JOINT REPRESENTATION IN INDUSTRIAL
PITCHED-ROOF PORTAL FRAMES

Jean-Pierre Jaspart
Department MSM, University of Liège
Belgium

Keywords: Structural Joints, Pitched-Roof Industrial Portal frames, Characterisation, Modelling, Classification, Idealisation, Static Behaviour

Abstract: Most of the research works, normative documents and design aids for engineers in the field of structural joints cover cases where the two connected elements form an angle of 0° (beam splices or column splices) or 90° (beam-to-column joints). In the present paper, the application of the available knowledge on structural joints to industrial pitched-roof portal frames where the connected members form an angle intermediate between these two extreme values is discussed. The modifications required by the extension of the available design rules to this area are seen to be limited.

1. INTRODUCTION

In building structures, the structural steel frame is usually made of beam and column elements connected perpendicularly so as to constitute what could be called rectangular frames, such as office buildings.
Most of the experimental, numerical and theoretical research works carried out in the field of structural joints relate to this type of structural configuration and Eurocode 3 Revised Annex J [1], which appears as the outcome of these research works, is not an exception even if nothing is explicitly said about that.
Pitched-roof industrial portal frames where the structural elements are not perpendicularly connected (Figure 1) represent however a quite important part of the steel construction activity where high profit of the new concepts introduced in Eurocode 3, and in particularly in its Revised Annex J, could be made.
This topic is covered in the present paper where the slight modifications to be made to the Eurocode 3 rules for joint representation are presented. In fact, the joint representation consists in four successive actions that have to be carried out by the
designer in order to "represent" the actual response of the joints into the frame analysis and design process. These four steps are [2]:

- **the joint characterisation**  
  i.e. the evaluation through appropriate means of the stiffness, resistance and ductility properties of the joints (full $M-\varphi$ curves or key values);

- **the joint modelling**  
  i.e. the way on how the joint is physically represented in view of the frame analysis;

- **the joint classification**  
  i.e. the tool providing boundary conditions for the use of conventional types of joint modelling (e.g. rigid or pinned);

- **the joint idealisation**  
  i.e. the derivation of a simplified moment-rotation curve so as to fit with specific analysis approaches (e.g. linear idealisation for an elastic analysis).

\[\text{(a) Beam-to-column joint} \quad \text{(b) Beam splice}\]

**Figure 1** Typical joints in a pitched-roof portal frame

## 2. JOINT CHARACTERISATION

The "component method" [3] is nowadays widely recognised as the most efficient procedure for joint characterisation. It has been adopted as the common procedure in Eurocodes for the evaluation of the mechanical properties of steel joints, composite joints and column bases. Three main aspects have to be considered:

- the external loading on the joint;
- the distribution of internal forces in the joint;
- the properties of the constitutive components.

They are successively introduced here below in the case of a beam-to-column joint with an end-plate connection; conclusions drawn are however valid whatever is the joint configuration and connection type (beam splice, haunched beam, ...).

### Loading

In rectangular frames, beam-to-column joints and beam splices are mainly subjected to in-plane bending moments and shear forces.
Axial forces in the beam remain rather low and are usually disregarded. In pitched-roof portal frames, the inclination of the rafters induce higher axial forces and a specific attention has, in all the cases, to be paid to their influence on the joint rotational response. Another aspect is the non-perpendicularity between the beam cross-sections and the connection cross-section, as indicated in Figure 2, what is again due to the inclination of the rafters.

![Beam and connection cross-sections](image)

**Figure 2** Beam and connection cross-sections

As a result, the internal forces $M_r$, $V_r$ and $N_r$ acting at the rafter extremity and obtained through a frame analysis have to be transformed into a bending moment $M$, a shear force $V$ and an axial force $N$ acting on the connection cross-section, as shown in Figure 3.

These forces are derived as follows:

$$M = M_r$$

$$V = V_r \cos \alpha + N_r \sin \alpha$$

$$N = N_r \cos \alpha - V_r \sin \alpha$$

Equations (1.a), (1.b), (1.c)

$M$, $N$ and $V$ are expressed at the level of the rafter neutral axis.

