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MARIE CURIE ACTIONS

Parameter Estimation and Structural Model Updating Using Modal Methods in the Presence of Nonlinearity

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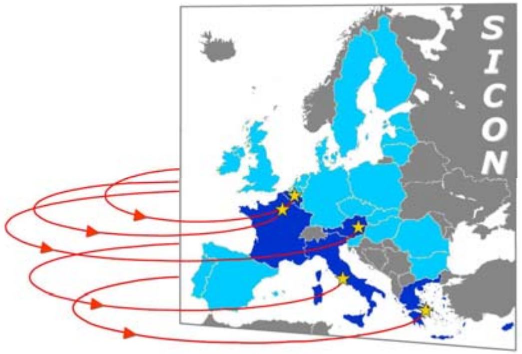


Outline

1. Introduction
2. Theoretical Modal Analysis of Nonlinear Systems
3. Nonlinear Experimental Modal Analysis
4. Model Parameter Estimation Techniques
5. Concluding Remarks

Introduction

Design of engineering structures relies on

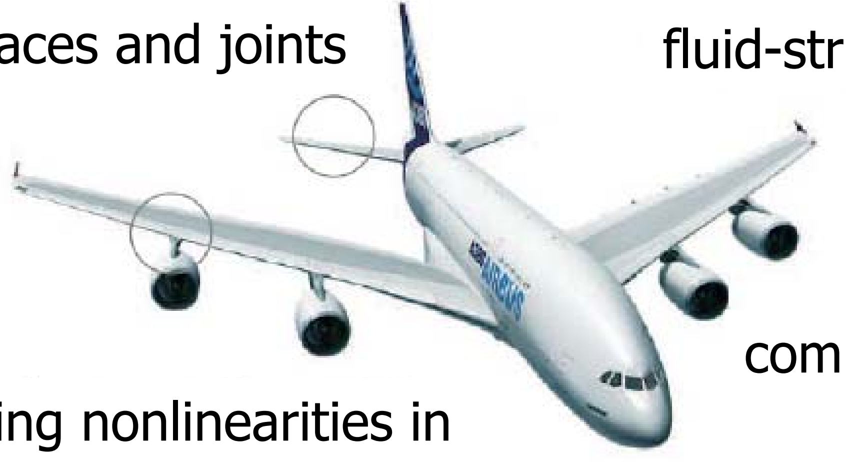
- Numerical predictions → modal analysis (FEM)
- Dynamic testing → experimental modal analysis (EMA)

In the case of **linear structures**, the techniques available for EMA are mature e.g.

- Eigensystem realization algorithm
- Stochastic subspace identification
- Polyreference least-squares complex exponentials frequency domain
- etc

Introduction

Nonlinearity in Engineering Applications



backlash and friction in control surfaces and joints

fluid-structure interaction

composite materials

hardening nonlinearities in engine-to-pylon connections

Many works are reported in the literature on dynamic testing and identification of nonlinear systems but very few address nonlinear phenomena during modal survey tests.

Introduction

Aim of this presentation

- To extend experimental modal analysis to a practical analogue using the nonlinear normal mode (NNM) theory.
- Validate mathematical models of non-linear structures against experimental data.

Why?

- NNMs offer a solid and rigorous mathematical tool.
- They have a clear conceptual relation to the classical LNMs.
- They are capable of handling strong structural nonlinearity.

Outline

1. Introduction
2. Theoretical Modal Analysis of Nonlinear Systems
   - Nonlinear Normal Modes (NNMs)
   - Numerical Computation of NNMs
   - Frequency-Energy Plot
3. Nonlinear Experimental Modal Analysis
4. Model Parameter Estimation Techniques
5. Concluding Remarks

Theoretical Modal Analysis

MDOF system

$$\mathbf{M}\,\ddot{\mathbf{x}}+\mathbf{C}\,\dot{\mathbf{x}}+\mathbf{K}\,\mathbf{x}+\mathbf{f}_{NL}(\mathbf{x},\dot{\mathbf{x}})=\mathbf{p}(t)$$

Vector of nonlinear forces

- **Dynamic analysis**

Prediction of the responses using a numerical integration procedure (e.g. Newmark’s schema)

- **Modal analysis of the MDOF system (with no damping)**

In the linear case

$$\mathbf{M}\,\ddot{\mathbf{x}}+\mathbf{K}\,\mathbf{x}=0$$

Structural eigenproblem

j th eigenvector

$$\mathbf{K}\,\mathbf{\Phi}_j=\omega_j^2\,\mathbf{M}\,\mathbf{\Phi}_j \qquad j=1,\cdots,n$$

j th natural frequency

In the nonlinear case

$$\mathbf{M}\,\ddot{\mathbf{x}}+\mathbf{K}\,\mathbf{x}+\mathbf{f}_{NL}\left(\mathbf{x},\dot{\mathbf{x}}\right)=0$$

Use of the concept of nonlinear normal modes (NNMs) which is a rigorous extension of the concept of eigenmodes to nonlinear systems.

Nonlinear Normal Modes

Definitions

Two definitions of an NNM in the literature:

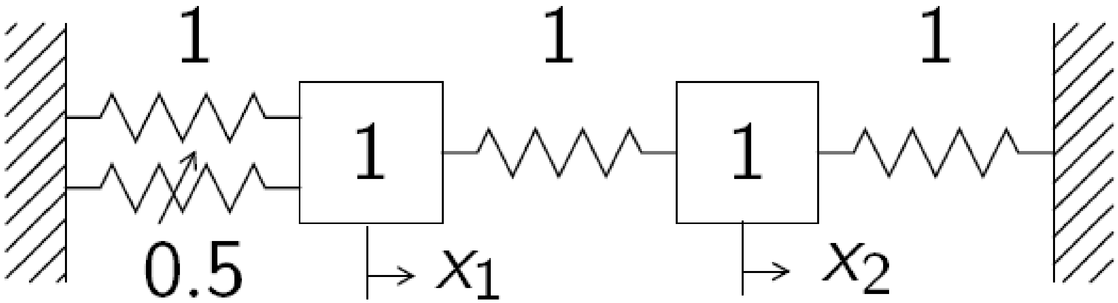
1. Targeting a straightforward nonlinear extension of the linear normal mode (LNM) concept, ***Rosenberg*** defined an NNM motion as a vibration in unison of the system (i.e., a synchronous periodic oscillation).
2. To provide an extension of the NNM concept to damped systems, ***Shaw and Pierre*** defined an NNM as a two-dimensional invariant manifold in phase space. Such a manifold is invariant under the flow (i.e., orbits that start out in the manifold remain in it for all time), which generalizes the invariance property of LNMs to nonlinear systems.

In the ***present study***, an NNM motion is defined as a (non-necessarily synchronous) periodic motion of the undamped mechanical system

→ this extended definition is particularly attractive when targeting a numerical computation of the NNMs.

