Social Choice and Welfare
https://doi.org/10.1007/s00355-021-01379-y

ORIGINAL PAPER

Childlessness, childfreeness and compensation

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Received: 14 January 2021 / Accepted: 16 November 2021

Abstract
We study the design of a fair family policy in an economy where parenthood is regarded either as desirable or as undesirable, and where there is imperfect fertility control, leading to involuntary childlessness/parenthood. Using an equivalent consumption approach in the consumption-fertility space, we show that the identification of the worst-off individuals depends on how the social evaluator fixes the reference fertility level. Adopting the ex post egalitarian criterion (giving priority to the worst off in realized terms), we study the compensation for involuntary childlessness/parenthood. Unlike real-world family policies, the fair family policy does not always involve positive family allowances, and may also include positive childlessness allowances. Our results are robust to assuming asymmetric information and to introducing Assisted Reproductive Technologies.

1 Introduction

Family policies are old, and date back, at least, to the Mercantilist epoch, that is, the first attempt to build a consistent national system of economic policies. For instance, there was, under the French Kingdom at the time of Colbert (seventeenth century), a public pension offered to fathers of at least 12 children. The goal was to increase the number of births, at a time where a sizable population was regarded as a necessary condition for military and economic power.

The authors would like to thank A. Baurin, D. de la Croix, J. Drouard, M. Fall, M. Fleurbaey, P. Gobbi, B. Le Maux, E. Malin, F. Maniquet, F. Moizeau, Y. Nishimura, T. Penard, L. Pensionero, M. Shrime, two anonymous reviewers and participants of seminars at Princeton, CREM (Rennes), IRES (Louvain) and Osaka (SOPE), for their suggestions and comments on this paper.

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Published online: 30 November 2021

More than three centuries after Colbert, it is still the case that, when economists and demographers evaluate the impact of family policies, they focus mainly on their effect on fertility, that is, on the quantity of births per woman (see Gauthier 2007; Thévenon and Gauthier 2011). While family policies are multidimensional (family allowances, parental leave, day-care policies, etc.), most studies analyzed the impact of family allowances on fertility, and often showed a positive effect of these allowances on the number of children.[1]

But family policies do not only have an impact on the number of births. These policies have also important distributive implications. Subsidizing births by means of uniform family allowances affects the distribution of income, to the extent that fertility behaviors vary with income. If children are inferior goods, subsidizing births redistributes resources towards lower income classes, whereas the opposite holds if children are superior goods.[2]

The goal of this paper is to reexamine family policies, and in particular family allowances, from the perspective of *fairness*. Instead of evaluating their impact on the size of the population (which is, in the light of the population ethics literature and its paradoxes, a quite questionable goal), we propose to consider family policies from a fairness perspective, that is, to regard family policies as instruments aimed at providing compensation to the disadvantaged.[3]

In the context of fertility, a major source of disadvantage consists of inequality in fecundity, i.e. the capacity to give birth to children. Humans are unequal in terms of their capacity to give birth to children. As shown by Leridon (1992) for France, about 10 % of each cohort remains childless because of purely biological reasons. That phenomenon is a case of *involuntary* childlessness. Since having children is, for many persons, a key component of their life-plans, victims of involuntary childlessness suffer from a welfare deprivation.

That source of deprivation being (largely) exogenous, there is a strong case for compensating individuals who suffer from involuntary childlessness. Indeed, following the Principle of Compensation (Fleurbaey 2008; Fleurbaey and Maniquet 2010), a government should abolish well-being inequalities that are due to circumstances. Given that involuntary childlessness is mainly due to biological circumstances, there is here a strong ethical argument supporting the compensation of the involuntary childless.

But the design of a fair family policy faces—at least—three challenges.

[1] See Gauthier and Hatzius (1997), D'Addio and Mira d'Ercole (2005), Luci-Greulich and Thévenon (2013). One exception is Kalwij (2010), who finds that family allowances have no significant impact on fertility in Western Europe.

[2] On distributive effects, see Balestrino et al. (2002) and Pestieau and Ponthiere (2013).

[3] On population ethics and its paradoxes, classical references are Parfit (1984) and Blackorby et al. (2005). Those pieces of work question all standard social criteria in the context of varying population size, and, as such, question also a purely "productivist" evaluation of family policies in terms of their impact on the number of births.

Firstly, the childlessness phenomenon includes not only involuntary childlessness, but, also, *voluntary* childlessness.[4] Childfreeness, i.e. women who make the choice to remain "free" from children, concerns about 5% of a cohort.[5] Heterogeneity in preferences complicates the design of policies: one must make the well-being of the involuntary childless (who regards children as desirable) and of the childfree (who regards children as undesirable) *comparable*.

Secondly, in real-world economies, it is difficult for governments to observe preferences, and to distinguish between the childless and the childfree. Asymmetric information makes compensation for childlessness even more challenging.

Thirdly, another challenge for the design of a fair family policy is the development, in the last decades, of Assisted Reproductive Technologies (ART) (Trappe 2017; Prag and Mills 2017). ART reduce the number of victims of involuntary childlessness, but those treatments are extremely costly. Governments should restrict access to ART to individuals who want children and cannot have children otherwise, two features that are hard to observe.

This paper revisits the design of optimal family policy, by paying a particular attention to the compensation of the involuntary childless persons in an economy peopled by individuals for whom parenthood is regarded either as a desirable or as undesirable, and where there is imperfect contraceptive and reproductive technology, leading to involuntary parenthood/childlessness. From that perspective, a key issue concerns the identification of the most disadvantaged persons, whose demands should have priority (Fleurbaey and Maniquet 2011, 2019). Identifying the worst-off requires first to measure and compare the well-being of the involuntary childless and of the involuntary parents.

In order to examine the design of a fair family policy, we proceed in four stages. First, we develop a model of binary fertility (either 0 or 1 child), with imperfect fertility control and heterogeneity of preferences. Second, we address the challenge of interpersonal well-being comparisons by using an equivalent consumption approach, which amounts to construct a preferences-based index of well-being that relies on reference achievements for non-monetary dimensions of well-being (Fleurbaey and Maniquet 2011). Third, adopting the ex post egalitarian social criterion (which consists of a maximin on consumption equivalents), we characterize the social optimum and study its decentralization (under perfect observability of preferences) by means of a mixed adoption/transfer system. Fourth, we study the robustness of the fair family policy to introducing asymmetric information on preferences and ART investments.

Our analysis of a fair family policy leads us to two main results.

[4] Childlessness as a whole concerns about 15 % of a women cohort, with variations across countries and epochs (Sobotka 2017). Childlessness is less widespread in France in comparison to Germany or the UK (Koppen et al. 2017; Berrington 2017; Kreyenfeld and Konietzka 2017).

[5] Toulemon (2001) shows, in the case of France, that about 5% of individuals state that childlessness is the most ideal living arrangement, whereas most men and women say that 2 or 3 is the ideal number of children. See also Kuhnt et al. (2017).

First, the design of the fair family policy is not robust to how the social evaluator fixes the reference fertility level, which is defined as the level of fertility at which interpersonal well-being comparisons can be made by merely comparing individual consumption levels. The reference fertility level, by determining who is the worst-off, affects the ex post egalitarian optimum as well as the conditions under which the ex post egalitarian optimum can be decentralized.

Second, our analysis shows that the fair family policy differs from existing policies. The fair family policy does not always involve positive family allowances, and may also involve positive childlessness allowances, unlike real-world family policies. That last result is robust to the reference fertility level, to the introduction of asymmetric information on individual preferences (even though the childlessness allowance is then reduced), and to the inclusion of ART. When ART treatments are available, the fair family policy involves additional allowances compensating individuals for the monetary and psychological costs of using ART as well as an additional allowance to ART users who did not succeed in becoming parents.

As such, this paper casts original light on the design of a fair family policy, and on how it differs from existing family policies, taking into account new societal realities such as the development of ART. Shifting the objective from raising the population size to compensating the disadvantaged affects the optimal family policy in a way that is robust to introducing asymmetric information.

This paper is related to several branches of the literature. First, it is related to the literature on childlessness. As far as demographers are concerned, Kreyenfeld and Konietzka (2017) synthesizes research on the determinants and dynamics of the childlessness phenomenon. On the economic side, Gobbi (2013) and Baudin et al. (2015), Baudin et al. (2019) study the determinants of childlessness in the U.S. and around the world, while (Etner et al. 2020) examine the impact of childlessness on long-run growth, while adopting a macroeconomic setting (with same preferences). We complement those studies by considering childlessness from a normative perspective, to characterize a fair family policy under heterogeneous preferences. Second, our work is related to the literature on optimal policy under varying fertility, such as Cigno (1983), Cigno (1986), Cremer et al. (2006), Cremer et al. (2008) and Pestieau and Ponthiere (2013). The specificity of our paper is to adopt a fairness perspective, and to consider family policy as an instrument aimed at compensating for involuntary childlessness/parenthood. Finally, our work is also related to the welfare economics literature on compensation (Fleurbaey 2008; Fleurbaey and Maniquet 2010). That literature has given rise to various applications, including the compensation for unequal lifetimes (Fleurbaey et al. 2014, 2016). This paper considers the other end of the demographic chain, that is, compensation for unequal fecundity.

The paper is organized as follows. Section 2 presents a model of imperfect fertility control and heterogeneous preferences. The identification of the worst-off is carried out in Sect. 3. The ex post egalitarian optimum is characterized in Sect. 4, which studies also its decentralization. Section 5 introduces asymmetric information. Section 6 considers ART treatments. Section 7 concludes.

2 The model

Let us consider an economy whose population of adults is a continuum of size 1. The adult population is composed of two categories of individuals: on the one hand, individuals who regard children as a *desirable* good (who are in a proportion $0 < x < 1$ in the population); on the other hand, individuals who regard children as an *undesirable* good (who are in a proportion $1 - x$).

Fertility is assumed to be binary and imperfectly controlled: individuals can either remain childless, or have one child, with a probability that depends on whether they consider children to be desirable or not (see below).

2.1 Preferences

Individuals who consider children as desirable have additively-separable preferences in consumption and fertility, given by:

$$u(c) + v(n) \tag{1}$$

where c is consumption and $n \in \{0, 1\}$ is the number of children. We assume that $u(\cdot)$ is increasing and concave. $v(\cdot)$ is increasing with $v(0) = 0$.[6]

The preferences of individuals who consider children as undesirable are also additively-separable in consumption and fertility:

$$U(c) - V(n) \tag{2}$$

where $U(\cdot)$ is increasing and concave. $V(\cdot)$ is increasing and satisfies $V(0) = 0$.