![External forces acting on the connection](image)

**Figure 3** External forces acting on the connection

*Distribution of internal forces*

The distribution of the external forces $M$, $N$ and $V$ into internal forces within the joint components relates to what is usually defined as the step number three «joint assembly» of the component method. The assembly procedures for joints in bending are described in Revised Annex J. Assembly procedures for joints subjected to axial
forces in addition to bending moments are available in the literature [4]. All the principles given in Revised Annex J and the procedures reported in Eurocode 3 and other publications [4] applies indifferently to «rectangular» joints and to «non-rectangular» ones.

Component properties

In this paragraph, the applicability to joints in pitched-roof portal frames of the rules included in the Eurocode 3 Revised Annex J for the characterisation of the mechanical properties of the basic individual components is contemplated. Through a quick examination of the components, it appears clearly that:
- some components are affected by the rafter inclination;
- some components are not affected at all;
- some components can no more be used as soon as the rafters are inclined.
In Table 1, the component available in Eurocode 3 Annex J are classified according to these three categories.
The influence of the rafter inclination is seen to be restricted to five components.

Table 1 Influence of rafter inclination on the component properties

<table>
<thead>
<tr>
<th>№</th>
<th>Components</th>
<th>Affected</th>
<th>Not affected</th>
<th>No more available</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Column web panel in shear</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Column web in compression</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Beam flange and web in compression</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Column flange in bending</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Column web in tension</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>End-plate in bending</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>Beam web in tension</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>Flange cleat in bending</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>Bolts in tension</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>Bolts in shear</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>Bolts in bearing</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>Plate in tension or compression</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* For welded connections only

In [4], detailed applications rules covering these five components are made available to the reader. This one will also get in [4] useful information on beam-to-column joints with haunched beams (Figure 1.a) and beam splices with stiffened end-plates (Figure 1.b) which are not presently covered by Revised Annex J of Eurocode 3.

3. JOINT MODELLING

Eurocode 3 Revised Annex J distinguishes between three different types of joint modelling: continuous, semi-continuous and simple joint modelling. For the semi-continuous modelling, reference is made to a so-called “rotational spring model” (Table 2). This approach applies similarly to rectangular or non-rectangular frames.
4. JOINT CLASSIFICATION

Eurocode 3 revised Annex J proposes criteria for stiffness, resistance and ductility classification of beam-to-column joints in rectangular frames. Only the stiffness classification is likely to be affected by the beam inclination.

In [5], the possibility to extend the actual Eurocode 3 stiffness classification boundaries to joints belonging to pitched-roof portal frames has been demonstrated. However, the length of the beam to be used when applying the Eurocode 3 classification formula is the developed length of the beam and not the distance between the column axes.

<table>
<thead>
<tr>
<th>JOINT MODELLING</th>
<th>BEAM-TO-COLUMN JOINTS</th>
<th>BEAM SPLICES</th>
<th>COLUMN BASES</th>
</tr>
</thead>
<tbody>
<tr>
<td>SIMPLE</td>
<td>![Diagram]</td>
<td>![Diagram]</td>
<td>![Diagram]</td>
</tr>
<tr>
<td>SEMI-CONTINUOUS</td>
<td>![Diagram]</td>
<td>![Diagram]</td>
<td>![Diagram]</td>
</tr>
<tr>
<td>CONTINUOUS</td>
<td>![Diagram]</td>
<td>![Diagram]</td>
<td>![Diagram]</td>
</tr>
</tbody>
</table>

Table 2 Modelling for joint

5. JOINT IDEALISATION

The elastic analysis of a steel frame with semi-rigid joints requires the linear idealisation of the behaviour laws for the beam and column elements as well as for the joints in bending. For the beam and column elements, the linear idealisation of the response is achieved through the definition of the so-called flexural rigidity $EI/L$ ($I$ is the inertia and $L$ the length of the considered element). For joints, a similar idealisation has to be contemplated.

In an elastic design procedure, two different approaches can be followed:

- **Elastic verification of the cross-section**
  Further to the elastic frame analysis, the maximum applied bending moment in each element is compared to the maximum elastic resistant moment $M_{el,rd}$ of the cross-section or of the joint.

- **Plastic verification of the cross-section**
  The plastic verification consists in limiting the value of the maximum applied bending moment in each element to the plastic moment resistance $M_{pl,rd}$ of the cross-section or of the joint.

