Relations between strong decay widths of the P_c pentaquarks in the SU(4) flavor-spin model

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(Received 23 June 2021; accepted 10 August 2021; published 29 September 2021)

In a previous work we have studied the isospin $1/2$ lowest positive and negative parity states of the pentaquark $uudc\bar{c}$, in a constituent quark model with a linear confinement and an SU(4) flavor-spin hyperfine interaction and we compared the results with the $P_c^+(4312)$, $P_c^+(4440)$ and $P_c^+(4457)$ pentaquarks observed at LHCb in 2019. Here we extend the previous work by calculating ratios of decay rates of the P_c pentaquarks to J/Ψ and η_c and similarly the ratio of decay rates to $\Lambda_c\bar{D}^*$ and $\Lambda_c\bar{D}$. Our predictions are based on the $\mathrm{SU}(4)\times\mathrm{SU}(2)$ structure of compact pentaquarks.

DOI: 10.1103/PhysRevD.104.054050

I. INTRODUCTION

The observation of the narrow structures $P_c^+(4312)$, $P_c^+(4440)$ and $P_c^+(4457)$ in the $\Lambda_b^0 \to J/\psi K^- p$ decay made by LHCb in 2019 and its interpretation as a pentaquark with flavor content $uudc\bar{c}$ [1] has stimulated considerable interest in further understanding the structure of these pentaquarks.

Although observed in the $J/\psi p$ channel, the proximity of the mass of the $P_c^+(4312)$ to the $\Sigma_c^+\bar{D}^0$ threshold (4318 MeV) and of the masses of $P_c^+(4440)$ and $P_c^+(4457)$ to the $\Sigma_c^+\bar{D}^{*0}$ threshold (4460 MeV), favored their interpretation as molecular S-wave of the $\Sigma_c^+ + \bar{D}^0$ and $\Sigma_c^+ + \bar{D}^{*0}$ systems respectively [2–15]. In such an interpretation, the binding arises via meson exchanges between point particles and in the elastic channel all resonances acquire a negative parity.

The spectrum of the $uudc\bar{c}$ pentaquark has also been analyzed in compact pentaquark models. An advantage with respect to molecular models is that they allow a classification of pentaquarks into multiplets [16]. A considerable amount of studies are based on the color-spin (CS) chromomagnetic interaction of the one gluon exchange model with quark/antiquark correlations [17] or without correlations, see, for example, Refs. [18,19].

In Ref. [20] we have studied the spectrum of the $uudc\bar{c}$ pentaquark and in Ref. [21] the spectrum of the isoscalar $udsc\bar{c}$ pentaquark within a model based on the SU(4) flavor-spin (FS) hyperfine interaction obtained as an extension of the meson exchange model between quarks [22,23] to both light and heavy meson exchange. The flavor-spin model provides a good description of low-lying nonstrange and strange baryons by correctly reproducing the order of positive and negative parity states in contrast to models based on the hyperfine color-spin chromomagnetic interaction.

The extension to SU(4) has been made in the spirit of the phenomenological approach of Ref. [24] where, in addition to Goldstone bosons of the hidden approximate chiral symmetry of QCD, the flavor exchange interaction was augmented by an additional exchange of D mesons between u, d and c quarks and of D_s mesons between s and c quarks.

The conclusion was that the lowest state of the $uudc\bar{c}$ pentaquark has negative parity for the CS interaction and positive parity for the FS interaction. The spin and parity of the narrow structures $P_c^+(4312)$, $P_c^+(4440)$ and $P_c^+(4457)$ are presently unknown experimentally.

The parity of the pentaquark is given by $\mathrm{P} = (-)^{\ell+1}$, where ℓ is the orbital angular momentum of the excited system. For the lowest positive parity states one way is to introduce an angular momentum $\ell = 1$ in the internal motion of the four-quark subsystem. According to the Pauli principle, the four-quark subsystem must be in a state of orbital symmetry $[31]_O$. Although the kinetic energy of $[31]_O$ is higher than that of the totally symmetric $[4]_O$ state of negative parity, the flavor-spin interaction overcomes this excess and generates a lower eigenvalue for the $[31]_O$ state with an s^3p configuration than for $[4]_O$ with an s^4 configuration [20].

The present work aims at understanding the role of the flavor-spin structure of the wave functions of the $uudc\bar{c}$ pentaquarks studied in Ref. [20] on some of their strong decay properties. We restrict our considerations to the lowest positive and negative parity states $J^P = 1/2^+$ and $J^P = 1/2^-$, respectively.