Nonlinear Normal Modes

Illustrative example: 2 DOF-system with a cubic stiffness

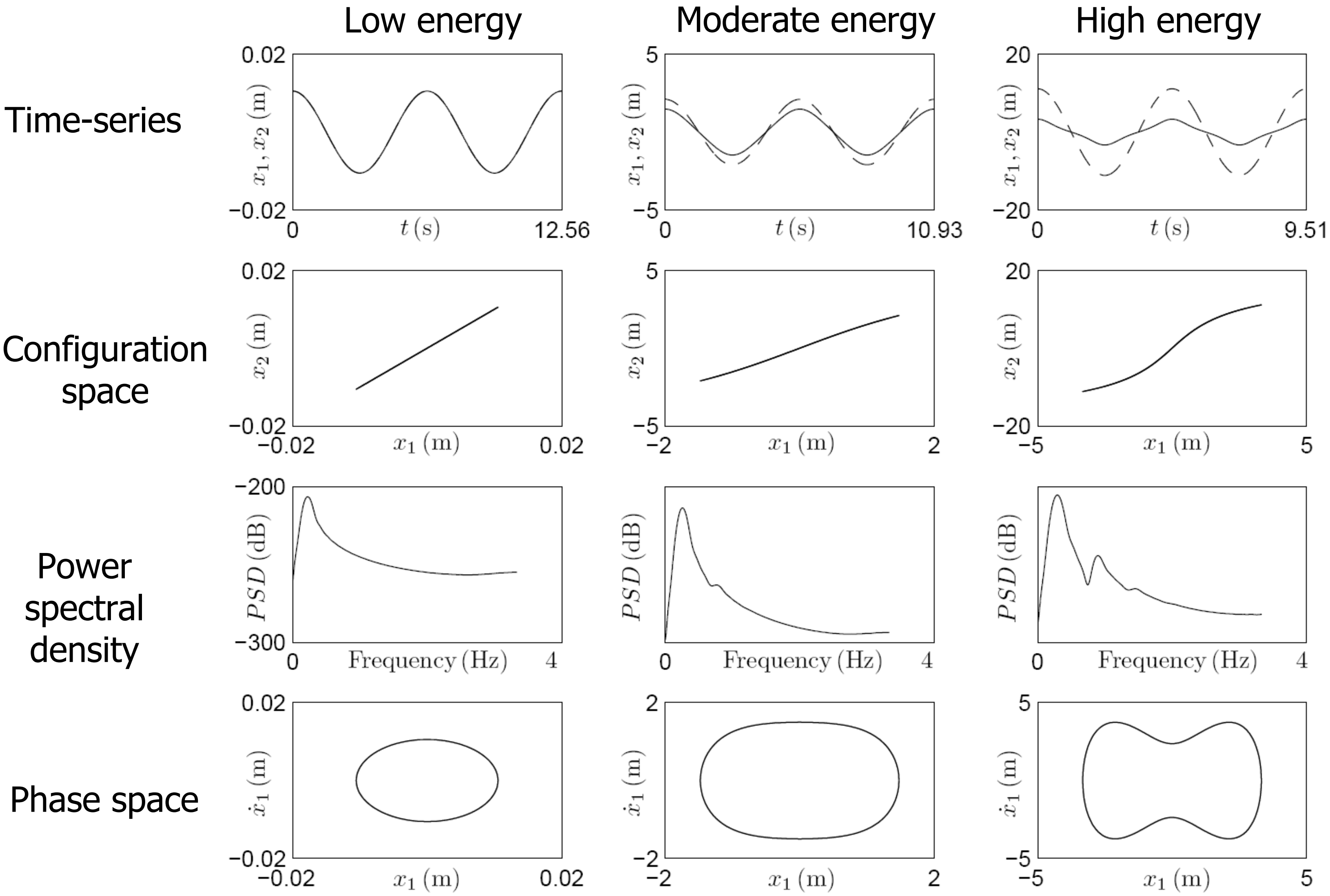


$$\ddot{x}_1 + \left(2\, x_1 - x_2\right) + 0.5\, x_1^3 = 0$$

$$\ddot{x}_2 + \left(2\, x_2 - x_1\right) = 0$$

Nonlinear Normal Modes

In-Phase NNMs for Increasing Energy



Numerical Computation of NNMs

General equation of the nonlinear system (with no damping)

$$\mathbf{M}\,\ddot{\mathbf{x}}+\mathbf{K}\,\mathbf{x}+\underset{\big\downarrow}{\mathbf{f}_{NL}}\left(\mathbf{x},\dot{\mathbf{x}}\right)=0$$

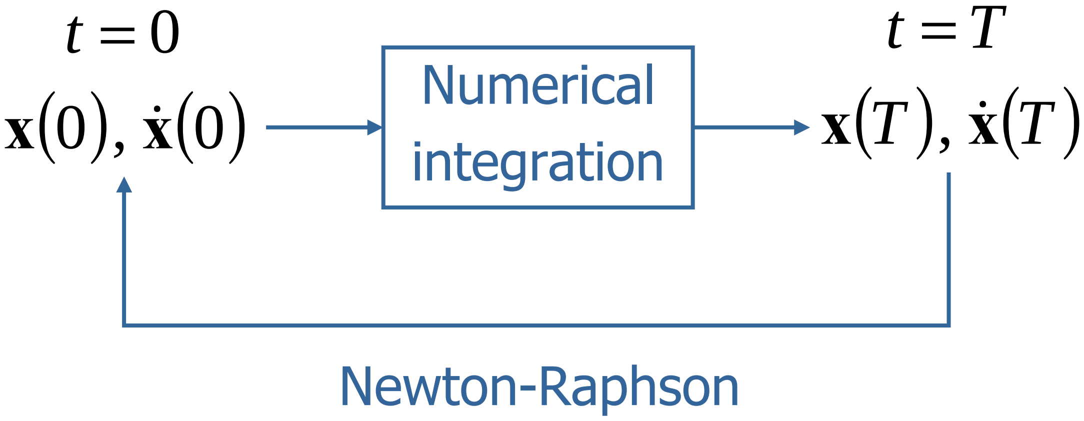
Vector of nonlinear forces

The numerical computation of NNMs relies on two main techniques, namely a ***shooting procedure*** and a method for the ***continuation of periodic solutions***.

Numerical Computation of NNMs

- **Shooting method**

The shooting method consists in finding, in an iterative way, the initial conditions $\mathbf{x}(0), \dot{\mathbf{x}}(0)$ and the period T inducing an isolated periodic motion (i.e., an NNM motion) of the conservative system.



