Experimental Characterization of Superharmonic Resonances Using Phase-Lock Loop and Control-Based Continuation

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Abstract

Experimental characterization of nonlinear structures usually focuses on fundamental resonances. However, there is useful information about the structure to be gained at frequencies far away from those resonances. For instance, non-fundamental harmonics in the system's response can trigger secondary resonances, including superharmonic resonances. Using the recently-introduced definition of phase resonance nonlinear modes, a phase-locked loop feedback control is used to identify the backbones of even and odd superharmonic resonances, as well as the nonlinear frequency response curve in the vicinity of such resonances. When the backbones of two resonances (either fundamental or superharmonic) cross, modal interactions make the phase-locked loop unable to stabilize some orbits. Control-based continuation can thus be used in conjunction with phase-locked loop testing to stabilize the orbits of interest. The proposed experimental method is demonstrated on a beam with artificial cubic stiffness exhibiting complex resonant behavior. For instance, the frequency response around the third superharmonic resonance of the third mode exhibits a loop, the fifth superharmonic resonance of the fourth mode interacts with the fundamental resonance of the second mode, and the second superharmonic resonance of the third mode exhibits a branch-point bifurcation and interacts with the fourth superharmonic resonance of the fourth mode.

Keywords: Control-Based, Superharmonic, Characterization

Introduction

The design of mechanical structures with nonlinear behavior is a challenging task. It is often necessary to build reliable models upon physical experiments. Usually, post-processing the experimental data is a long procedure. To accelerate the design iterations, characterization methods relying on control have been proposed that identify nonlinear modes or nonlinear responses without the need for a priori knowledge about the experiment. Two prominent methods are phase-locked loop (PLL) testing [1] and control-based continuation (CBC) [2]. They control two different experimental parameters, the phase lag and the amplitude of the response's first harmonic respectively.

Until now, PLL testing and CBC were used exclusively to characterize the primary resonance of structures, i.e. the resonance of the first harmonic. However, important information about the structure can be obtained through higher harmonics of the response. Typically, nonlinear modes can enter into resonance when the structure is excited at a fraction of the resonance frequency. These superharmonic resonances are included in the recent definition of phase resonance nonlinear modes (PRNMs) [3].

This work applies PLL testing to a structure possessing an artificial nonlinear stiffness presented in [4], to identify the backbone curves of superharmonic resonances and the response curve in their vicinity. In the presence of modal interactions, the PLL has been shown incapable of stabilizing certain unstable responses [5]. To improve the stabilization, PLL testing is coupled with online CBC as implemented in [4].

Methods

The structure is excited by a signal that is a sine wave at steady-state $f(t) = p\sin(\Omega t)$. This work focuses on periodic responses x of the structure, they can be truncated to N_{H} Fourier coefficients: $x(t) = \sum_{k=1}^{N_{\mathrm{H}}} \hat{x}_k \sin(k\Omega t + \phi_k)$. The value ϕ_k is the phase lag of the kth harmonic. The phase lag of each harmonic is estimated at every moment using adaptive filtering, as proposed in [4].

When applying PLL testing, the excitation frequency is output by a PI controller as proposed in [1]. The controller's input is the difference between a phase target ϕ^* and the phase lag ϕ_k: $\Omega(t) = \kappa_{\mathrm{p}}(\phi^* - \phi_k(t)) + \kappa_{\mathrm{i}} \int_0^t (\phi^* - \phi_k(\tau))\,\mathrm{d}\tau$. The controller, depending on its gains, cannot stabilize every response. To improve stabilization, the PLL can be coupled with CBC. The excitation signal is output by a PD controller whose input is the difference between a reference signal whose frequency is determined by the PLL and the response as proposed in [2]: $f(t) = k_{\mathrm{p}} \left[x^* \sin(\int_0^t \Omega(\tau)\,d\tau) - x(t)\right] + k_{\mathrm{d}} \frac{\mathrm{d}}{\mathrm{d}t} \left[x^* \sin(\int_0^t \Omega(\tau)\,d\tau) - x(t)\right]$. The amplitude x^* of the reference signal determines the amplitude of the excitation.

Frequency response curves (FRCs) are obtained with PLL testing by keeping the excitation amplitude p constant and performing a sweep on ϕ^*. PRNMs are obtained with PLL testing coupled with CBC by keeping $\phi^* = -\pi/2$ for odd superharmonic resonances (including primary resonances) [3] or $\phi^* = -3\pi/4$ and $\phi^* = -7\pi/4$ for even superharmonic resonances [3] and performing a sweep on x^*.

