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SPARE PARTS INVENTORY ROUTING PROBLEM WITH TRANSSHIPMENT AND SUBSTITUTIONS UNDER STOCHASTIC DEMANDS

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ABSTRACT
We study a two-level spare parts supply chain in which a manufacturer supplies a central warehouse (CW) with original equipment manufacturer (OEM) and replacement or pattern parts (PP). The CW, distantly located from the manufacturer, distributes both OEM parts and PP to a given number of depots facing stochastic demands. The demand for spare parts is intermittent, exhibiting an infrequent rate and extreme dispersal over time periods. Along with lateral transshipment, PP can be used as substitutes for the OEM parts to sidestep shortage at depots. Assuming that emergency shipments are significantly longer and more expensive, we aim at underlining the relative effectiveness of such a new spare parts inventory management policy. A mixed-integer linear programming model is pro- posed to solve the inventory routing problem with transshipment and substitution under stochastic demands. The objective is to minimise costs of holding inventory, transportation which includes regular shipment and transshipment, substitution and lost sales. To solve the problem, Sample Average Approximation method is used. Based on empirical goodness-of-fit tests, three demand patterns are studied: Poisson distribution, stuttering Poisson distribution and negative binomial distribution. The model is tested on well-known benchmark instances generated for multi-product multi-vehicle IRP. Computational experiments highlight the benefits of promoting transshipment and substitution on the overall supply chain

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performance. Results also suggest insights, which are of interest to professionals who are willing to develop new decision support models for the most efficient management of such items.

1. Introduction

Generally, in the first level of a spare parts supply chain, some producers or manufacturers supply their customers with original equipment manufacturer (OEM) parts and/or aftermarket parts also called replacement parts or pattern parts (PP). The latter are reverse-engineered OEM parts, designed to perform in the same way as OEM parts, less expensive, have a quality equivalent to or better than OEM parts, and are provided with a wider range of variety [1,2]. Spare parts distributors (or depots) and eventual re-sellers such as shops and car dealerships are on the second level of the supply chain. The key role of the depots is to guarantee that end-user demands are met while incurring the lowest logistical costs [3].

An efficient spare parts inventory management is argued to be the backbone for reliable plant operations, costs reduction, and service level maximisation [4]. Such items are at the greatest risk of obsolescence and may collectively account for up to 60% of the total stock value [5]. Moreover, their demand pattern is intermittent, exhibiting an infrequent rate and extreme dispersal over time periods which often hinders the reduction of lost sales at depots. This is particularly true in the aerospace, IT, and automotive industry contexts [6]. In this respect, the classical management models mainly designed to guarantee smooth replenishment often do not apply [7]. Thus, alternative methods to manage inventory within the distribution network must be investigated to minimise logistics costs and ensure a high customer service level commitment [8]. In this paper, we propose a new management policy consisting of promoting multi-sourcing options to mitigate shortages: (1) regular shipment from manufacturer, (2) inventory sharing, or so-called lateral transshipment (LT), among depots, and (3) the use of PP as substitutes for OEM parts. We focus particularly on the inventory routing problem (IRP) that arises in a two-level spare parts supply chain in which a manufacturer distributes via her/his central warehouse (CW) a set of spare parts to a given number of depots facing stochastic demand. Indeed, assuming that emergency supplies from the CW are significantly long and expensive, this paper promotes spare parts substitution along with LT whenever demand exceeds the available stock at the level of each depot. That is, LT and substitution can be considered by depots to meet expected demands with the use of the same part from the inventory of other depots (LT) or with the use of a PP as substitutes for OEM parts held at their inventory (substitution).

Different incentives can promote the use of both LT and substitutions among depots.

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Regardless of the type of item, promoting LT is known to be a viable solution for managers aiming to improve the system's wide service level by dynamically reallocating (excess) stocks while lowering the lost sales cost [9-11]. Another criterion that can promote substitution is the readily availability of PP, which increases the likelihood of the user switching more often. Not to mention the case where PP are of high quality. However, from the manufacturer's point of view, it would be beneficial if she/he sells more PP than OEM parts by relying on branding and pricing, especially in a market where there are fewer substitute PP, which allows a higher probability of earning greater profits. Moreover, multiple-sourcing is a promising research field for spare parts management (e.g., producing spare parts on-demand via additive manufacturing) [12,13]. With this respect, the use of PP can be viewed as one of more sourcing options available for spare parts management since PP is a less expensive and less reliable sourcing option. Another incentive for LT and substitution would be the slowness of the procedures regarding the control of the conformity of the imported spare parts with regards of local or international standards. For instance, most automotive spare part distributors, such as Moroccan companies, often procure spare parts from domiciled offshore suppliers [14]. The procedure of the quality control can take up to over 15 days and even longer if the Ministry's departments decide to rely on laboratories to analyse further the supplied parts [14,15]. Moreover, spare parts are often stored until the results are published, which increases storage costs and therefore renders emergency supplies significantly long and expensive [15].

On-demand distribution, parametric approaches rely upon a lead-time demand distributional assumption, and the use of an appropriate forecasting procedure for estimating the characteristics of a given distribution (i.e., means and variance) [6,7,16]. For the case of fast-moving items, the Normality assumption is typically sufficient [17]. However, Stock Keeping Units (SKUs) often exhibit intermittent or irregular demand patterns that may not be represented by the Normal distribution [7]. This is almost the case for spare parts since demands arise whenever a component fails or requires replacement instead of being generated according to buying behaviours of end-consumers [7,17]. We refer to [18] for further details on such non-Normal demand patterns. Demand for spare parts exhibits thus an infrequent rate, extreme dispersal over time periods, with some time periods having no demand. In the literature, intermittent demand patterns are built from compound elements, namely a demand inter-arrival time and distribution of the demand sizes, when demand occurs [6,7,16]. As such, [7] carry out an empirical analysis of the fitness of different compound distributions and their stock-control effects concerning inventories, demands and service levels in real-world contexts. According to the authors, the compound Poisson distribution (called the

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stuttering Poisson), a combination of a Poisson distribution for demand occurrence and a geometric distribution for demand size, outperforms in all considered data sets. In this paper, three compound distributions are considered. In addition to stuttering Poisson distribution, we conduct experiments for two other distributions, namely Poisson distribution for demand occurrence, combined with demands of constant size and negative binomial distribution.

This paper contributes to the literature in three main dimensions. We study a two-level spare parts supply chain in which a manufacturer supplies a CW with OEM and PP parts. The CW, distantly located from the manufacturer, distributes both OEM parts and PP to a given number of depots facing stochastic demands. These depots may thus form a virtual pool of their parts inventories, allowing LT. Unlike other research, our approach integrates LT decisions in the design of routes that carry out regular shipments from CW. Substitutions are also possible among parts for which waiting is not an option. In addition, parts are assumed to be substitutable only if they have the same shape, fit, and function. Substitution is also assumed to be not bi-directional. That is, part 1 is, for example, substituting part 2, and the inverse is not necessarily applied. Considering the aforementioned multi-sourcing options and assuming that emergency shipments are significantly longer and more expensive, we aim at underlining the relative effectiveness of the spare parts inventory management policy based on LT and substitutions. We model the problem as a two-stage stochastic multi-product multi-vehicle IRP considering LT and substitution as emergency measures to mitigate shortages. The objective is to minimise the total cost, including the inventory holding cost at the CW and depots, transportation which includes regular shipment and transshipment, substitution and lost demands. To solve the problem, the sample average approximation method (SAA) is used because of its good convergence properties. Based on empirical goodness-of-fit tests of [7], three different demand patterns are studied, namely Poisson distribution for demand occurrence, combined with demands of constant size, stuttering Poisson distribution and negative binomial distribution. The model is tested on well-known benchmark instances generated for multi-product multi- vehicle IRP. Computational experiments provide insights into the benefits of promoting transshipment and substitution on the overall supply chain performance. They also suggest findings that may interest practitioners willing to improve decision support models for the most effective management of such items.

The remainder of the paper is structured as follows. Section 2 presents related works. After a detailed description of the problem in Section 3, a mathematical formulation and a solution approach are provided in Section 4. Then, Section 5 provides computational results. Finally, Section 6 presents conclusions and perspectives.

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2. Related work

The classical IRP includes inventory management, vehicle routing, and delivery scheduling decision problems [19]. Such decisions can be streamlined by introducing a vendor managed inventory (VMI) approach, which incorporates replenishment and distribution processes, resulting in overall logistics cost reduction. The deterministic versions of the IRP have been widely studied. Applications can be found in [20-24]. The most closely aligned work to this paper is the one that studied the stochastic variant of the IRP. The Stochastic IRP (SIRP) is similar to the deterministic IRP except that the customer's demand is known in probabilistic sense [19]. More recent work on SIRP includes the one of [25], in which stock-outs and finite horizon SIRP is studied and solved using a dynamic programming model and a hybrid roll-out algorithm. A similar problem is addressed in a robust optimisation approach through MILP formulations by [26], who suggest a robust-based strategy for these demands that assumes a uniform random behaviour. In [27], the authors develop a modified ant colony optimisation metaheuristic for the multi-product SIRP. A robust inventory routing policy, considering stochastic customer demands and replenishment lead times, is addressed in [28]. Dynamic SIRP under Maximum Level (ML) and Order Up-To level (OU) policies is studied in [29]. The authors use a proactive and reactive approach to solve the problem. In [30], this work is extended by addressing the robustness of inventory replenishment and customer selection policies. In [31], a SIRP with split deliveries and service level constraints is addressed. In [32], a multi-period IRP with stochastic stationary demand through a deterministic equivalent approximation model is studied. Finally, in [33] the authors study a SIRP and incorporate constructive components in a simheuristic they use to solve the problem.

It is in the latter context that in [34], the authors introduce the concept of LT between customers within a deterministic inventory-routing (IRPT). In [34], a single-product, single-vehicle IRPT is studied and an Adaptive Large Neighbourhood Search heuristic (ALNS) is used to solve large scale instances. The authors assume that shortages are not permitted, and LT is performed by a carrier's vehicles and not by the supplier's. In [11], the authors extend the work of [34] by proposing new sets of valid inequalities to strengthen the linear relaxation. A multi-product IRPT is studied and solved using a Randomized Variable Neighbourhood Descent in [35]. On stochastic IRPT (SIRPT), [29] study the stochastic version of the problem addressed in [34]. Under the same assumptions, the authors propose a reactive and proactive policy to solve the single-product, single-vehicle SIRPT. In [36], a multi-product, multi-vehicle SIRPT is addressed and solved using a Variable Neighbourhood Search algorithm. In [37], a mathematical model that decides on proactive transshipment under stochastic demand to reduce total

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costs, as well as shortages in a blood supply chain, is developed. Finally, single-product SIRPT is studied in [38]. Lot sizing and perishability of the product are also considered. The authors propose a Lagrangian relaxation- based heuristic to solve the problem.

Based on this literature review, it can be stated that fewer papers on IRPT and SIRPT study a multi-product multi-vehicle version and take into account lost sales due to the shortages as a measure of the service quality. Moreover, in the design of vehicle routing, LT-related decisions are not integrated as LT is always assumed to be subcontracted or handled by another carrier's vehicles. Indeed, to simplify the optimization problem, authors identify just the nodes and time periods when LT may take place and manage stocks so LT may be performed. With this respect, this paper is thus intended to fill this gap. Furthermore, to the best of the authors' knowledge, none of the existing papers incorporates product substitution within the settings along with promoting LT between depots to avoid shortages of parts. We study in the following a multi- product multi-vehicle SIRPT and ML inventory policy. Therefore, we develop an appropriate model to underline the relative effectiveness of this new spare parts inventory management policy based on LT and substitutions and use the SAA method to solve it.

3. Problem description

We consider a spare parts supply chain with two levels. The CW distributes different parts to a certain number of depots (the second level). Compared to the distance from the CW, the depots are located at a negligible distance away from each other, and all hold low-demand spare parts. By allowing LT, these depots form a pool that can share inventories. In this paper, we assume that LT is not outsourced and is incorporated into the routes that carry out regular shipments. We also assume that PP can also be transshipped between depots. Given the demand distributions of depots, at the beginning of the planning horizon, the CW's manager needs to choose the routes, inventory levels and transshipment and substitution decisions. Once demand is realised, if it exceeds the available capacity at the level of each depot, spare part substitution and LT are used as a recourse. That is, when critical OEM parts are not available, transshipped parts and the compatible PP can be used to satisfy the demand at the depots. As is commonly the case, we assume that the vehicle capacity is expressed as a function of the demands to be satisfied [20,29,39]. We also assume that all PP made available to depots are reliable and of a good quality. Parts are substitutable only if they have the same shape, fit, and function. Furthermore, we assume that the substitution is not bi-directional. We also implicitly assume that emergency supplies from the CW are significantly long and expensive. Therefore, LT and substitutions is favoured over the use of emergency supplies. Transshipment and substitution can be then considered by

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depots to meet expected demands with the use of the same part from the inventory of other locations (transshipment) or with the use of a compatible part from their inventory (substitution).

The problem of concern can be addressed as a two-stage stochastic inventory-routing problem considering transshipment and substitution as emergency measures to mitigate shortages. We assume the routing (which depots to visit in each period) is the first-stage decision. The quantities to deliver including transshipment, the lost sales at each time period as well as the inventory levels and substituted quantities are adjusted to the scenario. The stochastic approach aims to find the solution that minimises the routing cost plus the expected cost of both inventory, transshipment, substitution, and shortage due to loss of sales.