$t = 0$

$\mathbf{x}(0), \dot{\mathbf{x}}(0)$

Numerical integration

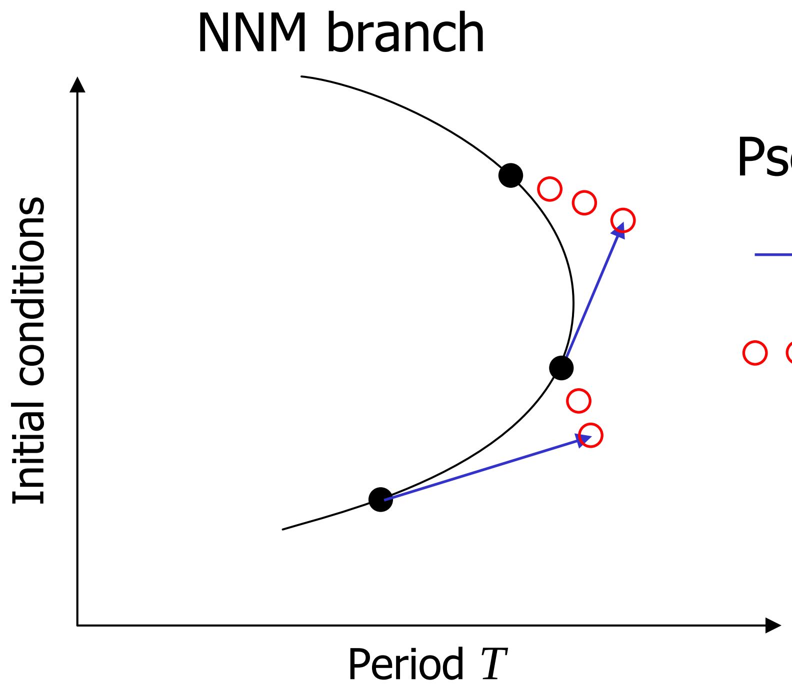
$t = T$

$\mathbf{x}(T), \dot{\mathbf{x}}(T)$

Newton-Raphson

Numerical Computation of NNMs

- **Pseudo-arclength continuation method**

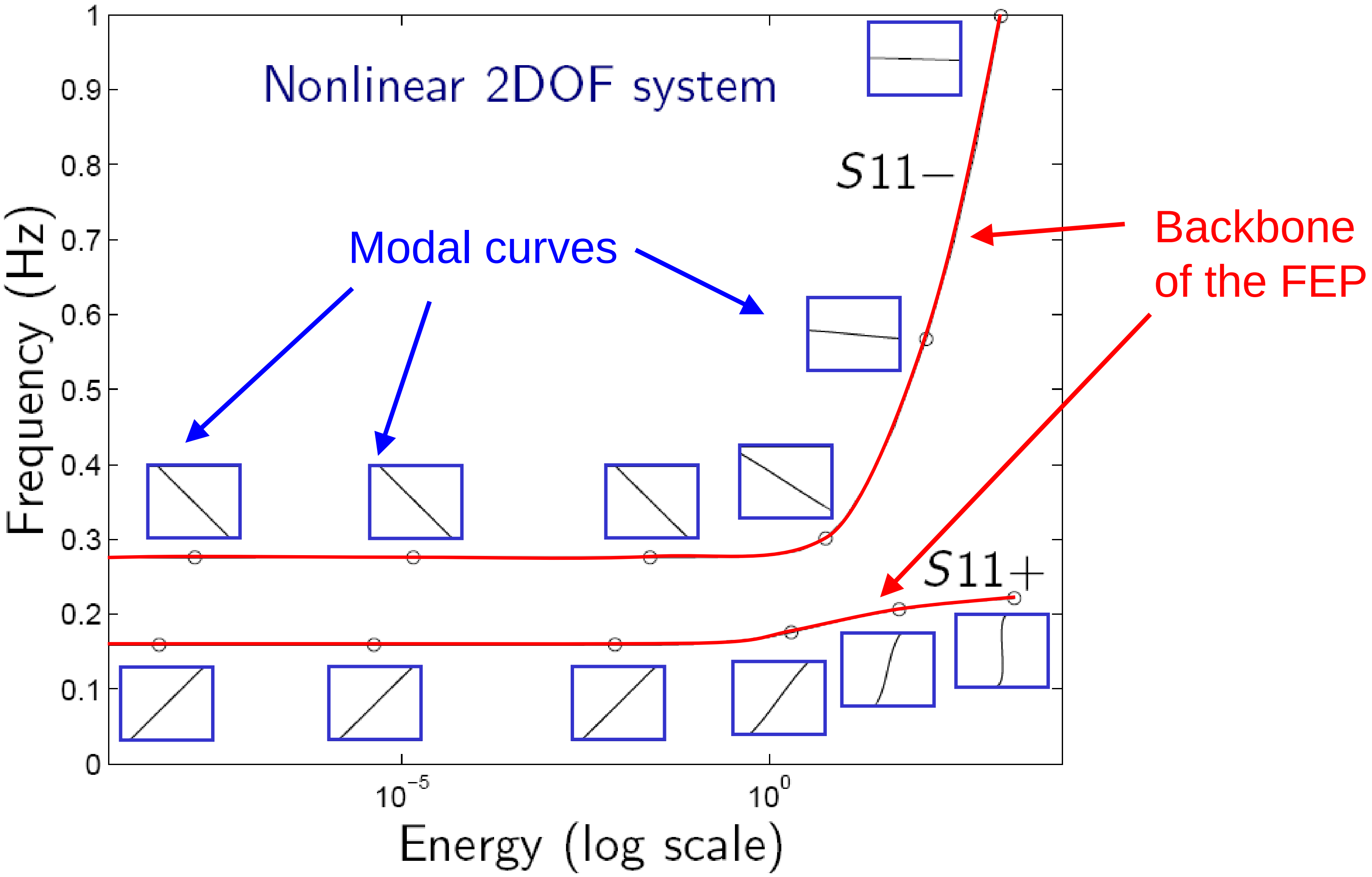


Pseudo-arclength continuation method:

→ predictor step tangent to the branch

○ ○ ○ corrector step perpendicular to the predictor step (shooting)

Frequency-Energy Plot (FEP)



Modal curves

Backbone of the FEP

Outline

1. Introduction

2. Theoretical Modal Analysis of Nonlinear Systems

3. Nonlinear Experimental Modal Analysis

   - Phase Separation Methods

     - Proper Orthogonal Decomposition

   - Phase Resonance Methods

     - Nonlinear Normal Mode Testing

4. Model Parameter Estimation Techniques

5. Concluding Remarks

Experimental Modal Analysis (EMA)

Linear systems

Theoretical Approach

Finite Element Model

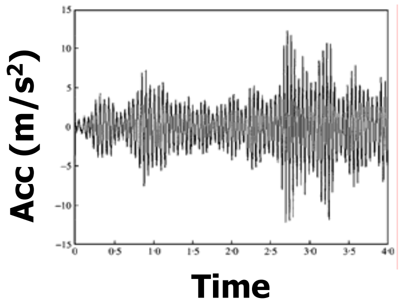
$$\mathbf{M}\,\ddot{\mathbf{x}} + \mathbf{K}\,\mathbf{x} = 0$$

Eigenvalue problem

$$\mathbf{K}\,\mathbf{\Phi}_j = \omega_j^2\,\mathbf{M}\,\mathbf{\Phi}_j$$

Experimental Approach

Response Measurements



Time series

Identification methods

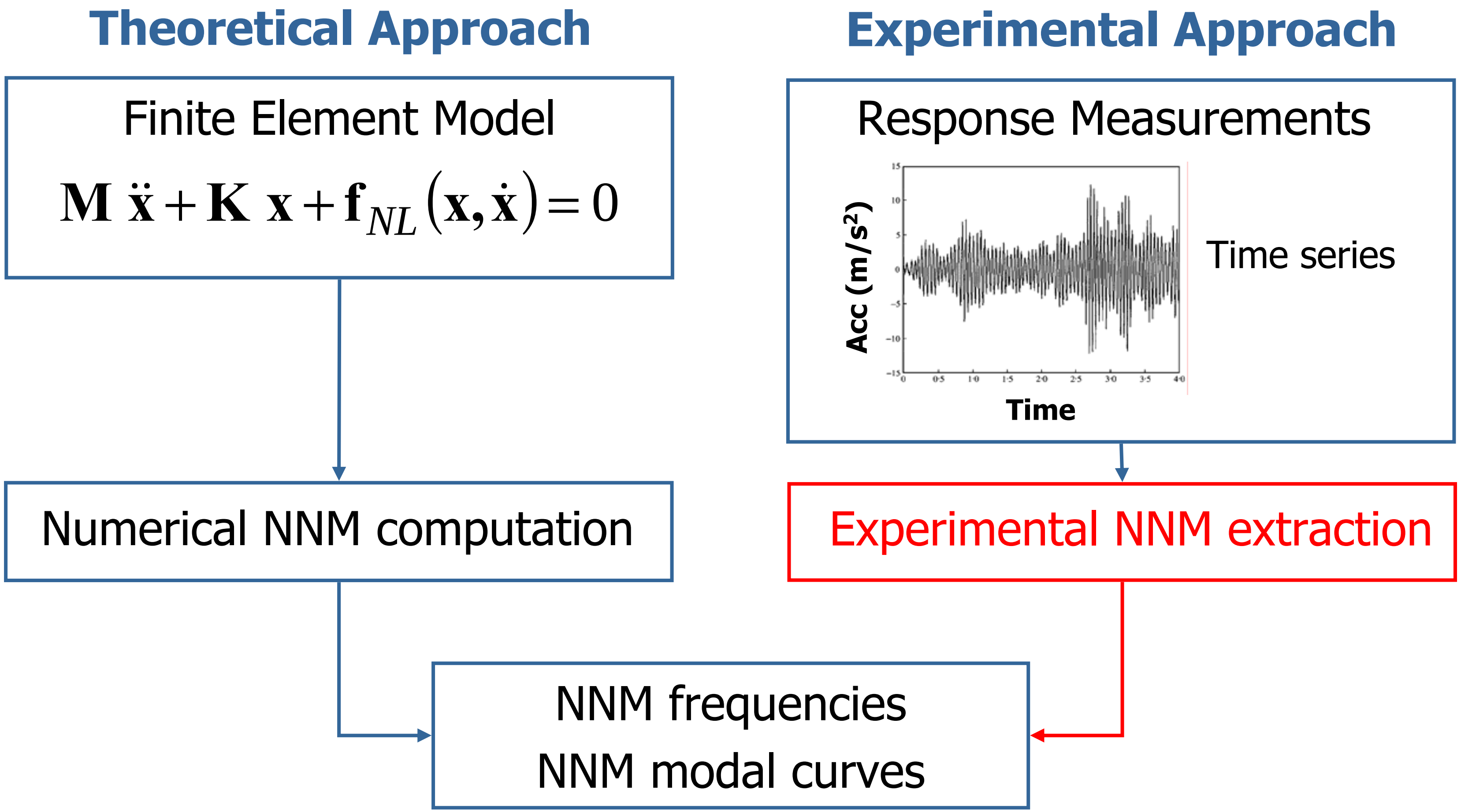
Natural frequencies (ω_j^2)

Mode shapes ($\mathbf{\Phi}_j$)

EMA for linear systems is now mature and widely used in structural engineering → well established techniques [1], [2].

Experimental Modal Analysis (EMA)

Nonlinear systems



Theoretical Approach

Finite Element Model

$$\mathbf{M}\,\ddot{\mathbf{x}}+\mathbf{K}\,\mathbf{x}+\mathbf{f}_{NL}\left(\mathbf{x},\dot{\mathbf{x}}\right)=0$$

Numerical NNM computation

Experimental Approach

Response Measurements

Time series

Experimental NNM extraction

NNM frequencies

NNM modal curves

EMA for nonlinear systems is still a challenge.