The study of strong decay properties of P_c pentaquarks is a present challenge. A list of meson-baryon systems into which the P_c pentaquarks of positive or negative parity can decay was presented in Ref. [25]. Some strong decay properties of the 2019 LHCb pentaquarks have been considered in the framework of the baryon-meson molecules scenario [26,27]. The decay widths of the LHCb pentaquark $P_c(4312)$ has also been analyzed, for example, in a chiral constituent quark model containing both chromomagnetic and meson exchange interactions [28].

Our work is similar in spirit to that of Ref. [26] where the molecular scenario was used to calculate ratios of rates of decays to various channels. Here we study the role of the flavor-spin structure of the pentaquark wave functions on the ratios of decays of the $uudc\bar{c}$ to J/ψ and to η_c and of the decays to $\Lambda_c\bar{D}$ and to $\Lambda_c\bar{D}^*$. This is achieved by calculating the overlap between the flavor-spin part of the pentaquark wave function and the flavor-spin part of the decay channel wave function.

The paper is organized as follows. In the next section we reproduce the flavor-spin Hamiltonian model generalized to SU(4) [20,21]. In Sec. III we describe the SU(4) symmetry structure of the lowest positive and negative parity states named $P_c(1/2^+)$ and $P_c(1/2^-)$ respectively. In Sec. IV we calculate the overlap between the pentaquark and the decay channel flavor-spin wave functions from which we derive the ratio of decay rates in a simple manner. The last section is devoted to conclusions. Appendix A is a remainder of the flavor states of baryons into which the pentaquark can decay. Appendix B describes the flavor states of the pentaquark obtained in an SU(4) classification [16]. In Appendix C we derive the spin part of the pentaquark wave function and in Appendix D we present some useful details of the flavor-spin wave function of negative parity states.

II. THE HAMILTONIAN

Here we closely follow the description of the model used to calculate the spectrum of the $uudc\bar{c}$ pentaquark in Ref. [20], extended to strange hidden charm pentaquarks in Ref. [21]. The Hamiltonian in its general SU(4) form is

$$H = \sum_i m_i + \sum_i \frac{\vec{p}_i^{\,2}}{2m_i} - \frac{(\sum_i \vec{p}_i)^2}{2\sum_i m_i} + \sum_{i<j} V_{\rm conf}(r_{ij}) + \sum_{i<j} V_\chi(r_{ij}), \tag{1}$$

with m_i and $\vec{p}_i$ denoting the quark masses and momenta respectively and r_{ij} the distance between the interacting quarks or quark-antiquark i and j. The Hamiltonian contains the internal kinetic energy and the linear confining interaction

$$V_{\rm conf}(r_{ij}) = -\frac{3}{8}\lambda_i^c \cdot \lambda_j^c C r_{ij}. \tag{2}$$

The hyperfine part $V_\chi(r_{ij})$ has a flavor-spin structure extended to SU(4) in Ref. [20] which has the following form

$$\begin{aligned} V_\chi(r_{ij}) = \Bigg\{ & \sum_{F=1}^{3} V_\pi(r_{ij})\lambda_i^F\lambda_j^F + \sum_{F=4}^{7} V_K(r_{ij})\lambda_i^F\lambda_j^F \\ & + V_\eta(r_{ij})\lambda_i^8\lambda_j^8 + V_{\eta'}(r_{ij})\lambda_i^0\lambda_j^0 \\ & + \sum_{F=9}^{12} V_D(r_{ij})\lambda_i^F\lambda_j^F + \sum_{F=13}^{14} V_{D_s}(r_{ij})\lambda_i^F\lambda_j^F \\ & + V_{\eta_c}(r_{ij})\lambda_i^{15}\lambda_j^{15}\Bigg\}\vec{\sigma}_i\cdot\vec{\sigma}_j, \end{aligned} \tag{3}$$

with the SU(4) generators λ_i^F $(F = 1, 2, ..., 15)$ and $\lambda_i^0 = \sqrt{2/3}\,\mathbf{1}$, where $\mathbf{1}$ is the 4×4 unit matrix. After integration in the flavor space, the two-body matrix elements containing contributions due to light, strange and charm quarks are