4. Mathematical formulation

4.1. Mathematical modelling

The SIRPTS is defined on a graph $G = (N, A)$, where N is the vertex set indexed by $i \in \{0, \ldots, n\}$ and $A = \{(i, j) : i, j \in N, i \neq j\}$ is the edge set. Vertex 0 represents the CW, and the set $N_0 = N \setminus \{0\}$ denotes the depots. The length of the planning horizon is H with discrete time periods t. Each depot $i \in N_0$ has demands for spare part $p \in \{1, . . . , m\}$ per period $t \in H = \{1, . . . , T\}$ which is a random variable D_{pit}. A scenario denoted ω is a set of potential demands that appear by the end of the horizon H. We denote the set of scenarios for the realisation of demands by Ω. Thus, $D_{pit}(\omega)$ denotes the demand of a depot i of a spare part p in period t in scenario $\omega \in \{1, . . . , |\Omega|\}$. Moreover, each depot and the CW, $i \in N$, incur unit inventory holding costs, h_{pi}, per period and per spare part $p \in \{1, . . . , m\}$, with inventory capacities K_i. Inventories are not allowed to exceed the holding capacity and must be positive. At the beginning of the planning horizon, at each location, $i \in N$, the current inventory levels I_{pi0} of the spare part p are known. A set of homogeneous vehicles $v \in V = \{1, . . . , k\}$ is available, each with a capacity Q in terms of the spare part without distinction between them, with a being a fixed transportation cost per km. Each vehicle is able to perform a route per period. A distance d_{ij} is associated for all $(i, j) \in A$. A transshipment can start from any depot, i.e., a depot can transfer to other locations as needed. Transshipments can occur when it is profitable to ship spares between depots. As in [34], we choose a transshipment cost per unit of $b_{ij} = 0.01\, a d_{ij}$. The unit cost of substituting a spare part p by $s \in P$ is c_{ps}. All possible combinations according to the parts' compatibility are represented by o_{ps}, which is equal to 1 if a spare part p is compatible with a spare part s, and 0 otherwise. The lost sales cost associated with a spare part p at the depot i is f_{pi}, and the quantity of spare parts shipped by the manufacturer (from the

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factory) to the CW during period t is g_{pt} . Finally, it is assumed that holding and vehicle capacities, and quantity shipped by the manufacturer are exogenous parameters and are not under the control of the CW. All of these notations as well as decision variables are summarised in Table 1:

The formulation of the SIRPTS can be written as:

$$\min \sum_{t\in\mathcal{H}}\sum_{v\in V}\sum_{i,j\in\mathcal{N}} a d_{ij} x_{ijvt} + \frac{1}{|\Omega|}\sum_{\omega\in\Omega}\left[\sum_{t\in\mathcal{H}}\sum_{i\in\mathcal{N}}\sum_{p\in\mathcal{P}} h_{pi} I_{pit}(\omega) + \sum_{p\in\mathcal{P}}\sum_{t\in\mathcal{H}}\sum_{v\in V}\sum_{i,j\in\mathcal{N}_0} b_{ij} y_{pijvt}(\omega) + \sum_{t\in\mathcal{H}}\sum_{i\in\mathcal{N}_0}\sum_{p\in\mathcal{P}} f_{pi} w_{pit}(\omega) + \sum_{t\in\mathcal{H}}\sum_{i\in\mathcal{N}_0}\sum_{p,s\in\mathcal{P}} c_{sp} z_{spit}(\omega)\right] \quad (1)$$

Subject to:

$$I_{pit}(\omega) = I_{pit-1}(\omega) + Q_{pit}(\omega) - D_{pit}(\omega) + w_{pit}(\omega) + \sum_{v\in V}\sum_{j\neq i\in\mathcal{N}_0}\left(y_{pjivt}(\omega) - y_{pijvt}(\omega)\right) + \sum_{s\neq p\in\mathcal{P}}\left(z_{spit}(\omega) - z_{psit}(\omega)\right) \quad \forall p\in\mathcal{P}, i\in\mathcal{N}_0, t\in\mathcal{H} \quad (2)$$

$$I_{p0t}(\omega) = I_{p0t-1}(\omega) - \sum_{i\in\mathcal{N}} Q_{pit}(\omega) + g_{pt} \quad \forall p\in\mathcal{P}, t\in\mathcal{H} \quad (3)$$

$$Q_{pjt}(\omega) = \sum_{v\in V}\sum_{i\neq j\in\mathcal{N}_0}\left(y_{pijvt}(\omega) - y_{pjivt}(\omega) + q_{pijvt}(\omega) - q_{pjivt}(\omega)\right) \quad \forall p\in\mathcal{P}, j\in\mathcal{N}_0, t\in\mathcal{H} \quad (4)$$

$$\sum_{p\in\mathcal{P}} q_{pi0vt}(\omega) = 0 \quad \forall i\in\mathcal{N}_0, v\in V, t\in\mathcal{H} \quad (5)$$

Table 1*- Notation summary.*

Sets	
$\mathcal{N}$	Set of nodes including CW
$\mathcal{N}_0$	Set of depots
$\mathcal{H}$	Planning horizon indexed by t
V	Set of vehicles indexed by v
$\mathcal{P}$	Set of spare parts indexed by p
Ω	Set of scenarios indexed by ω; $\omega\in\{1,\dots,|\Omega|\}$
Parameters	
a	Transportation cost per km
d_{ij}	Distance in km between $(i,j)\in\mathcal{A}$
h_{pi}	Unit inventory holding cost per period for spare part p at node $i\in\mathcal{N}$
b_{ij}	Unit transshipment cost between node $i\in\mathcal{N}$ and node $j\in\mathcal{N}$
K_i	Maximum inventory capacity at node $i\in\mathcal{N}$
I_{pi0}	Inventory level of a spare part p at location $i\in\mathcal{N}$ at the beginning of the planning horizon
Q	Vehicle capacity
c_{sp}	Unit cost associated with the substitution of a spare part p by a substitute s
o_{sp}	Equal to 1 if a substitute part s is compatible with a spare p, and 0 otherwise
Variables	
x_{ijvt}	Equal to 1 if $(i,j)\in\mathcal{A}$ is visited by the vehicle v in period t, 0 otherwise
u_{vt}	Equal to 1 if the vehicle v is used in period t, 0 otherwise
$I_{pit}(\omega)$	Inventory level of a spare part p at a location $i\in\mathcal{N}$ at the end of period t for a scenario ω
$Q_{pit}(\omega)$	Quantity of a spare part p delivered from the CW to the depot $i\in\mathcal{N}_0$ in a period t for a scenario ω
$q_{pijvt}(\omega)$	Quantity of a spare part p transported from a location $i\in\mathcal{N}$ to a location $j\in\mathcal{N}$ by vehicle v in a period t for a scenario ω. This quantity includes regular shipment from CW and transshipment between depots
$y_{pijvt}(\omega)$	Quantity of a spare part p transshipped from the depot $i\in\mathcal{N}_0$ to the depot $j\in\mathcal{N}_0$ by vehicle v to address a shortage of spare p in a period t for a scenario ω
$w_{pit}(\omega)$	Lost sales quantity of a spare part p at the depot $i\in\mathcal{N}_0$ in a period t for a scenario ω
$z_{spit}(\omega)$	Quantity of a spare part s substitute for spare part p used at the depot i in period t for a scenario ω to satisfy a part of the unsatisfied demand for $D_{pit}(\omega)$ and for all $o_{sp=1}$

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$$\sum_{p\in\mathcal{P}} I_{pit}(\omega) \leq K_i \quad \forall i \in \mathcal{N}, t \in \mathcal{H} \tag{6}$$

$$\sum_{p\in\mathcal{P}} q_{pijvt}(\omega) \leq Qx_{ijvt} \quad \forall i, j \in \mathcal{N}, v \in V, t \in \mathcal{H} \tag{7}$$

$$\sum_{p\in\mathcal{P}} q_{pijvt}(\omega) \leq Qu_{vt} \quad \forall i, j \in \mathcal{N}, v \in V, t \in \mathcal{H} \tag{8}$$

$$\sum_{v\in V} \sum_{j\neq i\in\mathcal{N}_0} y_{pijvt}(\omega) \leq I_{pit-1}(\omega) \quad \forall p \in \mathcal{P}, i \in \mathcal{N}_0, t \in \mathcal{H} \tag{9}$$

$$\sum_{i\neq j\in\mathcal{N}} x_{ijvt} = \sum_{i\neq j\in\mathcal{N}} x_{jivt} \quad \forall j \in \mathcal{N}, v \in V, t \in \mathcal{H} \tag{10}$$

$$\sum_{i\neq j\in\mathcal{N}} \sum_{v\in V} x_{ijvt} \leq 1 \quad \forall j \in \mathcal{N}_0, t \in \mathcal{H} \tag{11}$$

$$\sum_{j\in\mathcal{N}_0} x_{0jvt} = u_{vt} \quad \forall v \in V, t \in \mathcal{H} \tag{12}$$

$$\sum_{v\in V} u_{vt} \leq k \quad \forall t \in \mathcal{H} \tag{13}$$

$$y_{pijvt}(\omega) \leq x_{ijvt} K_j \quad \forall i, j \in \mathcal{N}, p \in \mathcal{P}, v \in V, t \in \mathcal{H} \tag{14}$$

The objective function minimises the total cost. The first term corresponds to the transportation costs, the second term to the inventory cost at the CW and depots' locations, the third term to the transshipment costs, the fourth term to lost sales costs at the depots' locations, and the last term corresponds to the cost of substitutions. Constraints (2) indicate that, for each depot *i* and each spare part *p,* the inventory level at period *t* is the inventory level at the previous period plus the delivered quantity of the spare part *p* and the lost sales w_{pit}, if any, minus the demand, plus the difference between the quantity of spare part *p* transshipped to and from *i,* plus the difference between the quantity of spare part *s* used as a substitute of *p* and the quantity of *p* used as a substitute of the other spare parts. Constraints (3) express the conservation conditions of inventory at the CW over successive periods. The conditions take into account quantities delivered to the CW and the depots. Constraints (4) express the flow conservation conditions at a depot *j*. Constraints (5) state that at the end of each period, vehicles must return empty to the CW. Constraints (6) guarantee that inventory levels do not exceed the maximal available inventory capacity. Constraints (7) and (8) state that the maximum capacity of the vehicle is not exceeded. Constraints (9) state that the quantity transshipped from a depot *i* at a period *t* does not exceed the initial inventory level at this period. Constraints (10) stipulate that if a vehicle *v* visits the depot *j,* it must leave *j* in the same period *t*. Constraints (11) ensure that, at most, one vehicle *v* visits a depot per period. Constraints

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(12) ensure that only vehicles carrying parts leave the CW. Constraints (13) stipulate that the sum of vehicles used in time period *t* is bounded by the number of available vehicles. Constraints (14) ensure that spare part *p* is transshipped from the node *i* to the node *j* by vehicle *v* if the arc *(i, j)* is being used by the vehicle *v* during the period *t*.

4.2. SAMPLE AVERAGE APPROXIMATION

Given their inherent analytical complexities and high computational requirements, solving large-scale stochastic optimisation problems is extremely challenging [40,41]. The SAA method's sound convergence properties, which have been thoroughly explored in the literature, are one of its most appealing features [42,43]. Regarding SAA estimators' consistency, which is often seen as a minimum criterion for any good estimator, [44] stress, in a somewhat general fashion, that the sequence of approximate objective function epi-converges to the optimal solution. This enables the inference of sets of optimal values with high consistency. An alternate approach, based on the epic convergence, has been followed. This approach draws from a strong consistency of optimal estimators, by constructing almost sure uniform convergence [45]. In [42], the authors examine the rates at which optimum SAA estimators converge in the almost sure and mean sense with their deterministic counterparts. Finally, in [46], the SAA method is applied to three classes of 2-stage stochastic routing problems. Through a considerable amount of experimentation, the authors proved the good convergence properties of SAA and the high quality of solutions to the stochastic programming problems under consideration. The interested reader is referred to this paper for further details. For all these reasons, SAA has been widely used to cover a large variety of applications such as stochastic supply chain design and optimisation problems of large scale [47]; stochastic knapsack problem [40] and reliability-based design of engineering systems [48].

In SAA, the objective function value of the stochastic problem is unknown and approximated using a random sample estimate [49,50]. For a given number of scenarios, the objective function is evaluated iteratively before the optimality gap falls below a certain threshold value. SAA provides a straightforward framework that is conducive to parallel implementation and reduction of variance techniques. It also possesses good convergence properties and well-developed statistical methods to validate solutions and perform error analysis. For this reason, SAA is used to solve the SIRP with transshipment and Substitution (SIRPTS).

SAA approximates the expected cost of the objective function by the average sample function. This expected cost is replaced by the mean value of a random set of samples $(\omega^1, \omega^2, \ldots, \omega^{|\Omega|})$ of size $|\Omega|$ obtained by the Monte Carlo method, where Ω is the set of ω-indexed scenarios. This is repeated *L* times with different samples, and each time results

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in a candidate solution. Thus, SAA method generates L separate sample sets Ω_l, $l \in \{1, \ldots, L\}$. For each scenarios set, Ω_l, the related SAA problem (where Ω is replaced by Ω_l in SIRPTS) is solved and generates a candidate solution. Therefore, the first-stage solution is fixed for each candidate solution, and the value of the objective function for a very large sample with l scenarios is computed. This value is computed in the two-stage model SIRPTS by solving a pure linear programming problem on the second-stage variables.

With a reasonable level of accuracy, SAA solves the real problem if certain conditions are met [40,51]. These requirements and justifications for how SIRPTS meets them are as follows:

1. a sample realisation of the random variable can be generated. For SIRPTS, this can be done for each random variable D_{pit} which represents demand each depot i has to satisfy for each spare part p and for each period t (see Section 5.1.2).
2. with a moderate sample size, the SAA problem can be solved effectively. In the computational experiments section, we will show that for most test instances, with a sample size of 20, we can solve SIRPTS in a reasonable amount of time.
3. the expected costs can be easily calculated by solving the model for a given first-stage solution and a given realisation of demand.
4. there is a complete recourse to the actual problem, i.e. every solution to the first stage problem is feasible to the second stage. In SIRPTS, this is made possible by assuming that transshipment and substitutions are always used when demand cannot be met with the related first stage variables.