Assuming additively-separable preferences is standard in the literature on fertility (Strulik 2017). However, despite its simplicity, that structure can account for various aspects of preferences. The functions $u(\cdot)$ and $U(\cdot)$ can be different: this allows the marginal utility of consumption to differ between individuals having distinct tastes for fertility, $U'(c) \gtrless u'(c)$.[7] The functions $v(\cdot)$ and $V(\cdot)$ can also differ, so that our modeling allows also for having an asymmetry between, on the one hand, the well-being gain from parenthood for individuals who regard children as desirable, and, on the other hand, the well-being loss from parenthood for individuals who regard children as undesirable.

2.2 Fertility technology

Individuals have an imperfect control on the number of children $n \in \{0, 1\}$. Our framework lies somewhere between the standard model of perfect fertility control

[6] The function $v(\cdot)$ incorporates any utility gain obtained from having a (desired) child. This includes the pure joy of having a (desired) child as well as all other fertility motives.

[7] The desire for children may reinforce the marginal utility of consumption (reinforcement effect: $u'(c) > U'(c)$ for a given c), or, alternatively, it may weaken the marginal utility of consumption (redundancy effect: $u'(c) < U'(c)$ for a given c).

(see Barro and Becker 1989) and a model of purely random fertility (no control at all).

Let us assume that individuals who consider children as desirable have a child with probability $0 < \pi < 1$, and are childless with probability $1 - \pi$. We suppose also that individuals who consider children as undesirable have a child with probability $0 < \varepsilon < 1$, and are childless with a probability $1 - \varepsilon$. We assume, without loss of generality, that the probability to have a child is larger among individuals who consider children as desirable than among individuals who consider children as undesirable, that is: $\pi > \varepsilon$.[8] The gap $\pi - \varepsilon$ depends on the efficiency of reproductive and contraceptive technologies. Perfect control occurs when $\pi = 1$ and $\varepsilon = 0$. In that case, there is no involuntary childlessness/parenthood, which are the object of this paper. We thus assume imperfect fertility control, i.e. $\pi < 1$ and $\varepsilon > 0$.

Thanks to the progress in contraceptive technologies, the risk of becoming involuntary parent ε is small. However, we assume that the risk of involuntary parenthood is not zero, because of two reasons. First, assuming $\varepsilon = 0$ would introduce an arbitrary asymmetry in our model, which would involve perfect control when desired fertility is 0 and imperfect control when desired fertility is 1. Second, assuming $\varepsilon > 0$ allows us to study the role played by adoption policies as instruments of social justice in the context of involuntary childlessness.[9] Having stressed this, it remains true that the prevalence of involuntary parenthood is limited. Hence, we assume that ε is bounded upwards:

$$0 < \varepsilon \leq \frac{(1 - \pi)x}{(1 - x)} \tag{3}$$

Under that condition, the number of non-desired children born due to imperfect fertility control cannot exceed the number of unborn—but desired—children.

Following (Barro and Becker 1989), we assume that having a child implies a monetary cost $g > 0$, as well as a time cost qw, where $w > 0$ is the hourly wage and $0 < q < 1$ is the fraction of time dedicated to raising the child. Note that the parameter g accounts for all monetary costs related to raising the child and for all costs related to the child's consumption (including the cost of clothes, food, etc.). As such, the parameter g reflects the fact that consumption needs increase with the number of children.

2.3 The laissez-faire economy

Once the fertility outcome has realized (i.e. ex post), the economy is composed of four types of individuals:[10]

[8] Note that a model with pure random fertility would involve $\pi = \varepsilon$ (same probability to have a child for all individuals, independently from their willingness to have a child), which is unrealistic.

[9] The assumption $\varepsilon > 0$ implies a positive number of children available for adoption.

[10] Under the Law of Large Numbers, probabilities π and ε will also determine, together with the parameter x, the proportions of the different types of individuals in the population.

- Type 1: individuals who want a child and have a child, in proportion $x\pi$
- Type 2: individuals who want a child and do not have a child, in proportion $x(1-\pi)$
- Type 3: individuals who do not want a child and have a child, in proportion $(1-x)\varepsilon$
- Type 4: individuals who do not want a child and do not have a child, in proportion $(1-x)(1-\varepsilon)$.

Let us compare the well-being W^i of these different groups at the laissez-faire, i.e. in the absence of family policy:

$$W^1 \equiv u(w(1-q)-g)+v(1)$$
$$W^2 \equiv u(w)$$
$$W^3 \equiv U(w(1-q)-g)-V(1)$$
$$W^4 \equiv U(w)$$

It is reasonable to assume that the involuntary childless (type 2) are worse-off than the lucky parents (type 1), so that $W^1 > W^2$:

Assumption 1 (A1) Individuals who want to have a child are better off with a child than without it: $u(w(1-q)-g)+v(1) > u(w)$.

We also have, given the monotonicity of $U(\cdot)$ and $V(\cdot)$, that individuals who have a child but do not want to have a child (type 3) are worse-off than those who do not want a child and do not have one (type 4), i.e. $W^3 < W^4$, since

$$U(w(1-q)-g)-V(1) < U(w)$$

Finally, concerning types 2 and 4, we assume that $W^4 > W^2$, that is:

Assumption 2 (A2) Individuals who do not want a child and have no child are better off than those who want a child and have no child: $U(c) > u(c)$.

Assumptions A1 and A2 are plausible: persons who achieve their life-goals are *better off* ex post than persons who do not achieve them. Note, however, that the extent of the deprivation faced by persons who do not realize their life-plans depends on the precise manner in which well-being is measured. Note also that A1 and A2 do not allow us to provide a *complete* ranking of individuals in terms of well-being (whether $W_1 \gtrless W_4$ or $W_2 \gtrless W_3$). The reason is that individuals do not share the same preferences: while types 1 and 2 want a child, types 3 and 4 prefer not to have one. As a consequence, it is difficult, without additional assumptions, to compare the situations of types 2 and 3, and to have a complete well-being ranking. The next section examines well-being comparisons by using equivalent consumption indexes.

3 Identifying the worst-off at the laissez-faire

Prior to the design of a fair family policy, a preliminary step consists of identifying worst-off individuals. How can we compare the situation of involuntary childless individuals (type 2) with the one of involuntary parents (type 3)?

To answer that question, this section builds on recent advances in welfare economics and makes interpersonal well-being comparisons by means of the equivalent income/consumption indexes (Fleurbaey and Maniquet 2011; Fleurbaey and Blanchet 2013; Fleurbaey 2016). The construction of the equivalent income/consumption index consists in building, from individual preferences, an inclusive index of well-being, which takes into account not only the material part of well-being, but, also, all non-monetary dimensions of well-being. Those non-monetary dimensions of well-being are included by fixing reference levels for all these dimensions, and by deriving the hypothetical income/consumption level which, combined with reference levels for all non-monetary dimensions, would make individuals as well-off as they are with their current situation.

In that approach, reference levels for non-monetary dimensions of well-being are ethical parameters, which allow for the interpersonal comparison of well-being across individuals who have different preferences. Situations are generally hard to compare when individuals have different preferences, but when individuals enjoy reference levels, it is sufficient, in order to rank their well-being levels, to compare the levels of income/consumption that they enjoy (Fleurbaey 2016; Fleurbaey and Maniquet 2011).

Selecting a level of reference for a non-monetary dimension of well-being is a difficult task. The literature generally assumes, for simplicity, that the reference level takes the *same* level for every person. In our setting, this would justify to fix the reference fertility to 0 or 1 for all individuals. However, there is also some intuitive support for fixing the reference level to the *ideal* level for each person, given her preferences (Fleurbaey and Maniquet 2019). This kind of neutrality towards preferences would lead us to fix reference fertility to 1 for individuals of types 1 and 2, and to 0 for individuals of types 3 and 4.[11]

In this paper, we will consider all these cases, without assuming *a priori* any "natural" or "benchmark" level for reference fertility. The reason for keeping this issue open is twofold. First, there exist various arguments supporting the reliance on one or the other reference fertility level.[12] Second, we consider that the goal of this paper is to explore various normative approaches to the design of fair

[11] By doing so, we would obtain, in a hypothetical world where children were costless and where each individual enjoyed his ideal fertility level, that there would be no justification for transfers across individuals. While this point may support the reliance on case R3 below, there exist other arguments justifying to consider also other cases (see *infra*).

[12] As we already noted, one argument supporting R1 and R2 is simplicity, whereas another argument supporting R3 concerns the implications in terms of transfers in a world where all individuals would have their ideal outcomes. But other arguments supporting some reference levels exist. As we will see, one possible argument for the selection of case R1 may lie in the incentive-compatible nature of the implied optimal first-best allocations (see Sect. 5).

family policies, without imposing from the start a particular ethical view. Hence, throughout this paper, we will consider three distinct approaches:

R1 The reference fertility level is fixed to 0 child for all individuals.
R2 The reference fertility level is fixed to 1 child for all individuals.
R3 The reference fertility level is fixed to 1 child for those who want children, and to 0 child for those who do not want children.

This section studies the sensitivity of the identification of the worst-off to the postulated reference fertility, by constructing equivalent consumption indexes for all individuals under reference fertility R1 to R3.

Construction of equivalent consumption under R1

When the reference fertility level is fixed to $\bar{n} = 0$, equivalent consumptions $\hat{c}^i$ satisfy:

$$
\begin{aligned}
u(\hat{c}^1) + v(0) = u(w(1-q) - g) + v(1) &\iff \hat{c}^1 > w(1-q) - g \\
u(\hat{c}^2) + v(0) = u(w) + v(0) &\iff \hat{c}^2 = w \\
U(\hat{c}^3) - V(0) = U(w(1-q) - g) - V(1) &\iff \hat{c}^3 < w(1-q) - g < w \\
U(\hat{c}^4) - V(0) = U(w) - V(0) &\iff \hat{c}^4 = w
\end{aligned}
$$

Using Assumption A1, we have: $\hat{c}^3 < \hat{c}^2 = \hat{c}^4 < \hat{c}^1$. The worst-off is type-3 (involuntary parent). Voluntary parents (type 1) are strictly better off than childfree (type 4) and childless (type 2) individuals.

Construction of equivalent consumption under R2

Let us now fix the reference fertility level to $\bar{n} = 1$. Equivalent consumptions $\tilde{c}^i$ satisfy:

$$
\begin{aligned}
u\left(\tilde{c}^1\right) + v(1) = u(w(1-q) - g) + v(1) &\iff \tilde{c}^1 = w(1-q) - g \\
u\left(\tilde{c}^2\right) + v(1) = u(w) + v(0) &\iff \tilde{c}^2 < w \\
U(\tilde{c}^3) - V(1) = U(w(1-q) - g) - V(1) &\iff \tilde{c}^3 = w(1-q) - g \\
U(\tilde{c}^4) - V(1) = U(w) - V(0) &\iff \tilde{c}^4 > w
\end{aligned}
$$

Under Assumption A1, we have $\tilde{c}^1 > \tilde{c}^2$. The last two lines yield that $\tilde{c}^4 > \tilde{c}^3$ and together with $\tilde{c}^1 = \tilde{c}^3$, we obtain: $\tilde{c}^2 < \tilde{c}^1 = \tilde{c}^3 < \tilde{c}^4$. Hence the worst-off is type-2 (involuntary childless). Childfree individuals (type 4) are here regarded as better off than voluntary and involuntary parents.