When a plastic analysis of the frame is contemplated, it is usually recommended, for beams and column cross-sections, to refer to the so-called «plastic hinge idealisation». A similar concept also applies to joints. In the present paper, only the linear idealisation
of the joint response in view of an elastic structural analysis is discussed as it is the only one which is likely to be affected by the beam inclination.

![Diagram](image)

Figure 4 Linear idealisations of the joint flexural response

In Figure 4, the two verification procedures are illustrated. For the elastic one, reference is made to the initial stiffness ($S_{j,ini}$) and to the elastic resistance ($2/3M_{Rd}$) of the joint while, for the plastic verification procedure, an equivalent reduced stiffness ($S_{j,eq}$) is used in combination with the plastic resistance of the joints ($M_{Rd}$). $S_{j,eq}$ is defined in Eurocode 3 Revised Annex J as a proportion of $S_{j,ini}$ ($S_{j,eq} = S_{j,ini} / \eta$). In [5], it is shown that the values of $\eta$ recommended for joints in rectangular frames may apply with a similar accuracy to joints in pitched-roof portal frames. But more precise expressions of $\eta$ than those suggested by Eurocode 3 are also proposed in [5].

CONCLUSIONS

Pitched-roof portal frames represent a significant part of the activity in steel construction and the design of their structural joints is of high economical importance. In the present paper, it is seen that the modifications to bring to the existing rules for joints belonging to rectangular frames are quite limited and are restricted to the characterisation of the mechanical joint properties (stiffness, resistance and ductility).

REFERENCES

JOINT REPRESENTATION IN INDUSTRIAL PITCHED-ROOF PORTAL FRAMES

Jean-Pierre Jaspart
Department MSM, University of Liège
Belgium

Keywords: Structural Joints, Pitched-Roof Industrial Portal frames, Characterisation, Modelling, Classification, Idealisation, Static Behaviour

Abstract: Most of the research works, normative documents and design aids for engineers in the field of structural joints cover cases where the two connected elements form an angle of 0° (beam splices or column splices) or 90° (beam-to-column joints). In the present paper, the application of the available knowledge on structural joints to industrial pitched-roof portal frames where the connected members form an angle intermediate between these two extreme values is discussed. The modifications required by the extension of the available design rules to this area are seen to be limited.

1. INTRODUCTION

In building structures, the structural steel frame is usually made of beam and column elements connected perpendicularly so as to constitute what could be called rectangular frames, such as office buildings.

Most of the experimental, numerical and theoretical research works carried out in the field of structural joints relate to this type of structural configuration and Eurocode 3 Revised Annex J [1], which appears as the outcome of these research works, is not an exception even if nothing is explicitly said about that.

Pitched-roof industrial portal frames where the structural elements are not perpendicularly connected (Figure 1) represent however a quite important part of the steel construction activity where high profit of the new concepts introduced in Eurocode 3, and in particularly in its Revised Annex J, could be made. This topic is covered in the present paper where the slight modifications to be made to the Eurocode 3 rules for joint representation are presented. In fact, the joint representation consists in four successive actions that have to be carried out by the
designer in order to “represent” the actual response of the joints into the frame analysis and design process. These four steps are [2]:

- **the joint characterisation**
  i.e. the evaluation through appropriate means of the stiffness, resistance and ductility properties of the joints (full $M$-$\varphi$ curves or key values);
- **the joint modelling**
  i.e. the way on how the joint is physically represented in view of the frame analysis;
- **the joint classification**
  i.e. the tool providing boundary conditions for the use of conventional types of joint modelling (e.g. rigid or pinned);
- **the joint idealisation**
  i.e. the derivation of a simplified moment-rotation curve so as to fit with specific analysis approaches (e.g. linear idealisation for an elastic analysis).

![Beam-to-column joint and Beam splice](image)

**Figure 1** Typical joints in a pitched-roof portal frame

## 2. JOINT CHARACTERISATION

The “component method” [3] is nowadays widely recognised as the most efficient procedure for joint characterisation. It has been adopted as the common procedure in Eurocodes for the evaluation of the mechanical properties of steel joints, composite joints and column bases.