Statistical estimates of the lower and upper bounds on the objective function value of the stochastic problem, as well as estimates of the variances of these bounds, can be computed in order to evaluate the quality of the SAA solution [40,51]. Let $\hat{X}^l$ be a candidate solution to the first stage with an objective function $\hat{F}^l$. To estimate the lower bound of the true objective function value, the mean $\bar{F}$ and the variance $\hat{\sigma}^2_{\Omega,L}$ of the objective function $\hat{F}^1, \ldots, \hat{F}^L$

$$\bar{F} = \frac{1}{L}\sum_{l=1}^{L}\hat{F}^l \quad (15)$$

$$\hat{\sigma}^2_{\Omega,L} = \frac{1}{L(L-1)}\sum_{l=1}^{L}(\hat{F}^l - \bar{F})^2 \quad (16)$$

The lower bound is then expressed as:

$$LB = \bar{F} - \chi_{\pi,\rho}\hat{\sigma}_{\Omega,L} \quad (17)$$

where $\chi_{\pi,\rho}$ is the π-critical value of the χ-distribution with p degrees of freedom, p — L - 1

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By assessing the solution with a huge scenario tree of size $|\Omega'|$ that is assumed to represent the true distribution of demand, the upper bound on each candidate solution's true objective function value is computed. As each scenario $\omega \in \{1,\dots, |\Omega'|\}$ is an i.i.d. random sample, the problem of assessing a candidate solution is broken down into Ω' sub-problems. The size of Ω', which is the scenario tree, is far larger than the one held in any SAA run [40]. The objective function value of a given sub-problem ω is denoted as $\psi_\omega(X^l)$, which is calculated as:

$$\psi_\omega(X^l) = \sum_{t\in\mathcal{H}}\sum_{i\in\mathcal{N}}\sum_{p\in\mathcal{P}} h_{pi} I_{pit}(\omega) + \sum_{p\in\mathcal{P}}\sum_{t\in\mathcal{H}}\sum_{v\in V}\sum_{i,j\in\mathcal{N}_0} b_{ij} y_{pijvt}(\omega) + \sum_{t\in\mathcal{H}}\sum_{i\in\mathcal{N}_0}\sum_{p\in\mathcal{P}} f_{pi} w_{pit}(\omega) + \sum_{t\in\mathcal{H}}\sum_{i\in\mathcal{N}_0}\sum_{p,s\in\mathcal{P}} c_{sp} z_{spit}(\omega) \tag{18}$$

It has be noted that $|\Omega'|$ can be very large since each subproblem is solved separately without creating a significant computation complexity. The approximation of the true objective value, denoted as $\psi(\chi^l)$, of the second stage problem is computed as:

$$\psi(X^l) = \frac{1}{|\Omega'|}\sum_{\omega=1}^{|\Omega'|}\psi_\omega(X^l) \tag{19}$$

The value of the true objective function, $\bar{F}^l$, for a candidate $\hat{X}^l$ and its variance $\hat{\sigma}^2_{\Omega'}(X^l)$ as computed then as follows:

$$\bar{F}^l = \psi(X^l) + \sum_{t\in\mathcal{H}}\sum_{v\in V}\sum_{i,j\in\mathcal{N}} ad_{ij} x_{ijvt} \tag{20}$$

$$\hat{\sigma}^2_{|\Omega'|}(X^l) = \frac{1}{|\Omega'|(|\Omega'|-1)}\sum_{\omega=1}^{|\Omega'|}(\psi_\omega(X^l) - \psi(X^l))^2 \tag{21}$$

The upper bound of the candidate $\hat{X}^l, \phi^l$ is then computed as:

$$\phi^l = \bar{F}^l - \tau_\pi \hat{\sigma}^2_{|\Omega'|}(X^l) \tag{22}$$

where τ_π is the π-critical value of the τ-distribution. The upper bound of the algorithm is the smallest ϕ^l and the candidate solution $\hat{X}$ refers to the solution with the smallest upper bound solution that results in the smallest optimality gap for all candidate solutions:

$$\hat{X} = argmin_{l\in\{1..|L|\}}(\phi^l) \tag{23}$$

5. Computational experiments

This section presents the experimental design adopted in this paper as well as the computational results.

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5.1. EXPERIMENTAL DESIGN

5.1.1. SAA Setting

To solve the SIRP, |Ω| = 20 scenarios is used to compute the expected costs of the second stage. Each scenario is repeated *L*= 20 times in order to compute the LB on the true expected value. This choice is based on a trade-off between the op-timality gap and computational time. For the UB on expected cost, all *L* candidate solutions are assessed using a scenario tree of |Ω'| = 800. Finally, all optimisation steps are carried out with a personal computer (MacBook Pro, macOS Big Sur, 3.3 GHz Quad-Core Intel Core i7 CPU with 8 GB of RAM), and with CPLEX 12.9 and Python 3.7. A maximum time limit of 1200 seconds is fixed.

5.1.2. Demand distributions

The demand for spare parts occurs when a component fails or needs replacement rather than being triggered by the end-user purchasing behaviours (and the way demand moves upstream in a supply chain). Then, it is possible to identify such items as sporadic and slow movers arising at irregular intervals and variable sizes. It is preferable to model spare parts demand from the constituent components, i.e. the size of demand and the inter-demand interval. Consequently, compound theoretical distributions (which specifically include the combination of size and interval) are widely used in such application contexts [6,7,52]. In this paper, demands each depot *i* has satisfy per spare part *p* and per period *t* are random variables D_{pit}. Different distributions have been studied based on empirical goodness-of-fit tests. Discrete distributions are chosen since they provide a better fit for intermittent demands compared to continuous ones. According to [7], these distributions are: (1) Poisson distribution (PD) for demand occurrence with demands of constant size; (2) stuttering Poisson distribution (SPD), with Poisson distribution for demand occurrence and geometric distribution for demand size; and (3) negative binomial distribution (NBD), with a Poisson distribution for demand occurrence and logarithmic distribution for demand size.

For β = 0,1,2 . the distribution functions of Poisson distribution occurrence $PD_\lambda(\beta)$ can be expressed as:

$$PD_\lambda(\beta) = \frac{\lambda^\beta \ e^\beta}{\beta!} \tag{24}$$

the stuttering Poisson distribution $SPD_{(\lambda,\varnothing)}(\beta)$ as

$$SPD_{(\lambda,\theta)}(\beta) = \sum_{1\le i\le \beta} e^\beta \frac{\lambda^\beta}{i!}\binom{\beta-1}{i-1}\theta^i(1-\theta)^{\beta-i} \tag{25}$$

where λ and $\varnothing$ are the Poisson and geometric distribution parameters, and the Negative

Binomial distribution *NBD(r,* μ*)(*β*)* as:

$$NBD(r, \mu)(\beta) = \binom{\beta + r - 1}{\beta} \mu^r (1 - \mu)^\beta \tag{26}$$

where *r* is the number of successes, and *i* is the probability of success.

To generate an independent and identical distributed (i.i.d.) random sample of |Ω| realisations of D_{pit} for each distribution under consideration, the Inverse Transform Sampling algorithm 1 is used.

Algorithm 1 Inverse Transform Sampling.

```
procedure ITS(F)                                          ▷ F is a distribution function
    α ← Generate random number from the standard uniform distribution in [0, 1];
    β ← F^{-1}(α)
end procedure
```

5.1.3. Other input data

First, the model is tested on randomly instances generated by [39] for multi-product multi-vehicle IRP. Following a brief description is provided, and the reader is referred to their paper for further details. The dataset can be downloaded from http://www.leandro-coelho.com/instances/. For each instance, the number of depots varies between 10 and 50, and the number of both products and vehicles varies between 1 and 5. Each instance contains 3, 5 and 7 periods. Product availability at the CW is a multiple of a number randomly generated according to a discrete uniform distribution in the interval [50,140], and the maximum inventory level is a multiple of a number drawn randomly from [150,200]. The initial inventory level is a randomly generated number in the interval [100, 150]. Holding costs are randomly generated from a continuous uniform distribution in the interval [0.02,0.2]. As in [29], shortage penalty cost equals 200 times the holding cost.

Secondly, to highlight the benefit of promoting substitutions along with transshipment on the overall supply chain performance, we use the same set of instances but this time for a number of products varying between 20 and 40. For each depot, period and spare part Poisson and geometric distribution parameters λ and *Ø,* as well as the NBD parameters *r* and μ*,* are random numbers generated between 0 and 1.

An instance name is referred to as n [number of depots] m [number of spare parts] k [number of vehicles] T [number of periods], e.g. n5m20k2T5 is an instance consisting of 5 depots, demands for a number of spare parts equal to 20 performed by 2 vehicles in a planning horizon that corresponds to 5 days.

5.2. COMPUTATIONAL RESULTS

This section present for each distribution, computational results obtained for all instances under consideration. Tables 2, 4, 6, 8, 10 and 12 report costs computed for the first and second stage (FSC, SSC), standard deviation regarding the upper and lower bound (UB, LB), and CPU time in second. Tables 3, 5, 7,9 , 11 and 13 provide for all instances under consideration the breakdown of costs namely: Transportation (T), Inventory (I), Lost sales (LS), substitution (S) and transshipment (Ts). All experiments are performed for four different models: SIRP, SIRP with Transshipment (SIRPT), SIRP with Substitution (SIRPS) and finally SIRPTS. They also report cost saving (SV) computed with regard to total cost (TC), which is expressed as follows:

$$SV = \frac{TC_{SIRP} - TC_{SIRP-with-X}}{TC_{SIRP}} \tag{27}$$

From Tables 2, 4, 6, 8, 10 and 12 we can notice that for all instances and demand patterns under study any reduction in costs is made possible by allowing LT between depots and substitution of spare parts. Indeed, from Tables 3, 5, 7, 9, 11 and 13, we can see that these emergency measures allow to reduce holding and transportation costs along with lost sales.

For all sets of instances and models, we can see that when substitution and LT are not considered, the supply chain seems to experience high transportation, inventory and lost sales costs for the three different distributions. When LT is allowed (SIRPT), the depots receiving the quantity latterly transshipped can satisfy even more (sometimes completely) demand and consequently reduce lost sales. The depots from which the LT is carried out are, in counterpart, allowed to lower their inventory holding costs. When the substitution is also allowed (SIRPS), compared to the SIRP model, we observe a reduction of the costs of lost sales and holding inventory. Indeed, quantities that might be latterly transshipped from a depot can be used at its level as a substitute to other spare parts, which allows reducing the lost sales. For SIRPTS, we observe that it allows reducing costs considerably compared to the other models. In addition to what can be received through LT, each depot can use the quantities of spare parts, if compatible, that could constitute idle stock (which leads to high holding cost) to meet other spares' demand. Moreover, PP can also be transshipped to substitute other spare parts through substitution based on our assumptions. Finally, when compared with SIRPS, the SIRPT provides, in general, the same results in terms of reduction of total costs. From Table 9 for example, we can see that for PD, on average SIRPS allows a saving of 16% while SIRPT allows a saving of 17%.

Concerning the demand patterns, we observe that a very low variability in demand size (as in the case of PD) can be less stressful, regardless of the average inter-demand interval. Indeed, when demand size is greater than the quantity available to promise, the emergency measures cannot be sufficient to mitigate any loss of sales.

***Table 2** Computational results for a number of products varying between 1 and 5 - number of depots equal to 10*

Instances	Model	PD					SPD					NBD				
		FSC	SSC	LB (%)	UB (%)	CPU (s)	FSC	SSC	LB (%)	UB (%)	CPU (s)	FSC	SSC	LB (%)	UB (%)	CPU (s)
n10m1k3T3	SIRP	1,590.2	2,512.6	0.01	0.09	30	2,116.6	2,921.7	0.06	0.07	36	2,409.4	3,868.3	0.04	0.09	45
	SIRPT	1,436.8	2,091.6	0.07	0.00	45	1,696.1	2,456.9	0.03	0.08	50	2,160.0	3,031.1	0.02	0.05	63
n10m1k3T5	SIRP	3,430.0	5,100.2	0.08	0.07	82	3,719.8	5,825.5	0.10	0.01	91	3,869.7	7,956.9	0.09	0.05	114
	SIRPT	2,995.1	4,511.5	0.07	0.01	75	3,105.2	5,339.7	0.01	0.05	90	3,480.2	6,273.7	0.03	0.04	121
n10m1k3T7	SIRP	4,820.3	7,291.1	0.07	0.01	116	5,741.6	8,961.6	0.01	0.04	129	7,043.9	9,908.9	0.05	0.02	161
	SIRPT	4,255.8	6,402.3	0.08	0.03	113	4,483.3	7,389.8	0.06	0.05	124	5,655.1	8,117.7	0.01	0.01	160
n10m3k3T3	SIRP	1,928.0	2,722.3	0.09	0.01	48	2,219.4	3,430.7	0.01	0.08	63	2,506.8	3,764.9	0.10	0.06	78
	SIRPT	1,572.1	2,427.1	0.03	0.04	34	1,914.3	2,896.8	0.04	0.09	37	2,385.0	3,378.7	0.08	0.10	46
	SIRPS	1,618.0	1,567.2	0.08	0.06	60	2,143.6	1,945.7	0.09	0.06	74	1,955.8	3,774.7	0.04	0.07	92
	SIRPTS	1,686.2	1,141.5	0.09	0.01	75	2,158.9	905.3	0.02	0.10	97	2,604.9	1,710.1	0.01	0.02	120
n10m3k3T5	SIRP	3,768.4	5,835.1	0.06	0.09	89	4,589.7	7,145.7	0.09	0.06	109	5,677.9	9,073.6	0.00	0.06	134
	SIRPT	3,401.3	4,953.7	0.07	0.07	68	4,112.0	6,031.0	0.03	0.09	84	4,500.6	7,549.3	0.01	0.08	102
	SIRPS	3,537.7	5,009.5	0.04	0.06	41	3,508.1	6,201.5	0.05	0.00	47	4,657.0	7,489.6	0.00	0.03	59
	SIRPTS	3,463.2	4,891.9	0.06	0.06	58	3,677.1	4,513.4	0.03	0.04	71	4,276.2	5,934.5	0.05	0.02	92
n10m3k3T7	SIRP	5,344.0	7,946.3	0.06	0.09	87	6,189.7	9,971.3	0.09	0.03	104	7,173.8	11,282.1	0.02	0.05	139
	SIRPT	4,649.3	7,311.9	0.06	0.01	111	5,194.3	8,489.4	0.08	0.01	133	5,751.7	9,916.2	0.05	0.09	178
	SIRPS	4,655.1	6,907.5	0.07	0.04	75	5,397.3	7,518.0	0.02	0.02	89	5,725.0	9,553.9	0.06	0.01	109
	SIRPTS	4,519.3	6,910.4	0.08	0.02	74	5,263.8	5,669.0	0.05	0.07	85	5,486.2	8,536.5	0.04	0.08	105
n10m5k3T3	SIRP	2,427.0	3,663.3	0.06	0.06	69	2,911.1	4,068.3	0.06	0.01	79	3,107.2	5,009.9	0.06	0.02	101
	SIRPT	2,212.8	3,268.5	0.02	0.01	63	2,654.7	3,939.2	0.04	0.03	71	2,873.5	4,870.3	0.02	0.03	89
	SIRPS	2,010.2	3,227.4	0.08	0.00	79	2,452.0	3,482.2	0.06	0.01	103	2,399.8	5,264.4	0.05	0.03	139
	SIRPTS	2,201.2	2,019.2	0.07	0.01	71	2,581.2	1,679.3	0.08	0.02	81	2,828.6	3,615.1	0.04	0.10	104
n10m5k3T5	SIRP	4,883.5	7,056.7	0.09	0.09	180	5,099.8	8,583.6	0.04	0.03	203	5,588.2	10,188.8	0.02	0.03	258
	SIRPT	4,094.7	6,412.7	0.09	0.00	90	4,888.8	7,688.5	0.04	0.04	116	6,039.0	8,406.0	0.09	0.01	156
	SIRPS	4,311.4	6,434.8	0.01	0.05	96	5,176.6	5,912.2	0.09	0.07	115	6,512.9	7,023.6	0.06	0.01	153
	SIRPTS	4,188.9	5,257.9	0.02	0.02	89	5,040.5	5,640.0	0.07	0.08	114	5,184.1	5,980.0	0.08	0.06	153
n10m5k3T7	SIRP	5,518.0	8,522.7	0.02	0.01	133	6,236.3	9,882.4	0.08	0.02	166	8,595.4	11,407.9	0.07	0.05	213
	SIRPT	5,221.5	7,415.2	0.09	0.09	130	6,147.2	8,751.4	0.04	0.10	168	5,789.0	12,538.1	0.06	0.04	225
	SIRPS	4,863.5	7,632.7	0.05	0.06	100	6,102.8	8,192.9	0.05	0.03	110	5,605.6	10,562.8	0.07	0.01	147
	SIRPTS	4,904.3	5,786.0	0.02	0.01	135	6,215.0	6,329.8	0.01	0.00	169	7,031.5	7,160.4	0.03	0.07	209