Experimental Modal Analysis (EMA)

There are two main techniques for EMA.

1. Phase separation methods

Several modes are excited at once using either broadband excitation (e.g., hammer impact and random excitation) or swept-sine excitation in the frequency range of interest.

→ in the nonlinear case, extraction of individual NNMs is not possible generally, because modal superposition is no longer valid.

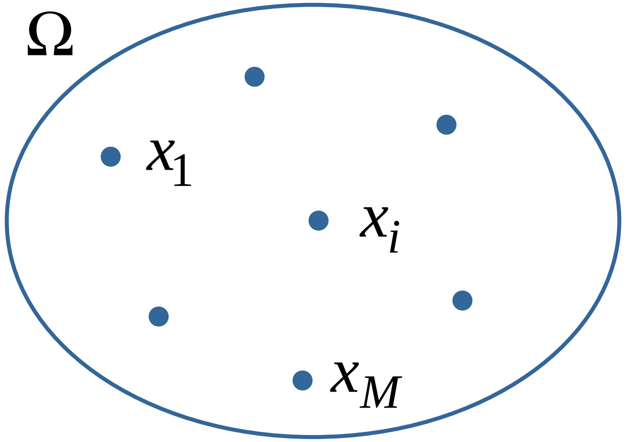
→ use of the proper orthogonal decomposition (POD) method to extract features from the time series .

Remark

- All structures encountered in practice are nonlinear to some degree.
- If a nonlinear structure is excited with a broadband excitation signal (e.g. random force), then the results will appear linear → experimental modal analysis will lead to an updated linearized model !

Proper Orthogonal Decomposition (POD)

Instrumented structure



$$\mathbf{X} = \left.\overbrace{\begin{bmatrix} x_1(t_1) & \cdots & x_1(t_N) \\ \vdots & \ddots & \vdots \\ x_M(t_1) & \cdots & x_M(t_N) \end{bmatrix}}^{N \text{ snapshots}}\right\} \begin{array}{c} M \\ \text{measurement} \\ \text{co-ordinates} \end{array}$$

$\mathbf{X} = [\mathbf{x}(t_1) \quad \ldots \quad \mathbf{x}(t_N)]$ is the observation matrix

Proper Orthogonal Decomposition (POD)

The M x M correlation matrix $\mathbf{R}$ is built

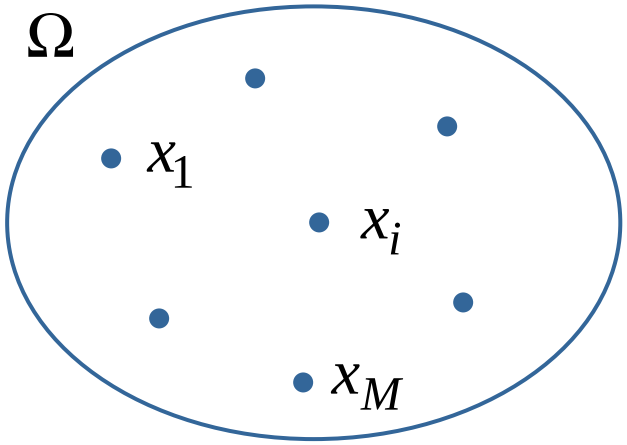
$$\mathbf{R} = \frac{1}{M}\,\mathbf{X}\,\mathbf{X}^T$$

The eigenvalue problem is solved

$$\mathbf{R}\,\underset{\text{Eigenvectors of } \mathbf{XX}^T \text{ (POMs)}}{\mathbf{u}} = \overset{\text{Eigenvalues (POVs)}}{\lambda}\,\mathbf{u}$$

Proper Orthogonal Decomposition (POD)

Computation of the POMs using SVD



M measurement co-ordinates

N time samples

$$\mathbf{X}_{M\,x\,N} = \begin{bmatrix} x_1(t_1) & \cdots & x_1(t_N) \\ \vdots & \ddots & \vdots \\ x_M(t_1) & \cdots & x_M(t_N) \end{bmatrix}$$

Using SVD

$$diag(\sqrt{\lambda_i}) \quad (\lambda_i \equiv \mathrm{POV})$$

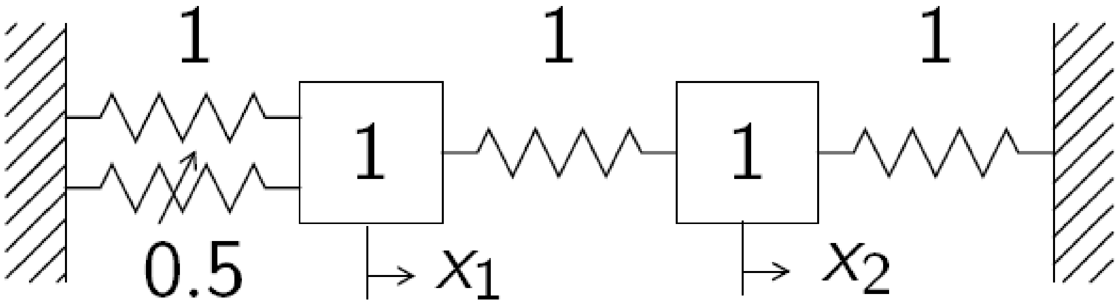
$$\mathbf{X}_{M\times N} = \mathbf{U}_{M\times M}\,\Sigma_{M\times N}\,\mathbf{V}^{\mathbf{T}}_{N\times N}$$

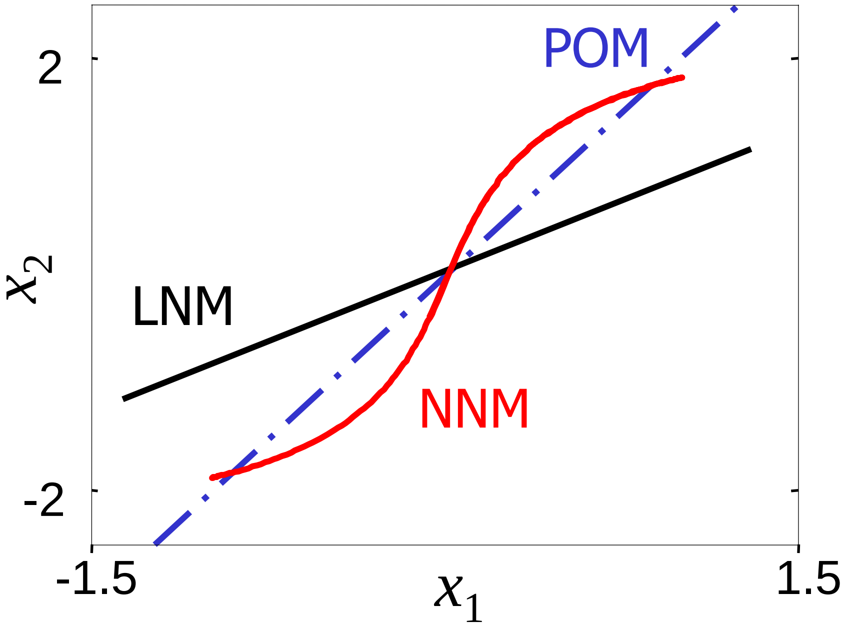
Eigenvectors of $\mathbf{XX}^T$ (POM)

Proper Orthogonal Decomposition (POD)

Geometric Interpretation of the POMs

Comparison of LNM, NNM and POM on the 2 DOF example





First mode

The POM is the best linear representation of the nonlinear normal mode.

Proper Orthogonal Decomposition (POD)

Key idea: Application of the POD to Features Extraction

Linear Systems

$$\mathbf{M}\,\ddot{\mathbf{x}}+\mathbf{C}\,\dot{\mathbf{x}}+\mathbf{K}\,\mathbf{x}=\mathbf{p}(t)$$

Deterministic approach

Eigenvalue problem :

$$(\mathbf{K}-\omega^2\,\mathbf{M})\,\mathbf{\Phi}=\mathbf{0}$$

Response :

Spatial information

$$\mathbf{x}(t)=\sum_{i=1}^{n}\eta_i(t)\,\mathbf{\Phi}_{(i)}$$

Natural frequencies

$$\eta_i=A_i\,\cos(\omega_i\,t)+B_i\,\sin(\omega_i\,t)$$

Nonlinear Systems

$$\mathbf{M}\,\ddot{\mathbf{x}}+\mathbf{C}\,\dot{\mathbf{x}}+\mathbf{K}\,\mathbf{x}+\mathbf{f}_{NL}(\mathbf{x},\dot{\mathbf{x}})=\mathbf{p}(t)$$

Statistical approach

Proper Orthogonal Decomposition :

$$\mathbf{X}=\mathbf{U}\,\Sigma\,\mathbf{V}^{\mathbf{T}}$$

Response :

Spatial information

$$\mathbf{x}(t)=\sum_{j=1}^{n}a_j(t)\,\mathbf{u}_{(j)}$$

Time information

→ **Instantaneous frequencies**

Experimental Modal Analysis (EMA)

2. Phase resonance methods (Normal mode testing)

One of the normal mode at a time is excited using multi-point sine excitation at the corresponding natural frequency. The modes are identified one by one.