$$V_{ij} = \vec{\sigma}_i\cdot\vec{\sigma}_j \begin{cases} V_\pi + \frac{1}{3}V_\eta^{uu} + \frac{1}{6}V_{\eta_c}^{uu}, & [2]_F,\ I=1 \\ 2V_K - \frac{2}{3}V_\eta^{us},\quad 2V_D^{uc} - \frac{1}{2}V_{\eta_c}^{uc} & [2]_F,\ I=\frac{1}{2} \\ 2V_{D_s}^{sc} - \frac{1}{2}V_{\eta_c}^{sc} & [2]_F,\ I=0 \\ \frac{4}{3}V_\eta^{ss} + \frac{3}{2}V_{\eta_c}^{cc} & [2]_F,\ I=0 \\ -2V_{D_s}^{sc} - \frac{1}{2}V_{\eta_c}^{sc} & [11]_F,\ I=0 \\ -2V_K - \frac{2}{3}V_\eta^{us},\quad -2V_D^{uc} - \frac{1}{2}V_{\eta_c}^{uc} & [11]_F,\ I=\frac{1}{2} \\ -3V_\pi + \frac{1}{3}V_\eta^{uu} + \frac{1}{6}V_{\eta_c}^{uu}, & [11]_F,\ I=0 \end{cases} \tag{4}$$

In Eqs. (4) the pair of quarks ij is either in a symmetric $[2]_F$ or in an antisymmetric $[11]_F$ flavor state and the isospin I is defined by the quark content. The upper index of V exhibits the flavor of the two quarks interchanging a meson specified by the lower index. Obviously, in every sum/difference of Eq. (4) the upper index is the same for all terms.

Thus the SU(4) version of the interaction (3) contains $\gamma = \pi, K, \eta, D, D_s, \eta_c$ and η' meson-exchange terms. Every $V_\gamma(r_{ij})$ is a sum of two distinct contributions: a Yukawa-type potential containing the mass of the exchanged meson and a short-range contribution of opposite sign, the role of which is crucial in baryon spectroscopy. For a given meson γ the meson exchange potential is

$$V_\gamma(r) = \frac{g_\gamma^2}{4\pi}\frac{1}{12m_i m_j}\left\{\theta(r-r_0)\mu_\gamma^2\frac{e^{-\mu_\gamma r}}{r} - \frac{4}{\sqrt{\pi}}\alpha^3\exp(-\alpha^2(r-r_0)^2)\right\}. \tag{5}$$

In the calculations of the spectrum of $uudc\bar{c}$ we used parameters of Ref. [23] to which we added the μ_D mass and the coupling constants $\frac{g^2_{Dq}}{4\pi}$. The contribution of V_{η_c} can be neglected. We have

$$\frac{g^2_{\pi q}}{4\pi} = \frac{g^2_{\eta q}}{4\pi} = \frac{g^2_{Dq}}{4\pi} = 0.67, \qquad \frac{g^2_{\eta' q}}{4\pi} = 1.206,$$
$$r_0 = 0.43 \text{ fm}, \qquad \alpha = 2.91 \text{ fm}^{-1}, \qquad C = 0.474 \text{ fm}^{-2},$$
$$\mu_\pi = 139 \text{ MeV}, \qquad \mu_\eta = 547 \text{ MeV},$$
$$\mu_{\eta'} = 958 \text{ MeV}, \qquad \mu_D = 1867 \text{ MeV}.$$

The meson masses correspond to the experimental values from the Particle Data Group [29]. The model has previously been used to study the stability of open flavor tetraquarks [30] and open flavor pentaquarks [31]. Accordingly, for the quark masses $m_{u,d}$ and m_c we take

$$m_{u,d} = 340 \text{ MeV}, \qquad m_c = 1350 \text{ MeV}. \tag{6}$$

They were adjusted to satisfactorily reproduce the average mass $\overline{M} = (M + 3M^*)/4 = 2008$ MeV of D mesons. Using the above parameters the calculated baryon masses are $m_N = 960$ MeV, $m_{\Lambda_c} = 2180$ MeV and $m_{\Sigma_c} = 2434$ MeV [20].

III. THE PENTAQUARK WAVE FUNCTION

The pentaquarks under discussion are denoted by $P_c(1/2^+)$ and $P_c(1/2^-)$, which are the lowest positive and negative parity states of the Hamiltonian introduced in Sec. II. The parity of a pentaquark is $P = (-1)^{\ell+1}$. The pentaquark wave functions showing the symmetry structure of the four-quark subsystem in terms of the orbital (O), color (C), flavor (F) and spin (S) degrees of freedom are

$$P_c(1/2^+) = ([31]_O[211]_C[1^4]_{OC}; [22]_F[22]_S[4]_{FS})\phi(\bar{c}), \tag{7}$$

$$P_c(1/2^-) = ([4]_O[211]_C[211]_{OC}; [211]_F[22]_S[31]_{FS})\phi(\bar{c}). \tag{8}$$