***Table 3** Breakdown of cost for a number of products varying between 1 and 5 - number of depots equal to 10.*

Instances	Model	PD						SPD						NBD					
		T	I	LS	S	Ts	SV (%)	T	I	LS	S	Ts	SV (%)	T	I	LS	S	Ts	SV (%)
n10m1k3T3	SIRP	1,590	1,340	1,173	0	0		2,117	2,029	893	0	0		2,409	2,442	1,427	0	0	
	SIRPT	1,437	739	541	0	812	14	1,696	835	624	0	998	18	2,160	1,677	192	0	1,162	17
n10m1k3T5	SIRP	3,430	3,325	1,775	0	0		3,720	4,314	1,511	0	0		3,870	5,474	2,483	0	0	
	SIRPT	2,995	1,819	1,077	0	1,615	12	3,105	2,328	1,159	0	1,852	12	3,480	3,465	596	0	2,213	18
n10m1k3T7	SIRP	4,820	4,284	3,007	0	0		5,742	5,465	3,497	0	0		7,044	5,540	4,369	0	0	
	SIRPT	4,256	2,831	1,350	0	2,222	12	4,483	2,566	1,997	0	2,827	19	5,655	4,828	645	0	2,645	19
n10m3k3T3	SIRP	1,928	1,376	339	1,007	0		2,219	2,188	1,243	0	0		2,507	2,189	1,576	0	0	
	SIRPT	1,572	1,084	534	0	809	14	1,914	1,184	596	0	1,117	15	2,385	1,832	291	0	1,257	8
	SIRPS	1,618	506	490	572	0	32	2,144	762	612	571	0	28	1,956	1,923	605	1,247	0	9
	SIRPTS	1,686	182	182	360	418	39	2,159	222	96	315	273	46	2,605	507	10	532	661	31
n10m3k3T5	SIRP	3,768	2,982	2,853	0	0		4,590	4,311	2,835	0	0		5,678	5,513	3,560	0	0	
	SIRPT	3,401	2,167	1,134	0	1,653	13	4,112	2,673	1,540	0	1,819	14	4,501	4,605	10	0	2,935	18
	SIRPS	3,538	2,385	1,067	1,557	0	11	3,508	2,478	1,682	2,042	0	17	4,657	4,629	725	2,135	0	18
	SIRPTS	3,463	842	526	1,598	1,925	13	3,677	848	305	1,633	1,728	30	4,276	1,831	45	2,081	1,978	31
n10m3k3T7	SIRP	5,344	4,217	3,729	0	0		6,190	7,214	2,757	0	0		7,174	8,738	2,544	0	0	
	SIRPT	4,649	2,260	2,219	0	2,833	10	5,194	3,327	1,872	0	3,290	15	5,752	5,842	462	0	3,612	15
	SIRPS	4,655	2,404	2,023	2,480	0	13	5,397	2,952	1,906	2,660	0	20	5,725	6,953	10	2,591	0	17
	SIRPTS	4,519	1,339	1,164	2,153	2,253	14	5,264	1,184	594	1,944	1,948	32	5,486	2,320	46	2,892	3,278	24
n10m5k3T3	SIRP	2,427	2,432	1,231	0	0		2,911	2,534	1,535	0	0		3,107	2,532	2,478	0	0	
	SIRPT	2,213	1,125	1,075	0	1,068	10	2,655	1,432	1,041	0	1,466	6	2,873	2,444	682	0	1,744	5
	SIRPS	2,010	1,421	859	948	0	14	2,452	1,395	1,004	1,084	0	15	2,400	3,320	1	1,944	0	6
	SIRPTS	2,201	381	170	672	796	31	2,581	407	190	566	517	39	2,829	971	2	1,318	1,324	21
n10m5k3T5	SIRP	4,884	4,156	2,900	0	0		5,100	5,575	3,009	0	0		5,588	5,458	4,731	0	0	
	SIRPT	4,095	2,753	1,511	0	2,149	12	4,889	2,701	1,841	0	3,146	8	6,039	4,311	875	0	3,219	8
	SIRPS	4,311	2,943	1,508	1,983	0	10	5,177	2,529	1,627	1,757	0	19	6,513	4,832	211	1,981	0	14
	SIRPTS	4,189	683	673	1,904	1,997	21	5,040	1,247	434	1,857	2,101	22	5,184	1,769	351	1,970	1,890	29
n10m5k3T7	SIRP	5,518	5,922	2,601	0	0		6,236	5,506	4,376	0	0		8,595	7,830	3,578	0	0	
	SIRPT	5,221	2,853	2,027	0	2,535	10	6,147	3,830	1,910	0	3,011	8	5,789	8,263	381	0	3,894	8
	SIRPS	4,864	3,457	1,521	2,655	0	11	6,103	3,981	1,835	2,377	0	11	5,606	6,178	852	3,533	0	19
	SIRPTS	4,904	817	754	1,916	2,299	24	6,215	1,237	654	2,396	2,043	22	7,031	1,741	54	2,581	2,784	29

Table 4 *Computational results for a number of products varying between 20 and 40 - number of depots equal to 10*

Instances	Model	PD					SPD					NBD				
		FSC	SSC	LB (%)	UB (%)	CPU (s)	FSC	SSC	LB (%)	UB (%)	CPU (s)	FSC	SSC	LB (%)	UB (%)	CPU (s)
n10m20k3T3	SIRP	2,798.9	3,847.7	0.63	0.37	1,128	3,666.0	4,370.0	0.27	0.60	1,167	3,695.6	7,296.6	0.63	0.47	1,118
	SIRPT	2,062.1	3,410.5	0.76	0.50	903	3,022.8	3,588.7	0.32	0.78	802	3,053.9	6,253.9	0.83	0.77	1,099
	SIRPS	2,227.2	3,548.8	0.54	0.66	1,125	2,524.1	4,104.4	0.83	0.84	992	3,047.7	5,781.1	0.78	0.78	844
	SIRPTS	1,758.3	2,750.2	0.44	0.33	857	2,003.1	2,831.9	0.70	0.71	1,025	2,527.9	4,311.6	0.90	0.88	1,043
n10m20k3T5	SIRP	4,870.5	9,247.0	0.39	0.79	1,148	5,888.8	9,106.9	0.22	0.70	814	6,612.1	11,648.2	0.82	0.66	1,191
	SIRPT	4,672.4	8,111.3	0.66	0.43	1,107	6,177.2	6,837.8	0.67	0.34	962	6,054.8	9,824.5	0.44	0.69	907
	SIRPS	4,964.4	7,269.2	0.34	0.54	868	5,533.4	7,868.9	0.28	0.52	1,137	6,145.8	8,749.4	0.85	0.86	1,056
	SIRPTS	4,176.7	5,929.0	0.49	0.43	864	5,480.6	6,978.1	0.36	0.38	1,151	4,895.5	7,869.9	0.80	0.63	813
n10m20k3T7	SIRP	8,088.0	13,555.1	0.75	0.49	1,048	10,820.1	14,337.1	0.75	0.29	1,078	10,218.2	20,347.8	0.34	0.53	838
	SIRPT	6,385.2	12,074.5	0.40	0.61	878	7,589.9	12,273.7	0.77	0.59	1,140	8,347.3	16,319.7	0.42	0.46	1,077
	SIRPS	6,810.9	10,353.7	0.53	0.78	1,115	8,856.4	10,447.0	0.71	0.88	1,014	7,459.2	13,505.3	0.86	0.51	1,143
	SIRPTS	7,067.9	7,483.0	0.55	0.53	1,063	8,226.6	9,571.5	0.69	0.61	865	10,104.9	7,933.0	0.96	0.63	959
n10m30k3T3	SIRP	3,084.7	5,039.4	0.74	0.68	1,019	3,652.6	5,924.3	0.44	0.43	837	4,148.2	6,388.2	0.58	0.84	1,055
	SIRPT	2,427.5	4,395.2	0.70	0.66	910	2,807.6	4,683.3	0.22	0.28	906	3,686.1	5,437.9	0.41	0.36	959
	SIRPS	2,436.1	4,574.2	0.34	0.41	986	3,886.3	3,442.3	0.48	0.47	982	3,658.3	5,247.0	0.54	0.40	1,105
	SIRPTS	2,517.8	2,584.6	0.49	0.53	934	3,840.3	1,905.5	0.55	0.41	1,024	3,447.7	3,542.7	0.56	0.60	989
n10m30k3T5	SIRP	6,083.3	8,859.7	0.71	0.36	939	8,542.0	12,100.8	0.58	0.87	990	8,419.3	16,112.4	0.77	0.47	978
	SIRPT	5,450.8	8,961.7	0.41	0.71	1,088	7,313.1	10,153.2	0.48	0.68	1,167	6,252.1	12,762.7	0.90	0.52	1,122
	SIRPS	4,988.7	8,626.8	0.49	0.62	934	5,867.1	8,881.7	0.72	0.79	1,199	7,295.5	11,908.3	0.64	0.36	1,111
	SIRPTS	6,067.4	5,052.7	0.49	0.53	805	6,743.4	9,266.6	0.70	0.67	881	6,615.5	9,690.9	0.73	0.84	900
n10m30k3T7	SIRP	8,798.7	12,306.3	0.47	0.41	900	10,931.3	15,491.9	0.33	0.58	1,140	11,103.5	19,367.1	0.95	0.55	1,129
	SIRPT	7,932.9	10,834.3	0.35	0.37	829	9,578.6	12,985.8	0.58	0.33	876	9,564.3	18,109.8	0.38	0.55	953
	SIRPS	7,455.6	11,160.1	0.65	0.69	808	8,764.8	12,597.2	0.75	0.34	1,014	9,748.9	17,789.0	0.86	0.80	1,160
	SIRPTS	7,834.8	8,698.5	0.34	0.53	991	10,454.3	8,178.8	0.22	0.51	954	9,244.8	15,013.5	0.53	0.45	949
n10m40k3T3	SIRP	3,473.6	6,124.7	0.69	0.44	864	5,565.5	7,374.5	0.26	0.55	1,032	3,971.1	11,407.6	0.97	0.77	812
	SIRPT	3,364.6	5,164.2	0.42	0.60	1,191	4,504.4	6,276.8	0.41	0.30	1,151	5,199.9	8,136.6	0.38	0.94	1,194
	SIRPS	2,880.7	5,394.8	0.73	0.37	850	4,368.9	6,194.1	0.51	0.50	1,038	3,925.1	6,697.6	0.53	0.41	971
	SIRPTS	3,765.6	3,010.0	0.60	0.44	1,191	4,803.6	5,964.5	0.80	0.23	841	5,337.6	4,413.5	0.91	0.53	1,063
n10m40k3T5	SIRP	8,881.1	12,360.5	0.74	0.37	830	8,084.9	13,411.8	0.21	0.21	1,154	8,370.2	16,800.2	0.48	0.36	1,172
	SIRPT	7,635.5	10,458.2	0.63	0.45	909	7,827.3	12,912.7	0.39	0.68	892	8,831.9	14,358.9	0.67	0.94	1,161
	SIRPS	6,751.0	11,775.4	0.44	0.74	893	8,893.0	11,758.6	0.66	0.69	894	10,033.7	13,440.4	0.79	0.74	895
	SIRPTS	6,209.1	11,597.3	0.49	0.47	1,114	8,662.1	9,869.4	0.47	0.36	1,115	8,085.0	13,356.6	0.82	0.88	1,096
n10m40k3T7	SIRP	8,772.5	15,911.0	0.66	0.46	980	11,418.3	15,469.0	0.28	0.72	911	12,939.7	21,166.0	0.33	0.44	877
	SIRPT	8,401.5	13,118.7	0.47	0.38	832	10,141.3	14,782.3	0.30	0.77	892	10,757.3	16,359.6	0.80	0.70	843
	SIRPS	7,626.9	13,291.7	0.62	0.56	856	8,672.7	15,714.3	0.85	0.31	830	9,820.5	17,386.1	0.72	0.88	836
	SIRPTS	8,290.9	10,418.8	0.46	0.60	1,093	11,441.5	10,550.0	0.22	0.68	1,111	9,247.3	16,225.4	0.71	0.96	1,037

Table 5 *Breakdown of cost for a number of products varying between 20 and 40 - number of depots equal to 10.*