→ can be extended to nonlinear structures according to the invariance property of NNMs:

> « *If the motion is initiated on one specific NNM,*
> *the remaining NNMs remain quiescent for all time.* »

Remark

- Expensive and difficult.
- Extremely accurate mode shapes → a way to identify NNMs (but still a research topic).

Nonlinear EMA

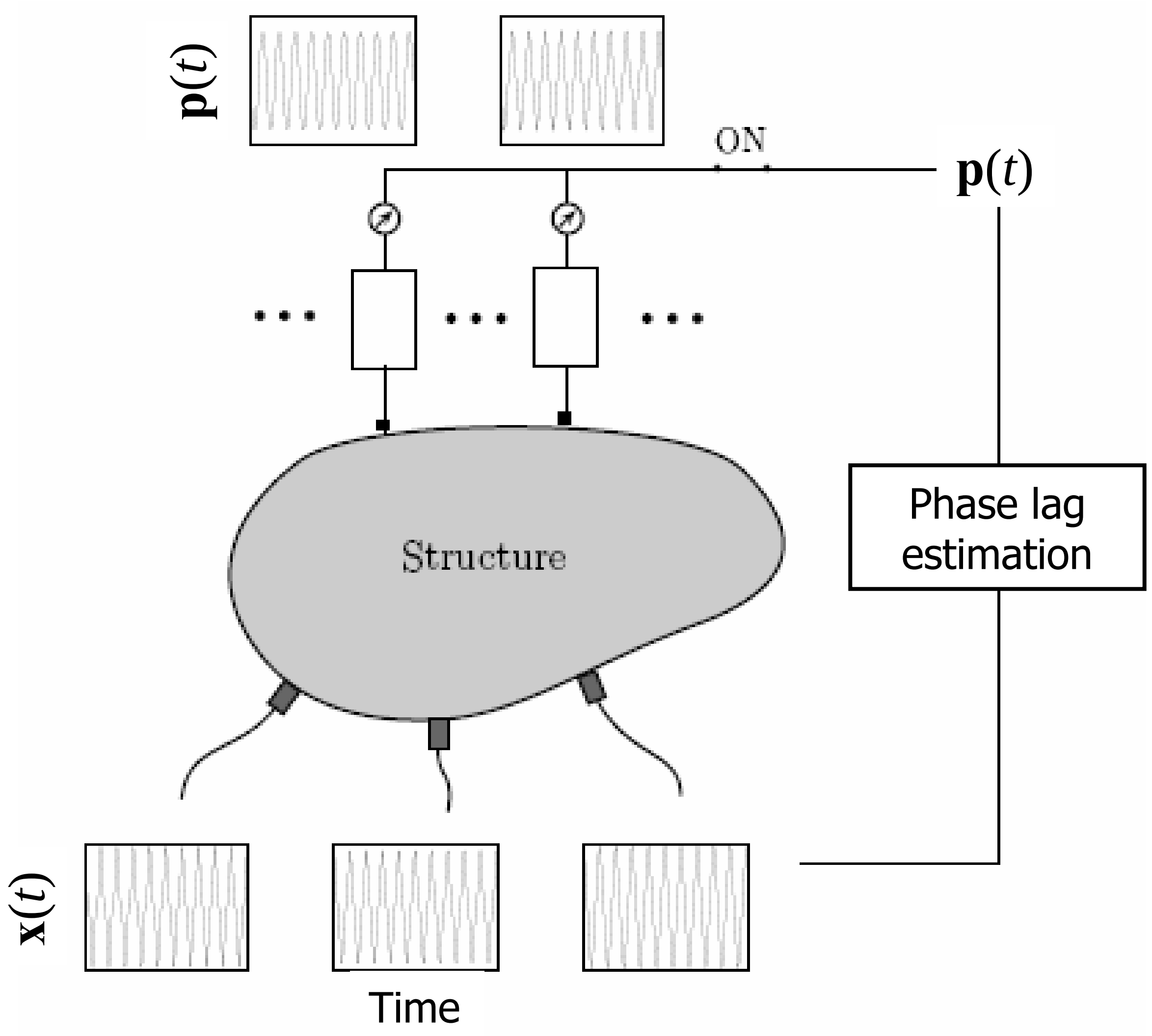
Fundamental properties

1. Forced responses of nonlinear systems at resonance occur in the neighborhood of NNMs [3].
2. According to the invariance property, motions that start out in the NNM manifold remain in it for all time [4].
3. The effect of weak to moderate damping on the transient dynamics is purely parasitic. The free damped dynamics closely follows the NNM of the underlying undamped system [5, 6, 7]

The proposed method for nonlinear EMA relies on a two-step approach that extracts the NNM modal curves and their frequencies of oscillation.

Step 1: NNM Force Appropriation

Objective: isolate a single NNM



Step 1: NNM Force Appropriation

Consider the forced response of a nonlinear structure with linear viscous damping

$$\mathbf{M}\,\ddot{\mathbf{x}}+\mathbf{C}\,\dot{\mathbf{x}}+\mathbf{K}\,\mathbf{x}+\mathbf{f}_{NL}(\mathbf{x})=\mathbf{p}(t)$$

It is assumed here that the nonlinear restoring force contains only stiffness nonlinearities.

Appropriate excitation

For a given NNM motion $\mathbf{x}_{nnm}(t)$ the equations of motion of the forced and damped system lead to the appropriate excitation

$$\mathbf{p}_{nnm}(t)=\mathbf{C}\,\dot{\mathbf{x}}_{nnm}(t)$$

This relationship shows that the appropriate excitation is periodic and has the same frequency components as the corresponding NNM motion (i.e., generally including multiharmonic components).