The $\phi(\bar{c})$ is the wave function of the antiquark with the same degrees of freedom. The antiquark is coupled to the q^4 subsystem in the color, flavor and spin spaces. The coupling in the flavor space to a definite flavor symmetry of $q^4\bar{q}$ is described in Appendix B and the coupling to a total spin $S = 1/2$ for both $P_c(1/2^+)$ and $P_c(1/2^-)$ is presented in Appendix C. The total angular momentum is $\vec{J} = \vec{L} + \vec{S}$, so that for $\ell = 1$ the pentaquark $P_c(1/2^+)$ can have quantum numbers $J^P = 1/2^+$ or $3/2^+$. The two states are degenerate so that the latter quantum number can be omitted in the discussion. The $q^4\bar{q}$ is in a color singlet state. The orbital parts of $P_c(1/2^+)$ and $P_c(1/2^-)$ are described in detail in Ref. [20]. They are defined in terms of the internal coordinates of five particles and are translationally invariant (no center of mass motion). In the following the expressions of the orbital wave functions are not necessary.

From Eqs. (7) and (8) one can see that the q^4 subsystem in $P_c(1/2^+)$ is in a symmetric flavor-spin state and that in $P_c(1/2^-)$ the subsystem q^4 is in an antisymmetric color-flavor-spin state. Together with the orbital part a totally antisymmetric state is obtained in both cases.

The pentaquark wave function results from the coupling of q^4 to $\bar{q}$ in the color, flavor and spin spaces. The expression of the flavor-spin part of the wave function of $P_c(1/2^+)$ is obtained from the Clebsch-Gordan coefficients [32] of the inner product $[22]_F \otimes [22]_S \to [4]_{FS}$ because after the coupling the flavor-spin wave function of q^4 remains symmetric under the permutation group S_4. Then the flavor-spin wave function of $P_c(1/2^+)$ has the following form

$$(\phi\chi)^5_E = \frac{1}{\sqrt{2}}(\phi^\rho_E\chi^\rho_{\text{SM}} + \phi^\lambda_E\chi^\lambda_{\text{SM}}), \tag{9}$$

in terms of flavor and spin states defined in Appendices B and C respectively. The upper indices ρ and λ indicate that the flavor or spin state is antisymmetric and symmetric respectively under interchange of the particles 1 and 2, like for the nucleon. Here and below the upper index 5 stands for five particles, i.e., the pentaquark.

In a similar way, the color-flavor-spin wave function of $P_c(1/2^-)$ can be obtained from the Clebsch-Gordan coefficients of the inner product $[211]_C \otimes [31]_{FS} \to [1^4]_{CFS}$ inasmuch as the flavor-spin-color wave function of q^4 remains antisymmetric under S_4. One has

$$(\phi\chi C)^5_{F_1} = \frac{1}{\sqrt{3}}[(\phi_{F_1}\chi_{\text{SM}})_1 C_1 - (\phi_{F_1}\chi_{\text{SM}})_2 C_2 + (\phi_{F_1}\chi_{\text{SM}})_3 C_3], \tag{10}$$

where C_i are the three independent basis vectors of the irreducible representation $[222]_C$ describing the color singlet $q^4\bar{q}$ system, which results from the SU(3)-color direct product decomposition $[211] \otimes [11]$. These are

$$C_1 = \begin{array}{|c|c|}\hline 1 & 2 \\ \hline 3 & 5 \\ \hline 4 & 6 \\ \hline\end{array}, \qquad C_2 = \begin{array}{|c|c|}\hline 1 & 3 \\ \hline 2 & 5 \\ \hline 4 & 6 \\ \hline\end{array}, \qquad C_3 = \begin{array}{|c|c|}\hline 1 & 4 \\ \hline 2 & 5 \\ \hline 3 & 6 \\ \hline\end{array}. \tag{11}$$

The only one which gives a nonvanishing overlap with the color wave function of the exit channel is C_3. Its explicit form is not needed in the calculation of O^{FS}. The corresponding flavor-spin state $(\phi_{F_1}\chi_{\text{SM}})_3$ with $S = 1/2$

TABLE I. Lowest positive and negative parity $uudc\bar{c}$ pentaquarks of quantum numbers S and J^P and symmetry structure defined in (7) and (8). Column 1 gives the name, column 2 the spin, column 3 the total angular momentum and parity, column 4 the mass calculated in Ref. [20] and columns 5–8 the value of the overlap O^{FS} for the indicated decay channel.