Construction of equivalent consumption under R3

A third approach consists in fixing reference to levels that correspond to the *preferred choice* of individuals based on their preferences, that is, to $\bar{n}_{1,2} = 1$ for individuals who want a child (types 1 and 2), and to $\bar{n}_{3,4} = 0$ for individuals who do not want a child (types 3 and 4). Equivalent consumptions $\bar{c}^i$ satisfy:

$$u(\bar{c}^1) + v(1) = u(w(1-q) - g) + v(1) \iff \bar{c}^1 = w(1-q) - g$$
$$u(\bar{c}^2) + v(1) = u(w) + v(0) \iff \bar{c}^2 < w$$
$$U(\bar{c}^3) - V(0) = U(w(1-q) - g) - V(1) \iff \bar{c}^3 < c^3 = w(1-q) - g$$
$$U(\bar{c}^4) - V(0) = U(w) - V(0) \iff \bar{c}^4 = c^4 = w$$

Assuming Assumption A1, we have: $\bar{c}^2, \bar{c}^3 < \bar{c}^1 < \bar{c}^4$. It is not possible here to identify the worst-off, since it depends on the forms of $u(\cdot)$, $v(\cdot)$, $U(\cdot)$ and $V(\cdot)$. However, given time and good costs of children, childfree individuals are better off than voluntary parents (like under R2).

Proposition 1 summarizes our results.

Proposition 1 *The identification of the worst-off individual is not robust to the postulated reference fertility level.*

Reference fertility	Worst-off individuals
R1 ($\bar{n} = 0$)	Involuntary parents
R2 ($\bar{n} = 1$)	Involuntary childless
R3 ($\bar{n}_{1,2} = 1, \bar{n}_{3,4} = 0$)	Involuntary childless/parents

Proof See above. □

Proposition 1 states a negative result. Indeed, if the identification of the worst-off were robust to the selected reference fertility level, one could base well-being comparisons on any of those reference levels, without any risk of lack of robustness. But Proposition 1 states that it is not the case. This result would not be problematic if there existed a salient candidate for the reference fertility level.[13] However, such a salient candidate does not exist here.

The lack of robustness of the identification of the worst off to the postulated reference fertility has a major corollary for the design of a fair family policy. Given that the reference fertility affects how well-being levels are ranked, policy analysis should rely not on one, but on several reference fertility levels, to avoid the arbitrariness due to the selected reference. The next section characterizes the fair family policy under cases R1, R2 and R3.

[13] For instance, when considering issues of health, the good health status is a salient reference level (Fleurbaey 2005). Similarly, when considering issues of life and death, the maximum lifespan is also a standard reference level (Fleurbaey et al. 2014).

4 The ex post egalitarian optimum

In our model of imperfectly controlled fertility, individuals can hardly be regarded as responsible for (not) having a child. These are, in our model of imperfect fertility control, pure circumstances that determine fertility outcomes, and, hence, lead to the well-being inequalities studied above.[14]

When well-being inequalities are due to pure circumstances, the Principle of Compensation states that the government should intervene, so as to abolish those inequalities (Fleurbaey 2008; Fleurbaey and Maniquet 2010). The underlying intuition is that well-being inequalities due to circumstances are ethically unacceptable. The Principle of Compensation requires to select an allocation of resources that leads to the compensation of individuals suffering from arbitrary well-being inequalities. In our model, this is the case for the involuntary childless, who cannot be regarded as responsible for being childless, as well as for the involuntary parents, who are not responsible for parenthood.[15]

As this is well-known in the literature, there is no unique way to do justice to the Principle of Compensation, which can be used as a foundation for various social welfare criteria, depending on the precise form of ethical concerns (Fleurbaey 2008). Throughout this paper, we adopt the ex post egalitarian social criterion, that is, a *maximin* defined on consumption equivalents.

Whereas providing axiomatic foundations to that social criterion goes beyond the scope of this paper, it should be stressed that the ex post egalitarian criterion exhibits three key features: (i) it is welfarist; (ii) it is defined from an ex post perspective; (iii) it gives priority to the worst-off.[16]

Our social criterion is *welfarist*: it relies on an informational basis that includes individual preferences, which are used to evaluate and compare the situation of each person. That preferences-based approach differs from other approaches abstracting from preferences, such as the dominance approach.[17]

Another key feature of our social criterion is its *ex post* nature. When comparing individual situations, the relevant perspective is not the ex ante one (when outcomes of the fertility lottery are not known), but the ex post one (when outcomes of the

[14] One may argue that progress in contraception is such that involuntary parents could be regarded as responsible for having a child (lack of prevention). Note, however, that *similar* preventive behaviors can give rise to *distinct* fertility outcomes, due to accidents or circumstances. Hence involuntary parents cannot be regarded as responsible for having a child.

[15] One could reply to this that involuntary parents knew about the imperfect reliability of contraception, a case of "option luck" instead of "brute luck" (Dworkin 2000). However, we adopt the ethical stance that involuntary parents should not be held responsible for the welfare loss due to having a child. The childfree and the involuntary parents are *ex ante* identical, and there is no good reason why they should have unequal welfare *ex post* (Fleurbaey 2010).

[16] Axiomatic foundations for maximin criteria defined on consumption equivalents can be found in Fleurbaey and Maniquet (2011). From an axiomatic perspective, each criterion based on distinct reference fertility levels (cases R1 to R3) would require a distinct characterization. The reliance on a reference fertility level is an ethical assumption on its own, which, under some conditions, follows from adopting a particular axiom defining socially desirable transfers among individuals who have distinct preferences but enjoy the reference fertility level(s).

[17] See Fleurbaey and Maniquet (2011) on the tensions between the welfarist approach and the dominance approach in social valuations.

fertility lottery are known). Ex post social valuation can be justified on the ground that preferences on lotteries are *uninformed*, unlike preferences on degenerate lotteries, which are fully *informed* (Fleurbaey 2010).

A third feature of our social criterion is that it gives priority to the persons who are the worst-off. Such an ethical view can at first glance be regarded as extreme, since it gives full weight to the most disadvantaged. However, as shown by Fleurbaey and Maniquet (2011), transfer axioms, when associated to Pareto efficiency and to certain independence conditions (weaker than Arrow independence), imply a social criterion that gives priority to the worst-off.[18]

As a preliminary step to the design of a fair family policy, this section characterizes the social optimum under the ex post egalitarian criterion.

4.1 The centralized solution

Let us first characterize the social optimum, by considering a benevolent planner who can allocate, within the population, not only material resources, but, also, children.[19] This section characterizes the *optimum optimorum*, where the social planner can control all variables without any constraint except: (1) the total number of children; (2) the resource constraint.[20] In particular, the social planner can reallocate, at no cost, children, from individuals who are involuntary parents to individuals who are involuntary childless.

Regarding the reallocation of children (adoption), two cases can arise depending on the parameters (π, x, ε), which determine the relative size of the "demand for adopted children" (coming from type-2 individuals) with respect to the "supply for adopted children" (coming from type-3 individuals).[21]

Excess demand for adopted children

That general case arises when $(1-\pi)x > \varepsilon(1-x)$. The reallocation of children from type-3 individuals to some type-2 individuals leads to an economy composed of voluntary parents of biological and adopted children (in proportion $x\pi + \varepsilon(1-x)$), involuntary childless individuals (in proportion $x(1-\pi) - \varepsilon(1-x)$) and childfree individuals (in proportion $1-x$). After reallocation of children, the social planner's

[18] For instance, in the consumption-lifetime space, Fleurbaey et al. (2014) provide a characterization of the maximin on equivalent consumptions, based on the Pareto Principle, Hanson Independence, and two transfer axioms: the Pigou–Dalton axiom for same preferences, and the Pigou–Dalton axiom for different preferences and reference lifetime.

[19] The reason why we allow for the reallocation of children in our benchmark case lies in the fact that adoption policies have been widely used across countries and epochs, as instruments allowing to reduce the prevalence of involuntary childlessness. For the sake of robustness to ethical foundations, Sect. 4.5 characterizes a constrained social optimum where the reallocation of children (adoption) is prohibited.

[20] Surrogacy is not allowed here, so that the social planner takes the total number of children as given. The consequences of introducing surrogacy are studied in Sect. 4.4.

[21] The case of excess supply of children is excluded under our assumption $0 < \varepsilon \leq \frac{(1-\pi)x}{(1-x)}$.

problem consists of selecting consumptions $\left\{c^1, c^2, c^4\right\}$ so as to maximize the equivalent consumption of the worst-off, subject to the resource constraint:

$$\max_{c^1,c^2,c^4} \min\left\{C^1, C^2, C^4\right\}$$
$$\text{s.t.} \begin{bmatrix} (x\pi + \epsilon(1-x))c_1 + (x(1-\pi) - \epsilon(1-x))c_2 + (1-x)c_4 \\ +(x\pi + \epsilon(1-x))g \end{bmatrix}$$
$$= (x\pi + \epsilon(1-x))w(1-q) + [(1-\pi)x + (1-x)]w$$

where

$$C^i = \begin{cases} \hat{c}^i & \text{when } \bar{n} = 0 \text{ (R1)} \\ \tilde{c}^i & \text{when } \bar{n} = 1 \text{ (R2)} \\ \bar{c}^i & \text{when } \bar{n}_{1,2} = 1, \bar{n}_4 = 0 \text{ (R3)} \end{cases}$$

Equality of demand and supply for adopted children

That specific case arises when $(1-\pi)x = \epsilon(1-x)$. All involuntary childless individuals are assigned a child, and all involuntary parents become childfree, so that the economy is left with only two categories, voluntary parents and childfree individuals. After reallocation of children, the social planner's problem consists of:

$$\max_{c^1,c^4} \min\left\{C^1, C^4\right\} \text{ s.t. } xc_1 + (1-x)c_4 + xg = xw(1-q) + (1-x)w$$

where C^i are defined above.

Those problems are solved in the Appendix. Proposition 2 summarizes our results for the case of an economy that is sufficiently productive. By "sufficiently productive", we mean that the wage rate w is sufficiently large so as to allow, at the optimum, for the perfect equalization of all consumption equivalents.[22]

Proposition 2 *Assume that the economy is sufficiently productive. At the ex post egalitarian optimum, equivalent consumption levels are equalized for all, and we have:*

Reference fertility	Excess demand	Equal demand and supply
R1 ($\bar{n} = 0$)	$c^1 < c^2 = c^4$	$c^1 < c^4$
R2 ($\bar{n} = 1$)	$c^4 < c^1 < c^2$	$c^4 < c^1$
R3 ($\bar{n}_{1,2} = 1, \bar{n}_{3,4} = 0$)	$c^1 = c^4 < c^2$	$c^1 = c^4$

[22] This assumption is elicited in Sect. 4.3 for particular forms for $u(\cdot)$, $U(\cdot)$, $v(\cdot)$ and $V(\cdot)$.