Step 1: NNM Force Appropriation

An NNM motion is now expressed as a Fourier cosine series

$$\mathbf{x}_{nnm}(t)=\sum_{k=1}^{\infty}\mathbf{X}_k^{nnm}\cos(k\,\omega t)$$

fundamental pulsation of the NNM motion

amplitude vector of the kth harmonic

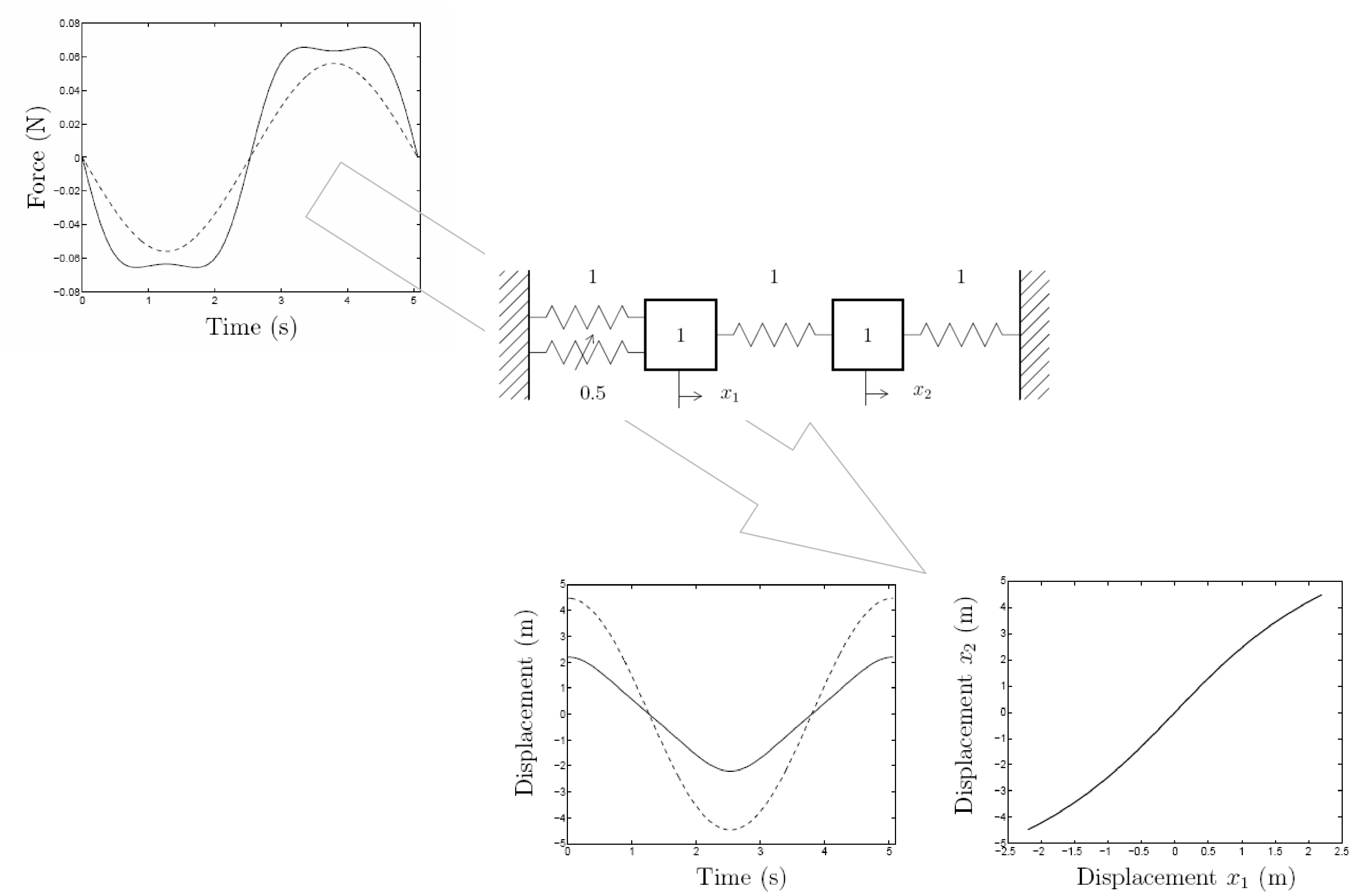
This type of motion is referred to as monophase NNM motion due to the fact that the displacements of all DOFs reach their extreme values simultaneously.

The appropriate excitation is given by

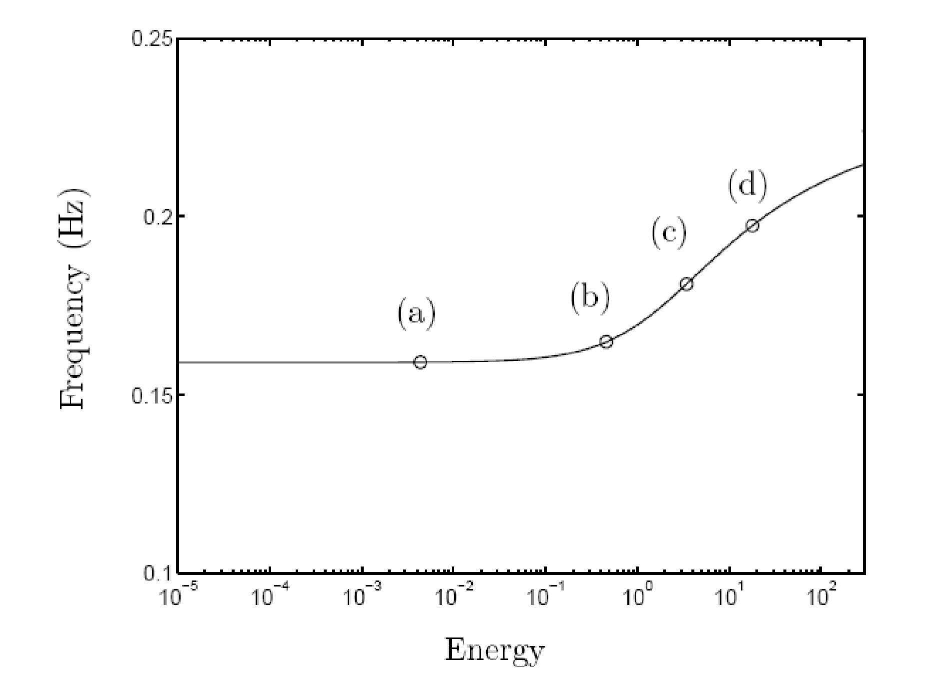
$$\mathbf{p}_{nnm}(t)=-\sum_{k=1}^{\infty}\mathbf{C}\,\mathbf{X}_k^{nnm}\,k\,\omega\sin(k\,\omega t)$$

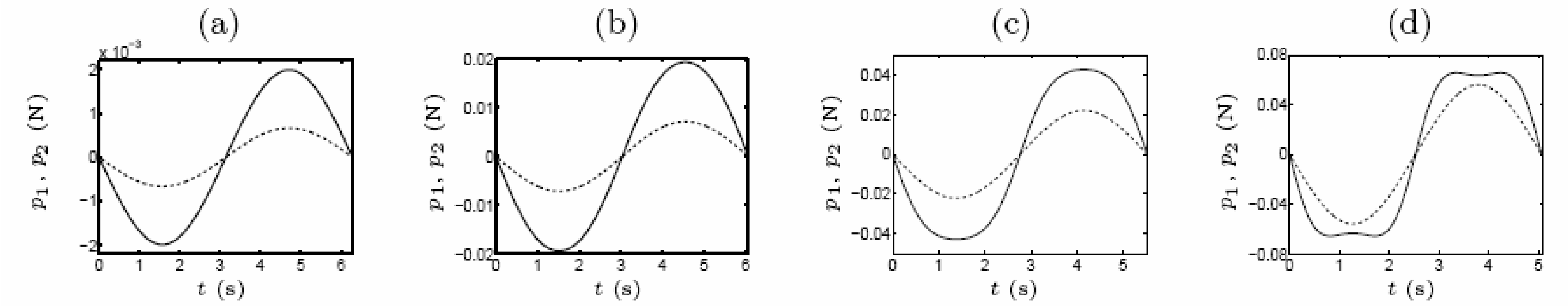
→ the excitation of a monophase NNM is thus characterized by a phase lag of 90◦ of each harmonics with respect to the displacement response.

Step 1: NNM Force Appropriation



Step 1: NNM Force Appropriation





Step 1: NNM Force Appropriation

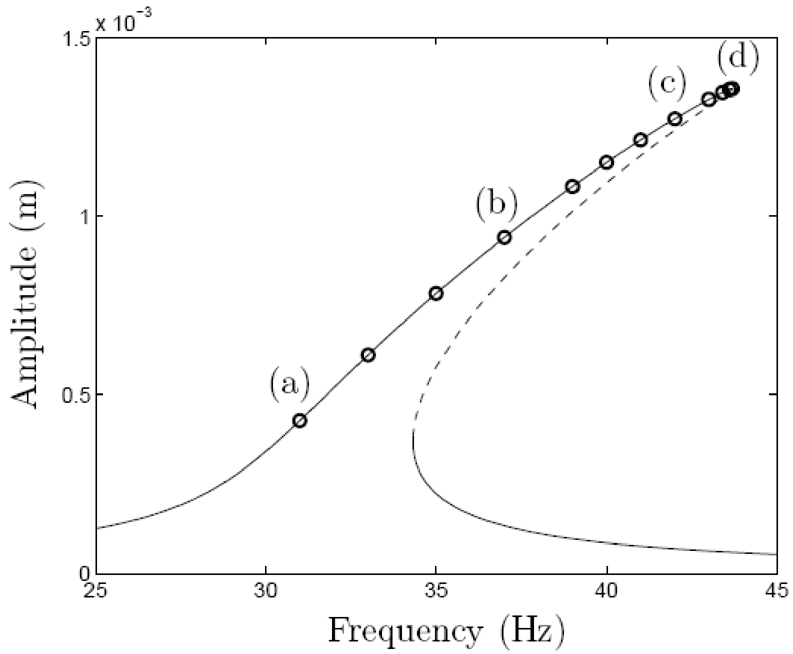
NNM Indicator

Phase lag quadrature criterion:

