Old age or dependence. Which social insurance? *

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Abstract

In recent decades, there has been an increase in life expectancy and a rapid increase of the very senior dependency ratio in developed countries. In this context, we examine the optimal levels of public pensions and public long-term care (LTC) insurance. According to the most reasonable estimates of correlations among individual incomes, risks of mortality and dependency, we show that it is always desirable for a utilitarian social planner to have a balanced budget increase in LTC benefits at the expense of public pension benefits, until the cost of LTC is fully covered. This is true with or without liquidity constraints. For a Rawlsian planner, the balance between the two schemes depends on a comparison of the ratio of the survival probability to the dependence risk of the poor with its population average.

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1 Introduction

In recent decades, there has been a huge increase in life expectancy in all OECD member countries. With a growing proportion of the population reaching the age of 80 years, there has also been an increase in life expectancy with good health, namely without major incapacity, although this has increased at a much slower pace. Indeed, more than one third of those aged over 80 years are dependent, as they experience difficulties in performing the activities of daily living independently. These two developments place pressure on governments to finance pensions and long-term care (LTC). Table 1 compares the relative importance of public LTC and pensions in a number of countries. For example, in Germany and France, public pensions account for 10.3% and 13.6% of gross domestic product (GDP), respectively, whereas public spending for LTC represents only 1.3% of GDP in both countries.

Countries	Public spending on LTC	Public Pensions
France	1.3	13.6
Germany	1.3	10.3
Italy	0.6	15.7
UK	1.2	5.6
USA	0.6	7

Table 1: Public spending on LTC and pensions as a % of GDP, 2019 (Source: OECD (2019a, 2019b))

By all accounts, these proportions of spending are not sufficient to meet the needs of aging populations and, in particular, compared with pensions, LTC appears to be neglected. One reason for this is that public pensions were introduced decades ago, just after the Second World War, whereas the need for LTC insurance appeared much later, with the rapid increase of the very senior dependency ratio, namely the growing fraction of the population aged over 80 years. A second reason is that, traditionally, LTC is provided informally by the family.[1] However, as the modern phenomenon in developed

[1] Measuring the importance of family in the provision of LTC is not easy. One can examine the proportion of elderly persons who rely on care provided by children or grandchildren. Bolin et al. (2008) used the data from 10 European countries in the Survey of Health, Ageing,

countries is for elderly individuals to live in isolation from their relatives, this accentuates the need for public provision of LTC care. Even in this context, it is tempting for budget-constrained governments not to meet their responsibilities for the provision of alternative care. As an example, the last three French presidents promised to develop full-fledged social insurance for LTC as priorities in their programs. However, no concrete policy actions followed.

Quite clearly, the scarcity of public funds gives the impression that there is a trade-off between financing public pensions and social LTC insurance. The purpose of this paper is to analyze such a trade-off from a normative viewpoint. The question raised is simple: given the increases in longevity and dependency, what are the optimal levels of public pensions and public LTC insurance?

The starting points of our paper are some stylized facts and a seminal paper on social insurance as a redistributive device. The stylized facts are that survival probabilities increase with income, whereas both conditional and unconditional probability of old age dependence decrease with income. Rochet (1991) proved that social insurance may be justified even when the insurance market is efficient. If there is a negative statistical dependence between probability of loss and labor productivity, social insurance should provide complete coverage for every household. The rationale behind Rochet's result is that redistribution through social insurance does not involve any distortion, unlike income taxation. Rochet's model implicitly assumes away liquidity constraints. Our paper deals with two risks, one positively correlated to income, namely that of having a long life, and the other negatively correlated to income, that is old age dependence. Without liquidity constraints, we show that it is always desirable to have a balanced budget increase in LTC benefits at the expense of public pension benefits. Introducing liquidity constraints complicates the analysis, but we show that when dependence involves a monetary cost, priority is given to LTC until the cost of LTC is fully covered.

To make our point, we use a simple model of a two-period economy with three states of nature: in the first period, people work

and Retirement in Europe (SHARE) to show that about one-third of single elderly individuals reported relying on some informal care. Another approach involves counting the number of hours devoted to informal LTC and assigning them a monetary value. According to Buckner and Yeandle (2011), based on the latter approach, informal LTC care amounts to 7.4% of GDP of the UK. See also Norton (2016, Section 3).

and save; in the second period, people either retire in good health or they may enter a state of dependency (Cremer et al. (2010), Cremer and Pestieau (2014), Leroux et al. (2019)). Society comprises a number of individuals who differ in their productivity and their probabilities of survival and dependence. There is strong evidence concerning the correlation between these probabilities. Lefebvre et al. (2018) used the data from the Survey of Health, Ageing, and Retirement in Europe (SHARE). They showed that there is a positive correlation between income and longevity and a negative correlation between income and dependence.