Proof See the Appendix. □

Provided the economy is sufficiently productive, the ex post egalitarian optimum involves an equalization of all equivalent consumption levels, leading to a full compensation of the childless and of the involuntary parents. The intuition behind that full compensation result comes from the fact that the postulated structure of preferences allows for some *substitutability* between consumption and parenthood. That substitutability allows to compensate involuntary childless individuals and involuntary parents by means of extra consumption.[23]

Proposition 2 shows that the ex post egalitarian optimum is sensitive to the postulated reference fertility level. Under excess demand for adopted children, all remaining involuntary childless (type 2) should receive higher consumption than other individuals (types 1 and 4). However, the ranking of consumptions between voluntary parents and childfree individuals depends on the reference fertility level. Under R3, the voluntary parents and the childfree obtain the same consumption (the cost of a child for the former being fully compensated). This is not the case under R1 and R2, where c^1 and c^4 differ.

4.2 Decentralization of the optimum

In order to decentralize the social optimum, we first assume the existence of a public adoption agency, which reallocates children at no cost. We also assume that the welfare associated to adopting a child is exactly equal to that of having a child of his own, and that leaving a child for adoption does not generate any welfare loss.[24] Once the reassignment of children from involuntary parents to involuntary childless has been made, we assume that the government can use monetary transfers to implement optimal allocations.

The timing of the decentralization is the following one:

Stage 1 Each parent of a child truthfully reveals whether his child was desired.
Stage 2 In case of a negative response, the parent sends his child to the adoption agency that takes care of all children available for adoption.
Stage 3 Each childless person truthfully reveals whether he wanted to have a child.
Stage 4 In case of an affirmative answer, the parent goes to the adoption agency and adopts a child provided a child is available for adoption.
Stage 5 After all possible adoptions have taken place, the government carries out monetary transfers aimed at equalizing equivalent consumption levels based on the postulated reference fertility.

In steps 1-4, the adoption agency reallocates children from involuntary parents (type 3) to the involuntary childless (type 2), which leads to Pareto-improvements in comparison

[23] Alternatively, if there were perfect complementarity between consumption and parenthood, transfers would not allow to achieve full compensation.

[24] Those assumptions are discussed below.

to the laissez-faire.[25] Yet, introducing an adoption agency is not enough to achieve the first-best optimum, because of two reasons. First, in case of excess demand for children, some individuals remain, after the reallocation, involuntary childless. Second, even when demand equals supply, there is no equalization of consumption equivalents since parents and childfree have different resources resulting from raising a child or not. Thus a system of monetary transfers is also needed. Proposition 3 summarizes our results.

Proposition 3 *Assume that the economy is sufficiently productive. The decentralization of the ex post egalitarian optimum is achieved as follows*:

1. If there is an excess demand for children, every child sent to adoption is adopted and all involuntary parent (type-3) become childfree (type-4). Transfers have the following form:

Reference fertility	Excess demand
R1 ($\bar{n} = 0$)	$b_1 < 0 < b_2 = b_4$
R2 ($\bar{n} = 1$)	$b_4 < 0 < b_1 < b_2$ or $b_4 < b_1 < 0 < b_2$
R3 ($\bar{n}_{1,2} = 1, \bar{n}_4 = 0$)	$b_4 < 0 < b_1 < b_2$ or $b_4 < b_1 < 0 < b_2$

2. If there is equality between supply and demand of children, there remain only voluntary parents of a child (either adopted or not) and childfree individuals. Transfers have the following form:

Reference fertility	Equal demand and supply
R1 ($\bar{n} = 0$)	$b_1 < 0 < b_4$
R2 ($\bar{n} = 1$)	$b_4 < 0 < b_1$
R3 ($\bar{n}_1 = 1, \bar{n}_4 = 0$)	$b_4 < 0 < b_1$

Proof See the Appendix. □

An important result of Proposition 3 is the lack of robustness of the fair family policy to the reference fertility. Depending on the reference fertility, policies decentralizing the ex post egalitarian optimum can take various forms.[26]

Another crucial result concerns the comparison of the fair family policy with real-world family policies. Departures are significant on two grounds.

First, Proposition 3 shows that it is far from being always the case that voluntary parents should obtain a positive child allowance (i.e. $b_1 > 0$). Under R1, voluntary

[25] Under Assumption A1, the involuntary childless are better off adopting a child, and involuntary parents are better off leaving their child (since $U(w(1-q)-g) - V(1) < U(w)$).

[26] Note that case R3, which accounts for some form of "neutrality" of the social planner towards preferences (Fleurbaey and Maniquet 2019), still involves transfers affecting voluntary parents and childfree individuals.

parents pay a child tax, while under R2 and R3, they receive a transfer or pay a tax. Proposition 3 reveals that a fair family policy would take a quite different form from actual ones where children are most often subsidized.

Second, the other interesting difference with respect to real-world family policies consists in the existence of allowances for *childless* individuals, which are always positive for the involuntary childless and can be positive or negative for the child-free, depending on the reference fertility level. This is a fundamental departure from actual policies, where only parents benefit from family policies.

This being said, one should also remind that our analysis makes some simplifying assumptions. First, it is assumed that the adoption system can be implemented at a zero cost. Yet, introducing a cost of adoption would not modify our results.[27] Second, we assume that involuntary parents leave their child to the adoption agency without any regret or guiltiness. Instead, one could add an extra psychological cost for leaving a child to the adoption agency. This would only affect the shape of the monetary transfers to be made. Finally, we assume that there is perfect substitutability between an adopted child and a biological child. Again, one could relax that assumption and provide additional monetary compensations for parents of an adopted child.

4.3 The quasi-linear case

Section 4.2 assumed that the economy is "sufficiently productive", so that resources are sufficiently large so as to allow for the decentralization of the ex post egalitarian optimum involving the full equalization of consumption equivalents. In order to make explicit the meaning of "sufficiently productive economy", Proposition 4 identifies, for the case of quasi-linear preferences, conditions on the hourly wage w that are necessary and sufficient for the decentralization of the social optimum with full equalization of consumption equivalents by means of a mixed adoption/monetary transfer system. For the sake of presentation, it focuses on the (realistic) case of excess demand for children, when reference fertility is fixed to 0 (R1) or 1 (R2).[28]

Proposition 4 *Assume quasi-linear preferences. Assume that there is excess demand for children. Define*

[27] Indeed, in case of an individual cost, this would require to give additional monetary transfers to involuntary childless who would engage in the process of adopting a child. If it is a cost to the society as a whole, this would enter the government budget constraint and modify the amounts of lump-sum transfers to be made.

[28] Similar conditions could be derived for the case R3.

$$\tilde{w}_1 \equiv \frac{g[x\pi + \varepsilon(1-x)] + [x(1-\pi) + (1-x)(1-\varepsilon)]v(1)}{1 - \varepsilon q(1-x) - qx\pi}$$

$$\tilde{w}_2 \equiv \frac{g(x\pi + \varepsilon(1-x)) - (1-x)V(1) + [x(1-\pi) - (1-x)\varepsilon]v(1)}{1 - \varepsilon q(1-x) - qx\pi}$$

- *Under R1, the social optimum with perfect equalization of consumption equivalents can be decentralized by a mixed system if and only if* $w > \tilde{w}_1$.
- *Under R2, the social optimum with perfect equalization of consumption equivalents can be decentralized by a mixed system if and only if* $w > \tilde{w}_2$.
- *Threshold hourly wage levels are such that* $\tilde{w}_2 < \tilde{w}_1$.

Proof See the Appendix. □

Proposition 4 states that the decentralization of the social optimum with the full equalization of consumption equivalents can only be achieved provided the hourly wage is above some threshold levels $\tilde{w}_1$ and $\tilde{w}_2$. These thresholds depend on the (dis) utility for parenthood (i.e. $v(1)$ and $V(1)$). The capacity of the economy to compensate individuals for parenthood or for childlessness depends on how much these individual value being involuntary parent or being involuntary childless.

Also, what "sufficiently large" means depends on the postulated reference fertility. The threshold wage level when $\bar{n}$ is fixed to 0 is *higher* than when $\bar{n}$ is fixed to 1. It is thus possible to think about an economy where the hourly wage lies between the two thresholds $\tilde{w}_2$ and $\tilde{w}_1$, with the corollary that this mixed adoption/transfer system could decentralize the social optimum with equal consumption equivalents when $\bar{n}$ is fixed to 1, but not when $\bar{n}$ is fixed to 0. As such, Proposition 4 shows the impact of reference fertility on the possibility to achieve full equalization of consumption equivalents.

4.4 Ethical aspects (1): surrogacy

Up to now, we considered a social planning problem where the total number of children is taken *as given*. By doing so, we implicitly assumed that surrogacy is not an ethically feasible option. Surrogacy is a legal agreement by which an individual agrees to conceive a child on behalf of another person, who will be the parent of the child later on. This option was not possible in this section.

The legal status of surrogacy varies around the world (from acceptance to prohibition). Discussing ethical arguments for or against surrogacy would lead us far beyond the scope of this paper. However, it is interesting to examine the robustness of the fair family policy to introducing institutionalized surrogacy as an additional instrument available for the social planner.

Our benchmark model can easily be adapted so as to allow for surrogacy. Under initial excess demand for children, institutionalizing surrogacy would allow the social planner, by asking some type-1 individuals to serve as surrogates, to achieve the equalization of demand and supply for children. Whatever the size of excess demand is, excess demand can be reduced to zero by asking some type-1 individuals to have more than one child, and by reallocating those children to the involuntary childless (type 2). Hence, introducing surrogacy would lead us back to the situation where the population is composed only of voluntary parents and childfree individuals (see *supra*). In our model, there is no cost of conceiving a child, only a cost of raising the child. Yet, if one added some costs of conceiving a child, the optimal policy would include additional transfers compensating the surrogates for the extra cost of conceiving a child.

Proposition 5 *Assume that surrogacy is an available instrument. Under an initial excess demand for children, the decentralization of the ex post egalitarian optimum is achieved by first asking some type-1 individuals to serve as surrogates for type-2 individuals. Once the equality of demand and supply for children is achieved, the fair family policy is the one in Proposition 3, bullet list item 2.*

Proof See above. □

Thus, under an initial excess demand for children, surrogacy can be part of a fair family policy, and be a substitute for childlessness allowances considered in Proposition 3. While that solution allows to equalize well-being levels at a higher level than in the absence of surrogacy, it should be reminded that allowing for surrogacy as an available instrument is itself an ethical position. We are thus not comparing a “first-best” with a “second-best” optimum, but two first-best optima characterized by different ethical standards.

