2D FEM Calculation of AC Losses in Twisted Superconductors with a Helicoidal Transformation

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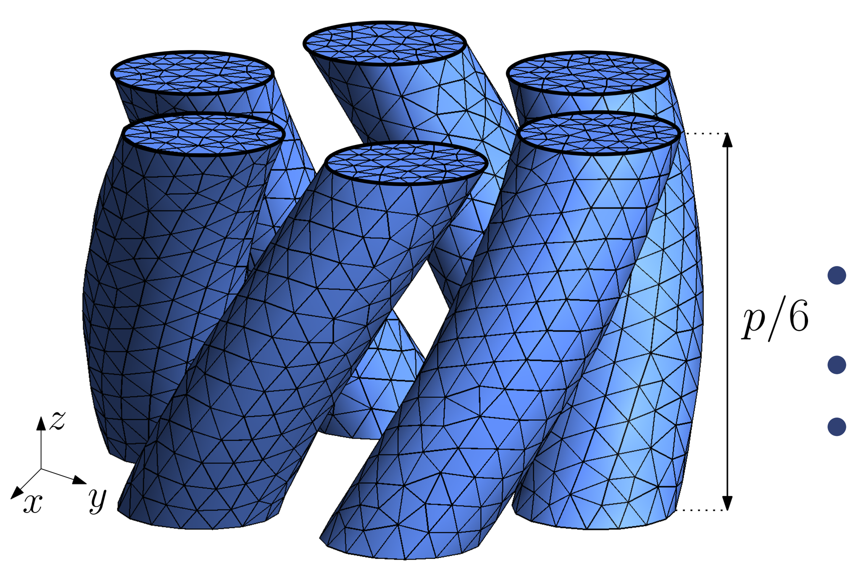
Introduction

- Computing accurately AC losses in **twisted superconducting (SC) wires** is an expensive task due to the strong **nonlinearity** of the e-j relation in the superconductor.
- When the system presents a **helicoidal symmetry**, an efficient **2D model** can be implemented.
- Objectives:
  - Model the magnetic response and compute AC losses for different types of excitations.
  - Account in 2D and 3D for coupling currents in wire matrix due to filaments twist.

3D verification model

Classical **3D h-formulation** (h-ϕ) on a fraction of the twist pitch length p with **periodic boundary conditions**.



$$\left(\partial_t(\mu \boldsymbol{h})\,,\boldsymbol{h}'\right)_\Omega + \left(\rho\, \textbf{curl}\, \boldsymbol{h}\,,\textbf{curl}\, \boldsymbol{h}'\right)_{\Omega_c} = 0$$

- **Power law** for SC: $\rho = \frac{e_c}{j_c}\left(\frac{\|\boldsymbol{j}\|}{j_c}\right)^{n-1}$, $n \in [10, 100]$, $e_c = 10^{-4}$ V/m.
- **Periodic cohomology functions** to impose net current intensities.
- Newton-Raphson method and adaptive time-stepping scheme.

$\Rightarrow$ **General model** for all types of excitations.

NB: Ω is the complete numerical domain. Ω_c is the conducting domain and Ω_c^C is its complementary.

2D model for helicoidally invariant boundary conditions

- The **geometrical transformation**, with $\alpha = 2\pi/p$,

$$\begin{cases} \xi_1 = x\cos(\alpha z) + y \sin(\alpha z),\\ \xi_2 = -x \sin(\alpha z) + y\cos(\alpha z),\\ \xi_3 = z, \end{cases}$$

  from $\boldsymbol{x} = (x, y, z)$ to $\boldsymbol{\xi} = (\xi_1, \xi_2, \xi_3)$, with Jacobian $\boldsymbol{J}^{\mathrm{T}} = \partial \boldsymbol{x}/\partial \boldsymbol{\xi}$, makes the geometry **$\xi_3$-invariant**.
- If **boundary conditions** are also ξ_3-invariant, then the problem is 2D in terms of (ξ_1, ξ_2). This is the case for
  - an applied transport current,
  - a longitudinal field (along z).