PRNMs are defined upon the observation that phase lag drops by a value π when the frequency response crosses a resonance. However, it is observed that ϕ_1 dropping by a value π entails that ϕ_3 drops by a value 3π and ϕ_5 by a value 5π. The odd harmonics therefore interfere with each other. A similar observation is made between even harmonics. Such interference is of no concern when the superharmonic resonances are well-separated but become important in the case of modal interaction. To counteract the interference of lower harmonics, the backbone of a 3rd superharmonic resonance is obtained by keeping $\phi_3 - 3\phi_1 = -\pi/2$ and the backbone of a 5th superharmonic resonance by keeping $\phi_5 - \phi_3 - (5-3)\phi_1 = -\pi/2$. Further analytical work is needed to confirm these assumptions and find similar expressions for even superharmonic resonances.

Results

By placing four equally spaced accelerometers from one quarter of the beam's length to its tip, the mode shape corresponding to the beam's response can be identified up to the fourth mode. The mode shape is decomposed in each harmonic. Each superharmonic resonance is associated with a mode, e.g. if the 5th harmonic follows the 4th mode shape, the superharmonic resonance is dubbed "H5M4".

Fig. 1 shows all the identified features. In plain black curves, FRCs at different excitation amplitudes p in N, identified using PLL testing by sweeping ϕ_1 near H1M2, ϕ_2 near H2M3, ϕ_3 near H3M3, and ϕ_5 near H5M4. The dashed black curves are obtained by an open-loop sine sweep to complete the FRC at $p = 1$ N. Plain blue curves in Fig. 1 show the PRNMs identified using PLL testing coupled with CBC and the phase lags controlled at the displayed values. In the presence of modal interaction, the PRNM does not always correspond to resonant responses, and resonances can occur at other phase lags. Imposing the phase lag ϕ_1 for the 1st harmonic, $\phi_3 - 3\phi_1$ for the third, and $\phi_5 - \phi_3 - (5-3)\phi_1$ for the fifth leads to the identification of more consistent superharmonic resonance backbones. They are shown in dash-dotted orange curves in Fig. 1.

Conclusion

This work is, to our knowledge, the first to identify nonlinear superharmonic resonance backbones and frequency responses with control-based methods. It is also the first one to propose a coupling of PLL testing and CBC to improve the stabilization of responses when the PI controller of the PLL is not sufficient, e.g. in the presence of modal interaction. Further work on the definition of PRNMs is envisioned to include modal interactions.

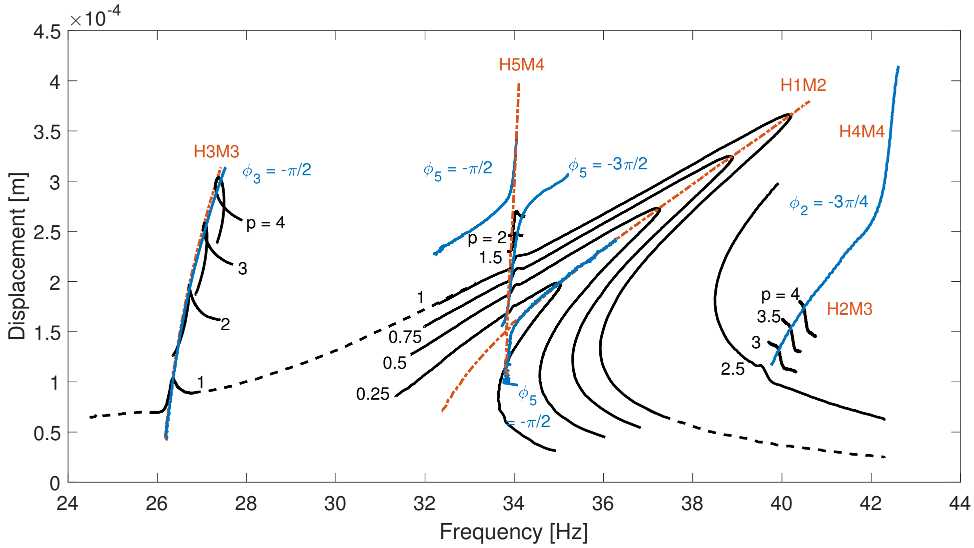

Fig. 1 Frequency response of the beam around its second primary resonance for different excitation amplitudes p in N obtained by PLL testing (plain black curves) and uncontrolled swept sine excitation (dashed black curves); PRNMs of resonances H1M2, H3M3, H5M4, and H2M3 obtained by PLL testing coupled with CBC (plain blue lines); backbone curves of the same resonances obtained by PLL testing coupled with CBC (dash-dotted orange curves)

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