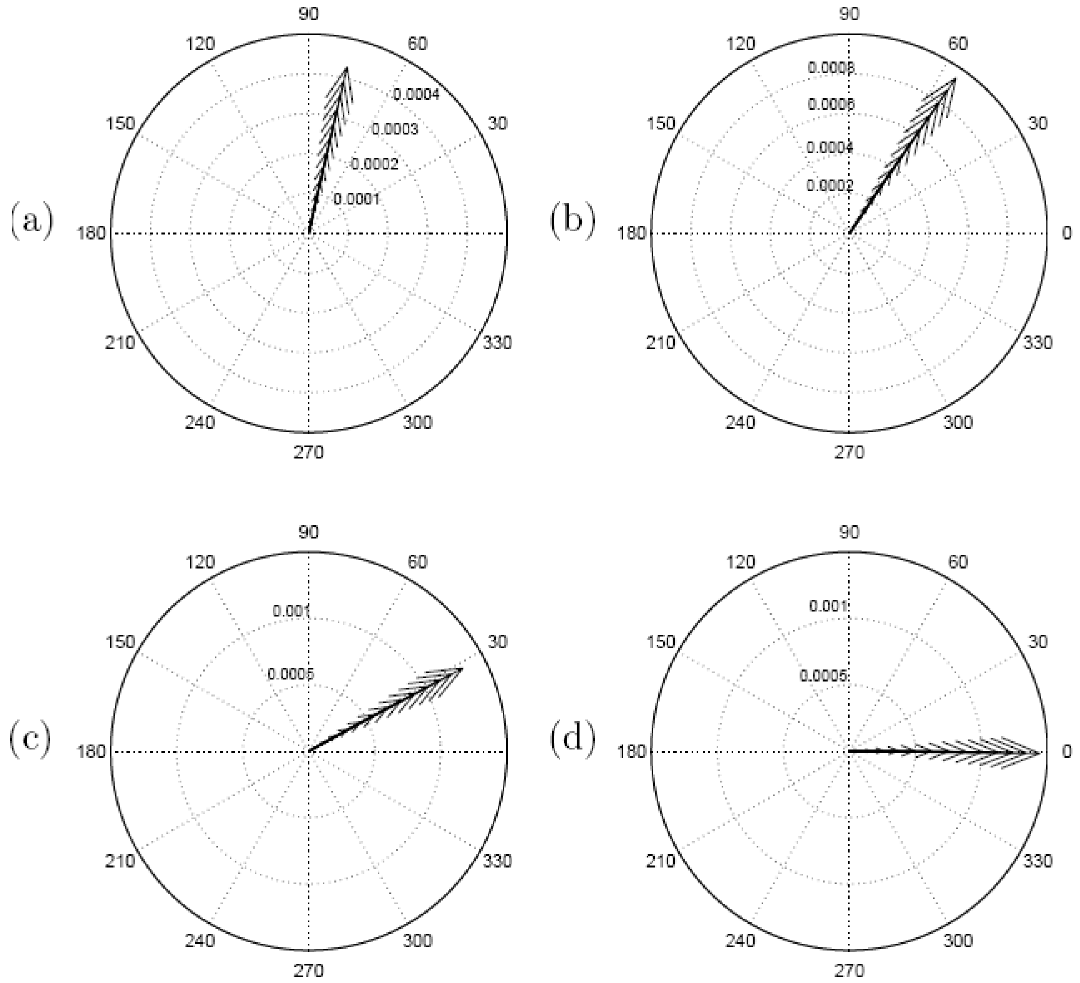
*A **linear** structure vibrates according to one of its **LNMs** if the degrees of freedom have a **phase lag of 90º** with respect to the excitation.*

*A **nonlinear** structure vibrates according to one of its **NNMs** if the degrees of freedom have a **phase lag of 90º** with respect to the excitation.*

Step 1: NNM Force Appropriation



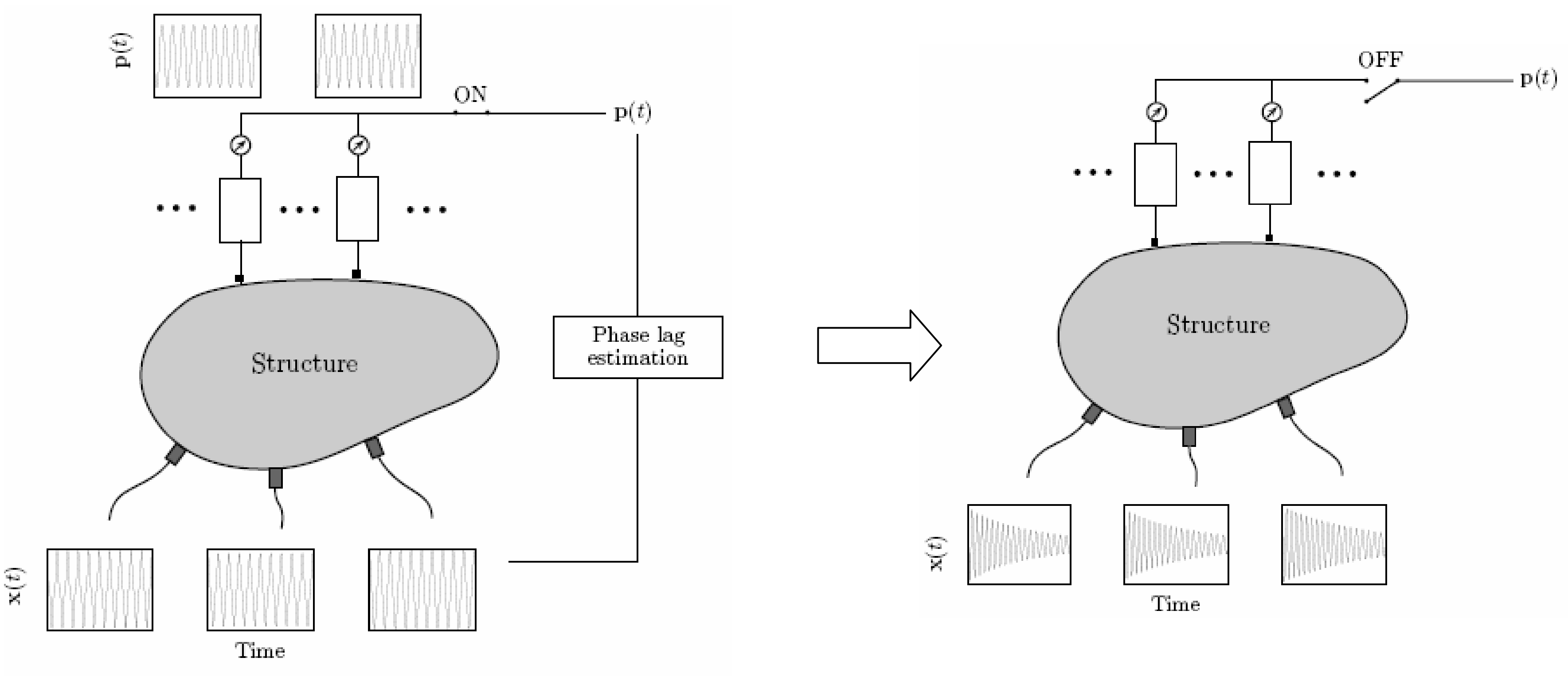
The excitation phase is 90º



Step 2: NNM Free Decay Identification

Turn off the excitation and track the NNM according to the invariance principle:

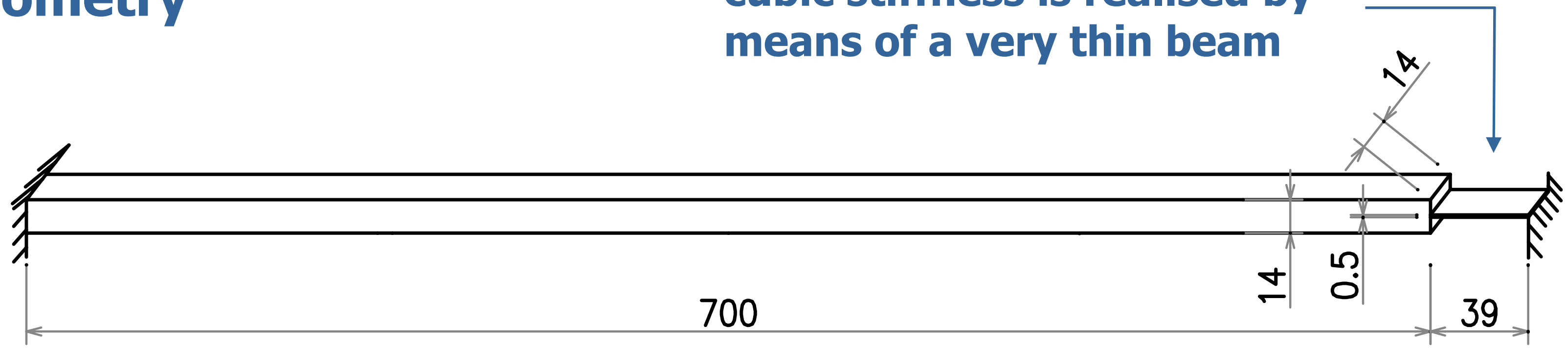
« *If the motion is initiated on one specific NNM,*
the remaining NNMs remain quiescent for all time. »



Nonlinear EMA (Illustrative Example)

Numerical experiments of a nonlinear beam (defined as benchmark in the framework of the European COST Action F3 « Structural Dynamics » [8]).