The h-formulation is expressed in the new system $\boldsymbol{\xi}$:

- The 1-form $\boldsymbol{h}$ transforms as $\boldsymbol{h}_{\boldsymbol{x}} = \boldsymbol{J}^{-\mathrm{T}}\, \boldsymbol{h}_{\boldsymbol{\xi}}$.
- The 2-form $\boldsymbol{j} = \textbf{curl}\, \boldsymbol{h}$ transforms as $\boldsymbol{j}_{\boldsymbol{x}} = (\boldsymbol{J}/\det \boldsymbol{J})\boldsymbol{j}_{\boldsymbol{\xi}}$.
- Current is not always along $\xi_3 \Rightarrow$ a **3-component h** is necessary: $\boldsymbol{h}(\xi_1,\xi_2) = \underbrace{\boldsymbol{h}_\parallel(\xi_1,\xi_2)}_{\text{in plane } (\xi_1,\xi_2)} + \underbrace{\boldsymbol{h}_\perp(\xi_1,\xi_2)}_{\text{in } \xi_3 \text{ direction}}$.

Modified **h-formulation** for $\boldsymbol{h} = \boldsymbol{h}_\parallel + \boldsymbol{h}_\perp$:

$$\left(\partial_t(\tilde{\mu} \boldsymbol{h})\,,\boldsymbol{h}'\right)_\Omega + \left(\tilde{\rho}\, \textbf{curl}\, \boldsymbol{h}\,,\textbf{curl}\, \boldsymbol{h}'\right)_{\Omega_c} = 0$$

with $\tilde{\mu} = \mu\, \underbrace{\boldsymbol{J}^{-1}\boldsymbol{J}^{-\mathrm{T}} \det(\boldsymbol{J})}_{=\boldsymbol{T}^{-1},\ \text{indep. of } \xi_3}$ and $\tilde{\rho} = \rho\, \underbrace{\boldsymbol{J}^{\mathrm{T}}\boldsymbol{J}/\det(\boldsymbol{J})}_{=\boldsymbol{T},\ \text{indep. of } \xi_3}$.

Function space for $\boldsymbol{h} = \boldsymbol{h}_\parallel + \boldsymbol{h}_\perp$:

- $\boldsymbol{h}_\parallel$: edge functions in Ω_c, gradient of nodal functions + cohomology basis in Ω_c^C.
- $\boldsymbol{h}_\perp$: "perpendicular edge functions" in Ω_c and constant per region in Ω_c^C.

$\Rightarrow \textbf{curl}\, \boldsymbol{h} = \boldsymbol{0}$ in Ω_c^C is **strongly** satisfied.

Note: the system (ξ_1, ξ_2, ξ_3) is not orthogonal. The components $\boldsymbol{h}_\parallel$ and $\boldsymbol{h}_\perp$ are coupled via $\tilde{\mu}$ and $\tilde{\rho}$.

- This 2D model is **equivalent** to the 3D model.
- Introducing the power law for ρ does not bring new difficulties.

2D model for non-helicoidally invariant boundary conditions

Example: a transverse field along y transforms as

$$\boldsymbol{h}_{\boldsymbol{x}} = \begin{pmatrix}0\\1\\0\end{pmatrix} \Rightarrow \boldsymbol{h}_{\boldsymbol{\xi}} = \begin{pmatrix}\sin\alpha\xi_3\\ \cos\alpha\xi_3\\ \alpha\xi_1\cos\alpha\xi_3 - \alpha\xi_2\sin\alpha\xi_3\end{pmatrix}$$

$\Rightarrow$ it is ξ_3-dependent. The problem is **no longer 2D**.

The solution $\boldsymbol{h}$ is expanded as a series.
The formulation is integrated along ξ_3: **quasi 3D model**.