4.5 Ethical aspects (2): no child reallocation

Another ethical aspect is worth being considered here. This other aspect is not about relaxing an ethical constraint (like for surrogacy), but about adding an extra ethical constraint. Up to now, we assumed that no ethical constraint regulates the reallocation of children. If one focuses only on the point of view of adults, such a reallocation can be easily defended, on the grounds of the Pareto-improvements that it allows. However, things become more complex once one wants to take into account the point of view of children.

Take, for instance, the case where an involuntary parent would like to become childfree, whereas his child does not want to be adopted by another parent. Reallocating that child to an involuntary childless person would no longer be a Pareto-improvement, since this would make the child worse-off.

To avoid such difficulties, one may impose a restriction on the reallocation of children. Given that a reallocation of children may lead to a worsening of the situation of some children, who are the most vulnerable individuals, but whose interests are not easily observable, one may argue in favor of introducing an ethical constraint on the family policy, requiring that *only monetary transfers are allowed, but not transfers of children*.

That "no child transfer" restriction is quite conservative, but can be justified on two grounds. First, from a consequentialist perspective—but including children—the social evaluator may want to avoid reallocations of children that could worsen the situation of those most vulnerable persons.[29] Second, one may depart from a consequentialist perspective, and regard the family policy as a process that has, in itself, an ethical value. From that perspective, the "no child transfer" condition could be a component of the fair process itself.

In the Appendix, we solve the social planner's problem under the "no child transfer" constraint, and compare the associated fair family policy with its form under the unconstrained case. Proposition 6 summarizes our results.

Proposition 6 *Assume the "no child transfer" condition.*

- *The (constrained) fair family policy depends on the reference fertility level, and involves, in general, two departures from real-world family policies:* (1) *voluntary parents may not obtain a positive allowance*; (2) *involuntary childless may receive a positive allowance.*
- *Under quasi-linear preferences, the wage threshold allowing for the equalization of all consumption equivalents is higher under the* "*no child transfer*" *system than under the mixed adoption/monetary transfer system. Hence, in some cases, the equalization of all consumption equivalents can be achieved by the mixed system, but not by the pure transfer system.*

Proof See the Appendix. □

Proposition 6 shows that imposing the "no child transfer" constraint does not qualitatively affect our results. However, a key difference is that, once the family policy involves only pure transfers, it becomes more difficult to equalize all consumption equivalents. Under the adoption/monetary transfer system, children's reallocation allowed to achieve Pareto-improvements among adults at zero cost, which saved resources available for compensating the remaining involuntary childless/parents. The "no child transfer" condition prevents this. Hence, it may be the case that the pure transfer system *cannot* equalize consumption equivalents, whereas the adoption/monetary transfer system can.

[29] Given that the interests of children are not easily observable, prohibiting *all* reallocations of children is a way to avoid reallocations that could worsen the situation of *some* of them.

5 Asymmetric information on preferences

Up to now, we assumed that the government can observe individual preferences, and, hence, can distinguish between voluntary and involuntary parents and between voluntary and involuntary childless individuals. This section examines how the fair family policies would be modified under the more realistic assumption of asymmetric information on individual preferences.[30]

Let us first study whether the allocations presented in Proposition 2 are incentive-compatible when the government cannot observe preferences.

It is tempting, at first glance, to believe that the mixed system satisfies incentive-compatibility constraints, since the adoption system (stages 1 to 4), *when taken separately*, would make individuals reveal their true type, and either obtain a child when they want one, or send the child to adoption when they do not want one.[31] However, if the structure of the mixed system is common knowledge, individuals anticipate, when announcing their types, that they may benefit from more advantageous transfers in stage 5 by lying on their type.

Under excess demand for children, the first-best allocation involves $c_2 \geq c_4$ for any reference fertility, so that childfree individuals (type 4) may have an incentive to pretend that they are involuntary childless (type 2). Indeed, if they do so, there is, under the mixed system, a probability $\left(1 - \frac{\varepsilon(1-x)}{(1-\pi)x}\right)$ that they will not be assigned a child, and benefit from compensatory transfers *as if* they were involuntary childless. Hence, under preferences satisfying the expected utility hypothesis, the incentive-compatibility constraint is:

$$U\left(c_4\right) \geq \frac{\varepsilon(1-x)}{(1-\pi)x}\left[U(c_1) - V(1)\right] + \left(1 - \frac{\varepsilon(1-x)}{(1-\pi)x}\right)U(c_2) \tag{4}$$

where on the LHS is the utility of a childfree when declaring his true type, and the RHS is his expected utility when pretending to be involuntary childless.

Under R1 where $c_1 < c_2 = c_4$, the first-best allocation satisfies the above incentive constraint. However, under R2 and R3 where $c_4 \leq c_1 < c_2$, nothing guarantees incentive-compatibility of the first-best allocations. Depending on the forms of $U(\cdot)$ and $V(\cdot)$, and on the probability $\frac{\varepsilon(1-x)}{(1-\pi)x}$ to be assigned a child, it may be welfare-improving, in expected terms, for a childfree individual to pretend to be involuntary childless. For instance, if there is a strong rationing of children available for adoption, so that $\frac{\varepsilon(1-x)}{(1-\pi)x} \to 0$, the incentive-compatibility constraint simplifies to $U(c_4) \geq U(c_2)$. Except under R1, the first-best allocations (where $c_4 < c_2$) violate this constraint.

[30] Quite realistically, we assume that having a child or not is observable, so that mimicking can never happen on this dimension.

[31] In other words, a pure adoption system (stages 1 to 4) would be incentive-compatible.

Proposition 7 *Assume asymmetric information on preferences. Under excess demand, first-best allocations under R1 are incentive-compatible. However, if the probability to adopt a child is sufficiently low, first-best allocations under R2 and R3 are not incentive-compatible.*

Proof See above. □

Under R2 and R3, the first-best allocations are, in general, not implementable under asymmetric information. This justifies the design of second-best allocations that would take into account incentive-compatibility constraints.

Let us now consider the decentralization under asymmetric information. When there is excess demand for children, type-4 individuals may be tempted, under R2 and R3, to declare to be type-2 individuals. Hence, the incentive compatibility constraint (Eq. 4) needs to be satisfied. This can be done by increasing c_4 and decreasing c_1 and c_2 with respect to the first-best optimum, so that the incentive-constraint is binding. Depending on preferences and on the probability that a type-4 obtains a child, two ranking are now possible: either $c_4 < c_1 < c_2$ or $c_1 < c_4 < c_2$.[32] This, however, prevents the equalization of all consumption equivalents. Indeed, in order to prevent mimicking from the childfree, one needs to leave them a rent, while decreasing the well-being of the involuntary childless and voluntary parents. As a result, at the second-best optimum, under R2, one now has $\tilde{c}_1$, $\tilde{c}_2 < \tilde{c}_4$ and $\tilde{c}_1 \gtrless \tilde{c}_2$.[33]

In sum, second-best allocations can be decentralized by using a mixed adoption/monetary transfer system, but full equalization of all consumption equivalents cannot be achieved at the second-best, since informational rents are left to the childfree so as to avoid mimicking.

6 Assisted reproductive technologies

Up to now, our analysis of a fair family policy assumed that the probability of becoming parent was *given*. One may want to relax that hypothesis, to allow individuals who want children to invest in assisted reproductive technologies (ART).[34] Such technologies, which are costly in terms of money and psychological strains, can increase the probability to have a child.

The decision of investing in ART concerns only the involuntary childless (type-2), since type-1 individuals already have a child, while other types do not want one. For simplicity, let us assume that there exist only two levels of investment in ART, either $e = 0$ (no investment) or $e = \ell$ (full investment) and that the price of ART investment is unitary. There is also a psychological cost of investing in ART, $\varphi(\ell)$, with $\varphi(\ell) > \varphi(0) = 0$.

At the laissez-faire, type-2 individuals invest in ART if and only if

[32] Under R3, since in the first-best, $c_1 = c_4$, $c_1 < c_4 < c_2$ is the unique second-best solution.

[33] The same rankings of consumption equivalents are obtained under R3.

[34] On ART, see Trappe (2017). This section abstracts from surrogacy (see Sect. 4.4).

$$p[u(w(1-q)-g-\ell)+v(1)-\varphi(\ell)]+(1-p)[u(w-\ell)-\varphi(\ell)] \geq u(w) \quad (5)$$

where $0 < p < 1$ is the probability of having a child when investing in ART. In the following, we assume that condition (5) is always satisfied, so that every involuntary childless individual invests in ART.

Introducing ART does not imply that involuntary childlessness disappears: some persons who invested in ART may turn out to be *unsuccessful* in having children. But, allowing for ART divides the type "voluntary parents" into two subtypes, depending on whether the child was obtained without ART or through the successful use of ART. For simplicity, we will abstract from involuntary parents.[35] This leads us to four ex-post types:[36]

- Type 1: individuals who want a child and have a child with no investment in ART, in proportion $x\pi$.
- Type 2: individuals who want a child, cannot have a child, invest in ART and are successful, in proportion $x(1-\pi)p$.
- Type 3: individuals who want a child, cannot have a child, invest in ART and are unsuccessful, in proportion $x(1-\pi)(1-p)$.
- Type 4: individuals who do not want a child and do not have a child, in proportion $1-x$.

At the laissez-faire, ex post welfare levels of type-i individuals rank as follows:

$$W^1 \equiv u(w(1-q)-g)+v(1)-\varphi(0) > W^2 \equiv u(w(1-q)-g-\ell)+v(1)-\varphi(\ell)$$
$$W^2 \equiv u(w(1-q)-g-\ell)+v(1)-\varphi(\ell) > W^3 \equiv u(w-\ell)-\varphi(\ell)$$
$$W^3 \equiv u(w-\ell)-\varphi(\ell) < W^4 \equiv U(w)$$

where $W^2 > W^3$ follows from condition (5) for a positive investment in ART. Assumption A2 implies that $U(w) > u(w-\ell)-\varphi(\ell)$, so that $W^4 > W^3$. Unsuccessful ART users (type 3) are the worst-off, but we do not have a complete well-being ranking due to heterogeneous preferences.

In the following, consumption equivalent indexes C^i are constructed by fixing reference fertility and ART levels to the *preferred* levels of n and e of each type of individuals. They satisfy: $\bar{n}_{1,2,3} = 1$ and $\bar{n}_4 = 0$, as well as $\bar{e}_{1,4} = 0$ and $\bar{e}_{2,3} = \ell$. Equivalent consumption indexes C^i are defined as follows:

$$u\left(C^1\right)+v(1)-\varphi(0) = u\left(c^1\right)+v(1)-\varphi(0)$$
$$u\left(C^2\right)+v(1)-\varphi(\ell) = u(c^2)+v(1)-\varphi(\ell)$$
$$u\left(C^3\right)+v(1)-\varphi(\ell) = u(c^3)+v(0)-\varphi(\ell)$$
$$U(C^4)-V(0)-\varphi(0) = U(c^4)-V(0)-\varphi(0)$$

[35] This amounts to assume that $\varepsilon = 0$ (i.e. perfect contraception).