				Decay channel			
Pentaquark	S	J^P	Mass (MeV)	$p + J/\psi$	$p + \eta_c$	$\Lambda_c + \bar{D}^*$	$\Lambda_c + \bar{D}$
$P_c(1/2^+)$	$\frac{1}{2}$	$\frac{1}{2}^+, \frac{3}{2}^+$	4273	$-\frac{\sqrt{3}}{4}$	$\frac{1}{4}$	$\frac{3}{4\sqrt{2}}$	$-\frac{3}{4\sqrt{6}}$
$P_c(1/2^-)$	$\frac{1}{2}$	$\frac{1}{2}^-$	4487	$\frac{3}{4\sqrt{6}}$	$-\frac{1}{4\sqrt{2}}$	$-\frac{1}{8}$	$\frac{1}{8\sqrt{3}}$

is written explicitly in Appendix D in terms of flavor and spin parts.

The exit channel is formed of a baryon B and a meson M. For S-waves decays, the corresponding flavor-spin state is defined by

$$|BM\rangle = \frac{1}{\sqrt{2}}[(\phi_B^\rho c\bar{c})(\chi_B^\rho \chi_M) + (\phi_B^\lambda c\bar{c})(\chi_B^\lambda \chi_M)], \quad (12)$$

where ϕ_B^ρ and ϕ_B^λ are the usual SU(3) octet baryon states of mixed symmetry [21], The corresponding spin states of the baryon are χ_B^ρ and χ_B^λ and χ_M is the spin state of the meson. The baryon and the meson spins are coupled together to a value equal to that of the pentaquark spin.

In Ref. [20] we have studied the mass spectrum of the $uudc\bar{c}$ pentaquark for several states including those here named $P_c(1/2^+)$ and $P_c(1/2^-)$. The calculated masses are in the range of the observed narrow structures $P_c^+(4312)$, $P_c^+(4440)$ and $P_c^+(4457)$. The SU(4) flavor-spin model described in Sec. II gives 4273 MeV for the mass of $P_c(1/2^+)$, see Table I, and it can tentatively be assigned to $P_c^+(4312)$.

The lowest negative parity state $P_c(1/2^-)$ has a mass of 4487 MeV, indicated in Table I, close to that of the $P_c^+(4440)$ and $P_c^+(4457)$ resonances. Therefore it supports the quantum number $J^P = \frac{1}{2}^-$ for one of them. In discussing the decay widths we shall take into account these assignments.

IV. DECAY WIDTHS

The decay width of a pentaquark resonance into a baryon + meson is proportional to the square of the transition amplitude matrix element between the initial and final states (see e.g., Ref. [33]). In the present model we suppose that the transition amplitude matrix element can be written as a product of two factors, one containing the orbital-color degrees of freedom and the other the flavor-spin degrees of freedom. In a simple estimate each factor is proportional to the overlap between the initial pentaquark and the baryon + meson channel wave functions in the corresponding degrees of freedom. In the following we shall denote them by O^{OC} and O^{FS} respectively. In the fall-apart mode considered here we think that O^{FS} is a good approximation to the flavor-spin part of the transition amplitude matrix element because it contains the basic quark interchange operator through the antisymmetrization of the four quark subsystem wave functions, as defined by Eqs. (7) and (8). The orbital-color part of the transition amplitude deserves a special discussion. Presently it is not needed because we are interested in ratios of decay rates of channels with the same symmetry in the flavor-spin space, where the orbital-color part simplifies.

Using Eqs. (9) and (12) the flavor-spin overlap of $P_c(1/2^+)$ can be written as

$$\begin{aligned} O^{FS} &= \langle \phi\chi \rangle_E^5 |BM\rangle \\ &= \frac{1}{2}(\langle \phi_E^\rho | \phi_N^\rho c\bar{c}\rangle \langle \chi_5^\rho | \chi_N^\rho \chi_{\rm SM}\rangle + \langle \phi_E^\lambda | \phi_N^\lambda c\bar{c}\rangle \langle \chi_5^\lambda | \chi_N^\lambda \chi_{\rm SM}\rangle). \end{aligned} \quad (13)$$

From Eqs. (B3) and (B4) associated to the pentaquark $uudc\bar{c}$ flavor wave functions and the spin wave functions of Appendix C we have obtained the overlap O^{FS} for the decay channels where $B = p$ and $B = \Lambda_c$, as indicated in Table I. One can see that the largest overlap is 0.5303 which corresponds to $\Lambda_c + \bar{D}^*$.

