What Formulation Should One Choose for Modeling a 3D HTS Magnet Motor Pole with a Ferromagnetic Material?

COMPUMAG 2021 January 16-20, 2022, Cancun - Mexico

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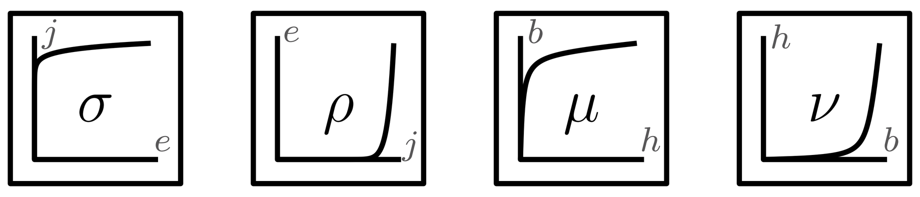
Introduction

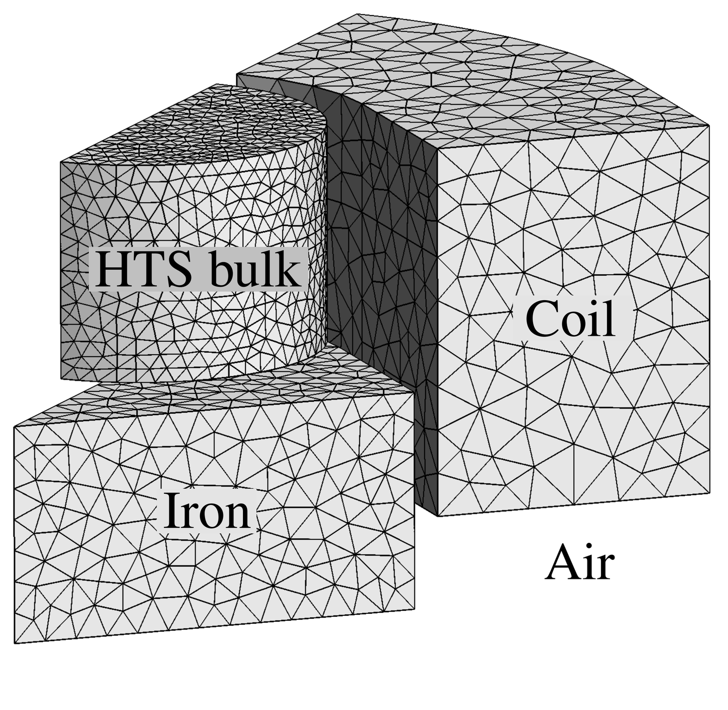
Modeling the magnetic behavior of systems with high-temperature superconductors (**HTS**) and ferromagnetic materials (**FM**) is an expensive task due the **strongly nonlinear** behaviors.



- Not all formulations yield the same efficiency. In this work, we compare the performance of different finite element formulations on a 3D problem.
- In particular, we propose a **coupled formulation** as an efficient choice for these kind of systems.

Problem definition

Four **HTS bulks** are put on an **iron substrate**, and are magnetized by an inducting coil. **One eighth** is modeled.



HTS bulk Ω_c

- Power law: $\rho = \frac{e_c}{j_c}\left(\frac{\|\boldsymbol{j}\|}{j_c}\right)^{n-1}$ with $e_c = 10^{-4}$ V/m.
- Kim's law: $j_c = j_c(\boldsymbol{b}) = 5\times 10^8/(1+\|\boldsymbol{b}\|/b_0)$ A/m^2, $b_0 = 0.5$ T.
- Power index: $n = n(\boldsymbol{b}) = n_1 + (n_0 - n_1)/(1+\|\boldsymbol{b}\|/b_0)$, $n_0 = 21$, $n_1 = 5$.

Iron substrate

- Saturation law (saturation at ≈ 2.2 T) extracted from measurements.