[36] We use, here again, the Law of Large Numbers.

6.1 Ex post egalitarian optimum and decentralization

Despite the introduction of ART, the laissez-faire involves well-being inequalities. Those inequalities are due to circumstances that lie beyond the control of individuals. Therefore, the Principle of Compensation applies also to that alternative setting. It is thus legitimate to consider the question of the compensation of individuals who are childless despite the use of ART, as well as the compensation of individuals who could become parent, but at the cost of ART investments that some other parents did not have to pay. We will, here again, rely on the ex post egalitarian social criterion, which does justice to the idea of compensating individuals for welfare inequalities due to circumstances.

A first interesting feature of the ex post egalitarian social optimum, is that, if the social planner were to choose both consumptions and ART investment for all individuals, the optimum would involve a zero level of ART investment. The intuition goes as follows. The worst-off is the one who invested in ART, but turned out to be unsuccessful in having children (type 3). From the perspective of maximizing the realized well-being of that individual, it would have been better not to invest in ART, since it involved costs, but did not allow him to have a child. Thus, if the social objective is to maximize the realized well-being of the worst-off, it is optimal to have a zero investment in ART.[37]

To examine the impact of ART on optimal policies, we assume that the government allows individuals to invest in ART, and selects the allocation that maximizes the realized well-being of the worst-off under that constraint.[38] Assuming the no child transfer condition, the problem of the planner is:

$$\max_{c^1,c^2,c^3,c^4} \min\left\{C^1, C^2, C^3, C^4\right\}$$

$$\text{s.t. } \pi x c_1 + x(1-\pi)pc_2 + x(1-\pi)(1-p)c_3 + (1-x)c_4 + \pi x g + x(1-\pi)pg$$
$$= [\pi x + x(1-\pi)p]w(1-q) + x(1-\pi)(1-p)w + (1-x)w + x(1-\pi)\ell$$

Our results are summarized in the following proposition:

Proposition 8 *Assume $\bar{n}_{1,2,3} = 1$ and $\bar{n}_4 = 0$, as well as $\bar{e}_{1,4} = 0$ and $\bar{e}_{2,3} = \ell$. Assuming that the economy is sufficiently productive, the ex post egalitarian optimum with ART leads to the equalization of the equivalent consumption levels for the four types, and to the ranking of consumptions $c^1 = c^2 = c^4 < c^3$. The decentralized solution can be achieved by a system of transfers b_i, such that $b_4 < b_1 < b_2 < b_3$, with $b_3 > 0$ and $b_4 < 0$ but $b_1, b_2 \gtrless 0$.*

Proof See the Appendix. □

[37] That result, which is close to the result of zero prevention (against mortality) in Fleurbaey and Ponthiere (2013), is due to the fact that there is a conflict between the goal of ex post compensation and the goal of investing in costly prevention that may be unsuccessful.

[38] If we fixed ART to zero, we would be left with three types: voluntary parents, childless individuals and childfree persons, which would be a reduced form of the model studied above.

Proposition 8 states that unlucky ART users should obtain higher consumption than the other types. The decentralization of the ex post egalitarian optimum requires to implement a positive allowance to ART users that would cover for the monetary and psychological costs associated to ART. Unsuccessful ART users always obtain a positive allowance, $b_3 > 0$, while ART users who turned out to be successful receive a (smaller) allowance b_2 that may be positive or negative depending on the government budget constraint requirements as well as on preferences. Those features contrast with actual policies, where ART is generally not (fully) covered by public allowances, and where there is no additional compensation scheme for unlucky ART users.

6.2 Second-best allocation

Suppose now that the government only observes who has a child and who uses ART. Under asymmetric information, if the social planner were to propose the first-best allocation, childfree individuals may have an interest in investing in ART and pretend to be involuntary childless, since $c_3 > c_4$. Hence, the second-best problem should include the incentive-compatibility constraint:[39]

$$U(c_4) \geq U(c_3) - \varphi(\ell)$$

where the LHS is the utility of type-4 individuals declaring honestly their type and the RHS is the utility they would obtain claiming to be a type 3.[40]

The first-best allocation (with $c_3 > c_4$) may not always satisfy this incentive constraint, so that type-4 individuals may be tempted to declare to be of type 3 if the first-best allocation were to be proposed. In that situation, for the incentive constraint to be satisfied, one needs to increase c_4 and decrease c_3 with respect to their first-best levels so that either $c_4 > c_3 > c_2 = c_1$ or $c_3 > c_4 > c_2 = c_1$. Compensation and incentive compatibility cannot be achieved at the same time and equivalent consumption levels are not equalized:

$$C_4 > C_2 = C_1 > C_3.$$

Childfree individuals (the potential mimickers) obtain a rent and are left better-off than any other type, and unsuccessful ART users (type-3) are left worse-off than the other categories, so as to avoid mimicking from the childfree.

Note, however, that the above second-best problem relies on the assumption that the government can observe the non-monetary cost of ART, so that all individuals

[39] For simplicity, we assume here that childfree individuals have the same disutility of the ART treatment as other individuals.

[40] One could oppose that the relevant incentive-compatibility constraint is:

$$U(c_4) \geq \tilde{p}[U(c_2) - V(1) - \varphi(\ell)] + (1 - \tilde{p})[U(c_3) - \varphi(\ell)]$$

where the RHS is the *expected utility* of investing in ART, and $\tilde{p}$ is the probability of successful ART for individuals who do not want the treatment to be successful. Assuming that a type 4 can make the treatment inoperative (leading to $\tilde{p} \to 0$), the two formulations are equivalent.

who receive ART must face the disutility cost $\varphi(\ell)$. If one assumes, on the contrary, that only the monetary cost of ART is observable, but not its psychological cost, individuals pretending to be needing ART could buy the treatment and throw it away, therefore not incurring the non-monetary cost of ART. In that case, the relevant incentive-compatibility constraint would be:[41]

$$U(c_4) \geq U(c_3).$$

The first-best allocation, with $c_3 > c_4$, never satisfies this constraint. Hence, for the second-best allocation to be incentive compatible, one should set $c_4 = c_3$, that is decrease c_3 and increase c_4 in comparison to their first-best levels so that $c_1 = c_2 < c_3 = c_4$. This goes against the idea of compensating (type 2 and type 3) ART users, and we now have $C_4 > C_2 = C_1 > C_3$.

Proposition 9 *Let us assume that preferences are not observable to the government, while having a child and the purchase of ART are.*

- *If the non-monetary cost of ART is observable to the government, the first-best allocation is not always implementable. The second-best allocation can require to increase consumption of the childfree and to decrease consumption of unsuccessful ART users as compared to first-best levels.*
- *If the non-monetary cost of ART is not observable to the government, the first-best allocation is never incentive compatible and consumption should be equalized between parents (whether they used ART or not) and between the childless (whether they used ART or not).*
- *In any case, consumption equivalents cannot be equalized.*

Proof See the Appendix. □

Whether the non-monetary cost of ART is observable or not, the first-best allocations are likely to be not implementable under asymmetric information.

7 Conclusion

This paper proposed to cast a new light on family policies, by considering, instead of their capacity to promote fertility, their capacity to serve social justice in an economy where individuals are unequal in terms of fecundity. Our results show that shifting the goal from producing more children to achieving fairness has a strong impact on the design of family policies.

[41] One could oppose that the incentive-compatibility constraint has to include the *expected utility* of investing in ART. If we assume that $\tilde{p} \to 0$ for type 4 (i.e. sabotage of ART), the two formulations are equivalent.

Our analysis first pointed out to a major difficulty when designing a fair family policy: the treatment of heterogeneity in preferences towards children. Children are desirable goods for some individuals, but undesirable goods for others. In a world of imperfect fertility control, this heterogeneity leads to potentially two distinct types of damages: involuntary childlessness and involuntary parenthood. Our analysis based on the construction of equivalent consumption indexes revealed that the identification of the worst-off depends on the reference fertility level, which plays a key role in interpersonal well-being comparisons.

We also showed that a fair family policy would differ strongly from family policies existing around the world. A fair family policy does not, in general, involve positive family allowances to voluntary parents, and may also, under some reference fertility, involve positive childlessness allowances, unlike real-world family policies. Note that, if one departs from that model and introduces surrogacy, it appears that institutionalizing surrogacy would make childlessness allowances unnecessary (since involuntary childlessness would not exist any more). If, instead, one allows for ART but not for surrogacy, a fair family policy involves an allowance compensating all ART costs, as well as an additional allowance for ART users who were unsuccessful in having children.

In sum, considering family policies as instruments towards fairness has major implications for the design of those policies. Moreover, the form of the fair family policy is sensitive to ethical judgements about available instruments for family policy, such as children's reallocation (adoption), ART and surrogacy.

Appendix

Proof of Proposition 2

Excess demand of children

Under R1, and once type 3 has disappeared thanks to the reallocation of children, the problem of the planner is:

$$\begin{aligned}
&\max_{c^1,c^2,c^4} \hat{c}^2 \\
&\text{s.t. } (\pi x + \varepsilon(1-x))c_1 + (x(1-\pi) - \varepsilon(1-x))c_2 + (1-x)c_4 + (\pi x + \varepsilon(1-x))g \\
&= (\pi x + \varepsilon(1-x))w(1-q) + (x(1-\pi) + (1-\varepsilon)(1-x))w \\
&\text{s.t. } \hat{c}^1 \geq \hat{c}^2,\ \hat{c}^4 \geq \hat{c}^2,\ \hat{c}^1 \geq \hat{c}^4
\end{aligned}$$

where consumption equivalents $\hat{c}_i$ are defined by

$$\begin{aligned}
u(\hat{c}^1) + v(0) = u(c^1) + v(1) &\iff \hat{c}^1 = u^{-1}\left(u(c^1) + v(1)\right) \\
u(\hat{c}^2) + v(0) = u(c^2) + v(0) &\iff \hat{c}^2 = c^2 \\
U(\hat{c}^4) - V(0) = U(c^4) - V(0) &\iff \hat{c}^4 = c^4
\end{aligned}$$

At the optimum, the egalitarian constraints are binding so that:

$$\hat{c}^1 = \hat{c}^2 \implies c^2 > c^1 \text{ and } \hat{c}^2 = \hat{c}^4 \implies c^2 = c^4.$$

Under R2, the problem of the planner is the same as under R1, except that equivalent consumptions are defined as $\tilde{c}^i$ and are given by:

$$\begin{aligned} u\left(\tilde{c}^1\right) + v(1) = u(c^1) + v(1) &\iff \tilde{c}^1 = c^1 \\ u\left(\tilde{c}^2\right) + v(1) = u(c^2) + v(0) &\iff \tilde{c}^2 = u^{-1}\left[u(c^2) - v(1)\right] \\ U(\tilde{c}^4) - V(1) = U(c^4) - V(0) &\iff \tilde{c}^4 = U^{-1}\left[U(c^4) + V(1)\right] \end{aligned}$$

At the optimum, the egalitarian constraints are binding so that:

$$\tilde{c}^1 = \tilde{c}^2 \implies c^2 > c^1 \quad \text{and} \quad \tilde{c}^4 = \tilde{c}^1 \implies c^1 > c^4.$$

Under R3, the problem of the planner is the same as under R1, except that equivalent consumptions are defined as $\bar{c}^i$ and satisfy:

$$\begin{aligned} u\left(\bar{c}^1\right) + v(1) = u(c^1) + v(1) &\iff \bar{c}^1 = c^1 \\ u\left(\bar{c}^2\right) + v(1) = u(c^2) + v(0) &\iff \bar{c}^2 = u^{-1}\left[u(c^2) - v(1)\right] \\ U(\bar{c}^4) - V(0) = U(c^4) - V(0) &\iff \bar{c}^4 = c^4 \end{aligned}$$

The egalitarian constraints are binding so that:

$$\bar{c}^1 = \bar{c}^2 \implies c^2 > c^1 \quad \text{and} \quad \bar{c}^1 = \bar{c}^4 \implies c^1 = c^4.$$

Equal supply and demand for children

The economy is, after reallocation of children, composed of voluntary parents and of childfree individuals. The resource constraint is: $xc_1 + (1-x)c_4 + xg = xw(1-q) + (1-x)w$. Under R1, we can show that the optimal allocation is $c^4 > c^1$. Under R2, the optimal allocation is $c^1 > c^4$. Under R3, the optimal allocation is $c^1 = c^4$.