To provide the intuition behind our results, assume for a moment that there are no liquidity constraints; namely, individuals can purchase negative amounts of either annuities or LTC insurance. Keeping a balanced government budget, low-income people will always prefer to have LTC benefits over pensions because the ratio of their dependency risk to their probability of survival is higher than the average for the rest of society. In other words, a redistributive LTC scheme brings more utility to the poor than does a redistributive public pension system. One dollar devoted to LTC benefits them more than a dollar spent on public pensions. When there are liquidity constraints, the optimal level of public pensions can be attained with positive saving, whereas at least some individuals do not buy LTC insurance (Theorem 1). When dependence involves a monetary cost, priority is given to the public LTC until the cost of LTC is fully covered. This is true with or without liquidity constraints (Theorem 2).

When the objective of the government is Rawlsian, then the desirability of a public LTC scheme depends on the comparison between the ratio of the survival probability to the dependence risk of the poor with its population average. However, the superiority of LTC benefits over public pensions is more limited than in the utilitarian case because both programs are required by the poorest individuals in society.

The remainder of the paper is organized as follows. Section 2 presents the basic model. Section 3 deals with the utilitarian case. Section 4 is devoted to numerical simulation. Sections 5 deals with the Rawlsian case. Finally, Section 6 concludes.

2 The Model

2.1 Longevity, Dependency Risk, and Individual Preferences

Consider a two-period model, where individuals work and save in the first period and retire in the second. In the second period, people face different risks of mortality and dependence. There are $I+1$ types of individuals. The proportion of type i ($i = 0, 1, ..., I$) individuals is denoted by n_i, with the total number of individuals born in the first period being normalized to unity: $\sum_{i=0}^{I} n_i = 1$. Each individual of type i has three characteristics: (i) w_i (her/his labor productivity in the first period), (ii) π_i (the probability of being alive in the second period), and (iii) p_i (the probability of becoming dependent in the second period). Using data from SHARE, Lefebvre et al. (2018) showed that the following relations hold:

- Longevity (π_i) increases with income.
- The probability of dependency ($\pi_i p_i$) decreases with income.

Consistent with these facts, we postulate that $cov(w_i, \pi_i) > 0$ and $cov(w_i, \pi_i p_i) < 0$.

Let c_i denote individual i's first-period consumption, $\ell_i \in [0, \bar{\ell}]$ denote labor supply, d_i denote the second period consumption if she/he is healthy, and let m_i denote LTC expenditures in the case of dependency. An individual's expected lifetime utility is given by:

$$U_i = u\left(c_i - v(\ell_i)\right) + \pi_i(1 - p_i)u(d_i) + \pi_i p_i H(m_i).$$

In the following, we use the notation $x_i \equiv c_i - v(\ell_i)$. We assume that $u' > 0$, $u'' < 0$, $v' > 0$, $v'' > 0$, $H' > 0$, $H'' < 0$, $v'(0) = 0$, and $v'(\bar{\ell}) > \max_{i=1,...,I} w_i$. In addition, we assume that $H(y) < u(y)$ and $H'(y) > u'(y)$ for all $y > 0$ to reflect the costly needs of dependency. The last assumption requires some explanation. We follow the tradition of merging preferences for daily life consumption and preferences for LTC expenditures (including its health services component).[2] This approach has been adopted in a series of recent papers on LTC. Under this approach, dependency results in higher

[2]In a recent paper, De Donder and Leroux (2021) discussed disentangling preferences for daily life consumption from preferences for LTC expenditures. However, in this paper, m_i comprises both usual consumption and LTC.

marginal utility than under autonomy and, thus, leads to full insurance when it is available at actuarially fair terms. To date, the empirical literature on state-dependent preferences in the context of the loss of autonomy has failed to reach a consensus. In part, this is due to specification differences. On the one hand, Lillard and Weiss (1997) and Ameriks et al. (2020) found that marginal utility is higher when dependent than when autonomous. On the other hand, Finkelstein et al. (2013) and Koijen et al. (2016) found that the marginal utility of consumption decreases in the case of poor health. We adopt the former specification in which, in the case of severe dependency, the LTC component dominates in the marginal utility.

2.2 Government's Policy and Individual Optimization

Private saving is invested in a perfect annuity market with a zero interest rate. From saving s_i, type i has a return s_i/π_i. In addition, there is a private insurance market insuring against dependency. From the insurance purchase P_i, type i receives $P_i/(\pi_i p_i)$, where the return is inversely proportional to the individual's risk of dependency.

The government's policy comprises three instruments: (i) a linear income tax ($\tau \geq 0$), (ii) a flat-rate pension ($b \geq 0$), and (iii) a uniform LTC benefit ($g \geq 0$). Individuals choose labor supply (ℓ_i), annuitized savings (s_i), and private insurance (P_i), while taking the government's scheme as given:

$$\begin{aligned} U_i &= u\left((1-\tau)w_i\ell_i - s_i - P_i - v(\ell_i)\right) \\ &\quad + \pi_i(1-p_i)u(s_i/\pi_i + b) + \pi_i p_i H(s_i/\pi_i + P_i/(\pi_i p_i) + g + b). \end{aligned} \tag{1}$$

In Section 3, we assume that $x_i = c_i - v(\ell_i) > 0$ holds for all i at the optimum of (1). Thus, individuals would not transfer all their first-period income to the second period through P_i and s_i. Here, we assume that the wage rates are sufficiently high, and/or the expected cost of dependency is sufficiently low. The case where there is an individual with $w_i = 0$ will be discussed in Section 5.

