Extreme depolarisation for any spin

arXiv:2106.11680

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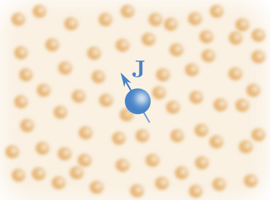
Motivations

- How does the decoherence rate scale with the spin number j ?
- Which states are more prone to (super)decoherence ?
- How fast does non-classicality of a spin state fade over time ?



Depolarisation master equation

Master equation

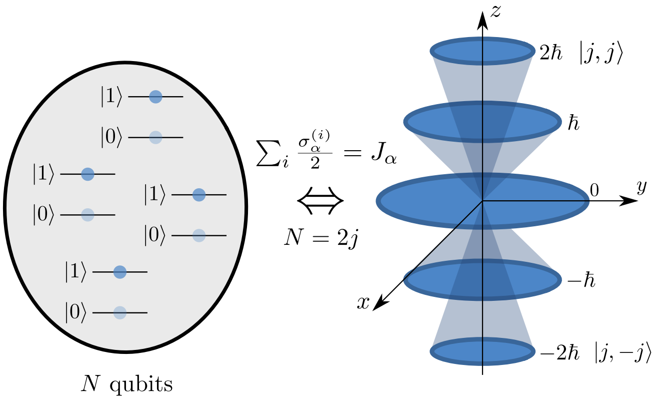
$$\dot{\rho}(t) = \frac{i}{\hbar}[\rho(t), H] + \sum_{\alpha=x,y,z} \mathcal{D}_\alpha[\rho(t)]$$

$$H = \hbar\omega J_z \qquad \mathcal{D}_\alpha[\rho] = \gamma_\alpha \left(2J_\alpha\rho J_\alpha - J_\alpha^2\rho - \rho J_\alpha^2\right)$$

For two non-zero decay rates γ_α, the purity $R = \mathrm{Tr}(\rho^2)$ of the state decreases monotonically and the unique stationary state is the maximally mixed state with minimal purity.

Isotropic depolarisation	Anisotropic depolarisation
$\gamma_x = \gamma_y = \gamma_z$	$\gamma_x = \gamma_y \neq \gamma_z$
Noisy magnetic field	High temperature bath

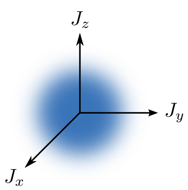
There exists an isomorphism between the Hilbert space of a single spin system and the symmetric subspace of an ensemble of qubits (spin-1/2). Example : $|\mathrm{W}\rangle = |j, -j+1\rangle$



Anticoherent spin states

An anticoherent state[1] to order q is an isotropic state for which the moments of $\mathbf{J}\cdot\mathbf{n}$ up to order q are independent of the direction $\mathbf{n}$, i.e.

$$\mathrm{Tr}\left[\rho\left(\mathbf{J}\cdot\mathbf{n}\right)^q\right] \neq f(\mathbf{n})$$



Anticoherence measure to order 1

$$\mathcal{A}_1 = 1 - \tfrac{1}{j^2}|\langle\psi|\mathbf{J}|\psi\rangle|^2$$

Anticoherent states to order 1

$$|\mathrm{GHZ}\rangle = \tfrac{1}{\sqrt{2}}\left(|j,j\rangle + |j,-j\rangle\right) \qquad |\mathrm{DB}\rangle = |j,0\rangle$$

Highest-Order Anticoherent Pure (HOAP) state :

Pure anticoherent spin state with the highest q for a given j.

Isotropic depolarisation

Observation 1 : HOAP states are the most superdecoherent states. They lead to the minimal purity at any time.

Observation 2 : For pure states, the purity loss rate depends only on the anticoherence measure to order 1 :

$$\dot{R}_{|\psi_0\rangle} = -2\gamma\left(N + \frac{N^2}{2}\mathcal{A}_1\right)$$

Under isotropic depolarisation ($\gamma_\alpha \equiv \gamma$, $\forall\alpha$) the purity loss rate can be written in terms of the purities of the state ρ and its $N-1$ reduced density matrix ρ_{N-1}.

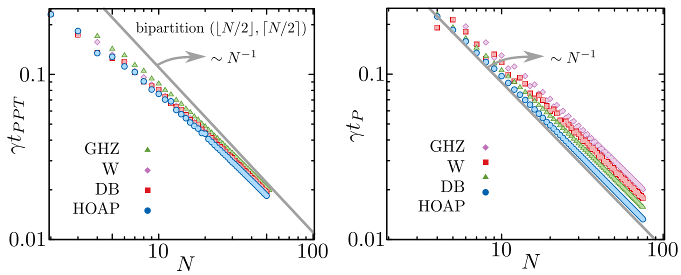
$$\dot{R}(\rho) = -2\gamma R(\rho) N - 2\gamma\left[R(\rho) - R(\rho_{N-1})\right] N^2$$

Any separable state necessarily verifies $R(\rho) < R(\rho_{N-1})$.

Observation 3 : Superdecoherence (purity loss rate scaling as N^2) cannot occur without entanglement.

In order to study the entanglement decay, we define two times :

- t_{PPT} : time at which the PPT criterion (necessary condition of separability) is fullfilled.
- t_P : time at which the P-function[2] of the state becomes everywhere positive (sufficient condition of separability).



Observation 4 : The time at which the entanglement of an initial pure state is lost due to depolarisation scales as N^{-1}.

Anisotropic depolarisation

The purity loss rate for pure states is directly related to the variances of the spin components.

$$\dot{R}_{|\psi_0\rangle} = -4\left[\gamma_z\Delta J_z^2 + \gamma_{\perp z}\left(\Delta J_x^2 + \Delta J_y^2\right)\right]$$

Observation 5 : An appropriate squeezing of the initial state can reduce decoherence on short time scales.

Perspectives

- Conditions of superdecoherence for anisotropic depolarisation
- Experimental investigation of large spin depolarisation (e.g. with Dysprosium[3]).
- Usefulness of anticoherent states for quantum sensing[4] or quantum cryptography

References

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3. T. Satoor, A. Fabre, J.-B. Bouhiron, A. Evrard, R. Lopes and S. Nascimbene, arXiv:2104.14389.
4. F. Gebia *et al.*, Phys. Rev. A **101**, 032112 (2020).