Three main aspects have to be considered:

- the external loading on the joint;
- the distribution of internal forces in the joint;
- the properties of the constitutive components.

They are successively introduced here below in the case of a beam-to-column joint with an end-plate connection; conclusions drawn are however valid whatever is the joint configuration and connection type (beam splice, haunched beam, ...).

**Loading**

In rectangular frames, beam-to-column joints and beam splices are mainly subjected to in-plane bending moments and shear forces.
Axial forces in the beam remain rather low and are usually disregarded. In pitched-roof portal frames, the inclination of the rafters induce higher axial forces and a specific attention has, in all the cases, to be paid to their influence on the joint rotational response. Another aspect is the non-perpendicularity between the beam cross-sections and the connection cross-section, as indicated in Figure 2, what is again due to the inclination of the rafters.

![Beam and connection cross-sections](image)

*Figure 2 Beams and connection cross-sections*

As a result, the internal forces $M_r$, $V_r$ and $N_r$ acting at the rafter extremity and obtained through a frame analysis have to be transformed into a bending moment $M$, a shear force $V$ and an axial force $N$ acting on the connection cross-section, as shown in Figure 3.

These forces are derived as follows:

\[
M = M_r, \quad V = V_r \cos \alpha + N_r \sin \alpha, \quad N = N_r \cos \alpha - V_r \sin \alpha.
\]

(1.a) \hspace{2cm} (1.b) \hspace{2cm} (1.c)

$M$, $N$ and $V$ are expressed at the level of the rafter neutral axis.

![External forces acting on the connection](image)

*Figure 3 External forces acting on the connection*

**Distribution of internal forces**

The distribution of the external forces $M$, $N$ and $V$ into internal forces within the joint components relates to what is usually defined as the step number three «joint assembly» of the component method. The assembly procedures for joints in bending are described in Revised Annex J. Assembly procedures for joints subjected to axial
forces in addition to bending moments are available in the literature [4]. All the principles given in Revised Annex J and the procedures reported in Eurocode 3 and other publications [4] applies indifferently to «rectangular» joints and to «non-rectangular» ones.

**Component properties**

In this paragraph, the applicability to joints in pitched-roof portal frames of the rules included in the Eurocode 3 Revised Annex J for the characterisation of the mechanical properties of the basic individual components is contemplated. Through a quick examination of the components, it appears clearly that:

- some components are affected by the rafter inclination;
- some components are not affected at all;
- some components can no more be used as soon as the rafters are inclined.

In Table 1, the component available in Eurocode 3 Annex J are classified according to these three categories.

The influence of the rafter inclination is seen to be restricted to five components.

**Table 1 Influence of rafter inclination on the component properties**

<table>
<thead>
<tr>
<th>Nº</th>
<th>Components</th>
<th>Affected</th>
<th>Not affected</th>
<th>No more available</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Column web panel in shear</td>
<td></td>
<td></td>
<td>x</td>
</tr>
<tr>
<td>2</td>
<td>Column web in compression</td>
<td>x</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Beam flange and web in compression</td>
<td>x</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Column flange in bending</td>
<td>x*</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Column web in tension</td>
<td>x*</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>End-plate in bending</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>Beam web in tension</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>Flange cleat in bending</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>Bolts in tension</td>
<td>x</td>
<td></td>
<td>x</td>
</tr>
<tr>
<td>10</td>
<td>Bolts in shear</td>
<td>x</td>
<td></td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>Bolts in bearing</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>Plate in tension or compression</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* For welded connections only

In [4], detailed applications rules covering these five components are made available to the reader. This one will also get in [4] useful information on beam-to-column joints with haunched beams (Figure 1.a) and beam splices with stiffened end-plates (Figure 1.b) which are not presently covered by Revised Annex J of Eurocode 3.