Neglecting kinematical differences and other common factors one can easily find that the ratio of the decay rates for $B = p$ is

$$\frac{\Gamma(P_c(1/2^+) \to \eta_c p)}{\Gamma(P_c(1/2^+) \to J/\psi p)} = \frac{1}{3}, \quad (14)$$

and for $B = \Lambda_c$ the ratio is

$$\frac{\Gamma(P_c(1/2^+) \to \Lambda_c \bar{D})}{\Gamma(P_c(1/2^+) \to \Lambda_c \bar{D}^*)} = \frac{1}{3}. \quad (15)$$

The above ratios are equal for the following reason. The flavor part contribution to O^{FS} cancels out in the numerator and the denominator when the scalar and vector mesons have the same flavor content. This is obviously the case for $\bar{D}$ and $\bar{D}^*$.

The quark content of η_c is not known experimentally. Here we have assumed that it has the same content as J/ψ, compatible with an ideal mixing. The value of $1/3$ corresponds to the square of the ratio of the mixing coefficients in the spin wave function of the pentaquark as given by Eq. (C5).

If $P_c(1/2^+)$ is identified with the observed $P_c^+(4312)$ resonance we can make a comparison with the results of Ref. [26] based on the molecular scenario. We note that the ratio (14) is the inverse of the value predicted from the molecular scenario when considered as a $J^P = 1/2^-$ $\Sigma_c\bar{D}^*$ molecule.

After integration in the color space, the overlap O^{FS} of the lowest negative parity pentaquark $P_c(1/2^-)$ described by the wave function (10) becomes

$$O^{FS} = \frac{1}{\sqrt{3}}\langle(\phi_{F_1}\chi_{\mathrm{SM}})_3|BM\rangle. \tag{16}$$

The presence of the factor $1/\sqrt{3}$ is due to the norm of (10) and as mentioned above only the third term of this wave function contributes to fall-apart decays. The integration in the flavor-spin space is made using Appendix D. The largest overlap is 0.3062 and corresponds to the $p + J/\Psi$ channel for the lowest negative parity pentaquark.

From the resulting expression of the overlap we find that the ratio of the decay rates for $B = p$ is

$$\frac{\Gamma(P_c(1/2^-) \to \eta_c p)}{\Gamma(P_c(1/2^-) \to J/\psi p)} = \frac{1}{3}, \tag{17}$$

i.e., the same as for $P_c(1/2^+)$, for the same reason. In Ref. [26] the pentaquark P_{c1} with $J^P = 1/2^-$ is assigned to the observed $P_c^+(4440)$ resonances. The ratio of decay rates to $\eta_c p$ and $J/\psi p$ is at variance with our result.

V. CONCLUSIONS

The present study relies on a compact pentaquark picture of the $uudc\bar{c}$ pentaquark based on the flavor-spin model extended to SU(4). An important feature is that the model introduces an isospin dependence of pentaquarks, necessary to discriminate between decay channels.

The spin part of the pentaquark state is a mixed state of spin 0 and spin 1 mesons. Thus spin 0 and spin 1 mesons can be produced in the decay of the pentaquark. The ratio of the flavor-spin overlap between the pentaquark and the exit channel states depends on the recoupling coefficient in the spin part of the pentaquark wave function, Appendix C, and thus it depends only on the meson spin when the exit baryon is fixed. The light quark masses used in the model were adjusted to reproduce the experimental values of the involved baryons. The charm quark mass reproduces the average mass of D mesons. Accordingly, the J/ψ and η_c mesons are degenerate, like in the heavy-quark spin symmetry limit used in baryon-meson molecular scenario [26]. When the kinematical difference between two final channels, for example, $\eta_c p$ and $J/\psi p$, are neglected, we obtain for the ratio of the decay rates of either $P_c(1/2^+)$ and $P_c(1/2^-)$ values which are at variance with those of Ref. [26].

The present work is a first attempt toward estimating ratios of decay widths in the SU(4) flavor-spin model, based on the approximation of the flavor-spin part of the transition matrix element between the initial and final states by the overlap of the corresponding wave functions. The approximation is satisfactory for describing ratios of decay rates of pentaquarks with the same parity. The ratio of decay rates of pentaquarks of different parities is affected by the orbital-color part of the transition matrix element between the initial and final states, not needed in this simple approach. It will be a challenge to estimate the orbital-color part because a deeper understanding of the decay mechanism is necessary. Further experimental information about the parity of pentaquarks would stimulate more theoretical work.