Proof of Proposition 3

Let us consider four lump-sum transfers b_i given to our types $i = \{1, 2, 3, 4\}$.

Under excess demand for children, there remain, after the reallocation of children, three types: voluntary parents who receive b_1, involuntary childless with b_2 and voluntary childless with b_4. The budget constraint is:

$$b_1(x\pi + \varepsilon(1-x)) + (x(1-\pi) - \varepsilon(1-x))b_2 + (1-x)b_4 = 0. \tag{6}$$

Under R1, the decentralization requires $c^{4,D} = c^{2,D} > c^{1,D}$, where D stands for Decentralization, so as to achieve $\hat{c}^1 = \hat{c}^2 = \hat{c}^4$. Transfers b_i satisfy:

$$u(w(1-q) - g + b_1) + v(1) = u(w + b_2) \tag{7}$$

$$w + b_2 = w + b_4 \tag{8}$$

The first equation ensures that $c^{1,D} = w(1-q) - g + b_1$ and $c^{2,D} = w + b_2$ are set such that $\hat{c}^1 = \hat{c}^2$. The second equality ensures that $\hat{c}^2 = \hat{c}^4$. Equation (8) yields $b_2 = b_4 = b$. Using equation (7) together with Assumption A1, we obtain that $b_1 < b$. For the budget constraint to be satisfied, only one solution is possible: $b_1 < 0 < b$.

Under R2, the ranking of consumptions at the decentralized optimum is: $c^{2,D} > c^{1,D} > c^{4,D}$, so as to achieve $\tilde{c}^1 = \tilde{c}^2 = \tilde{c}^4$. Transfers satisfy:

$$u(w(1-q) - g + b_1) + v(1) = u(w + b_2) \tag{9}$$

$$w(1-q) - g + b_1 = U^{-1}[U(w + b_4) + V(1)] \tag{10}$$

so that there is equalization of consumption equivalents. Equation (9) yields: $b_1 < b_2$ and Eq. (10) that $b_4 < b_1$. Together with the budget constraint, two sets of solutions $b_4 < 0 < b_1 < b_2$ and $b_4 < b_1 < 0 < b_2$ are possible.

Under R3, the ranking of consumptions is $c^{1,D} = c^{4,D} < c^{2,D}$, so that we achieve $\bar{c}^1 = \bar{c}^2 = \bar{c}^4$. Transfers b_1, b_2 and b_4 satisfy:

$$w(1-q) - g + b_1 = w + b_4 \implies b_1 > b_4$$
$$u(w(1-q) - g + b_1) + v(1) = u(w + b_2) \implies b_1 < b_2$$

Together with the budget constraint, only two solutions are possible: $b_4 < 0 < b_1 < b_2$ or $b_4 < b_1 < 0 < b_2$.

When there is equality between supply and demand of children, there remain only two types of individuals: voluntary parents who receive b_1 and the childfree who receive b_4. These transfers satisfy: $xb_1 + (1-x)b_4 = 0$. The solutions are: under R1, $b_1 < 0 < b_4$; under R2, $b_4 < 0 < b_1$; under R3, $b_4 < 0 < b_1$.

Proof of Proposition 4

Assume quasi-linear preferences: $u(x) = x$ and $U(x) = x - \alpha$. Take Proof 9.2 of Proposition 3.

Consider case R1. Using the budget constraint and $b_2 = b_4 = b$, we have:

$$b = \frac{-[x\pi + \varepsilon(1-x)]b_1}{[x(1-\pi) + (1-x)(1-\varepsilon)]}.$$

This defines locus A in the (b_1, b) space. Using condition (7), we obtain locus B, defined by

$$b = -qw - g + b_1 + v(1)$$

This is a 45° line with strictly positive value at $b_1 = 0$ since under Assumption 1, $-qw - g + v(1) > 0$. We also have $b = 0$ at $b_1 = qw + g - v(1) < 0$.

The decentralization by a mixed adoption-transfer scheme is possible only if the two loci A and B intersect. One can see that, when an intersection takes place between locus A and B, it must be for $b_1 < 0, b > 0$. In addition, consumptions cannot be negative, so that $-\left[w(1-q)-g\right] < b_1$ and $b > -w$.

If locus B is above locus A at $b_1 = -\left[w(1-q)-g\right]$, the decentralization of the optimum through the mixed system does not hold. That case arises when:

$$\begin{aligned} &-qw-g-\left[w(1-q)-g\right]+v(1) > \frac{-[x\pi+\varepsilon(1-x)]\left[-\left[w(1-q)-g\right]\right]}{[x(1-\pi)+(1-x)(1-\varepsilon)]} \\ &\iff w < \frac{g[x\pi+\varepsilon(1-x)]+v(1)[x(1-\pi)+(1-x)(1-\varepsilon)]}{1-q\varepsilon(1-x)-qx\pi} \equiv \tilde{w}_1 \end{aligned} \tag{11}$$

Thus only when $w \geq \tilde{w}_1$, the mixed system can decentralize the social optimum.

Consider now case R2. Using Eq. (6), we obtain: $b_4 = -b_1\frac{[x\pi+(1-x)\varepsilon]}{(1-x)} - \frac{[x(1-\pi)-(1-x)\varepsilon]}{(1-x)}b_2$. Moreover, Eq. (9) can be rewritten as

$$b_2 = -wq - g + b_1 + v(1)$$

which defines the locus I in the $\left(b_1, b_2\right)$ space. It is a 45 degree line which crosses the x axis at $b_1 = wq + g - v(1) < 0$.

Also, replacing for the expression of b_4 into Eq. (10), we obtain that:

$$\begin{aligned} w(1-q)-g+b_1 &= w - b_1\frac{[x\pi+(1-x)\varepsilon]}{(1-x)} - \frac{[x(1-\pi)-(1-x)\varepsilon]}{(1-x)}b_2 + V(1) \\ \rightarrow b_2 &= -b_1\frac{1-x(1-\pi)+(1-x)\varepsilon}{[x(1-\pi)-(1-x)\varepsilon]} + \frac{(1-x)(V(1)+wq+g)}{[x(1-\pi)-(1-x)\varepsilon]} \end{aligned}$$

That equation defines locus II.

Consumption cannot be negative so that $-(w(1-q)-g) < b_1$. Moreover, $b_2 > -w$. Hence, the transfer system does not decentralize the optimum when the locus I remains above the locus II at $b_1 = -(w(1-q)-g)$, that is, when:

$$w < \frac{g(x\pi+(1-x)\varepsilon)-(1-x)V(1)+v(1)[x(1-\pi)-(1-x)\varepsilon]}{1-(1-x)\varepsilon q-qx\pi} \equiv \tilde{w}_2 \tag{12}$$

The mixed system decentralizes the optimum only if $w > \tilde{w}_2$.

Let us finally show that $\tilde{w}_1 > \tilde{w}_2$. We do so by comparing the RHS of (11) and (12) and acknowledging that $v(1) > 0 > -V(1)$.

Proof of Proposition 6

Under R1, the social planner's problem is:

$$\max_{c^1,c^2,c^3,c^4} \hat{c}^3$$

$$\text{s.t. } x\pi c_1 + x(1-\pi)c_2 + (1-x)\varepsilon c_3 + (1-x)(1-\varepsilon)c_4 + \pi xg + \varepsilon(1-x)g$$
$$= \pi xw(1-q) + (1-\pi)xw + \varepsilon(1-x)w(1-q) + (1-\varepsilon)(1-x)w$$
$$\text{s.t. } \hat{c}^1 \geq \hat{c}^3, \hat{c}^2 \geq \hat{c}^3, \hat{c}^4 \geq \hat{c}^3, \hat{c}^2 \geq \hat{c}^1$$

Assuming that the egalitarian constraints are binding, we have:

$$\hat{c}^1 = \hat{c}^2 \implies c^2 > c^1 \text{ and } \hat{c}^4 = \hat{c}^3 \implies c^3 > c^4 \text{ and } \hat{c}^2 = \hat{c}^4 \implies c^2 = c^4$$

At the optimum under R1, one should implement: $c^3 > c^4 = c^2 > c^1$, so that $\hat{c}^1 = \hat{c}^2 = \hat{c}^3 = \hat{c}^4$. Similar proofs can be carried out for cases R2 and R3.

