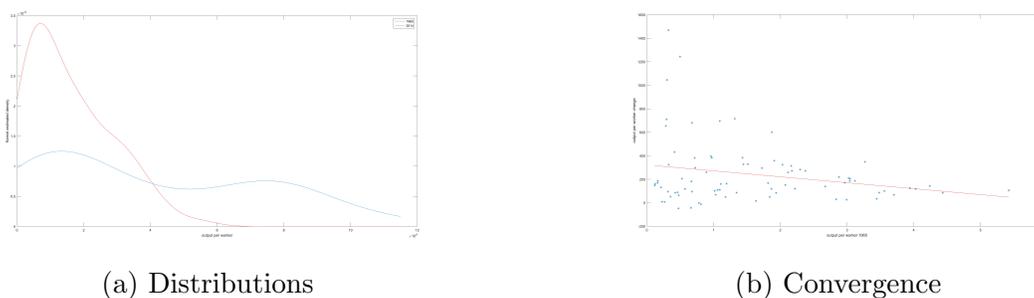


Figure 1(a) highlights a positive shift of labor productivity over time, but also reveals a transformation of the world labor productivity distribution from a uni- to a multi-modal distribution. Figure 1(b) shows that higher output per worker, on average, is associated with lower change. These findings are confirmed by statistical tests. First, we use the nonparametric Kolmogorov–Smirnov test that confirms the improvement between the two years ( $p$ -value of 0.000).<sup>5</sup> Next, we use Silverman’s calibrated test for multimodality due to Hall and York (2001).<sup>6</sup> While the hypothesis remains that the 1965 labor productivity distribution has more than one mode ( $p$ -value of 0.7167), it is easily rejected for 2014 ( $p$ -value of 0.0153). Finally, we find a negative and significant slope coefficient worth  $-0.0051$  for our regression line ( $p$ -value of 0.000).<sup>7</sup>

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that they under-(over) invest and that more (fewer) resources are needed to reach the steady state. In other words, our findings may be interpreted as revealing that countries have not reached their steady states yet.

<sup>4</sup>In particular Penn World Table 9.0 is used. See Feenstra et al. (2015) for more detail. Data can be freely downloaded at [www.ggd.net/pwt](http://www.ggd.net/pwt).

<sup>5</sup> $H_0$ : 2014 and 1965 labor productivity distributions are equal;  $H_1$ : 2014 labor productivity distribution is larger than 1965 labor productivity distribution.

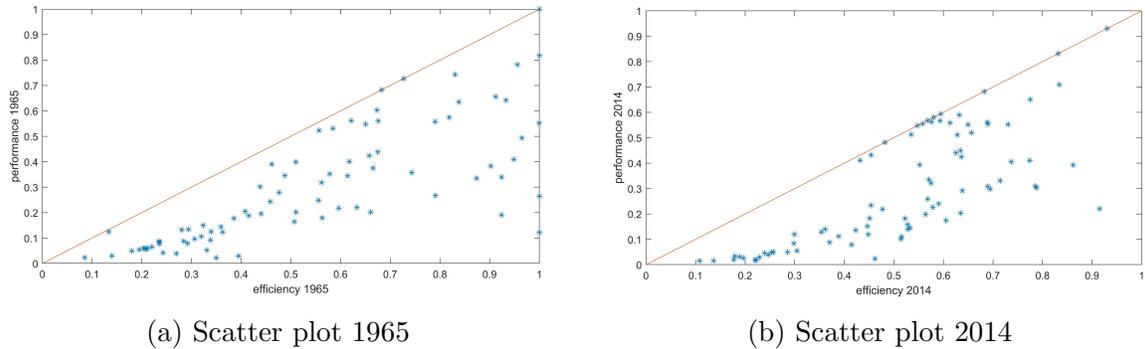
<sup>6</sup> $H_0$ : the distribution has one mode;  $H_1$ : the distribution has more than one mode.

<sup>7</sup> $H_0$ : the slope coefficient equals zero;  $H_1$ : the slope coefficient is negative.

To sum up, these preliminary investigations not only support the convergence but also highlight the existence of two clubs or groups of countries. Quah (1996, 1997) refers to this stylized fact as “twin-peak” convergence. A direct implication is the suspicion of empirical analyses based on the first moment (or even higher moments) of the labor productivity distribution, and the potential consequences of choosing a specific functional form for the technology (Kumar and Russell, 2002; Henderson and Russell, 2005). Moreover, the use of more sophisticated statistical methods often requires relatively large samples and, given the limited number of countries in the world, such techniques can ‘ask a lot of the available’. These various points argue in favour of adopting a nonparametric approach.

Figure 2 provides the scatter plots between efficiency and performance in 1965 and 2014; the diagonal red lines represent the situation where efficiency equals performance, i.e. the resource constraint ratio is one. All points below (above) these lines correspond to under-investment (unexploited capacity) situations, i.e. the resource constraint ratio is smaller (larger) than one.