Instances	Model	PD						SPD						NBD					
		T	I	LS	S	Ts	SV (%)	T	I	LS	S	Ts	SV (%)	T	I	LS	S	Ts	SV (%)
n10m20k3T3	SIRP	2,799	3,016	832	0	0		3,666	3,016	1,354	0	0		3,696	6,006	1,291	0	0	
	SIRPT	2,062	1,543	533	0	1,334	18	3,023	1,543	744	0	1,301	18	3,054	2,517	1,050	0	2,687	15
	SIRPS	2,227	1,836	614	1,099	0	13	2,524	1,836	749	1,519	0	18	3,048	2,899	1,030	1,852	0	20
	SIRPTS	1,758	423	315	1,003	1,010	32	2,003	423	253	1,068	1,087	40	2,528	465	171	1,796	1,879	38
n10m20k3T5	SIRP	4,871	5,818	3,429	0	0		5,889	5,818	3,288	0	0		6,612	7,950	3,698	0	0	
	SIRPT	4,672	3,017	2,580	0	2,514	9	6,177	3,017	1,513	0	2,308	13	6,055	4,966	1,291	0	3,568	13
	SIRPS	4,964	3,625	1,210	2,434	0	13	5,533	3,625	1,362	2,882	0	11	6,146	4,214	1,713	2,823	0	18
	SIRPTS	4,177	1,151	1,007	1,967	1,804	28	5,481	1,151	456	2,515	2,857	17	4,896	1,016	493	3,369	2,992	30
n10m20k3T7	SIRP	8,088	9,040	4,516	0	0		10,820	9,040	5,298	0,	0		10,218	13,865	6,483	0	0	
	SIRPT	6,385	6,426	1,873	0	3,776	15	7,590	6,426	1,835	0	4,014	21	8,347	6,318	2,861	0	7,140	19
	SIRPS	6,811	5,861	1,031	3,462	0	21	8,856	5,861	1,328	3,258	0	23	7,459	5,761	2,584	5,160	0	31
	SIRPTS	7,068	1,812	584	2,601	2,486	33	8,227	1,812	969	3,541	3,250	29	10,105	1,547	594	2,540	3,252	41
n10m30k3T3	SIRP	3,085	4,434	605	0	0		3,653	4,434	1,490	0	0		4,148	4,914	1,474	0	0	
	SIRPT	2,428	2,345	478	0	1,572	16	2,808	2,345	595	0	1,743	22	3,686	2,734	518	0	2,186	13
	SIRPS	2,436	1,822	1,009	1,742	0,	14	3,886	1,822	446	1,174	0	23	3,658	2,310	778	2,159	0	15
	SIRPTS	2,518	400	390	970	825	37	3,840	400	154	610	741	40	3,448	613	148	1,424	1,357	34
n10m30k3T5	SIRP	6,083	8,053	807	0	0		8,542	8,053	4,048	0	0		8,419	12,186	3,927	0	0	
	SIRPT	5,451	5,015	580	0	3,367	4	7,313	5,015	1,775	0	3,364	15	6,252	5,245	2,660	0	4,857	22
	SIRPS	4,989	3,408	2,769	2,449	0	9	5,867	3,408	2,032	3,442	0	29	7,296	6,050	1,139	4,719	0	22
	SIRPTS	6,067	1,230	54	1,813	1,955	26	6,743	1,930	1,028	2,937	3,370	22	6,616	1,872	626	3,307	3,886	34
n10m30k3T7	SIRP	8,799	12,109	198	0	0		10,931	12,109	3,383	0	0		11,104	13,166	6,201	0	0	
	SIRPT	7,933	6,650	682	0	3,503	11	9,579	6,650	1,607	0	4,729	15	9,564	7,773	2,396	0	7,941	9
	SIRPS	7,456	5,643	1,699	3,818	0	12	8,765	5,643	3,095	3,859	0	19	9,749	7,978	2,993	6,819	0	10
	SIRPTS	7,835	1,972	402	3,201	3,124	22	10,454	1,972	679	2,694	2,834	29	9,245	2,858	1,537	5,063	5,556	20
n10m40k3T3	SIRP	3,474	4,672	1,452	0	0		5,565	4,672	2,702	0	0		3,971	7,715	3,693	0	0	
	SIRPT	3,365	2,565	679	0	1,920	11	4,504	2,565	1,378	0	2,334	17	5,200	4,443	887	0	2,806	13
	SIRPS	2,881	2,506	1,296	1,593	0	14	4,369	2,506	1,497	2,191	0	18	3,925	3,195	637	2,865	0	31
	SIRPTS	3,766	765	45	1,069	1,131	29	4,804	1,065	614	1,831	2,455	17	5,338	1,116	218	1,452	1,628	37
n10m40k3T5	SIRP	8,881	9,927	2,433	0	0		8,085	9,927	3,484	0	0		8,370	13,368	3,432	0	0	
	SIRPT	7,636	5,959	941	0	3,559	15	7,827	5,959	2,086	0	4,868	4	8,832	7,925	1,447	0	4,987	8
	SIRPS	6,751	5,054	3,273	3,449	0	13	8,893	5,054	3,024	3,680	0	4	10,034	5,577	2,529	5,335	0	7
	SIRPTS	6,209	1,740	2,284	3,551	4,022	16	8,662	1,740	1,032	2,978	4,119	14	8,085	1,890	684	5,267	5,516	15
n10m40k3T7	SIRP	8,773	12,146	3,765	0	0		11,418	12,146	3,323	0	0		12,940	17,386	3,780	0	0	
	SIRPT	8,401	7,614	265	0	5,240	13	10,141	7,614	1,959	0	5,209	7	10,757	7,854	1,998	0	6,508	20
	SIRPS	7,627	8,633	748	3,910	0	15	8,673	8,633	2,173	4,908	0	9	9,820	7,786	2,693	6,907	0	20
	SIRPTS	8,291	1,549	1,108	4,086	3,676	24	11,441	1,549	784	4,118	4,100	18	9,247	3,505	717	6,320	5,684	25

Table 6 *Computational results for a number of products varying between 1 and 5 - number of depots equal to 20*

Instances	Model	PD					SPD					NBD				
		FSC	SSC	LB (%)	UB (%)	CPU (s)	FSC	SSC	LB (%)	UB (%)	CPU (s)	FSC	SSC	LB (%)	UB (%)	CPU (s)
n20m1k3T3	SIRP	2,148.3	3,144.3	0.06	0.04	100	2,525.4	3,497.6	0.05	0.08	111	2,528.8	4,313.3	0.03	0.05	137
	SIRPT	1,996.8	3,119.3	0.02	0.03	128	2,246.2	3,576.0	0.07	0.02	168	2,753.3	2,326.5	0.00	0.04	220
n20m1k3T5	SIRP	4,536.8	7,234.9	0.05	0.07	144	5,379.9	8,993.3	0.00	0.06	187	6,875.9	10,831.9	0.07	0.06	236
	SIRPT	4,472.6	6,487.0	0.05	0.01	124	5,575.6	7,696.5	0.09	0.02	164	6,850.8	9,341.1	0.07	0.02	200
n20m1k3T7	SIRP	6,348.9	10,001.5	0.01	0.07	316	7,810.5	10,812.6	0.00	0.07	376	9,019.3	13,682.2	0.09	0.07	500
	SIRPT	5,295.2	7,601.0	0.02	0.04	217	6,068.0	8,466.1	0.06	0.09	267	6,322.1	11,755.4	0.09	0.07	326
n20m3k3T3	SIRP	2,652.3	3,765.1	0.02	0.03	128	2,750.6	4,501.1	0.00	0.03	143	3,973.9	4,996.5	0.07	0.00	189
	SIRPT	2,400.8	3,598.1	0.10	0.08	147	2,939.4	4,295.2	0.08	0.07	172	3,501.5	4,847.3	0.00	0.09	212
	SIRPS	1,955.6	2,773.8	0.02	0.08	108	2,144.3	4,110.0	0.05	0.04	132	2,304.5	5,111.1	0.09	0.01	170
	SIRPTS	2,239.7	1,577.8	0.03	0.03	102	2,588.1	2,358.0	0.00	0.09	120	3,396.0	2,300.5	0.01	0.03	148
n20m3k3T5	SIRP	4,959.3	7,621.3	0.05	0.06	124	5,500.8	8,727.8	0.05	0.07	162	6,523.8	9,668.4	0.00	0.06	198
	SIRPT	4,595.6	7,185.0	0.01	0.01	104	5,317.5	8,007.7	0.07	0.05	125	5,656.9	9,659.5	0.05	0.06	154
	SIRPS	4,230.8	6,453.1	0.10	0.05	126	5,034.0	7,113.6	0.03	0.03	155	5,755.0	9,465.9	0.08	0.05	200
	SIRPTS	4,548.8	4,546.9	0.03	0.09	135	4,660.8	6,878.4	0.01	0.03	153	5,766.7	8,644.0	0.00	0.09	197
n20m3k3T7	SIRP	6,981.2	10,960.7	0.01	0.10	114	8,305.1	12,130.7	0.08	0.06	141	10,103.6	13,091.1	0.05	0.02	174
	SIRPT	6,263.4	9,286.3	0.07	0.06	331	7,270.0	12,487.7	0.07	0.02	420	7,333.1	13,881.8	0.10	0.08	517
	SIRPS	6,175.2	9,087.3	0.04	0.10	339	7,560.4	11,121.0	0.06	0.02	400	7,406.2	14,030.1	0.09	0.09	532
	SIRPTS	6,359.1	7,213.9	0.03	0.08	217	7,372.6	10,100.7	0.02	0.02	276	8,893.1	9,708.5	0.03	0.08	367
n20m5k3T3	SIRP	3,105.4	4,446.6	0.00	0.07	126	3,473.6	5,754.9	0.05	0.10	150	4,602.8	6,499.1	0.06	0.02	190
	SIRPT	2,503.4	4,019.3	0.00	0.09	250	3,065.1	4,475.1	0.04	0.03	325	3,256.0	6,184.4	0.09	0.08	400
	SIRPS	2,352.4	3,409.0	0.08	0.05	446	2,482.6	3,901.1	0.08	0.03	535	3,408.2	4,392.6	0.03	0.04	664
	SIRPTS	2,669.0	2,160.6	0.03	0.05	403	2,977.6	2,712.6	0.04	0.03	467	3,605.7	2,999.3	0.10	0.09	580
n20m5k3T5	SIRP	5,997.8	8,808.0	0.09	0.02	345	6,959.6	10,052.3	0.01	0.06	414	8,993.2	11,455.2	0.01	0.05	505
	SIRPT	5,398.3	8,261.3	0.09	0.05	209	6,166.3	10,334.4	0.08	0.01	261	6,956.9	12,134.5	0.07	0.06	332
	SIRPS	5,684.1	5,608.3	0.01	0.03	322	5,881.3	10,183.4	0.08	0.02	406	7,367.6	11,990.3	0.04	0.08	527
	SIRPTS	5,578.5	3,821.4	0.02	0.05	326	6,435.7	8,533.8	0.10	0.05	375	6,785.8	11,658.9	0.10	0.00	484
n20m5k3T7	SIRP	6,319.8	9,265.4	0.03	0.01	250	8,162.5	11,007.3	0.00	0.02	310	7,638.9	15,211.5	0.07	0.05	378
	SIRPT	6,443.6	8,236.7	0.01	0.07	341	6,705.2	11,776.6	0.08	0.03	443	7,264.6	14,008.0	0.04	0.07	545
	SIRPS	7,512.9	7,106.5	0.05	0.09	336	8,900.1	9,108.0	0.08	0.03	407	11,253.2	9,571.9	0.10	0.04	516
	SIRPTS	6,681.8	6,504.0	0.06	0.03	148	7,428.1	8,579.4	0.05	0.01	181	10,132.2	9,171.0	0.05	0.07	226

Table 7 *Breakdown of cost for a number of products varying between 1 and 5 - number of depots equal to 20*

Instances	Model	PD						SPD						NBD					
		T	I	LS	S	Ts	SV (%)	T	I	LS	S	Ts	SV (%)	T	I	LS	S	Ts	SV (%)
n20m1k3T3	SIRP	2,148	1,903	1,241	0	0		2,525	2,263	1,234	0	0		2,529	2,473	1,840	0	0	
	SIRPT	1,997	994	991	0	1,134	3	2,246	1,665	589	0	1,322	3	2,753	1,465	98	0	763	26
n20m1k3T5	SIRP	4,537	4,961	2,274	0	0		5,380	5,143	3,850	0	0		6,876	6,399	4,432	0	0	
	SIRPT	4,473	2,574	1,836	0	2,077	7	5,576	3,284	2,025	0	2,388	8	6,851	5,821	8	0	3,512	9
n20m1k3T7	SIRP	6,349	5,995	4,007	0	0		7,811	7,174	3,638	0	0		9,019	7,189	6,494	0	0	
	SIRPT	5,295	2,688	2,043	0	2,870	21	6,068	3,233	2,456	0	2,778	22	6,322	7,602	256	0	3,897	20
n20m3k3T3	SIRP	2,652	1,888	1,877	0	0		2,751	3,370	1,132	0	0		3,974	3,831	1,165	0	0	
	SIRPT	2,401	1,102	1,058	0	1,439	7	2,939	1,729	1,012	0	1,554	0	3,501	2,702	486	0	1,659	7
	SIRPS	1,956	1,028	848	898	0	26	2,144	1,539	1,165	1,406	0	14	2,304	3,069	450	1,592	0	17
	SIRPTS	2,240	239	180	560	599	41	2,588	327	177	889	966	32	3,396	528	121	758	893	36
n20m3k3T5	SIRP	4,959	3,944	3,677	0	0		5,501	5,488	3,240	0	0		6,524	5,901	3,768	0	0	
	SIRPT	4,596	2,846	1,948	0	2,391	6	5,318	3,796	1,404	0	2,808	6	5,657	5,509	434	0	3,716	5
	SIRPS	4,231	2,345	1,762	2,347	0	15	5,034	2,989	1,795	2,330	0	15	5,755	5,717	712	3,037	0	6
	SIRPTS	4,549	1,020	454	1,334	1,740	28	4,661	1,108	695	2,594	2,482	19	5,767	2,715	196	2,714	3,019	11
n20m3k3T7	SIRP	6,981	5,487	5,474	0	0		8,305	8,411	3,719	0	0		10,104	8,480	4,611	0	0	
	SIRPT	6,263	3,527	2,151	0	3,609	13	7,270	4,086	3,302	0	5,100	3	7,333	8,479	312	0	5,090	9
	SIRPS	6,175	3,982	2,299	2,806	0	15	7,560	4,484	2,909	3,728	0	9	7,406	8,048	1,880	4,102	0	8
	SIRPTS	6,359	1,068	953	2,467	2,725	24	7,373	2,348	1,139	3,142	3,471	14	8,893	2,901	92	3,454	3,261	20
n20m5k3T3	SIRP	3,105	2,859	1,588	0	0		3,474	3,835	1,920	0	0		4,603	3,383	3,116	0	0	
	SIRPT	2,503	1,516	1,206	0	1,298	14	3,065	1,618	1,216	0	1,641	18	3,256	4,063	35	0	2,087	15
	SIRPS	2,352	1,372	804	1,233	0	24	2,483	1,584	886	1,432	0	31	3,408	2,879	8	1,505	0	30
	SIRPTS	2,669	388	264	649	859	36	2,978	436	328	1,005	944	38	3,606	801	76	989	1,133	41
n20m5k3T5	SIRP	5,998	5,985	2,823	0	0		6,960	5,944	4,108	0	0		8,993	7,849	3,606	0	0	
	SIRPT	5,398	3,066	2,320	0	2,875	8	6,166	3,769	2,906	0	3,659	3	6,957	7,236	726	0	4,173	7
	SIRPS	5,684	1,891	1,835	1,882	0	24	5,881	4,003	2,813	3,368	0	6	7,368	7,215	701	4,074	0	5
	SIRPTS	5,579	791	377	1,301	1,352	37	6,436	1,357	664	3,023	3,490	12	6,786	2,755	798	3,992	4,114	10
n20m5k3T7	SIRP	6,320	5,617	3,649	0	0		8,162	8,145	2,862	0	0		7,639	11,092	4,119	0	0	
	SIRPT	6,444	3,466	1,754	0	3,017	6	6,705	4,566	3,100	0	4,111	4	7,265	8,392	564	0	5,051	7
	SIRPS	7,513	2,808	1,703	2,595	0	6	8,900	4,226	1,482	3,401	0	6	11,253	5,071	995	3,505	0	9
	SIRPTS	6,682	1,013	987	2,264	2,240	15	7,428	1,663	1,236	2,651	3,029	16	10,132	2,932	92	2,741	3,406	16