**3. JOINT MODELLING**

Eurocode 3 Revised Annex J distinguishes between three different types of joint modelling: continuous, semi-continuous and simple joint modelling. For the semi-continuous modelling, reference is made to a so-called “rotational spring model” (Table 2). This approach applies similarly to rectangular or non-rectangular frames.
4. JOINT CLASSIFICATION

Eurocode 3 revised Annex J proposes criteria for stiffness, resistance and ductility classification of beam-to-column joints in rectangular frames. Only the stiffness classification is likely to be affected by the beam inclination.
In [5], the possibility to extend the actual Eurocode 3 stiffness classification boundaries to joints belonging to pitched-roof portal frames has been demonstrated. However the length of the beam to be used when applying the Eurocode 3 classification formulae is the developed length of the beam and not the distance between the column axes.

Table 2 Modelling for joint

<table>
<thead>
<tr>
<th>JOINT MODELLING</th>
<th>BEAM-TO-COLUMN JOINTS MAJOR AXIS BENDING</th>
<th>BEAM SPLICES</th>
<th>COLUMN BASES</th>
</tr>
</thead>
<tbody>
<tr>
<td>SIMPLE</td>
<td><img src="SimpleDiagram" alt="Diagram" /></td>
<td><img src="SimpleDiagram" alt="Diagram" /></td>
<td><img src="SimpleDiagram" alt="Diagram" /></td>
</tr>
<tr>
<td>SEMI-CONTINUOUS</td>
<td><img src="Semi-ContinuousDiagram" alt="Diagram" /></td>
<td><img src="Semi-ContinuousDiagram" alt="Diagram" /></td>
<td><img src="Semi-ContinuousDiagram" alt="Diagram" /></td>
</tr>
<tr>
<td>CONTINUOUS</td>
<td><img src="ContinuousDiagram" alt="Diagram" /></td>
<td><img src="ContinuousDiagram" alt="Diagram" /></td>
<td><img src="ContinuousDiagram" alt="Diagram" /></td>
</tr>
</tbody>
</table>

5. JOINT IDEALISATION

The elastic analysis of a steel frame with semi-rigid joints requires the linear idealisation of the behaviour laws for the beam and column elements as well as for the joints in bending. For the beam and column elements, the linear idealisation of the response is achieved through the definition of the so-called flexural rigidity $EI/L$ ($I$ is the inertia and $L$ the length of the considered element). For joints, a similar idealisation has to be contemplated.
In an elastic design procedure, two different approaches can be followed:
- **Elastic verification of the cross-section**
  Further to the elastic frame analysis, the maximum applied bending moment in each element is compared to the maximum elastic resistant moment $M_{el,ld}$ of the cross-section or of the joint.
- **Plastic verification of the cross-section**
  The plastic verification consists in limiting the value of the maximum applied bending moment in each element to the plastic moment resistance $M_{pl,ld}$ of the cross-section or of the joint.

When a plastic analysis of the frame is contemplated, it is usually recommended, for beams and column cross-sections, to refer to the so-called «plastic hinge idealisation». A similar concept also applies to joints. In the present paper, only the linear idealisation
of the joint response in view of an elastic structural analysis is discussed as it is the only one which is likely to be affected by the beam inclination.

![Diagram](image)

(a) Elastic verification \(S_j = S_{j, ini}\)  
(b) Plastic verification \(S_j = S_{j, eqn}\)

**Figure 4** Linear idealisations of the joint flexural response

In Figure 4, the two verification procedures are illustrated. For the elastic one, reference is made to the initial stiffness \(S_{j, ini}\) and to the elastic resistance \(2/3M_{Rd}\) of the joint while, for the plastic verification procedure, an equivalent reduced stiffness \(S_{j, eqn}\) is used in combination with the plastic resistance of the joints \(M_{Rd}\). \(S_{j, eqn}\) is defined in Eurocode 3 Revised Annex J as a proportion of \(S_{j, ini} (S_{j, eqn} = S_{j, ini} / \eta)\). In [5], it is shown that the values of \(\eta\) recommended for joints in rectangular frames may apply with a similar accuracy to joints in pitched-roof portal frames. But more precise expressions of \(\eta\) than those suggested by Eurocode 3 are also proposed in [5].

**CONCLUSIONS**

Pitched-roof portal frames represent a significant part of the activity in steel construction and the design of their structural joints is of high economical importance. In the present paper, it is seen that the modifications to bring to the existing rules for joints belonging to rectangular frames are quite limited and are restricted to the characterisation of the mechanical joint properties (stiffness, resistance and ductility).

**REFERENCES**