The first-order conditions (FOCs) with respect to ℓ_i, s_i, and P_i

are:

$$u'(x_i)\left((1-\tau)w_i - v'(\ell_i)\right) = 0, \tag{2}$$
$$-u'(x_i) + (1-p_i)u'(d_i) + p_i H'(m_i) \leq 0, \tag{3}$$
$$-u'(x_i) + H'(m_i) \leq 0. \tag{4}$$

Let the solution values be ℓ_i^*, s_i^*, and P_i^*, respectively. The first condition is written with an equal sign, implying an interior solution for labor. The other two solutions are not necessarily interior, implying that some individuals may be constrained to have nonnegative levels of saving or LTC insurance. Formally: $s_i^* \geq 0; P_i^* \geq 0$. In the case of interior solutions, we have:

$$u'(c_i) = u'(d_i) = H'(m_i).$$

Concerning the solutions, we distinguish two cases in which they are interior:

- Given the parameters of the model, all the solutions are interior. This will be the case when both g and b are small, or alternatively when the tax rate is low for some reason (owing to political decisions or tax distortions).
- Liquidity constraints are assumed away, implying that individuals can have negative savings or insurance premiums.

Now, we turn to the optimal level of public benefits chosen by a government that is utilitarian or Rawlsian.

3 Utilitarian Case

3.1 The Tax-Reform Analysis

The Lagrangian expression associated with the utilitarian social welfare maximization is given by:

$$\begin{aligned} \pounds \; = \; & \sum n_i \{ u\left((1-\tau)w_i\ell_i^* - s_i^* - P_i^* - v(\ell_i^*)\right) \\ & + \pi_i(1-p_i)u(s_i^*/\pi_i + b) + \pi_i p_i H(s_i^*/\pi_i + P_i^*/(\pi_i p_i) + g + b) \} \\ & + \mu \sum n_i (\tau w_i \ell_i^* - \pi_i b - \pi_i p_i g). \end{aligned} \tag{5}$$

For simplicity, the star notations with respect to x_i, d_i, m_i, and ℓ_i are dropped in the remainder of the paper. The FOCs on g and b are as follows:

$$\frac{\partial \pounds}{\partial g} = \sum n_i \pi_i p_i H'(m_i) - \mu \overline{\pi p}, \tag{6}$$

$$\frac{\partial \pounds}{\partial b} = \sum n_i \pi_i \{(1-p_i)u'(d_i) + p_i H'(m_i)\} - \mu \overline{\pi}, \tag{7}$$

where the bar denotes the population average of the respective parameter.

We first adopt the viewpoint of tax reform; that is, we consider that the tax is given and not necessarily optimal, and we examine the welfare incidence of increasing g at the expense of b while keeping a balanced budget. This is given by:

$$\begin{aligned}\frac{\partial \pounds^c}{\partial g} &\equiv \frac{\partial \pounds}{\partial g} + \frac{\partial \pounds}{\partial b}\frac{\partial b}{\partial g}\Big|_{d\tau=0} \\ &= \left(1 - \frac{\overline{\pi p}}{\bar{\pi}}\right)\sum n_i \pi p_i H'(m_i) - \frac{\overline{\pi p}}{\bar{\pi}}\sum n_i \pi_i (1-p_i) u'(d_i),\end{aligned}$$

or:

$$\frac{\partial \pounds^c}{\partial g} = \left(1 - \frac{\overline{\pi p}}{\bar{\pi}}\right)\overline{\pi p}\left(cov\left(H'(m_i), \frac{\pi_i p_i}{\overline{\pi p}}\right) - cov\left(u'(d_i), \frac{\pi_i(1-p_i)}{\overline{\pi(1-p)}}\right) + \Delta\right), \tag{8}$$

where $\Delta \equiv \sum n_i \{H'(m_i) - u'(d_i)\}$.

Regarding the second-period consumption (m_i and d_i), the following property holds (regardless of the liquidity constraints). The proof is given in the Appendix:

Lemma 1 $\partial m_i / \partial w_i \geq 0$ *and* $\partial d_i / \partial w_i \geq 0$ *with strict inequality when* $s_i^* > 0$.

We have to ascertain that the wage gap is wider than the probability gap. To show why this is important, assume for a moment that the wage dispersion is negligible, whereas the variance of the dependence risk is very high. Then, it is not impossible that low-income individuals save more than high-income ones. Excluding such an unreasonable situation, we assume that w_i varies sufficiently across individuals so that $cov(w_i, \pi_i p_i) < 0$ implies that $cov(H'(m_i), \pi_i p_i / \overline{\pi p}) > 0$, and $cov(w_i, \pi_i) < 0$ implies that $cov(u'(d_i), \pi_i(1-p_i)/\overline{\pi(1-p)}) < 0$.