Expansion:

$$\boldsymbol{h}(\xi_1,\xi_2,\xi_3) = \sum_{k=-\infty}^{\infty} \left(\boldsymbol{h}_{\parallel,k}(\xi_1,\xi_2) f_k(\xi_3) + \boldsymbol{h}_{\perp,k}(\xi_1,\xi_2) f_k(\xi_3)\right)$$

with
$$\begin{cases} f_k(\xi_3) = \sqrt{2}\cos(\alpha k \xi_3), & k<0,\\ f_0(\xi_3) = 1, & \\ f_k(\xi_3) = \sqrt{2}\sin(\alpha k \xi_3), & k>0, \end{cases}$$

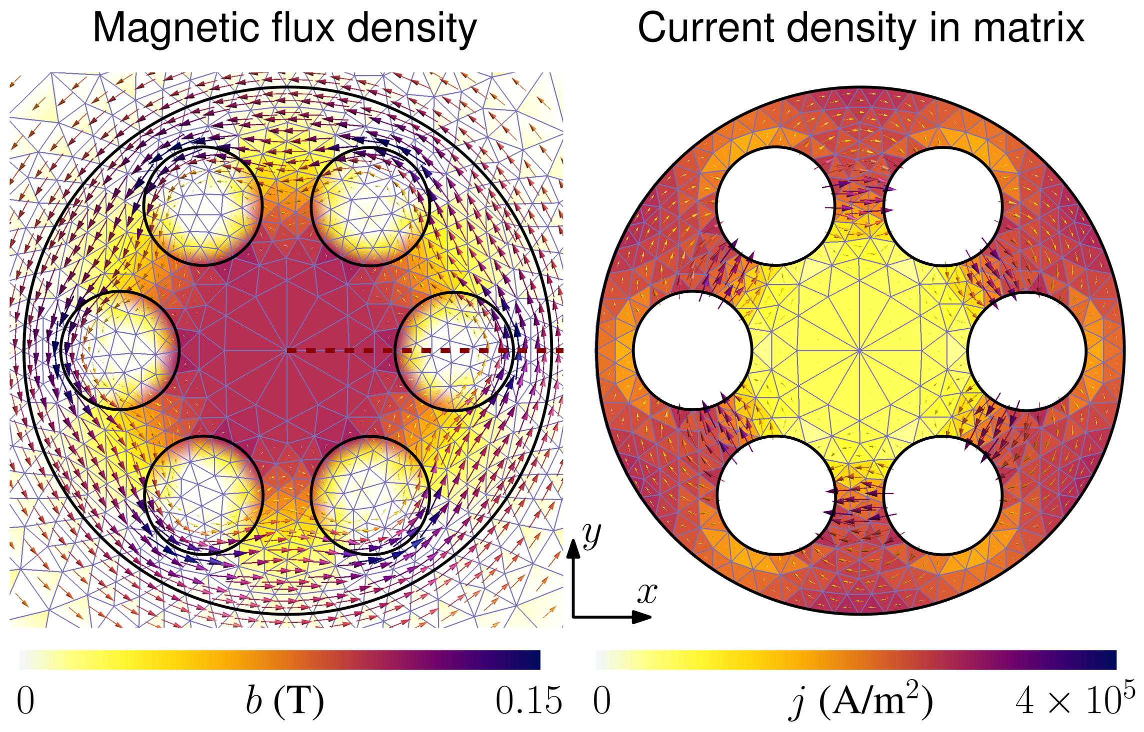
chosen so that $\langle f_{k_1}, f_{k_2}\rangle = \frac{1}{p}\int_0^p f_{k_1} f_{k_2}\, d\xi_3 = \delta_{k_1 k_2}$.

- In Ω_c^C, $\boldsymbol{h}$ can be strongly made curl-free by linking the $\boldsymbol{h}_{\parallel,\pm k}$ DOFs with the $\boldsymbol{h}_{\perp,\mp k}$ DOFs.
- For **linear materials**, all modes are decoupled. For a constant transverse field, only $\boldsymbol{h}_{\parallel,\pm 1}$ and $\boldsymbol{h}_{\perp,\pm 1}$ are nonzero. It is therefore **equivalent** to the 3D model.
- For **superconductors**, the nonlinearity **couples** the modes. The model is an **approximation** when we truncate the series $\rightarrow$ *further work*.