Table 8 *Computational results for a number of products varying between 20 and 40 - number of depots equal to 20*

Instances	Model	PD					SPD					NBD				
		FSC	SSC	LB (%)	UB (%)	CPU (s)	FSC	SSC	LB (%)	UB (%)	CPU (s)	FSC	SSC	LB (%)	UB (%)	CPU (s)
n20m20k3T3	SIRP	2,958.4	5,631.5	0.43	0.40	1,043	3,941.8	6,339.4	0.53	0.41	1,026	4,141.4	7,880.2	0.51	0.81	915
	SIRPT	2,847.9	4,215.2	0.40	0.42	997	4,378.4	4,839.6	0.27	0.41	1,160	4,385.2	6,461.1	0.56	0.58	989
	SIRPS	2,903.9	4,111.3	0.31	0.45	1,040	3,603.2	4,955.5	0.62	0.43	913	3,234.0	6,601.8	0.35	0.67	1,193
	SIRPTS	2,474.4	3,327.2	0.68	0.34	893	2,764.5	4,440.3	0.40	0.23	1,154	3,706.0	5,141.0	0.49	0.66	1,142
n20m20k3T5	SIRP	6,431.9	11,908.3	0.70	0.71	1,064	8,452.6	13,265.3	0.22	0.76	1,049	12,678.7	17,566.3	0.98	0.83	1,062
	SIRPT	6,810.6	9,601.5	0.75	0.52	1,037	9,215.2	11,263.1	0.65	0.23	1,064	11,569.0	15,167.8	0.36	0.45	1,050
	SIRPS	6,429.0	9,611.4	0.55	0.45	984	6,422.1	13,581.1	0.54	0.85	1,022	6,994.6	19,494.3	0.78	0.85	903
	SIRPTS	4,156.7	7,126.4	0.74	0.52	832	4,706.3	13,948.7	0.46	0.30	952	5,961.3	9,926.9	0.51	0.90	1,162
n20m20k3T7	SIRP	9,519.6	17,213.3	0.69	0.50	880	12,545.3	17,568.2	0.21	0.50	831	13,339.4	21,507.5	0.89	0.36	1,138
	SIRPT	8,323.2	16,419.8	0.41	0.77	1,147	9,833.0	16,418.6	0.66	0.84	991	12,108.8	17,657.2	0.51	0.84	1,117
	SIRPS	7,561.6	16,707.5	0.66	0.31	814	10,024.3	16,485.0	0.89	0.40	886	9,176.0	15,847.3	0.70	0.31	1,127
	SIRPTS	9,367.0	11,725.0	0.49	0.42	822	12,722.8	7,919.8	0.86	0.42	981	14,022.7	4,459.0	0.51	0.44	1,054
n20m30k3T3	SIRP	3,865.2	6,922.4	0.70	0.50	1,020	5,119.3	10,657.4	0.75	0.39	916	5,673.0	9,271.6	0.45	0.99	1,190
	SIRPT	3,591.7	5,168.4	0.72	0.51	1,173	5,201.9	8,232.8	0.68	0.88	912	4,817.0	8,449.3	0.50	0.69	1,066
	SIRPS	2,893.2	5,127.8	0.36	0.32	1,045	3,626.6	8,380.9	0.24	0.64	1,074	3,533.0	6,821.6	0.31	0.43	1,166
	SIRPTS	3,765.5	1,862.5	0.41	0.62	1,136	5,915.4	4,997.5	0.38	0.42	854	5,255.2	2,946.4	0.49	0.87	991
n20m30k3T5	SIRP	8,553.3	13,802.4	0.56	0.42	955	8,965.1	16,477.4	0.79	0.74	852	10,691.1	18,017.6	0.56	0.42	1,188
	SIRPT	7,106.7	11,930.8	0.48	0.44	814	10,080.7	13,902.5	0.38	0.45	913	9,861.3	15,739.2	0.64	0.94	1,163
	SIRPS	7,058.2	12,448.4	0.75	0.32	857	7,966.4	15,502.3	0.68	0.70	1,197	10,895.0	14,619.9	0.63	0.48	1,130
	SIRPTS	8,094.3	8,072.9	0.40	0.64	1,068	8,906.2	12,059.4	0.76	0.73	1,019	9,515.6	12,191.2	0.65	0.50	868
n20m30k3T7	SIRP	11,753.1	20,219.4	0.75	0.36	1,174	14,677.0	21,269.6	0.67	0.89	845	12,368.5	23,142.5	0.39	0.78	1,143
	SIRPT	10,508.4	14,153.4	0.39	0.65	916	12,985.2	18,517.0	0.83	0.26	1,051	11,922.3	21,399.1	0.79	0.64	1,171
	SIRPS	9,329.5	14,052.7	0.39	0.32	993	12,807.9	16,615.3	0.53	0.74	929	16,166.1	17,379.6	0.88	0.50	830
	SIRPTS	10,583.6	10,877.8	0.34	0.66	856	12,787.6	16,006.5	0.34	0.36	1,058	14,361.1	14,776.5	0.79	0.90	808
n20m40k3T3	SIRP	4,880.4	6,817.6	0.40	0.61	904	5,762.6	8,320.1	0.88	0.34	1,109	7,379.8	11,326.9	0.91	0.38	1,155
	SIRPT	3,431.4	6,646.2	0.55	0.53	1,187	4,722.1	6,588.3	0.56	0.67	868	5,462.6	10,907.1	0.82	0.39	1,194
	SIRPS	3,647.1	6,464.2	0.31	0.60	875	4,339.8	7,516.6	0.41	0.71	1,193	4,749.6	11,274.8	0.71	0.67	1,099
	SIRPTS	3,808.1	4,723.2	0.62	0.36	938	5,666.3	3,084.1	0.70	0.66	1,195	5,832.3	8,265.7	0.99	0.43	944
n20m40k3T5	SIRP	8,509.2	14,958.0	0.45	0.38	1,200	12,493.7	18,195.8	0.87	0.60	975	11,228.6	23,267.8	0.86	0.86	1,051
	SIRPT	8,576.5	13,276.0	0.53	0.39	1,197	11,335.0	14,257.7	0.41	0.59	1,080	12,095.7	20,161.6	0.98	0.62	849
	SIRPS	8,638.7	12,430.2	0.40	0.37	1,020	10,451.9	13,837.9	0.78	0.69	908	11,663.8	20,689.9	0.49	0.33	953
	SIRPTS	8,629.7	10,531.8	0.60	0.41	1,137	11,789.2	9,291.2	0.58	0.82	1,171	11,253.4	17,526.9	0.94	0.99	1,028
n20m40k3T7	SIRP	9,473.6	18,049.9	0.57	0.41	966	14,880.7	19,701.5	0.86	0.56	985	15,051.5	25,279.4	0.34	0.60	1,142
	SIRPT	10,390.7	14,999.2	0.44	0.60	916	13,507.1	18,614.4	0.25	0.28	816	12,992.1	20,278.4	0.67	0.71	935
	SIRPS	12,174.2	13,635.4	0.78	0.44	963	17,114.2	15,116.6	0.25	0.40	1,186	13,608.4	19,856.6	0.65	0.55	1,049
	SIRPTS	10,488.8	11,805.5	0.63	0.78	1,077	12,096.0	15,890.4	0.28	0.55	1,186	13,406.7	16,509.7	0.92	0.96	893

Table 9 *Breakdown of cost for a number of products varying between 20 and 40 - number of depots equal to 20*

Instances	Model	PD						SPD						NBD					
		T	I	LS	S	Ts	SV (%)	T	I	LS	S	Ts	SV (%)	T	I	LS	S	Ts	SV (%)
n20m20k3T3	SIRP	2,958	4,971	661	0	0		3,942	4,971	1,369	0	0		4,141	5,175	2,705	0	0	
	SIRPT	2,848	1,945	959	0	1,311	18	4,378	1,945	1,106	0	1,789	10	4,385	2,838	1,176	0	2,447	10
	SIRPS	2,904	1,990	526	1,596	0	18	3,603	1,990	1,133	1,833	0	17	3,234	3,557	724	2,321	0	18
	SIRPTS	2,474	960	139	1,128	1,100	32	2,764	960	534	1,375	1,571	30	3,706	1,162	433	1,704	1,843	26
n20m20k3T5	SIRP	6,432	10,186	1,722	0	0		8,453	10,186	3,079	0	0		12,679	12,866	4,701	0	0	
	SIRPT	6,811	5,139	1,119	0	3,343	11	9,215	5,139	1,987	0	4,137	6	11,569	7,243	2,251	0	5,674	12
	SIRPS	6,429	6,046	868	2,697	0	13	6,422	6,046	2,667	4,867	0	8	6,995	10,121	2,369	7,004	0	12
	SIRPTS	4,157	1,732	272	2,640	2,482	38	4,706	2,832	962	5,387	4,768	14	5,961	1,723	618	3,870	3,717	47
n20m20k3T7	SIRP	9,520	13,723	3,491	0	0		12,545	13,723	3,846	0	0		13,339	14,468	7,039	0	0	
	SIRPT	8,323	7,845	3,479	0	5,095	7	9,833	7,845	2,927	0	5,646	13	12,109	8,385	2,656	0	6,616	15
	SIRPS	7,562	6,949	4,689	5,069	0	9	10,024	6,949	3,003	6,533	0	12	9,176	6,953	3,242	5,653	0	28
	SIRPTS	9,367	1,781	1,023	4,351	4,570	21	12,723	1,781	566	2,455	3,118	31	14,023	663	295	1,720	1,781	47
n20m30k3T3	SIRP	3,865	6,552	370	0	0		5,119	8,252	2,405	0	0		5,673	6,498	2,774	0	0	
	SIRPT	3,592	3,216	176	0	1,776	19	5,202	3,816	1,409	0	3,007	15	4,817	3,368	1,809	0	3,273	11
	SIRPS	2,893	3,051	47	2,030	0	26	3,627	3,451	1,669	3,260	0	24	3,533	3,453	901	2,468	0	31
	SIRPTS	3,766	584	45	639	595	48	5,915	1,284	490	1,608	1,617	31	5,255	345	141	1,193	1,267	45
n20m30k3T5	SIRP	8,553	10,901	2,901	0	0		8,965	10,901	5,576	0	0		10,691	14,084	3,933	0	0	
	SIRPT	7,107	6,040	1,693	0	4,198	15	10,081	6,040	2,182	0	5,681	6	9,861	8,142	1,921	0	5,676	11
	SIRPS	7,058	7,779	182	4,488	0	13	7,966	7,979	2,718	4,806	0	8	10,895	7,550	2,197	4,873	0	11
	SIRPTS	8,094	2,560	29	2,298	3,186	28	8,906	2,660	1,087	4,195	4,117	18	9,516	2,537	621	4,629	4,403	24
n20m30k3T7	SIRP	11,753	13,379	6,841	0	0		14,677	13,379	7,891	0	0		12,368	15,281	7,862	0	0	
	SIRPT	10,508	8,487	31	0	5,636	23	12,985	8,487	2,581	0	7,449	12	11,922	9,829	3,085	0	8,485	6
	SIRPS	9,330	8,897	352	4,803	0	27	12,808	8,897	2,237	5,481	0	18	16,166	8,491	2,164	6,724	0	6
	SIRPTS	10,584	3,271	44	3,620	3,943	33	12,788	3,671	853	4,896	6,587	20	14,361	1,805	320	6,181	6,471	18
n20m40k3T3	SIRP	4,880	6,174	643	0	0		5,763	6,174	2,146	0	0		7,380	7,370	3,956	0	0	
	SIRPT	3,431	3,112	1,184	0	2,351	14	4,722	3,112	1,029	0	2,448	20	5,463	4,677	2,385	0	3,845	12
	SIRPS	3,647	3,186	701	2,577	0	14	4,340	3,186	1,427	2,903	0	16	4,750	5,409	1,591	4,274	0	14
	SIRPTS	3,808	789	437	1,847	1,650	27	5,666	789	191	1,043	1,062	38	5,832	868	285	3,501	3,612	25
n20m40k3T5	SIRP	8,509	12,515	2,443	0	0		12,494	12,515	5,681	0	0		11,229	19,296	3,972	0	0	
	SIRPT	8,577	5,211	3,284	0	4,781	7	11,335	5,211	3,070	0	5,977	17	12,096	9,463	3,164	0	7,534	6
	SIRPS	8,639	6,429	1,989	4,012	0	10	10,452	6,429	2,590	4,818	0	21	11,664	8,078	4,167	8,446	0	6
	SIRPTS	8,630	1,453	1,879	4,017	3,183	18	11,789	1,453	493	3,645	3,700	31	11,253	3,938	1,145	6,376	6,068	17
n20m40k3T7	SIRP	9,474	12,376	5,673	0	0		14,881	12,376	7,325	0	0		15,051	19,263	6,016	0	0	
	SIRPT	10,391	7,464	2,427	0	5,109	8	13,507	7,464	3,847	0	7,304	7	12,992	9,915	2,639	0	7,724	18
	SIRPS	12,174	6,872	2,448	4,316	0	6	17,114	6,872	3,443	4,801	0	7	13,608	8,403	4,426	7,027	0	17
	SIRPTS	10,489	2,875	840	4,087	4,003	19	12,096	2,875	1,524	5,371	6,121	19	13,407	2,521	1,311	6,993	5,684	26