The argument so far corresponds to Rochet (1991, Proposition 1): social insurance should be nil if the correlation between risk and the wage level is positive, and it can be as high as possible if this correlation is negative.[3] The current model deals with the two risks (having a long life and old age dependence) and we conclude the following: as long as $\Delta = 0$ ($H'(m_i) - u'(d_i) = 0$ for all i), it is always desirable to increase g at the expense of b. The LTC social insurance realizes targeted expenditures but the public pension favors the productive individuals who also live longer. Note that we assume that the tax distortions are independent of the type of insurance. As discussed above, $\Delta = 0$ holds when g and b (and τ) are small, or when liquidity constraints are assumed away.

To obtain $\dfrac{\partial \pounds^c}{\partial g} \leq 0$, Δ has to be negative. For illustrative purposes, suppose that $H(y) = u(y - L)$, where $L > 0$ denotes the resources required to compensate for the dependency. In other words, L reflects the cost of LTC beyond standard consumption. In this case, we have $H'(m_i) \geq u'(d_i) \iff P_i^*/(\pi_i p_i) + g - L \leq 0$. In the case of liquidity constraints for P_i, we have $H'(m_i) \geq u'(d_i) \iff g - L \leq 0$. We have $\Delta < 0$ only if $g > L$ (i.e., the government fully compensates for the resources needed for dependency).

In the numerical example presented below, we obtain an optimal level of public LTC that tends to be higher than that of public pensions, and everyone saves. In general, the optimal levels of g and b are given by:

$$\frac{\partial \pounds}{\partial g} = cov(H'(m_i), \pi_i p_i) + \overline{\pi p}(\overline{H'(m)} - \mu) = 0, \tag{9}$$

$$\frac{\partial \pounds}{\partial b} = cov(H'(m_i), \pi_i p_i) + \overline{\pi p}(\overline{H'(m)} - \mu) + cov(u'(d_i), \pi_i(1 - p_i)) + \overline{\pi(1-p)}(\overline{u'(d)} - \mu) \leq 0. \tag{10}$$

The covariance term in (9) is positive, and the interior optimum for the public LTC is given at the point where the m_i's are sufficiently high ($H'(m_i)$'s are sufficiently low: $\overline{H'(m)} - \mu < 0$). If (10) holds in

[3] Rochet (1991) added the risk of illness to the income taxation model à la Mirrlees (1971) and Sheshinski (1972). He proved that if there is a negative statistical relation between the probability of illness and labor productivity, then the utilitarian social optimum implies that social insurance provides complete coverage of risk for every household. Our model can be seen as an extension of Rochet's paper. Assuming away liquidity constraints, and assuming an actuarially fair market for annuities and LTC, our two-period model equates to his one-period model.

equality, it must be the case that $\overline{H'(m)} < \mu < \overline{u'(d)}$.[4] Indeed, (10) can be attained even with positive saving by everyone. However, if (10) holds with equality at its interior optimum, (3) and $\overline{H'(m)} < \overline{u'(d)}$ imply that $H'(m_i) < u'(x_i)$ ($P_i^* = 0$ from (4)) must be the case, at least for some individuals. This leads us to our first theorem.

Theorem 1 *(a) When there is no liquidity constraint, it is always desirable to have a balanced budget increase in LTC benefits at the expense of public pension benefits.*

(b) When there are liquidity constraints, it is socially desirable to substitute public pension benefits for LTC benefits as long as the liquidity constraints term does not dominate the two covariance terms. The optimal level of public pensions can be attained with positive saving, whereas at least some individuals do not purchase LTC insurance.

3.2 Optimal Tax and Social Insurance

Combining (6) and the revenue-side optimization, we obtain:

$$\begin{aligned}\frac{\partial \pounds^c}{\partial \tau} &= \frac{\partial \pounds}{\partial \tau} + \frac{\partial \pounds}{\partial g}\frac{\partial g}{\partial \tau}\Big|_{dr=0} \\ &= \sum n_i u_i'(x_i)(-w_i\ell_i) + \sum n_i \pi_i p_i H'(m_i)\frac{\overline{y} + \tau \partial \overline{y}/\partial \tau}{\overline{\pi p}} \qquad (11) \\ &= -cov(u'(x_i), y_i) + cov(H'(m_i), \frac{\pi_i p_i}{\overline{\pi p}})\overline{y} - \mathit{\Gamma}\overline{y} + \mu\tau\frac{\partial \overline{y}}{\partial \tau},\end{aligned}$$

where $y_i \equiv w_i\ell_i$ and $\mathit{\Gamma} \equiv \sum n_i\{u'(x_i) - H'(m_i)\} \geq 0$. This yields the following optimal tax formula:

$$\tau^* = \frac{-cov\left(u'(x_i), \dfrac{y_i}{\overline{y}}\right) + cov\left(H'(m_i), \dfrac{\pi_i p_i}{\overline{\pi p}}\right) - \mathit{\Gamma}}{-\mu \partial \overline{y}/\partial \tau \cdot 1/\overline{y}} > 0. \qquad (12)$$

The denominator is the conventional efficiency term. It is positive. The first term of the numerator is the traditional equity term $-cov(u'(x_i), y_i) > 0$. These two terms correspond to the conventional optimal tax formula (e.g., see Sheshinski (1972) and Hellwig (1986)). This redistributive impact of the conventional first term

[4] As mentioned above, we have $\overline{H'(m)} - \mu < 0$. When $\partial \pounds/\partial b = 0$, (10) and $cov(u'(d_i), \pi_i(1-p_i)) < 0$ imply that $\overline{u'(d)} - \mu > 0$. Otherwise, $\partial \pounds/\partial b|_{b=0} < 0$, i.e., $b = 0$ at the social optimum.

of the numerator is reinforced by the second term, which is positive and reflects the redistributive impact of the LTC benefit. Note that if the tax proceeds were used for pensions instead of LTC, the second term of the numerator would be negative, reflecting the fact that pensions tend to benefit high-income individuals. The last term of the numerator represents the cost of the binding liquidity constraints in (4).