Results

Applied current (helicoidally invariant BC): $I(t) = 0.5\, I_c\, \sin(2\pi t/T)$, $T = 0.1$ s.
Nb-Ti filaments ($n = 50$, $j_c = 7 \times 10^9$ A/m²) in a Cu matrix ($\rho = 1.81 \times 10^{-10}$ Ωm, value for 1 T).
Filament radius: 35 μm. Matrix radius: 155 μm. Twist pitch length $p = 1$ mm.

Solution of the 2D equivalent model at $t = T/4$:



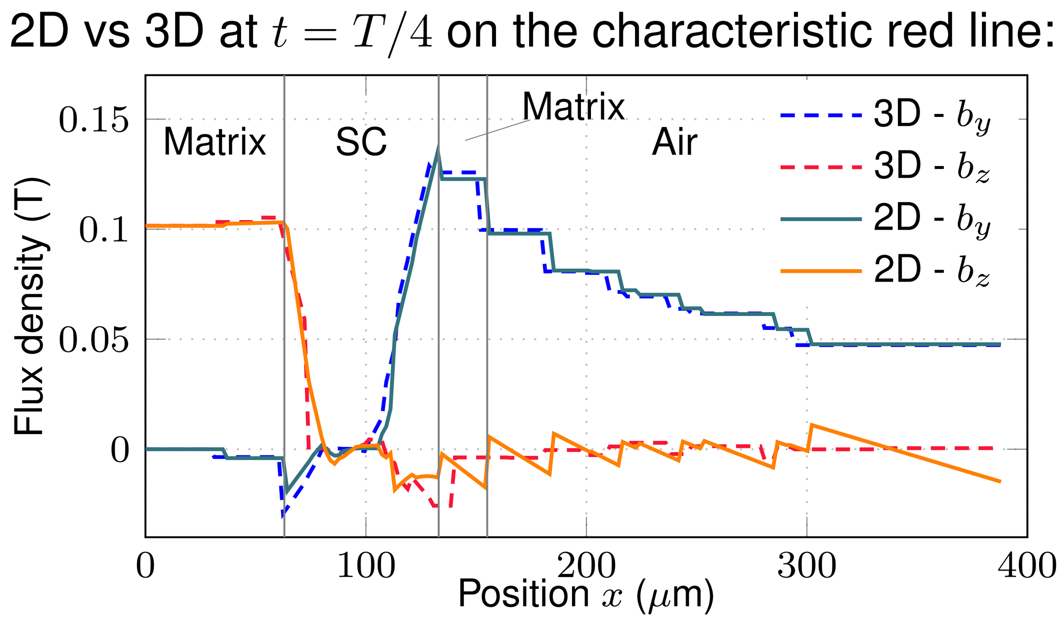
Arrows: x-y-components. Colored elements: z-component.

For this (coarse) resolution with similar accuracy:

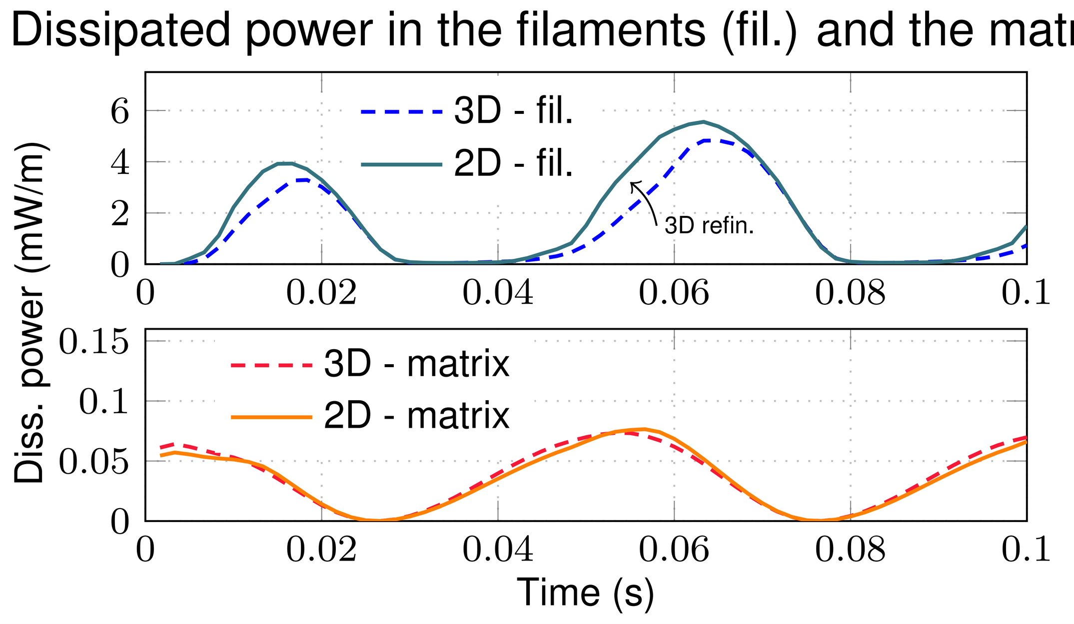
	3D	2D
# DOFs	22.5k	**2.3k**
CPU time	33 min	**1 min 30**