Magnetizing coil Ω_s

- $I_s(t) = \begin{cases} I_{max}\ \sin 2\pi t/T, & t < T, \\ I_{max}\ e^{-(t-T)/\tau}, & t > T, \end{cases}$ $I_{max} = 2$ kA, $T = 2$ ms, $\tau = 10$ ms.
- Number of turns: 55.5. Constant current density $\|\boldsymbol{j}_s\|$, no eddy currents.

h-conform formulations

Classical h-conform formulation, with Ω the full domain, Ω_c the conducting domain, Ω_c^C its complementary:

$$\left(\partial_t(\mu \boldsymbol{h})\,,\boldsymbol{h}'\right)_\Omega + \left(\rho\, \mathbf{curl}\, \boldsymbol{h}\,, \mathbf{curl}\, \boldsymbol{h}'\right)_{\Omega_c} = 0$$

It involves the **permeability** μ and the **resistivity** ρ.
The source current I_s is **strongly** imposed via a precomputed source field $\boldsymbol{h}_s$ satisfying $\mathbf{curl}\, \boldsymbol{h}_s|_{\Omega_s} = \boldsymbol{j}_s$.
Two main possibilities for the function space:

- Full h-formulation, with edge elements in Ω and a spurious **finite resistivity** in the non-conducting domain. This formulation is widely implemented in commercial softwares (e.g. Comsol) [1].
- Hybrid h-ϕ-formulation, where $\boldsymbol{h} = \boldsymbol{h}_s + \mathbf{grad}\, \phi$ in Ω_c^C so that $\mathbf{curl}\, (\boldsymbol{h} - \boldsymbol{h_s})|_{\Omega_c^C} = \mathbf{0}$ is strongly satisfied. Currents are imposed via cohomology basis functions.

b-conform formulations

Classical b-conform formulation, with Ω_s the coil volume, $\boldsymbol{a}$ the magnetic vector potential ($\boldsymbol{b} = \mathbf{curl}\, \boldsymbol{a}$, $\boldsymbol{e} = -\partial_t \boldsymbol{a}$):

$$\left(\nu\, \mathbf{curl}\, \boldsymbol{a}\,, \mathbf{curl}\, \boldsymbol{a}'\right)_\Omega + \left(\sigma\, \partial_t \boldsymbol{a}\,, \boldsymbol{a}'\right)_{\Omega_c} = \left(\boldsymbol{j}_s\,, \boldsymbol{a}'\right)_{\Omega_s}$$

It involves the **reluctivity** $\nu = 1/\mu$ and the **conductivity** $\sigma = 1/\rho$.
The source current I_s is **weakly** imposed via the right-hand side term.
The vector potential $\boldsymbol{a}$ is not unique in Ω_c^C:

- It can be made unique by introducing a spurious **non-zero conductivity** in the non-conducting domain. This approach is sometimes used in commercial softwares. We denote is as the $\bar{a}$-formulation.
- It can be gauged with a **tree-cotree** technique. We denote this method as the a-formulation.

Coupled formulations

The HTS nonlinearity is easier to handle with ρ.
By contrast, the saturation law is easier with ν.

$\Rightarrow$ Motivation for a **coupled formulation**.

Domain decomposed in two parts: Ω_a and Ω_h.

HTS domain $\subset \Omega_h$ and FM $\subset \Omega_a$

Depending on where we place the remaining domain, we end up with two different formulations:

- h-a-formulation: the rest is placed in Ω_a. The vector potential is gauged.
- h-ϕ-a-formulation: the rest is placed in Ω_h. The ϕ potential is introduced, as well as the source field.

