Physics in Collision - Stanford, California, June 20-22, 2002

THE VALENCE QUARK DISTRIBUTION OF THE PION[1]

J. P. Lansberg, F. Bissey, J. R. Cudell, J. Cugnon, M. Jaminon and P. Stassart
Université de Liège, Département de Physique B5a,
Sart Tilman, B-4000 LIEGE 1, Belgium

ABSTRACT

The pion structure function is investigated in a simple model, where the pion and its constituent quark fields are coupled through the simplest pseudoscalar coupling. The imaginary part of the forward $\gamma^\star\pi \to \gamma^\star\pi$ scattering amplitude is evaluated and related to the structure functions. It is shown that the introduction of non-perturbative effects, linked to the size of the pion, allows a connection with the quark distribution. It is predicted that higher-twist terms become negligible for Q^2 larger than $\sim$2 GeV^2, that quarks in the pion have a momentum fraction smaller than in the proton case, and that the momentum sum rule is violated for the pion.

1 The Model

We consider [1] an isospin triplet pion field $\vec{\pi} = (\pi^+, \pi^0, \pi^-)$ interacting with quark fields ψ through the Lagrangian density

$$\mathcal{L}_{int} = ig(\overline{\psi}\ \vec{\tau}\gamma_5\psi).\ \vec{\pi}, \tag{1}$$

where $\vec{\tau}$ is the isospin vector operator. The imaginary part of the amplitude for $\gamma^\star\pi \to \gamma^\star\pi$ is considered at lowest order (αg^2) and obtained using Cutkosky rules.

[1]Presented by J. P. Lansberg at the 22nd Physics in Collision Conference (PIC 2002), Stanford, California, June 20-22, 2002.

The coupling constant g (see Fig. 1.a) is chosen to fulfill the number of particles sum rule ($\int F_1(x)dx = \frac{5}{18}$). The effects of the pion size are mimicked by the introduction of a cut-off (Λ) on an observable quantity, *i.e.* t for the process $\gamma^\star\pi \rightarrow q\bar{q}$, –thus in a gauge invariant way–.

2 Results

As seen in Fig. 1, g exhibits plateaux for $Q^2 > 1.5$ GeV^2. These plateau values are in reasonable agreement with those extracted in the NJL model [2]. A similar behaviour is obtained for the momentum fraction $2\langle x\rangle$ derived from $\int F_2 dx$.

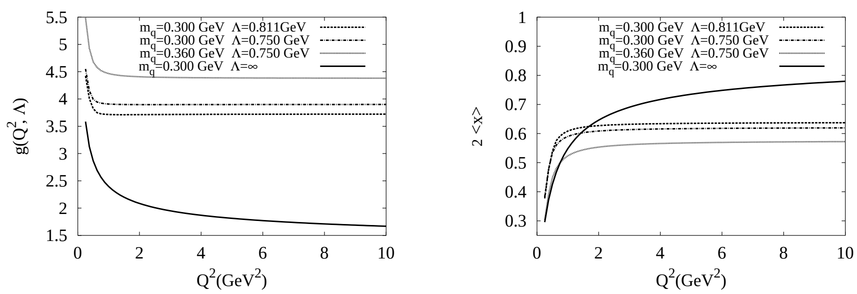

Figure 1: *Evolution of g and $2\langle x\rangle$ as functions of Q^2.*

The structure function F_2 is shown in Fig. 2, which illustrates the effect of the parameters Λ and m_q, and the evolution for increasing Q^2.

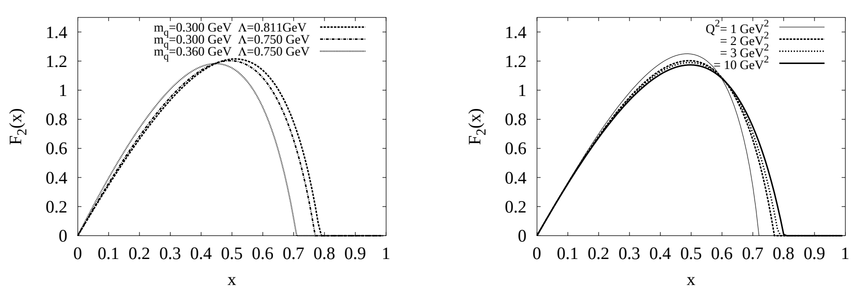

Figure 2: *Evolution of F_2 as a function of x for varying Λ and m_q (left) and Q^2 (right).*

The most striking feature of these distributions is the vanishing of F_2 for x larger than some value x_{max}. This effect originates from the kinematical cuts. Indeed for a finite Q^2, when x is large, there is no way to put the cut quarks on their mass shell: this requires at least an energy of $2m_q$.

The vanishing at large x is not obtained in similar works, in particular in Ref. [3]. In this reference, the Bjorken limit is taken first and the kinematical constraint is not applied. This procedure does not seem correct for evaluating cross-sections at finite Q^2.

3 Discussion and Conclusion

We have presented the simplest model allowing to relate virtual photon-pion forward elastic scattering to quark distributions. The introduction of a cut-off due to the pion size makes crossed diagrams appear as higher twists and thus allows us to define quark distributions. However, the introduction of the cut-off breaks the momentum sum rule ($2\langle x\rangle = 1$) at $Q^2 = \infty$ because, in the case of the pion, constituent quarks can never be considered as free.

We motivated the cut-off as a manifestation of the pion size. The value $\Lambda^{-1} = (0.75 \text{ GeV})^{-1}$ is close to the hard core rms radius of the chiral bag model, 0.35 fm [4]. The cut-off has been imposed on the relative quark momentum. This procedure is at variance with the double-subtraction Pauli-Villars procedure proposed in Ref. [5], or with other ones introduced in similar works based on NJL models.

In addition, there is no need in our approach to consider additional diagrams with local pion-pion-quark-quark interactions. Yet, the pion-quark-quark coupling constant turns out to be the same in our approach and in NJL models. This may not be too surprising, as in both cases this coupling is determined by the requirement that the pion appears as made of two constituent quarks.

Our main conclusion is that pions are different from other hadrons, in the sense that the quark momentum fraction should be smaller, and that higher twist terms disappear for $Q^2 \sim 2 \text{ GeV}^2$.

References

1. F. Bissey, J. R. Cudell, J. Cugnon, M. Jaminon, J. P. Lansberg and P. Stassart, arXiv:hep-ph/0207107, to appear in Phys. Lett. B.

2. M. Jaminon, R. Mendez Galain, G. Ripka and P. Stassart, Nucl. Phys. A **537** (1992) 418.

3. T. Shigetani, K. Suzuki and H. Toki, Phys. Lett. B **308** (1993) 383 [arXiv:hep-ph/9402286].

4. G. E. Brown, M. Rho and W. Weise, Nucl. Phys. A **454** (1986) 669.

5. H. Weigel, E. Ruiz Arriola and L. P. Gamberg, Nucl. Phys. B **560** (1999) 383 [arXiv:hep-ph/9905329].