Figure 2: Scatter plots and frontiers for efficiency and performance



The main findings from these scatter plots are, first, that under-investment situations are observed for both time periods, while unexploited capacity never is. Second, more countries lie on the red line in 2014 meaning that fewer countries present an under-investment situation in 2014. In 1965, Luxembourg, Trinidad and Tobago, and the United States lie on the 45 degree line. Note that, for that year, only the United States have an unity values for both efficiency and performance. In 2014, Belgium, Cyprus, Ireland, Italy, the Netherlands, Norway, and Singapore are on the 45 degree line. Note that Norway has the best results with efficiency and performance at 0.9. In other words, as compared to 1965, countries in 2014 are less constrained by their

resources. Third, fewer countries lie in the upper-right in 2014, showing that fewer ones present high efficiency and performance scores for that year. In other words, two groups of countries can be identified in 2014: those that have moved to the (closer to) diagonal and those keeping close to their 1995 levels.

Next, we present the results by providing descriptive statistics for the efficiency and performance measurements and for our resource constraint ratio in Table 1. We give the averages and the standard deviations for both the levels and growths (denoted  $g$ ) of each variable. The averages indicate whether countries have improved their situation (absolute comparison), while the standard deviations show whether they are becoming more or less homogeneous. (relative comparison) We also distinguish several groups of countries in the same Table.

Table 1: Averages and standard deviations

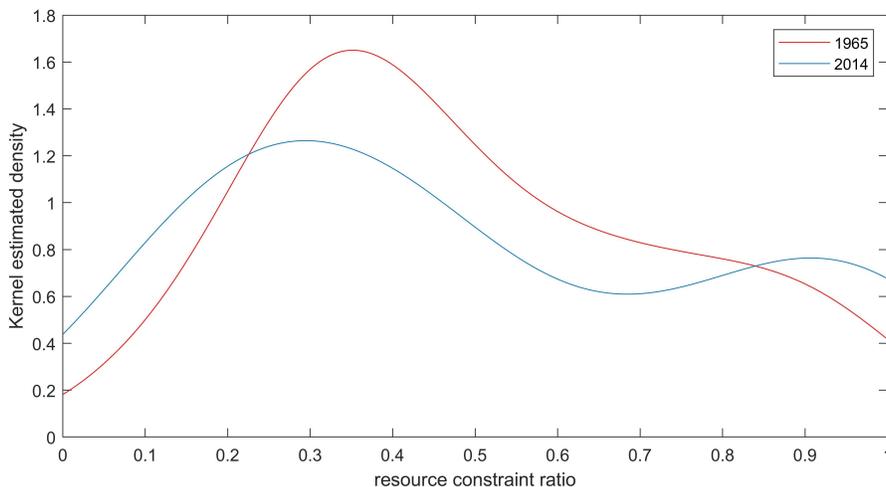
<i>Average</i>	<b>All</b>	<b>OECD</b>	<b>Latin America</b>	<b>Africa</b>	<b>Asia</b>	<b>Asian Tigers</b>
$e_{2014}$	0.53	0.61	0.53	0.35	0.54	0.58
$e_{1965}$	0.55	0.66	0.55	0.42	0.48	0.51
$g_{e_{2014-1965}}$	10.89	3.41	4.02	12.98	20.41	37.11
$p_{2014}$	0.29	0.52	0.18	0.12	0.19	0.43
$p_{1965}$	0.30	0.50	0.26	0.15	0.19	0.17
$g_{p_{2014-1965}}$	17.21	31.44	-17.94	-21.07	12.15	168.32
$r_{2014}$	0.52	0.83	0.34	0.25	0.31	0.69
$r_{1965}$	0.49	0.72	0.45	0.38	0.33	0.36
$g_{r_{2014-1965}}$	3.92	20.80	-20.37	-25.31	-6.93	104.17
<i>Std</i>	<b>All</b>	<b>OECD</b>	<b>Latin America</b>	<b>Africa</b>	<b>Asia</b>	<b>Asian Tigers</b>
$e_{2014}$	0.18	0.11	0.16	0.19	0.19	0.12
$e_{1965}$	0.25	0.22	0.21	0.27	0.23	0.24
$g_{e_{2014-1965}}$	57.31	42.99	36.96	83.49	36.42	52.24
$p_{2014}$	0.21	0.16	0.08	0.12	0.12	0.18
$p_{1965}$	0.22	0.21	0.15	0.17	0.10	0.10
$g_{p_{2014-1965}}$	91.51	92.01	34.37	71.06	42.81	92.22
$r_{2014}$	0.32	0.18	0.08	0.17	0.11	0.24
$r_{1965}$	0.25	0.18	0.17	0.25	0.07	0.16
$g_{r_{2014-1965}}$	49.42	31.13	16.21	37.51	27.11	49.34

Efficiency, on average, shows a slow decrease between 1965 and 2014, while performance remains fairly stable. Asian Tigers present the largest growths for both

dimensions, while OECD countries reach the highest levels. Latin American and African countries have a negative growth for the performance indicator. Next, there are improvements for the resource constraint ratios for the OECD group and the Asian Tigers. That is, under-investment is decreasing over time for these countries. The largest negative growths have been found for the Latin American and African countries, which reveals that under-investment is only getting worse for them. In terms of homogeneity, we find more similarities between Latin American and Asian countries for all variables.