Let (τ^*, g^*, b^*) be the social optimum. With respect to g^*, it is characterized by $\partial \pounds^c / \partial g = 0$ or $\partial \pounds^c / \partial g|_{b=0} > 0$. From (8), we conclude the following:

Theorem 2 *Suppose that $H(y) = u(y - L)$. If $g^* \leq L$, then $b^* = 0$.*

When $H(y) = u(y - L)$, if $b^* > 0$, then $g^* > L$, so that individuals do not purchase any LTC insurance ($P_i^* = 0$ for all i). This is consistent with Theorem 1.(b). Also, Theorem 2 is easily extended to the case where the private saving and private insurance comprise loading costs, with the loading costs of P_i being higher than those of s_i. With sufficiently high loading costs, some individuals may prefer not to buy insurance, in which case the role of the public LTC insurance is strengthened. See the Appendix for the proof.

Evaluated at $g \leq L$ and $b = 0$, and taking account of the government budget balance, the total effect of the tax increase for the increase of g is given by (11). Whether an optimum exists at $g^* \leq L$ depends on the sign of (11) at $g \leq L$. The qualitative features are as follows. Other things being equal, (11) is lower (and the optimal LTC social insurance g^* is lower) when the distribution of income and the risk of dependency are more equal, or when the tax distortions are high.[5]

Between g and b, priority is given to g until $g = L$. This is true with or without liquidity constraints.

4 Numerical Example

To illustrate the above results, we provide a numerical example to illustrate the incidence of parameter changes on the final outcome.

[5] When $g \leq L$ and $b = 0$, we can show that $\Gamma = 0$. Dividing (11) by $\overline{y}$, it is increasing in $-cov(u'(x_i), y_i/\overline{y})$ and decreasing in $\partial \overline{y}/\partial \tau \cdot 1/\overline{y}$. Evaluated at the left side of the Laffer curve ($\overline{y} + \tau \partial \overline{y}/\partial \tau > 0$), (11) is also increasing in $cov(H'(m_i), \pi_i p_i / \overline{\pi p})$.

The society comprises two types of individuals. We use the following specification:

$$u(x) = ax - 0.5x^2; v(\ell) = \frac{\gamma/k}{1+\gamma}\ell^{\frac{1+\gamma}{\gamma}}.$$

The initial values of the parameters that make our benchmark scenario are:

$$a = 400,\ n_1 = n_2 = 0.5,\ w_1 = 10,\ w_2 = 20,\ \pi_1 = 0.8,\ \pi_2 = 0.9,$$

$$p_1 = 0.6,\ p_2 = 0.45,\ L = 50, \gamma = 1,\ k = 2.77.$$

The values of π_i and p_i in this benchmark scenario came from those in the Health and Retirement Study (HRS). We segment the population into half with respect to income levels and take the average probability of π_i and p_i. We have $\pi_1 p_1 > \pi_2 p_2$. Given these parameters, social welfare is maximized at $b^* = 68.48$ and $g^* = 73.58 \approx 1.47L$ for a tax rate $\tau^* = 0.155$. Here, we have $b^* < g^*$ but, in terms of the size of the expenditure, we have $\sum n_i \pi_i b^* = 58.21 > \sum n_i \pi_i p_i g^* = 32.56$: public pensions require a greater amount of expenditure than the public LTC. This result, as well as other results following changes in parameters, are presented in Table 2. Note that in all the scenarios studied except in the case of a lower w_2, individuals do not purchase any LTC insurance at the social optimum.

	b^*	g^*	τ^*	s_1^*	y_1^*	s_2^*	y_2^*
benchmark	68.48	73.58	0.155	7.22	234.03	149.85	936.11
(A) $\gamma = 2 > 1$ (k=0.40)	22.57	72.62	0.086	1.96	133.08	138.18	1064.67
(B) $w_2 = 14 < 20$	0	35.31	0.040	53.63	266.01	115.76	521.37
(C) $\pi_1 = 0.68 < 0.8$	70.05	73.70	0.144	6.98	237.15	154.11	948.59
(D) $n_1 = 0.4 < 0.5$	77.55	72.43	0.149	4.16	235.80	148.65	943.20
(E) $p_1 = 0.8 > 0.6$	34.03	112.40	0.149	7.30	235.81	160.76	943.25
(F) $p_2 = 0.5 > 0.45$	71.00	65.71	0.155	8.16	233.95	149.83	935.78