For positive parity the procedure can be easily extended to the study of excited pentaquark states with J^P values up to $5/2^+$, allowed by the symmetry of the wave functions [20].

ACKNOWLEDGMENTS

This work has been supported by the Fonds de la Recherche Scientifique - FNRS, Belgium, under the Grant No. 4.4503.19.

APPENDIX A: BARYONS

For convenience here we reproduce the flavor wave functions of the baryons needed in this study with phase conventions consistent with the permutation group S_3 [32]. For the proton we have

$$|\phi_p^\rho\rangle = \frac{1}{\sqrt{2}}(udu - duu),$$
$$|\phi_p^\lambda\rangle = -\frac{1}{\sqrt{6}}(udu + duu - 2uud), \tag{A1}$$

for Λ_c they are

$$|\phi_{\Lambda_c}^\rho\rangle = \frac{1}{\sqrt{12}}(2udc - 2duc + cdu - cud + ucd - dcu),$$
$$|\phi_{\Lambda_c}^\lambda\rangle = -\frac{1}{2}(cud - cdu + ucd - dcu). \tag{A2}$$

APPENDIX B: FLAVOR STATES OF $uudc\bar{c}$ PENTAQUARKS

In the following we reproduce the flavor wave functions obtained in Ref. [16] for $uudc\bar{c}$ pentaquarks P_c needed in this study. They form an octet and have isospin $1/2$.

The flavor states of a pentaquark $q^4\bar{q}$ are basis states of SU(4) irreducible representations appearing in the decomposition of the direct product of [22] or [211] of q^4 states and the irreducible representation of $\bar{q}$, namely [111]. Using the notation of Ref. [16] one has

$$E\colon\ [22]_{q^4} \otimes [111]_{\bar{q}} = [331]_{q^4\bar{q}} \oplus [3211]_{q^4\bar{q}} \tag{B1}$$

and

$$F_1\colon\ [211]_{q^4} \otimes [111]_{\bar{q}} = [322]_{q^4\bar{q}} \oplus [3211]_{q^4\bar{q}} \oplus [2221]_{q^4\bar{q}}. \tag{B2}$$

The flavor part of the state (7) is obtained from combinations of basis states of $[331]_{q^4\bar{q}}$ and $[3211]_{q^4\bar{q}}$. and the flavor part of the state (8) is obtained from combinations of basis states of $[322]_{q^4\bar{q}}$ and $[3211]_{q^4\bar{q}}$.

In this way the pentaquark wave functions (7) containing the symmetry $[22]_F$ of the four-quark subsystem become

$$|\phi_E^\rho\rangle = -\frac{1}{2\sqrt{2}}(ucud + uduc - cuud - duuc - ucdu - udcu + cudu + ducu)\bar{c}, \tag{B3}$$

$$|\phi_E^\lambda\rangle = -\frac{1}{2\sqrt{6}}(2uucd + 2uudc - ucud - uduc - cuud - duuc + 2cduu + 2dcuu - ucdu - udcu - cudu - ducu)\bar{c}, \tag{B4}$$

where the subscripts ρ and λ indicate that the quarks 1 and 2 are in an antisymmetric and symmetric state respectively. Similarly the pentaquark wave functions (8) containing the symmetry $[211]_F$ of the four-quark subsystem are

$$|\phi_{F_1}^\rho\rangle = -\frac{1}{4\sqrt{3}}(2dcuu - 2cduu - udcu + ducu + ucdu - cudu + 3uduc - 3duuc - 3ucud + 3cuud)\bar{c}, \tag{B5}$$

$$|\phi_{F_1}^\lambda\rangle = -\frac{1}{4}(2uudc - uduc - duuc - 2uucd + ucud + cuud + udcu + ducu - ucdu - cudu)\bar{c}, \tag{B6}$$

$$|\phi_{F_1}^\eta\rangle = -\frac{1}{\sqrt{6}}(dcuu - cduu - ducu + udcu + cudu - ucdu)\bar{c}, \tag{B7}$$

corresponding to the three basis states of $[211]_F$.