Finally, Figure 3 displays the (kernel) distribution for the resource constraint ratio. We see that two modes are present for 2014 (only one in 1965), and that there are more extreme countries that year. This suggests that under-investment plays a role in economic divergence (or, at least, not a positive one for economic convergence). The above findings are confirmed by statistical tests. The Kolmogorov–Smirnov test confirms the lack of improvement ( $p$ -value of 0.4384); the calibrated Silverman’s test presents a large  $p$ -values in 1965 (0.3563) and a very small one in 2014 (0.0582).

Figure 3: Kernel distribution for the resource constraint ratio

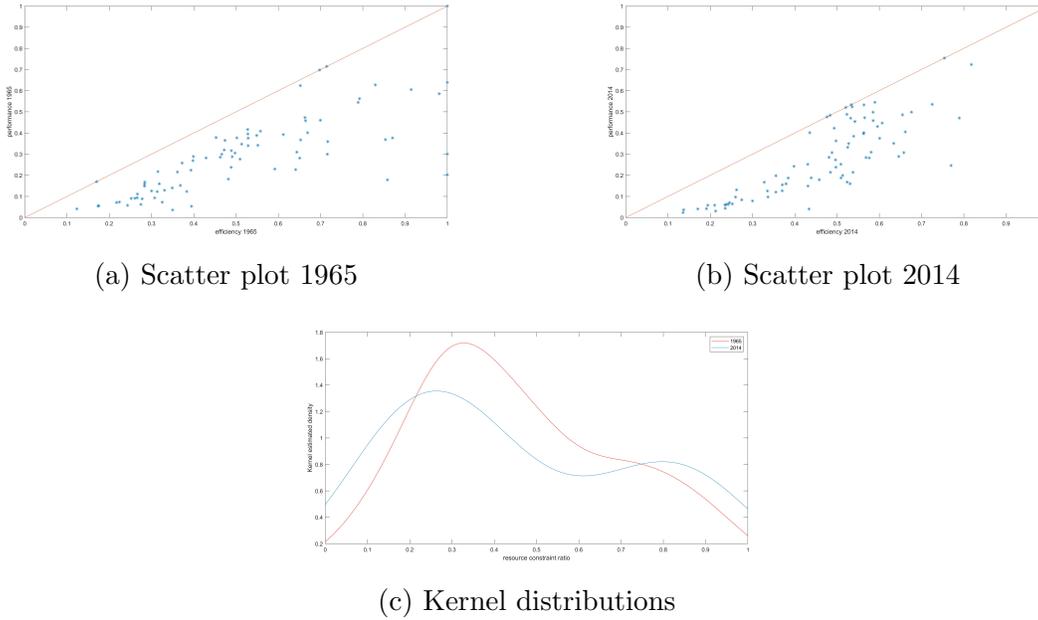


As a final step, we reevaluate the countries’ under-investment behavior when adding human capital as a third production factor. The role of human capital in economic growth has been well documented in the literature (e.g. Galor and Tsiddon, 1997; Castello and Domenech, 2002; Henderson and Russell, 2005). In practice, we compute (bias-corrected and robust)  $e_t(\hat{y}_{it}, \hat{k}_{it})$  and  $p_t(\hat{y}_{it})$  (and then  $r_{it}(\hat{y}_{it}, \hat{k}_{it})$ ) where  $\hat{y}_{it}$  and  $\hat{k}_{it}$  are output and capital per efficiency unit of labor (i.e. divided by

the product of labor and human capital), respectively. Human capital is measured by returns to education (Barro and Lee, 2013).<sup>8</sup>

For brevity's sake, we only focus our discussion on the descriptive statistics when pooling all countries together (to be compared with the first column in Table 1), the scatter plots (to be compared with Figures 2(a) and (b)), and the kernel distributions (to be compared with Figure 3). The averages for the efficiency, performance, and resource ratio are 0.51, 0.29, and 0.54 in 1965; 0.46, 0.28, and 0.54 in 2014; and we obtain 1.80, 11.65, and 5.98 for the growth rates. In words, efficiency is worse than when excluding human capital, while performance and the resource ratio present similar patterns. Next, Figure 4 reveals that under-investment is present for the initial and final time period, while fewer countries are constrained by their resources over time, and bi-modality is remains.<sup>9</sup> All in all, our previous findings remain valid.

Figure 4: Scatter plots and kernel distributions with human capital



<sup>8</sup>Data can be freely downloaded at <http://www.barrolee.com/>.

<sup>9</sup>Algeria, Luxembourg, Venezuela, and Zambia are on the diagonal in 1965. In 2014, Belgium, Norway, and Singapore no longer lie on the diagonal (they have efficiency and performance scores close to 0.95) whereas Spain and Portugal now are.

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