Table 2: Numerical Examples

The values of k in the first row (the benchmark case) and the second row were chosen so that $\tau = 0.033$ at $g = L$ and $b = 0$. Now, we interpret this table by examining the effects of changes in the parameters. An increase in labor elasticity implies an increase of the denominator of the optimal tax formula (12), which implies lower tax and, consequently, lower LTC benefits and pensions. Having a lower wage inequality (w_2 is lower than in the benchmark case) results in quite a lower tax rate, and we have $g^* = 35.31 < L$: see the interpretation on the sign of (11) at $g \leq L$ in Section 3. Consistent with Theorem 2, we have $b^* = 0$. Decreasing π_1 from 0.8 to 0.68 leads to lower fiscal burdens for the LTC benefits and pensions. We obtain the values of b^* and g^*, which are similar to the benchmark case, but the tax rate to finance them becomes lower. If the relative number of type 1 individuals decreases from 0.5 to 0.4 (n_2 increases from 0.5 to 0.6), then the utilitarianism prefers a lower optimal tax rate because greater social weight is given to type 2 individuals. Accordingly, b^* is higher and g^* is lower than the benchmark case. As for the composition of public expenditure, a greater proportion is spent on public pensions than in the benchmark case. Finally, we examine changes in the risk of dependence. If p_1 increases from 0.6 to 0.8, then the covariance term in (9) is increased, and the LTC benefits substantially increase. This change reduces public pensions substantially. On the other hand, if p_2 goes from 0.45 to 0.5, the expenditures on LTC benefits decrease even though the number of dependent people has increased. The decrease of g^* in this case is due to the decreased covariance between $\pi_i p_i$ and w_i. In cases (C), (D), and (F), we have $\sum n_i \pi_i b^* > \sum n_i \pi_i p_i g^*$ (public pensions require a greater amount of expenditure than public LTC).

5 Rawlsian Case

Suppose that there is an individual 0 whose wage is $w_0 = 0$. For this individual, we have $s_0^* = 0$ and $P_0^* = 0$. Suppose that the government's social objective is to maximize the second-period utility of individual 0. The Lagrangian expression is:

$$\pounds = \pi_0(1 - p_0)u(b) + \pi_0 p_0 H(b + g) + \mu \sum n_i(\tau w_i \ell_i^* - \pi_i b - \pi_i p_i g). \quad (13)$$

As individual 0 does not pay payroll tax, the optimal tax rate under this social objective is the peak of the Laffer curve: $\tau^* = \dfrac{\overline{y}}{-\partial \overline{y}/\partial \tau}$. The issue here is how to allocate the tax revenue between g and b.

The FOCs with respect to g and b are:

$$\frac{\partial \pounds}{\partial g} = \pi_0 p_0 H'(b+g) - \mu\overline{\pi p}, \quad (14)$$

$$\frac{\partial \pounds}{\partial b} = \pi_0\{(1-p_0)u'(b) + p_0 H'(b+g)\} - \mu\bar{\pi}. \quad (15)$$

From these FOCs, we have:

$$\frac{\partial \pounds^c}{\partial g} = \left(1 - \frac{\overline{\pi p}}{\bar{\pi}}\right)\pi_0 p_0 H'(b+g) - \frac{\overline{\pi p}}{\bar{\pi}}\pi_0(1-p_0)u'(b) \quad (16)$$

In other words, a compensated increase of LTC benefits, g, is desirable if and only if:

$$H'(b+g) > u'(b)\Phi, \quad (17)$$

where $\Phi = \frac{1-p_0}{p_0}\frac{\overline{\pi p}/\bar{\pi}}{1-\overline{\pi p}/\bar{\pi}} < 1$.

The inequality $\Phi < 1$ can be expressed as:

$$\frac{\pi_0}{p_0\pi_0} < \frac{\bar{\pi}}{\overline{p\pi}}.$$

This inequality compares the ratio of the survival probability to the dependence risk of the poor with the same ratio for the whole society. The survival probability π_0 corresponds to the benefit b, whereas the probability $\pi_0 p_0$ corresponds to the benefit $b+g$. Assume that p_0 increases, with all other probabilities being constant. Then, Φ decreases, which implies a decrease of the social marginal utility of $d = b$ relative to $m = b+g$. Note that in the case of $\pi_i = \overline{\pi}$ for all i, $\Phi < 1$ because $p_0 > \overline{p}$.

Clearly, in the Rawlsian case, the superiority of LTC benefits over public pensions is more limited than in the utilitarian case because both programs are required by the poorest individuals.