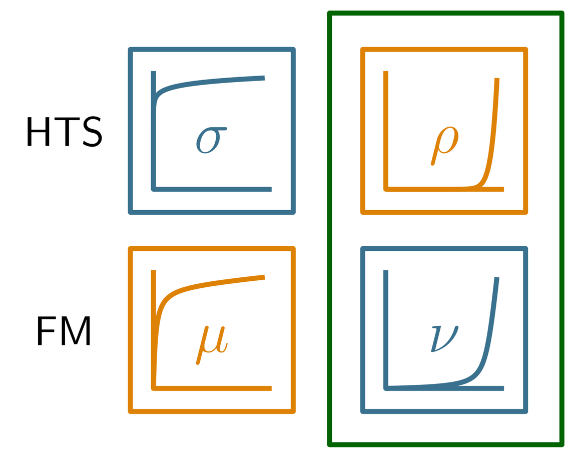
Coupling via the common boundary Γ_m:

$$\begin{aligned}\left(\partial_t(\mu \boldsymbol{h})\,,\boldsymbol{h}'\right)_{\Omega_h} + \left(\rho\, \mathbf{curl}\, \boldsymbol{h}\,, \mathbf{curl}\, \boldsymbol{h}'\right)_{\Omega_{h,c}} &= \langle \partial_t \boldsymbol{a} \times \boldsymbol{n}_{\Omega_a}\,, \boldsymbol{h}' \rangle_{\Gamma_m} \\ \left(\nu\, \mathbf{curl}\, \boldsymbol{a}\,, \mathbf{curl}\, \boldsymbol{a}'\right)_{\Omega_a} - \left(\boldsymbol{j}_s\,, \boldsymbol{a}'\right)_{\Omega_s} &= \langle \boldsymbol{h} \times \boldsymbol{n}_{\Omega_a}\,, \boldsymbol{a}' \rangle_{\Gamma_m}\end{aligned}$$

The formulation is **mixed**. To avoid instability and satisfy the **inf-sup condition**, one of the fields (**$\boldsymbol{h}$ or $\boldsymbol{a}$**) has to be enriched on Γ_h [2].

Summary and comparison

Formulation	Function space	Support of DOFs	$\sigma \neq 0$ in Ω_c^C?
h-formulation	$\mathcal{H}(\Omega) = \{\boldsymbol{h} \in H(\mathbf{curl};\Omega)\}$	Edges in Ω	Yes
h-ϕ-formulation	$\mathcal{H}_\phi(\Omega) = \{\boldsymbol{h} \in H(\mathbf{curl};\Omega) \mid \mathbf{curl}\, \boldsymbol{h} = \mathbf{0}$ in $\Omega_c^C\}$	Edges in Ω_c, nodes in Ω_c^C	No
$\bar{a}$-formulation	$\bar{\mathcal{A}}(\Omega) = \{\boldsymbol{a} \in H(\mathbf{curl};\Omega)\}$	Edges in Ω	(Yes)
a-formulation	$\mathcal{A}(\Omega) = \{\boldsymbol{a} \in H(\mathbf{curl};\Omega) \mid$ co-tree gauge in $\Omega_c^C\}$	Edges in Ω_c, facets in Ω_c^C	No
h-a-formulation	$\boldsymbol{h} \in \mathcal{H}(\Omega_c)$, $\boldsymbol{a} \in \mathcal{A}(\Omega_c^C)$	Edges in Ω_c, facets in Ω_c^C	No
h-ϕ-a-formulation	$\boldsymbol{h} \in \mathcal{H}_\phi(\Omega_a)$, $\boldsymbol{a} \in \mathcal{A}(\Omega_a)$	Edges in $\Omega_{h,c}$, nodes in $\Omega_{h,c}^C$, facets in Ω_a	No



Results

The problem is solved by the **six formulations** on the **same mesh**. They yield results of **similar accuracy**.
The model is implemented in GetDP, and time integrated with an implicit Euler method and adaptive time steps.