Table 10 *Computational results for a number of products varying between 1 and 5 - number of depots equal to 50*

Instances	Model	PD					SPD					NBD				
		FSC	SSC	LB (%)	UB (%)	CPU (s)	FSC	SSC	LB (%)	UB (%)	CPU (s)	FSC	SSC	LB (%)	UB (%)	CPU (s)
n50m1k3T3	SIRP	2,714.2	4,007.4	0.04	0.03	123	2,942.9	4,894.5	0.07	0.08	137	3,562.7	5,850.0	0.05	0.07	183
	SIRPT	2,879.7	3,341.1	0.08	0.08	814	3,345.0	3,123.4	0.04	0.03	952	4,495.1	3,523.0	0.10	0.00	1,138
n50m1k3T5	SIRP	5,987.4	8,727.2	0.04	0.08	837	7,364.0	10,676.1	0.02	0.00	937	7,139.9	13,822.7	0.08	0.09	1,172
	SIRPT	4,566.5	6,612.3	0.07	0.01	910	5,247.4	9,435.5	0.08	0.06	1,147	5,371.1	13,063.1	0.04	0.08	1,114
n50m1k3T7	SIRP	7,779.3	12,495.2	0.05	0.03	1,022	9,267.2	14,271.5	0.02	0.09	1,018	9,898.4	19,266.0	0.05	0.05	1,153
	SIRPT	5,650.9	8,406.0	0.04	0.02	1,037	6,167.3	12,209.0	0.01	0.04	1,196	6,555.2	12,179.3	0.05	0.05	1,198
n50m3k3T3	SIRP	3,221.0	4,608.3	0.07	0.06	1,020	3,315.0	5,485.1	0.02	0.10	1,153	4,724.5	8,950.0	0.05	0.05	1,133
	SIRPT	2,987.7	4,690.8	0.06	0.09	1,044	3,518.1	3,749.9	0.02	0.08	1,105	4,984.4	6,267.0	0.06	0.01	1,131
	SIRPS	2,252.0	3,234.0	0.02	0.08	1,031	2,402.3	4,235.8	0.01	0.03	1,155	2,923.3	5,334.6	0.02	0.06	1,136
	SIRPTS	2,784.4	1,338.5	0.01	0.02	733	3,490.3	1,796.3	0.07	0.01	814	3,708.1	3,672.4	0.03	0.05	1,066
n50m3k3T5	SIRP	7,491.9	10,749.9	0.01	0.01	944	8,989.5	13,411.5	0.09	0.07	1,046	9,362.7	16,107.2	0.01	0.07	1,157
	SIRPT	6,773.8	10,190.3	0.04	0.00	1,018	7,451.0	11,996.2	0.00	0.06	1,022	9,074.0	14,842.8	0.05	0.01	1,051
	SIRPS	5,775.1	8,327.7	0.07	0.05	1,021	6,314.3	10,862.8	0.09	0.09	1,044	8,842.2	10,877.2	0.01	0.02	1,175
	SIRPTS	5,666.7	8,104.1	0.07	0.03	900	5,867.6	10,368.2	0.07	0.05	990	7,959.9	10,351.1	0.04	0.04	1,138
n50m3k3T7	SIRP	9,468.7	14,394.0	0.02	0.04	931	10,872.8	17,762.5	0.01	0.05	1,192	12,500.6	11,775.8	0.07	0.01	1,154
	SIRPT	6,830.4	9,652.2	0.01	0.08	1,020	7,477.7	19,999.4	0.06	0.00	1,065	8,971.0	13,885.1	0.05	0.06	1,156
	SIRPS	8,163.8	8,593.3	0.05	0.01	1,107	9,693.2	18,173.9	0.05	0.02	1,051	12,214.8	8,257.9	0.02	0.04	1,114
	SIRPTS	6,977.8	8,589.6	0.08	0.08	1,100	8,249.6	13,200.2	0.01	0.02	1,009	11,887.5	6,989.1	0.00	0.02	1,141
n50m5k3T3	SIRP	4,322.9	6,098.8	0.06	0.03	1,103	5,296.7	7,365.7	0.04	0.08	1,157	6,257.0	8,355.4	0.03	0.09	1,047
	SIRPT	3,165.3	4,661.9	0.02	0.09	1,149	3,642.2	6,774.0	0.09	0.09	1,167	4,529.3	6,120.4	0.04	0.09	1,136
	SIRPS	2,792.4	4,121.3	0.06	0.02	1,031	2,942.8	6,772.9	0.07	0.08	1,189	3,887.5	5,610.5	0.07	0.01	1,088
	SIRPTS	3,543.3	1,676.8	0.04	0.08	1,111	4,330.4	2,977.6	0.04	0.01	1,189	5,735.5	1,484.6	0.01	0.03	1,027
n50m5k3T5	SIRP	8,734.7	13,474.1	0.04	0.08	1,035	10,493.1	16,357.3	0.01	0.00	1,142	11,211.8	20,767.1	0.02	0.05	1,002
	SIRPT	6,379.8	13,192.1	0.07	0.00	1,032	7,388.0	16,391.8	0.01	0.05	1,142	10,282.9	19,097.9	0.03	0.02	1,031
	SIRPS	8,089.8	10,919.6	0.08	0.01	1,138	10,541.2	10,990.3	0.00	0.10	1,134	9,170.9	18,917.7	0.03	0.01	1,164
	SIRPTS	6,270.5	9,713.4	0.02	0.01	1,037	8,190.7	11,149.8	0.06	0.03	1,038	8,765.5	13,727.5	0.01	0.09	1,093
n50m5k3T7	SIRP	6,842.1	16,925.0	0.00	0.09	1,110	7,953.2	18,420.6	0.04	0.03	1,143	8,572.8	16,798.0	0.06	0.04	1,079
	SIRPT	9,156.9	11,862.0	0.01	0.03	1,063	10,719.7	14,406.8	0.08	0.06	1,144	12,886.1	9,144.9	0.00	0.08	1,067
	SIRPS	9,966.5	10,169.7	0.07	0.08	1,012	10,551.7	15,399.7	0.06	0.10	1,126	13,102.6	8,271.4	0.08	0.10	984
	SIRPTS	8,516.5	11,309.5	0.01	0.03	1,025	9,476.7	13,754.1	0.05	0.07	1,171	10,727.6	8,792.5	0.00	0.06	1,127

Table 11 *Breakdown of cost for a number of products varying between 1 and 5 - number of depots equal to 50*

Instances	Model	PD						SPD						NBD					
		T	I	LS	S	Ts	SV (%)	T	I	LS	S	Ts	SV (%)	T	I	LS	S	Ts	SV (%)
n50m1k3T3	SIRP	2,714	2,513	1,495	0	0		2,943	3,524	1,371	0	0		3,563	3,799	2,051	0	0	
	SIRPT	2,880	1,256	809	0	1,276	7	3,345	1,332	688	0	1,103	17	4,495	2,352	20	0	1,151	15
n50m1k3T5	SIRP	5,987	6,024	2,704	0	0		7,364	6,859	3,817	0	0		7,140	8,158	5,665	0	0	
	SIRPT	4,567	2,778	1,252	0	2,582	24	5,247	4,437	1,735	0	3,264	19	5,371	4,253	305	3,671	4,835	12
n50m1k3T7	SIRP	7,779	8,067	4,428	0	0		9,267	9,062	5,209	0	0		9,898	12,912	6,354	0	0	
	SIRPT	5,651	2,785	2,295	0	3,326	31	6,167	5,335	2,970	0	3,904	22	6,555	7,353	122	0	4,705	36
n50m3k3T3	SIRP	3,221	2,548	2,060	0	0		3,315	3,208	2,277	0	0		4,725	6,655	2,295	0	0	
	SIRPT	2,988	2,186	975	0	1,530	2	3,518	1,306	915	0	1,529	17	4,984	4,075	65	0	2,127	18
	SIRPS	2,252	1,236	971	1,027	0	30	2,402	1,986	675	1,575	0	25	2,923	3,324	443	1,567	0	40
	SIRPTS	2,784	269	158	466	446	47	3,490	228	181	659	728	40	3,708	1,083	107	1,069	1,413	46
n50m3k3T5	SIRP	7,492	6,435	4,315	0	0		8,990	7,413	5,999	0	0		9,363	10,507	5,600	0	0	
	SIRPT	6,774	4,141	2,748	0	3,301	7	7,451	5,054	2,147	0	4,795	13	9,074	8,581	575	0	5,686	6
	SIRPS	5,775	3,725	2,191	2,412	0	23	6,314	4,811	2,489	3,563	0	23	8,842	7,309	353	3,215	0	23
	SIRPTS	5,667	1,780	841	2,382	3,101	25	5,868	2,137	1,461	3,240	3,530	28	7,960	2,804	208	3,653	3,686	28
n50m3k3T7	SIRP	9,469	8,841	5,553	0	0		10,873	12,443	5,320	0	0		12,501	6,682	5,094	0	0	
	SIRPT	6,830	3,729	2,659	0	3,264	31	7,478	8,653	4,191	0	7,156	4	8,971	7,435	1,998	0	4,452	6
	SIRPS	8,164	4,035	2,084	2,474	0	30	9,693	7,550	4,954	5,670	0	3	12,215	5,296	87	2,875	0	16
	SIRPTS	6,978	1,467	1,116	2,573	3,434	35	8,250	1,965	1,362	4,656	5,217	25	11,888	1,997	303	2,494	2,195	22
n50m5k3T3	SIRP	4,323	3,590	2,509	0	0		5,297	5,268	2,098	0	0		6,257	5,102	3,254	0	0	
	SIRPT	3,165	1,982	926	0	1,754	25	3,642	2,647	1,716	0	2,411	18	4,529	4,107	14	0	1,999	27
	SIRPS	2,792	1,402	1,305	1,414	0	34	2,943	2,919	1,686	2,169	0	23	3,888	2,868	1,179	1,564	0	35
	SIRPTS	3,543	338	213	491	634	50	4,330	637	246	984	1,111	42	5,735	521	0	478	485	51
n50m5k3T5	SIRP	8,735	7,944	5,530	0	0		10,493	9,567	6,790	0	0		11,212	13,719	7,048	0	0	
	SIRPT	6,380	5,066	3,415	0	4,711	12	7,388	7,331	4,083	0	4,978	11	10,283	10,756	2,085	0	6,256	8
	SIRPS	8,090	4,452	2,532	3,935	0	14	10,541	4,199	3,002	3,788	0	20	9,171	12,209	452	6,256	0	12
	SIRPTS	6,270	1,856	1,241	3,176	3,440	28	8,191	2,735	926	3,717	3,771	28	8,766	5,194	313	3,816	4,404	30
n50m5k3T7	SIRP	6,842	10,509	6,416	0	0		7,953	12,213	6,208	0	0		8,573	9,850	6,948	0	0	
	SIRPT	9,157	3,996	3,592	0	4,274	12	10,720	5,453	3,467	0	5,486	5	12,886	5,779	5	0	3,361	13
	SIRPS	9,966	3,657	3,614	2,898	0	15	10,552	7,517	3,343	4,540	0	2	13,103	4,725	674	2,873	0	16
	SIRPTS	8,517	2,107	1,279	3,927	3,997	17	9,477	2,742	2,203	4,382	4,427	12	10,728	2,231	88	3,178	3,296	23

Table 12 *Computational results for a number of products varying between 20 and 40 - number of depots equal to 50*