Theorem 3 *In the Rawlsian case, a balanced budget increase in LTC benefits is desirable as long as LTC benefits are not too high relative to pension benefits and if the dependence probability of the poorest is higher than that of the average population.*

6 Conclusion

This paper has studied the design of both a social LTC insurance scheme and a public pension system. Both benefits were uniform, as well as the payroll tax rate. Under the realistic assumption of a positive correlation between income and the survival probability, and of a negative correlation between income and the dependency probability, we show that, as an extension of Rochet (1991), a utilitarian government should give priority to the LTC scheme relative to the pension program. Even with liquidity constraints, priority is given to LTC until the cost of LTC is fully covered. When the government adopts a Rawlsian criterion, both programs are required and the relative advantage of one over the other will depend on a comparison between the ratio of the survival probability to the dependence risk of the poor with its population average. In this paper, we use linear instruments. In a related paper, Nishimura and Pestieau (2016) used nonlinear instruments but with only two types of individuals. In the optimal nonlinear scheme, our stylized facts determined the features of the optimal policies regarding public pensions and LTC schemes.

It is fair to recognize that one of the reasons why LTC has a negligible role in most social insurance schemes is that it is mainly supplied by families, which is not the case for old-age support. Introducing family solidarity would clearly modify our results, but only partially, if we take into account the possibility of solidarity default. In this paper, we have made a number of assumptions to keep the presentation simple. We have assumed a zero interest rate, quasi-linear preferences, no time preference, and a pure Beveridgian public pension. Relaxing any of the assumptions would not change our basic results.

Appendix

Proof of Lemma 1: Let $f(\tau, w_i) \equiv (1-\tau)w_i\ell_i^* - v(\ell_i^*)$. From the Envelope Theorem, $\partial f/\partial w_i = (1-\tau)\ell_i^* > 0$. When $s_i^* > 0$ and $P_i^* > 0$, (3) and (4) imply that $u'(f(\tau, w_i) - s_i^* - P_i^*) = H'(m_i) = u'(s_i^*/\pi_i + b)$, so $x_i = \dfrac{s_i^*}{\pi_i} + b = \dfrac{f(\tau, w_i) - P_i^* - b}{1+\pi_i} + b$. Differentiating

$$-u'\left(\frac{f(\tau, w_i) - P_i^* - b}{1+\pi_i} + b\right) + H'\left(\frac{f(\tau, w_i) - P_i^* - b}{1+\pi_i} + \frac{P_i^*}{\pi_i p_i} + g + b\right) =$$

0, we obtain:

$$\frac{\partial P_i^*}{\partial w_i} = \frac{(u''(x_i) - H''(m_i))/(1+\pi_i) \cdot \partial f/\partial w_i}{(u''(x_i) - H''(m_i))/(1+\pi_i) + H''(m_i)/(\pi_i p_i)}. \tag{18}$$

The denominator of (18) is negative because of the second-order condition with respect to P_i^*. Then, $\frac{\partial x_i}{\partial w_i} = \frac{1}{1+\pi_i}\left(\frac{\partial f}{\partial w_i} - \frac{\partial P_i^*}{\partial w_i}\right) = \frac{H''(m_i)/(\pi_i p_i)\partial f/\partial w_i}{u''(x_i) - H''(m_i) + H''(m_i)(1+\pi_i)/(\pi_i p_i)} > 0$. As $H''(m_i)\frac{\partial m_i}{\partial w_i} = u''(d_i)\frac{\partial d_i}{\partial w_i} = u''(x_i)\frac{\partial x_i}{\partial w_i}$, we have $\frac{\partial m_i}{\partial w_i} > 0$ and $\frac{\partial d_i}{\partial w_i} > 0$.

When the individual optimum faces the liquidity constraint for P_i^*, then (3) and (4) are characterized by $-u'(f(\tau, w_i) - s_i^*) + (1 - p_i)u'(s_i^*/\pi_i + b) + p_i H'(s_i^*/\pi_i + g + b) \leq 0$ and $P_i^* = 0$. When $s_i^* > 0$, differentiating the former equation, we obtain $\frac{\partial s_i^*}{\partial w_i} = \frac{u''(x_i)}{u''(x_i) + (1-p_i)u''(d_i)/\pi_i + p_i H''(m_i)/\pi_i}\frac{\partial f}{\partial w_i} > 0$. When $s_i^* = 0$ and $P_i^* = 0$, $\partial m_i/\partial w_i = 0$.

When $s_i^* = 0$ and $P_i^* > 0$, then differentiating $-u'\left(f(\tau, w_i) - P_i^*\right) + H'\left(\frac{P_i^*}{\pi_i p_i} + g + b\right) = 0$, $\frac{\partial x_i}{\partial w_i} = \frac{\partial f}{\partial w_i} - \frac{\partial P_i^*}{\partial w_i} = \frac{H''(m_i)/(\pi_i p_i)}{u''(x_i) + H''(m_i)/(\pi_i p_i)}\frac{\partial f}{\partial w_i} > 0$. As $H''(m_i)\frac{\partial m_i}{\partial w_i} = u''(x_i)\frac{\partial x_i}{\partial w_i}$, we have $\frac{\partial m_i}{\partial w_i} > 0$. *Q.E.D.*