The 3D model with unstructured mesh is **less accurate** at the same discretization level.

2D vs 3D at $t = T/4$ on the characteristic red line:

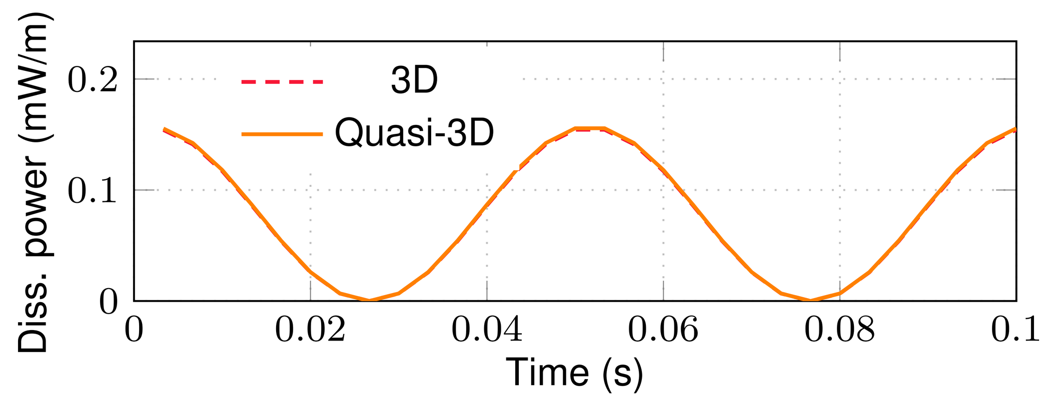


Dissipated power in the filaments (fil.) and the matrix.



Transverse field (non-heliciodally invariant BC):
$\boldsymbol{b}_{\text{app}}(t) = b_{\max} \sin(2\pi t/T)\, \hat{\boldsymbol{e}}_y$, $b_{\max} = 0.5$ T, $T = 0.1$ s.
Cu fil. ($\rho = 1.81 \times 10^{-10}$ Ωm) in a matrix ($\rho = 10^{-8}$ Ωm).

	3D	Quasi 3D
# DOFs	44k	**7k**
CPU time	3 min 45	**25 s**



Models are implemented in GetDP.

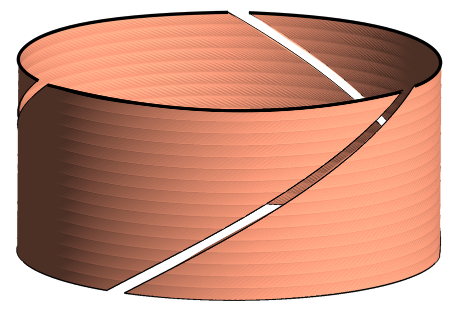
Conclusions

In the purely helicoidal case, the 2D model does not introduce any approximation, and **strongly reduces** the computational work.

In the non-helicoidally invariant case with **linear materials**, the same conclusions hold.

Further work:

- Implement the **transverse case** for **nonlinear** materials. First investigations suggest that the first two modes will already provide a good approximation.
- Consider **different geometries**, such as CORC cables. Apply the approach on the t-a-formulation.



References

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