Lemma 1 *Assuming that the economy is sufficiently productive, the ex post egalitarian optimum involves equal equivalent consumption levels for the four types. Under R1 ($\bar{n} = 0$), we have: $c^1 < c^2 = c^4 < c^3$. Under R2 ($\bar{n} = 1$), we have $c^4 < c^1 = c^3 < c^2$. Under R3 ($\bar{n}_{1,2} = 1, \bar{n}_{3,4} = 0$), we have $c^1 = c^4 < c^2, c^3$.*

Proof See above. □

Let us now consider the decentralization by means of 4 monetary transfers b_i given to types $i = \{1, 2, 3, 4\}$. The government's budget constraint is:

$$b_1 x\pi + x(1-\pi)b_2 + b_3(1-x)\varepsilon + (1-x)(1-\varepsilon)b_4 = 0 \tag{13}$$

Under R1, the decentralization of the optimum requires: $c^{3,D} > c^{4,D} = c^{2,D} > c^{1,D}$ where D stands for Decentralization, so as to achieve $\hat{c}^1 = \hat{c}^2 = \hat{c}^3 = \hat{c}^4$. Transfers b_i satisfy:

$$u(w(1-q) - g + b_1) + v(1) = u(w + b_2) \tag{14}$$

$$U(w(1-q) - g + b_3) - V(1) = U(w + b_4) \tag{15}$$

$$w + b_2 = w + b_4 \tag{16}$$

Equation (14) ensures that $c^{1,D} = w(1-q) - g + b_1$ and $c^{2,D} = w + b_2$ are set such that $\hat{c}^1 = \hat{c}^2$. Equation (15) ensures that $c^{3,D} = w(1-q) - g + b_3$ and $c^{4,D} = w + b_4$ are set such that $\hat{c}^3 = \hat{c}^4$, while the third equality ensures that $\hat{c}^2 = \hat{c}^4$. Together with (14) and (15), this implies that $\hat{c}^1 = \hat{c}^3$ so that all consumption equivalents are equalized. Equation (16) yields: $b_2 = b_4 = b$. Using Eq. (14) with Assumption A1, we obtain that $b_1 < b$ and using Eq. (15), we have that $b < b_3$. Let us now find the signs of $\{b, b_1, b_3\}$. Situations where $0 < b_i \forall i$ or $b_i < 0 \forall i$ would not be possible as they do not satisfy (13). Yet, using Eqs. (14), (15) and (16), both $b > 0$ or $b < 0$ are possible solutions.

Similar proofs exist for cases R2 and R3. Lemma 2 summarizes our results.[42]

[42] When the sign of the transfer is ambiguous, its sign depends on the specific forms of $u(.)$, $U(.)$, $v(.)$, $V(.)$ and on parameter values.

Lemma 2 *Assume that the economy is sufficiently productive. The decentralization of the ex post egalitarian optimum can be achieved by means of the following instruments:*

Reference fertility	Monetary transfers
R1 ($\bar{n} = 0$)	$b_1 < 0 < b_2 = b_4 < b_3$ or $b_1 < b_2 = b_4 < 0 < b_3$
R2 ($\bar{n} = 1$)	$b_4 < 0 < b_3 = b_1 < b_2$ or $b_4 < b_3 = b_1 < 0 < b_2$
R3 ($\bar{n}_{1,2} = 1, \bar{n}_{3,4} = 0$)	$b_4 < b_1 < 0 < b_2, b_3$ or $b_4 < 0 < b_1 < b_2, b_3$

Proof See above. □

Finally, let us consider the quasi-linear case. In case R1, transfers b_1, b_2, b_3, b_4 satisfy Eqs. (13)–(16), where we have replaced for the quasi-linear utilities. Equation (16) leads to $b_2 = b_4 = b$ so that the budget constraint leads to:

$$b_3 = \frac{-x\pi b_1 - [(1-x)(1-\varepsilon) + x(1-\pi)]b}{(1-x)\varepsilon}$$

Equation (14) leads to: $b = -qw - g + b_1 + v(1)$. This defines the locus I, i.e. the set of pairs (b_1, b). This can be represented by an increasing line, with slope 1 and with a positive intercept (when $b_1 = 0$) at $v(1) - qw - g > 0$ (Assumption A1).

Equation (15) together with the budget constraint

$$b = -\frac{x\pi}{1-\pi x} b_1 - \frac{V(1) + qw + g}{\frac{1-\pi x}{\varepsilon(1-x)}}$$

It defines the locus II, i.e. the set of pairs (b_1, b) such that $\hat{c}_3 = \hat{c}_4$ and the budget constraint is satisfied. Since $\pi x < 1$, it has a negative slope, less than the 45°line. When $b_1 = 0$, we have $b = -\frac{V(1)+wq+g}{\frac{1-\pi x}{\varepsilon(1-x)}} < 0$. In addition, consumptions cannot be negative, so that $-[w(1-q) - g] < b_1$ and $b > -w$.

Non-existence arises when locus I is above locus II at $b_1 = -[w(1-q) - g]$, that is, when:

$$w < \frac{g(x\pi + \varepsilon(1-x)) + v(1)(1-\pi x) + V(1)\varepsilon(1-x)}{1 - x\pi q - \varepsilon q(1-x)} \equiv \bar{w}_1 \tag{17}$$

The decentralization through a pure transfer system exists if and only if $w \geq \bar{w}_1$.

Consider now case R2. The proof is similar to the one of case R1. Since under R2, $b_1 = b_3 = b$, the (im)possibility to decentralize the optimum with equal consumption equivalent can be studied by examining the (non)intersection of two loci in the $\left(b, b_2\right)$ space:

$$b_2 = b - wq - g + v(1) \text{ (locus I)}$$
$$b_2 = -\frac{1-x+x\pi}{x(1-\pi)}b + \frac{(1-x)(1-\varepsilon)}{x(1-\pi)}(V(1) + wq + g) \text{ (locus II)}$$

Consumptions cannot be negative, so that $c_1 > 0$ implies $b \geq -w(1-q) + g$. Moreover, $c_2 > 0$ implies $b_2 \geq -w$. Decentralization cannot take place when the locus I is above the locus II at $b = (-w(1-q) + g)$, that is, when:

$$w < \frac{g(x\pi + \varepsilon(1-x)) + v(1)x(1-\pi) - (1-x)(1-\varepsilon)V(1)}{1 - qx\pi - q\varepsilon(1-x)} \equiv \bar{w}_2 \tag{18}$$

Thus the decentralization of the constrained optimum is possible only if $w \geq \bar{w}_2$.

Comparing the RHS of (17) with (18), we can show that $\bar{w}_1 > \bar{w}_2$, since $v(1) > 0 > -V(1)$. We can also show that $\bar{w}_1 > \tilde{w}_1$ by comparing the RHS of (11) and (17), and that $\bar{w}_2 > \tilde{w}_2$ by comparing the RHS of (12) and (18). Finally, comparing the RHS of (11) and (18), we obtain that $\tilde{w}_1 > \bar{w}_2$ so that: $\bar{w}_1 > \tilde{w}_1 > \bar{w}_2 > \tilde{w}_2$. Lemma 3 sums up our findings.

Lemma 3 *Assume quasi-linear utility and excess demand for children. Define* $\bar{w}_1 \equiv \frac{g(x\pi+\varepsilon(1-x))+v(1)(1-\pi x)+V(1)\varepsilon(1-x)}{1-\varepsilon q(1-x)-qx\pi}$; $\bar{w}_2 \equiv \frac{g(x\pi+\varepsilon(1-x))+v(1)x(1-\pi)-(1-x)(1-\varepsilon)V(1)}{1-\varepsilon q(1-x)-qx\pi}$. *Threshold wage levels satisfy*: $\tilde{w}_2 < \bar{w}_2 < \tilde{w}_1 < \bar{w}_1$.

Under R1, (*i*) *if* $w > \bar{w}_1 > \tilde{w}_1$, *equalizing* $\hat{c}^i$ *can be done by a pure transfer system or a mixed system*; (*ii*) *if* $\bar{w}_1 > w > \tilde{w}_1$, *equalizing* $\hat{c}^i$ *can only be done by a mixed system*; (*iii*) *if* $\bar{w}_1 > \tilde{w}_1 > w$, *equalizing* $\hat{c}^i$ *cannot be done.*

Under R2, (*i*) *if* $w > \bar{w}_2 > \tilde{w}_2$, *equalizing* $\tilde{c}^i$ *can be done by a pure transfer system or a mixed system*; (*ii*) *if* $\bar{w}_2 > w > \tilde{w}_2$, *equalizing* $\tilde{c}^i$ *can only be done by a mixed system*; (*iii*) *if* $\bar{w}_2 > \tilde{w}_2 > w$, *equalizing* $\tilde{c}^i$ *cannot be done.*

Proof See above. □

Proof of Proposition 8

Assuming $\bar{n}_{1,2,3} = 1$ and $\bar{n}_4 = 0$, as well as $\bar{e}_{1,4} = 0$ and $\bar{e}_{2,3} = \ell$, we have:

$$u\left(C^1\right) + v(1) = u\left(c^1\right) + v(1) \implies C^1 = c^1$$
$$u\left(C^2\right) + v(1) - \varphi(\ell) = u(c^2) + v(1) - \varphi(\ell) \implies C^2 = u^{-1}\left[u\left(c^2\right)\right] = c^2$$
$$u\left(C^3\right) + v(1) - \varphi(\ell) = u(c^3) + v(0) - \varphi(\ell) \implies C^3 = u^{-1}\left[u(c^3) - v(1)\right]$$
$$U(C^4) - V(0) = U(c^4) - V(0) \implies C^4 = c^4$$

If egalitarian constraints are binding at the optimum (i.e., a sufficiently productive economy), we have $C^1 = C^2 = C^3 = C^4$, so that $c^1 = c^2 = c^4 < c^3$.

As to the decentralized solution, consumption equivalents can be written as:

$$u\left(C^1\right) = u\left(w(1-q) - g - \ell + b_1\right) \text{ and } u\left(C^2\right) = u\left(w(1-q) - g - \ell + b_2\right)$$
$$u\left(C^3\right) = u(w - \ell + b_3) - v(1) \text{ and } U(C^4) = U(w + b_4)$$

From $C^1 = C^2 = C^4$, we obtain: $b_2 > b_1 > b_4$. We also have that $b_3 > b_2$, by contradiction as we show now. When condition (5) is satisfied and starting from $b_2 = b_3 = 0$, we have: $u\left(w(1-q) - g - \ell + b_2 - \ell\right) > u(w - \ell + b_3) - v(1)$. If b_2 increases, $b_2 > b_3$ and the LHS becomes larger than the RHS, so that equivalent consumption levels cannot be equalized. Hence, the only possible solution to ensure that $C_2 = C_3$, and thus that $u\left(w(1-q) - g - \ell + b_2 - \ell\right) = u(w - \ell + b_3) - v(1)$, consists in setting $b_3 > b_2$.

Furthermore, the budget constraint of the government is balanced when $x\pi b_1 + x(1-\pi)pb_2 + x(1-\pi)(1-p)b_3 + (1-x)b_4 = 0$. This implies, with the relations above, that $b_3 > 0$ and $b_4 < 0$. b_2 and b_3 can be positive or negative depending on the value of $v(1)$ and on $\{\pi, x, p\}$.

Transfers must ensure that individuals who cannot have children but want one decide to invest in ART ($e = \ell$) after the State's intervention, that is:

$$\begin{aligned} &p[u(w(1-q) - g - \ell + b_2) + v(1) - \varphi(\ell)] \\ &+ (1-p)[u(w - \ell + b_3) - \varphi(\ell)] \geq u(w + b_4) \end{aligned} \tag{19}$$

where the RHS is the utility the individual if he does not invest in ART and remains childless with probability 1. Using the definitions of C_2, C_3 and C_4, we show that this condition is satisfied whenever $v(1) \geq \varphi(\ell)$, which is always the case when the involuntary childless invests in ART (see condition 5).

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