Proof of Theorem 2 (with loading costs): Suppose that the private saving and the private LTC insurance accrue loading costs. That is, type i's return from saving s_i is $\lambda^s s_i/\pi_i$, and the return of the private insurance P_i is $\lambda^P P_i/(\pi_i p_i)$, where λ^s and λ^P represent the loading factors of the private savings and the private LTC, respectively. It is reasonable to assume that $0 < \lambda^P \leq \lambda^s \leq 1$, i.e., the loading costs of the private LTC are greater than those of the private savings. With $f(\tau, w_i) \equiv (1-\tau)w_i \ell_i^* - v(\ell_i^*)$, the FOCs of individual optimization (3) and (4) are modified to:

$$-\frac{u'(f(\tau, w_i) - s_i - P_i)}{\lambda^s} + (1-p_i)u'\left(\frac{\lambda^s s_i}{\pi_i} + b\right) + p_i H'\left(\frac{\lambda^s s_i}{\pi_i} + \frac{\lambda^P P_i}{\pi_i p_i} + g + b\right) \leq 0, \tag{3'}$$

$$-\frac{u'(f(\tau, w_i) - s_i - P_i)}{\lambda^P} + H'\left(\frac{\lambda^s s_i}{\pi_i} + \frac{\lambda^P P_i}{\pi_i p_i} + g + b\right) \leq 0. \tag{4'}$$

When $\lambda^s < 1$ and $\lambda^P < 1$, some individuals may prefer not to have private savings or private insurances.

Suppose that $g \le L$. If (4') holds in equality, then (3') implies that $-\frac{\lambda^P}{\lambda^s}H'(m_i) + (1-p_i)u'(d_i) + p_i H'(m_i) = (1-p_i)(u'(d_i) - H'(m_i)) + \frac{\lambda^s - \lambda^P}{\lambda^s}H'(m_i) \le 0$ for all i. When $g \le L$ and $P_i^* = 0$, $u'(d_i) = u'(\lambda^s s_i^*/\pi_i + b) \le H'(\lambda^s s_i^*/\pi_i + b + g) = H'(m_i)$ for all i. In both cases, we have $u'(d_i) \le H'(m_i)$ for all i when $g \le L$. Therefore, we have $\Delta \ge 0$. From (8), we have the desired result. *Q.E.D.*

References

[1] Ameriks, J., Briggs, J., Caplin, A., Shapiro, M. D., & Tonetti, C. (2020). Long-term-care utility and late-in-life saving. Journal of Political Economy, 128(6), 2375–2451.

[2] Bolin, K., Lindgren, B., & Lundborg, P. (2008). Informal and formal care among single-living elderly in Europe. Health Economics, 17(3), 393–409.

[3] Buckner, L., & Yeandle, S. (2011). *Valuing carers 2011 — Calculating the value of carers' support*. London: Carers UK. https://www.carersuk.org/component/cck/?task=download&file=policy_file&id=5458

[4] Cremer, H., Lozachmeur, J.-M., & Pestieau, P. (2010). Collective annuities and redistribution. Journal of Public Economic Theory, 12(1), 23–41.

[5] Cremer, H., & Pestieau, P. (2014). Social long-term care insurance and redistribution. International Tax and Public Finance, 21(6), 955–974.

[6] De Donder, P., & Leroux, M.-L. (2021). Long term care insurance with state-dependent preferences. Working Paper No. 1061. Toulouse: Toulouse School of Economics.

[7] Finkelstein, A., Luttmer, E., & Notowidigdo, M. (2013). What good is wealth without health? The effect of health on the marginal utility of consumption. Journal of the European Economic Association, 11(S1), 221–258.

[8] Hellwig, M. F. (1986). The optimal linear income tax revisited. Journal of Public Economics, 31(2), 163–179.

[9] Koijen, R., Van Nieuwerburgh, S., & Yogo, M. (2016). Health and mortality delta: Assessing the welfare cost of household insurance choice. Journal of Finance, 71(2), 957–1009.

[10] Lefebvre, M., Perelman, S., & Schoenmaeckers, J. (2018). Inégalités face à la mort et au risque de dépendance. Revue Française d'Économie, 33(2), 75–111.

[11] Leroux, M.-L., Pestieau, P., & Ponthiere, G. (2019). Fair long-term care insurance. CESifo Working Paper No. 7660. Munich: CESifo GmbH.

[12] Lillard, L., & Weiss, Y. (1997). Uncertain health and survival: effects on end-of-life consumption. Journal of Business & Economic Statistics, 15(2), 254–268.

[13] Mirrlees, J. (1971). An exploration in the theory of optimum income taxation. Review of Economic Studies, 38(2), 175–208.

[14] Nishimura, Y., & Pestieau, P. (2016). Efficient taxation with differential risks of dependence and mortality. Economics Bulletin, 36(1), 52–57.

[15] Norton, E. (2016). Health and long-term care. In Piggott, J., & Woodland, A. (eds.), *Handbook of the economics of population aging*, Volume 1B, 951–989. New York, NY: Elsevier Science B.V.

[16] OECD (2019a). *Pensions at a glance 2019*. Paris: OECD.

[17] OECD (2019b). *Health at a glance 2019*. Paris: OECD.

[18] Rochet, J.-C. (1991). Incentives, redistribution and social insurance. Geneva Papers on Risk and Insurance Theory, 16(2), 143–165.

[19] Sheshinski, E. (1972). The optimal linear income-tax. Review of Economic Studies, 39(3), 297–302.