Instances	Model	PD					SPD					NBD				
		FSC	SSC	LB (%)	UB (%)	CPU (s)	FSC	SSC	LB (%)	UB (%)	CPU (s)	FSC	SSC	LB (%)	UB (%)	CPU (s)
n50m20k3T3	SIRP	4,189.7	7,894.2	0.60	0.34	1,140	6,486.8	7,651.9	0.62	0.68	884	5,681.2	10,979.3	0.82	0.94	929
	SIRPT	5,310.2	5,234.5	0.70	0.32	1,136	5,750.2	5,095.7	0.69	0.29	827	7,075.3	7,787.1	0.60	0.86	1,154
	SIRPS	3,097.9	7,374.1	0.33	0.66	1,043	4,360.7	6,286.9	0.70	0.24	811	3,956.6	10,154.3	0.94	0.47	1,178
	SIRPTS	3,245.0	5,371.6	0.75	0.66	1,043	4,627.2	3,995.4	0.23	0.24	1,070	4,455.3	5,757.7	0.75	0.50	1,091
n50m20k3T5	SIRP	8,704.8	16,059.9	0.70	0.72	1,142	12,703.4	15,818.0	0.49	0.73	1,055	10,128.5	21,336.3	0.75	0.32	1,175
	SIRPT	7,578.3	10,173.6	0.71	0.63	931	9,572.6	14,180.8	0.30	0.32	866	10,322.3	16,579.6	0.97	0.65	924
	SIRPS	7,760.2	11,846.2	0.70	0.74	1,152	7,831.4	11,172.2	0.71	0.66	1,118	8,818.3	15,315.0	0.71	0.56	986
	SIRPTS	5,039.3	7,919.1	0.66	0.58	948	6,299.6	7,467.0	0.63	0.24	934	6,197.2	10,926.8	0.67	0.98	913
n50m20k3T7	SIRP	12,111.8	20,854.5	0.59	0.65	1,030	19,547.9	23,480.8	0.65	0.62	983	19,948.0	32,285.5	0.60	0.74	1,150
	SIRPT	8,820.1	15,357.8	0.78	0.36	810	13,174.9	21,122.2	0.66	0.35	898	13,650.1	28,966.6	0.68	0.76	1,049
	SIRPS	8,544.7	15,066.0	0.64	0.47	852	12,009.2	23,327.8	0.33	0.58	1,195	13,411.9	31,137.1	0.33	0.43	1,096
	SIRPTS	12,904.1	7,019.0	0.71	0.50	1,021	15,048.4	13,507.6	0.56	0.39	1,026	16,133.6	22,243.0	0.92	0.99	1,107
n50m30k3T3	SIRP	5,320.9	7,918.4	0.55	0.50	803	5,063.2	8,392.1	0.60	0.24	926	6,508.0	12,247.1	0.79	0.78	953
	SIRPT	4,664.1	6,921.1	0.77	0.34	1,084	5,803.2	5,516.8	0.71	0.28	1,164	6,491.8	10,613.0	0.34	0.56	827
	SIRPS	3,385.1	7,601.1	0.63	0.72	988	5,113.1	6,151.8	0.64	0.62	1,054	4,943.0	12,376.9	0.81	0.44	1,112
	SIRPTS	4,268.5	4,524.2	0.33	0.75	834	5,505.4	4,363.7	0.47	0.89	970	5,809.2	10,019.4	0.80	0.56	1,166
n50m30k3T5	SIRP	11,174.6	17,775.2	0.45	0.56	824	13,835.3	20,124.6	0.28	0.55	930	18,103.9	25,551.5	0.53	0.85	8,74
	SIRPT	10,564.5	15,562.8	0.79	0.55	904	14,373.3	15,844.5	0.42	0.39	1,127	14,554.8	24,931.9	0.62	0.34	1,008
	SIRPS	9,375.0	15,826.6	0.75	0.43	920	12,109.5	18,837.7	0.76	0.58	806	12,168.8	27,337.1	0.86	0.95	930
	SIRPTS	8,392.5	14,147.0	0.76	0.70	831	11,184.4	15,221.6	0.65	0.83	875	11,221.4	21,276.1	0.44	0.82	935
n50m30k3T7	SIRP	13,231.7	24,757.7	0.33	0.46	1,069	19,379.8	28,412.5	0.42	0.74	1,072	17,471.9	36,856.2	0.74	0.87	827
	SIRPT	9,669.3	22,944.7	0.51	0.32	1,097	14,361.6	29,783.9	0.59	0.40	1,061	12,877.2	36,526.7	0.58	0.97	895
	SIRPS	13,086.7	19,611.9	0.49	0.77	851	14,319.7	29,980.9	0.81	0.33	1,062	20,766.6	30,057.2	0.65	0.63	954
	SIRPTS	11,635.0	15,735.0	0.71	0.61	1,180	16,101.0	21,221.6	0.58	0.55	1,058	16,069.3	29,056.5	0.47	0.68	1,107
n50m40k3T3	SIRP	6,394.7	30,832.4	0.79	0.33	1,052	9,187.8	11,325.3	0.27	0.65	1,029	8,283.4	14,219.7	0.60	0.39	1,123
	SIRPT	4,597.6	29,865.3	0.68	0.31	1,095	6,156.1	9,211.1	0.47	0.78	938	7,267.8	10,602.4	0.54	0.90	1,136
	SIRPS	4,353.9	30,514.5	0.46	0.77	1,090	4,723.4	10,529.2	0.54	0.74	897	5,821.6	11,018.4	0.91	0.92	1,173
	SIRPTS	5,749.8	22,620.0	0.57	0.38	829	6,654.8	5,941.2	0.52	0.41	1,178	7,115.2	7,282.9	0.80	0.78	1,030
n50m40k3T5	SIRP	15,555.0	21,666.9	0.79	0.36	1,178	19,721.5	28,662.9	0.63	0.38	1,012	18,428.7	35,519.6	0.43	0.60	892
	SIRPT	9,705.8	25,022.4	0.77	0.44	805	12,491.4	27,053.8	0.29	0.61	931	15,974.7	26,064.0	0.62	0.35	806
	SIRPS	13,219.6	20,256.1	0.49	0.39	951	14,697.8	24,748.8	0.90	0.83	923	15,173.7	28,807.3	0.92	0.35	881
	SIRPTS	8,657.2	15,958.0	0.51	0.66	1,085	15,280.9	19,615.0	0.56	0.86	923	13,711.9	21,692.0	0.44	0.95	912
n50m40k3T7	SIRP	11,093.0	38,880.1	0.79	0.45	896	15,387.4	29,067.2	0.86	0.86	985	14,070.6	27,968.7	0.99	0.55	805
	SIRPT	14,619.5	21,612.2	0.78	0.66	1,059	16,179.7	25,074.0	0.52	0.30	1,089	19,213.9	18,481.0	0.79	0.70	810
	SIRPS	16,061.8	20,402.0	0.52	0.52	1015	19,098.4	22,608.4	0.33	0.25	856	23,291.7	14,112.8	0.72	0.65	1,123
	SIRPTS	12,113.8	20,498.2	0.73	0.42	956	17,007.5	20,894.2	0.67	0.59	965	18,137.7	12,541.0	0.46	0.63	864

Table 13 *Breakdown of cost for a number of products varying between 20 and 40 - number of depots equal to 50*

Instances	Model	PD						SPD						NBD					
		T	I	LS	S	Ts	SV (%)	T	I	LS	S	Ts	SV (%)	T	I	LS	S	Ts	SV (%)
n50m20k3T3	SIRP	4,190	5,274	2,620	0	0		6,487	5,274	2,377	0	0		5,681	8,769	2,210	0	0	
	SIRPT	5,310	2,046	1,300	0	1,889	13	5,750	2,046	1,122	0	1,927	23	7,075	4,078	1,032	0	2,677	11
	SIRPS	3,098	2,550	2,286	2,538	0	13	4,361	2,550	1,321	2,416	0	25	3,957	3,987	2,002	4,165	0	15
	SIRPTS	3,245	828	714	1,815	2,015	29	4,627	828	310	1,258	1,600	39	4,455	955	425	2,387	1,991	39
n50m20k3T5	SIRP	8,705	10,013	6,047	0	0		12,703	10,013	5,805	0	0		10,129	15,134	6,202	0	0	
	SIRPT	7,578	6,588	183	0	3,403	28	9,573	6,588	1,894	0	5,699	17	10,322	7,895	2,653	0	6,032	15
	SIRPS	7,760	5,896	2,218	3,733	0	21	7,831	5,896	1,666	3,611	0	33	8,818	8,128	2,162	5,025	0	23
	SIRPTS	5,039	1,591	493	2,928	2,907	48	6,300	1,591	403	2,784	2,689	52	6,197	1,423	477	4,483	4,543	46
n50m20k3T7	SIRP	12,112	15,706	5,148	0	0		19,548	15,706	7,774	0	0		19,948	23,191	9,095	0	0	
	SIRPT	8,820	8,345	1,392	0	5,621	27	13,175	8,345	4,107	0	8,671	20	13,650	13,005	4,073	0	11,888	18
	SIRPS	8,545	10,404	18	4,643	0	28	12,009	11,404	4,841	7,082	0	18	13,412	14,582	5,657	10,898	0	15
	SIRPTS	12,904	1,833	473	2,424	2,289	40	15,048	1,833	997	5,209	5,469	34	16,134	2,990	1,525	8,490	9,238	27
n50m30k3T3	SIRP	5,321	6,493	1,425	0	0		5,063	6,493	1,899	0	0		6,508	10,406	1,841	0	0	
	SIRPT	4,664	2,261	2,237	0	2,423	12	5,803	2,261	1,296	0	1,960	16	6,492	4,989	1,985	0	3,639	9
	SIRPS	3,385	2,813	2,207	2,581	0	17	5,113	2,813	1,279	2,060	0	16	4,943	5,807	1,402	5,167	0	8
	SIRPTS	4,269	623	639	1,554	1,707	34	5,505	623	280	1,723	1,737	27	5,809	1,607	560	4,253	3,600	16
n50m30k3T5	SIRP	11,175	15,411	2,364	0	0		13,835	15,411	4,713	0	0		18,104	20,257	5,294	0	0	
	SIRPT	10,564	6,851	2,661	0	6,051	10	14,373	6,851	2,586	0	6,407	11	14,555	11,552	3,342	0	10,038	10
	SIRPS	9,375	9,491	131	6,204	0	13	12,110	9,891	3,274	5,672	0	9	12,169	12,838	4,726	9,773	0	10
	SIRPTS	8,393	2,831	2,394	4,312	4,611	22	11,184	2,831	1,191	5,978	5,223	22	11,221	3,365	1,631	6,976	9,304	26
n50m30k3T7	SIRP	13,232	19,213	5,545	0	0		19,380	19,213	9,200	0	0		17,472	26,817	10,040	0	0	
	SIRPT	9,669	15,055	926	0	6,964	14	14,362	15,055	3,554	0	11,175	8	12,877	19,994	4,040	0	12,492	9
	SIRPS	13,087	12,128	63	7,421	0	14	14,320	13,628	6,933	9,420	0	7	20,767	12,952	5,158	11,948	0	6
	SIRPTS	11,635	4,099	3	5,926	5,707	28	16,101	4,505	1,307	7,058	8,351	22	16,069	5,168	2,116	10,452	11,320	17
n50m40k3T3	SIRP	6,395	7,801	23,032	0	0		9,188	7,801	3,524	0	0		8,283	9,288	4,931	0	0	
	SIRPT	4,598	3,837	14,578	0	11,450	7	6,156	3,837	1,768	0	3,607	25	7,268	5,260	994	0	4,349	21
	SIRPS	4,354	4,575	13,788	12,151	0	6	4,723	4,575	2,177	3,777	0	26	5,822	5,353	1,703	3,962	0	25
	SIRPTS	5,750	1,311	3,606	8,849	8,853	24	6,655	1,311	529	2,183	1,918	39	7,115	1,493	389	2,499	2,902	36
n50m40k3T5	SIRP	15,555	18,339	3,328	0	0		19,721	18,339	10,324	0	0		18,429	23,539	11,981	0	0	
	SIRPT	9,706	11,191	6,087	0	7,744	7	12,491	11,191	5,393	0	10,470	18	15,975	10,019	5,002	0	11,043	22
	SIRPS	13,220	9,977	2,249	8,030	0	10	14,698	9,977	6,102	8,670	0	18	15,174	12,815	4,740	11,252	0	18
	SIRPTS	8,657	2,973	1,353	5,616	6,016	34	15,281	2,973	1,230	7,650	7,762	28	13,712	3,173	1,399	9,228	7,892	34
n50m40k3T7	SIRP	11,093	22,341	16,539	0	0		15,387	22,341	6,726	0	0		14,071	19,103	8,866	0	0	
	SIRPT	14,620	9,245	4,464	0	7,904	27	16,180	9,245	5,436	0	10,393	7	19,214	7,052	3,508	0	7,921	10
	SIRPS	16,062	10,078	3,783	6,541	0	27	19,098	10,078	3,508	9,023	0	6	23,292	7,161	1,719	5,233	0	11
	SIRPTS	12,114	2,640	3,469	6,533	7,857	35	17,008	2,640	1,222	8,314	8,719	15	18,138	2,220	615	4,461	5,245	27

Therefore, the benefits of promoting transshipment and substitution rely on the extent to which demand variability is considered as a stressful scenario. The computational experiments also stress that the benefit of transshipment and/or substitution can be less notable for some instances, as in the case of instances n20m1k3T3 and n20m20k3T7 in Table 7 with a saving equal to 7% for SIRPT model. As LT depends on the travelling distances and is incorporated in route decisions, the related cost cannot sometimes be offset by the savings it brings in reducing inventory and lost sales costs. In such context, the substitution would be given higher priority than LT as we could substitute spares at the level of the depot itself, and thus no further shipment would be required. Furthermore, the combination of LT and substitution, compared with the other configurations, always reduces total costs. On the other hand, the two alternatives help mitigate lost sales, but they can lead to higher costs. Thus, both LT and substitution can be of such interest as long as the costs they incur can be offset by the savings they enable.

As for SAA performance, from Table 8 for example, we can notice a low variability in the solutions of the SAA runs (based on the standard deviation computed for the UB and LB) which shows the sampling stability of the different runs. We note, however, that the maximum time limit of each SAA run (1200s) is reached for some instances and models under consideration (see Table 8, for example), which means that some of the SAA runs might not have been resolved to optimality, hindering the quality of the candidate solutions and, in turn, the optimality gap. However, there is no incentive to increase the number of scenarios retained in the SAA problem due to the low variability in the solutions across various instances under the current setting. Moreover, CPU time for some instances is small meaning that optimality is reached (see Tables 2 and 6). For future studies, because of the combinatorial complexity of the problem, we suggest the hybridisation of SAA and metaheuristics such as Genetic Algorithm to enhance further the quality of solutions and within a reasonable amount of time.

6. Conclusions & perspectives

In this paper, we consider a two-level supply chain. At the first level, a manufacturer-owned central warehouse distributes spare parts to a given number of depots (the second level). Spare parts demand arises when a component fails or requires replacement instead of being generated according to end-consumer buying behaviours. We model the problem as a shared inventory-routing problem considering the two flexible instruments of transshipment and substitution to mitigate shortages. We assume that lost sales are allowed when a shortage occurs.

Based on empirical goodness-of-fit tests, three discrete distributions are chosen since they provide a better fit for intermittent demands compared to continuous ones. These distributions are the Poisson distribution for demand occurrence, combined with demands of constant size; the stuttering Poisson distribution; and the negative binomial distribution.

To solve the problem, we have used the SAA method because of its good convergence properties. For the different demand patterns under consideration, computational results highlight that allowing transshipment and substitution is beneficial as they both allow reducing holding and transportation costs along with lost sales. In addition, experiments show the impact of transshipment and substitution on the overall performance depends on the size variability of demands, regardless of the average inter-demand interval. Moreover, they stress that transshipment and substitution can only be of such interest as long as the costs they incur can be offset by the savings they enable. Due to combinatorial complexity, metaheuristics must be developed for future studies to enhance the quality of the candidate solutions. It is also possible to extend this paper to examine a multi-echelon of either centralised or decentralised supply chains. Other policies and non-parametric methods for demand may be investigated, whereby the distribution of empirical distribution is instead directly constructed from the data. It is also possible to investigate stochastic lead time and production along with demands. Moreover, the reliability of PP can be integrated into the model as it affects the customers' future requirements for spare parts. Finally, using metaheuristics and solving the problem in large experimental data sets are necessary to strengthen the present analysis and generalise findings that can be applied to more complex supply chains.

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