APPENDIX C: THE SPIN PART

Here we write the spin wave functions χ_{SM} of $q^4\bar{q}$ in terms of products of baryon and meson wave functions, needed in the calculation the flavor-spin overlap O^{FS}. For $S = 1/2$ they are the basis vectors of the irreducible representation [32] of SU(2). The first step is to couple the antiquark of spin 1/2 to q^4 of spin s_q. One has

$$\chi_{SM} = \sum_{m_q, m_{\bar{q}}} C^{s_q 1/2 S}_{m_q m_{\bar{q}} M} \chi^{[f]}_{s_q m_q} \chi^{[1]}_{1/2 m_{\bar{q}}}, \tag{C1}$$

where $[f]$ and $[1]$ stand for the SU(2) irreducible representations associated to q^4 and $\bar{q}$ respectively. For the states (7) and (8) we have $[f]$ = [22], thus $s_q = 0$.

The following step is to decouple one quark q from q^4 which amounts to write

$$\chi^{[f]}_{s_q m_q} = \sum_{m_1, m_2} C^{s_1 1/2 s_q}_{m_1 m_2 m_q} \chi^{[f_1]}_{s_1 m_1} \chi^{[1]}_{1/2 m_2}, \tag{C2}$$

which is a linear combination of q^3 and q states described by $[f_1]$ and [1] respectively. Then one has to couple q to $\bar{q}$ to form a meson of a given spin $s = 0$ or 1

$$\chi^{[1]}_{1/2 m_2} \chi^{[1]}_{1/2 m_{\bar{q}}} = \sum_{s, m_s} C^{1/2 1/2 s}_{m_2 m_{\bar{q}} m_s} \chi^{[f']}_{s m_s}, \tag{C3}$$

where $[f'] = [11]$, $s = 0$ for scalar and $[f'] = [2]$, $s = 1$ for vector mesons respectively. Combining together the coupling and the decoupling one obtains

$$\chi_{SM} = \sum_s [(2s+1)(2s_q+1) W(s_1 1/2 S; s_q s) \left(\chi^{[f_1]}_{s_1 m_1} \chi^{[f']}_{s m_s}\right)^{[32]}_{SM}, \tag{C4}$$

in terms of the Racah coefficient $W(s_1 1/2 S; s_q s)$. Note that the baryon $\chi^{[f]}_{s_1 m_1}$ and the meson $\chi^{[f']}_{s m_s}$ states are coupled together to a given spin and projection SM. Here we have $[f_1] = [21]$ thus octet baryons with $s_1 = 1/2$. Implementing the Racah coefficient for the two possible values of the meson state, $s = 0$ and $s = 1$ we obtain the pentaquark spin state as

$$\chi_{1/2M} = -\frac{1}{2}\left(\chi^{[21]}_{1/2M} \chi^{[11]}_{00}\right)^{[32]}_{1/2M} + \frac{\sqrt{3}}{2}\left(\chi^{[21]}_{1/2 m_1} \chi^{[2]}_{1 m_s}\right)^{[32]}_{1/2M}, \tag{C5}$$

which is a linear combination containing scalar and vector meson states. There are two independent five-quark spin states for which the above formula applies. Each is defined by a Young tableau. Thus the left-hand side of Eq. (C5) can be associated

$$\text{either to}\quad \begin{array}{|c|c|c|}\hline 1&2&5\\\hline 3&4\\\cline{1-2}\end{array}\quad \text{or to}\quad \begin{array}{|c|c|c|}\hline 1&3&5\\\hline 2&4\\\cline{1-2}\end{array}\,. \tag{C6}$$

corresponding to χ_5^λ and χ_5^ρ respectively, introduced in Eq. (13).

APPENDIX D: FLAVOR-SPIN WAVE FUNCTION FOR NEGATIVE PARITY STATES

The flavor-spin parts of the negative parity state denoted by $(\phi_{F_1}\chi_{1/2M})_i$ $(i = 1, 2, 3)$ in Eq. (10) can be obtained by writing the spin-flavor state for the subsystem of four quarks followed by the coupling of the antiquark. Here we present only the case $i = 3$, the only one which is needed. The q^4 flavor-spin state can be written in terms of its spin and flavor parts with the help of isoscalar factors of the permutation group S_4 [32,34]. After the coupling of the antiquark one obtains

$$(\phi_{F_1}\chi_{1/2M})_3 = -\frac{1}{\sqrt{2}}\ \begin{array}{|c|c|c|}\hline 1&2&5\\\hline 3&4\\\cline{1-2}\end{array}\ \phi_{F_1}^\lambda - \frac{1}{\sqrt{2}}\ \begin{array}{|c|c|c|}\hline 1&3&5\\\hline 2&4\\\cline{1-2}\end{array}\ \phi_{F_1}^\rho\ , \tag{D1}$$

where the Young tableaux correspond to spin basis vectors of the pentaquark.

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