Linearization:

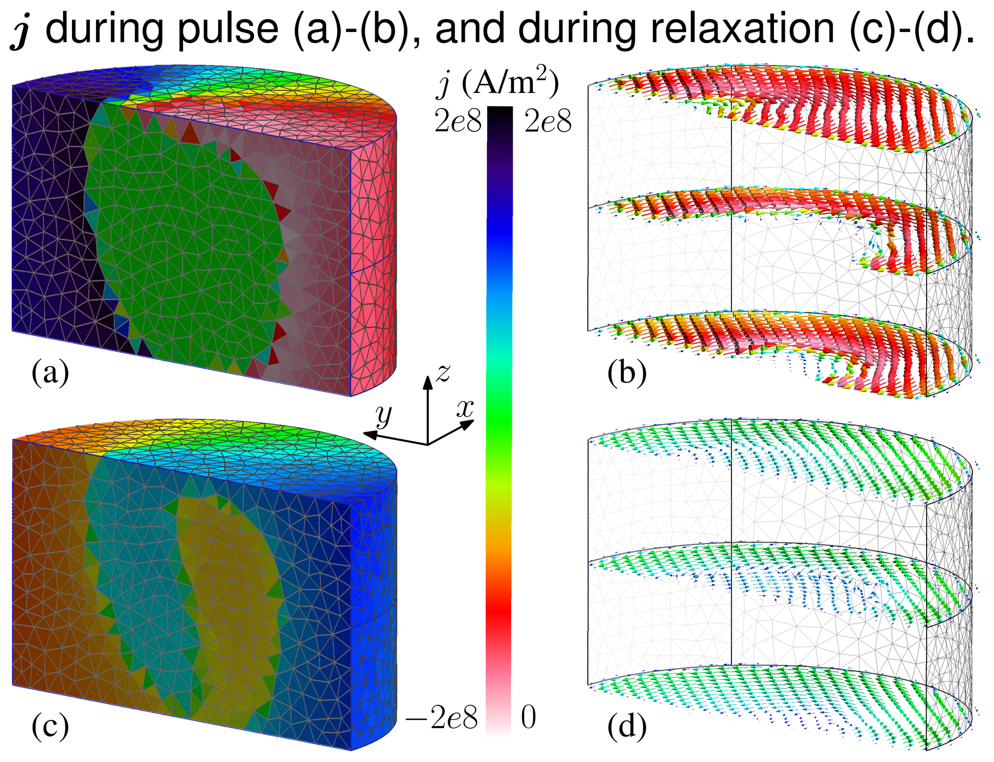
- **Newton-Raphson** (N-R) for ρ and ν.
- For σ or μ, N-R is **not robust**, even with relaxation factors.
  Fixed point method instead $\to$ slower convergence.
  This is the main motivation for a **coupled formulation**.

Performance:

Formulation	# DOFs	# iterations	CPU time
h-formulation	35,532	4,057	5h58
h-ϕ-formulation	12,172	3,937	3h38
$\bar{a}$-formulation	29,010	2,955	4h45
a-formulation	26,964	3,147	3h07
h-a-formulation	32,045	1,124	1h25
h-ϕ-a-formulation	16,070	1,108	1h16

Solution of the h-ϕ-a-formulation:

$\boldsymbol{j}$ during pulse (a)-(b), and during relaxation (c)-(d).



$\Rightarrow$ When possible, it is beneficial to introduce a scalar potential for $\boldsymbol{h}$ and to gauge $\boldsymbol{a}$.

$\Rightarrow$ The h-ϕ-a-formulation is **robust** and is the most efficient method. Compared to the h-a-formulation, it requires many less DOFs.

Conclusions

We presented a **coupled formulation** that is suited for modeling 3D hybrid systems with HTS and FM.

It proved to be **more robust and efficient** than other classical formulations, as was the case in 2D.

Further work: find an alternative to the fixed point method for σ and μ.

References

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Acknowledgements

Computational resources are provided by the Consortium des Équipements de Calcul Intensif (CÉCI), funded by the Fonds de la Recherche Scientifique de Belgique (F.R.S.-FNRS) under Grant No. 2.5020.11.