Geometry

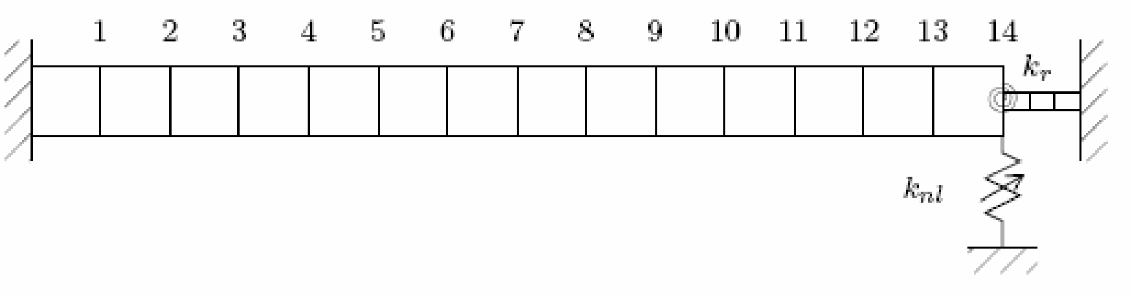


cubic stiffness is realised by means of a very thin beam

For weak excitation, the system behaviour may be considered as linear. When the excitation level increases, the thin beam exhibits large displacements and a nonlinear geometric effect is activated resulting in a stiffening effect at the end of the main beam.

Nonlinear EMA (Illustrative Example)

Finite Element model



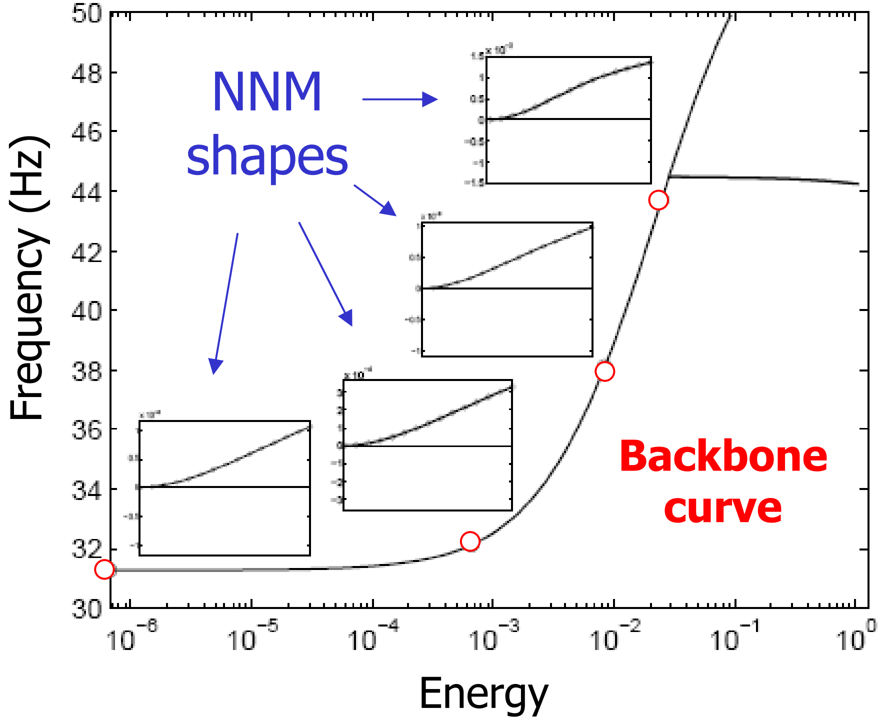
The thin beam is represented by two equivalent grounded springs: one in translation ($k_{n\ell} + k_{\ell}$) and one in rotation (k_r).

Young's modulus (N/m^2)	Density (kg/m^3)	Nonlinear parameter k_{nl} (N/m^3)
2.05 10^{11}	7800	8 10^9

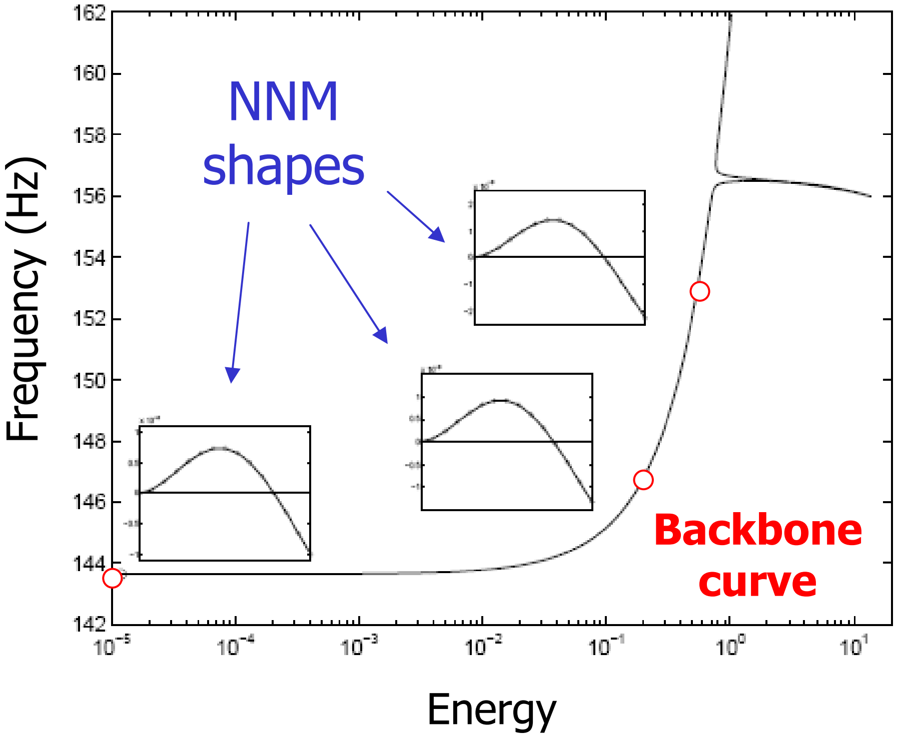
Nonlinear EMA (Illustrative Example)

Theoretical frequency-energy plots

First NNM



Second NNM



Nonlinear EMA (Illustrative Example)

Simulated experiments

Linear proportional damping is considered.

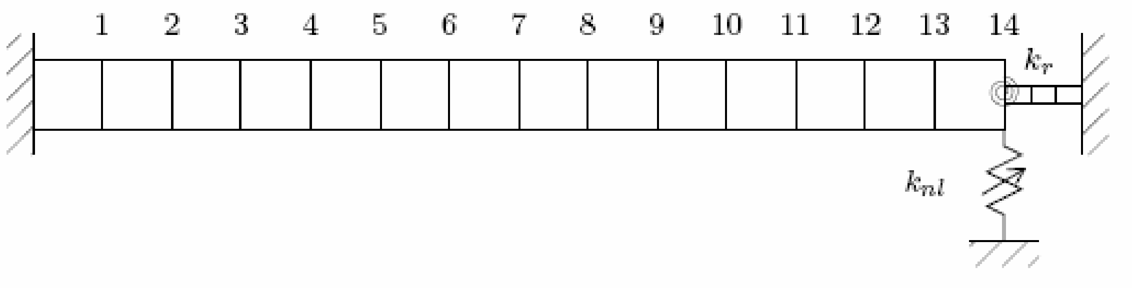
$$\mathbf{C} = 3\,10^{-7}\,\mathbf{K} + 5\,\mathbf{M}$$

It corresponds to moderate damping; for instance, the modal damping ratio is equal to 1.28% for the first linear normal mode.

Imperfect force appropriation

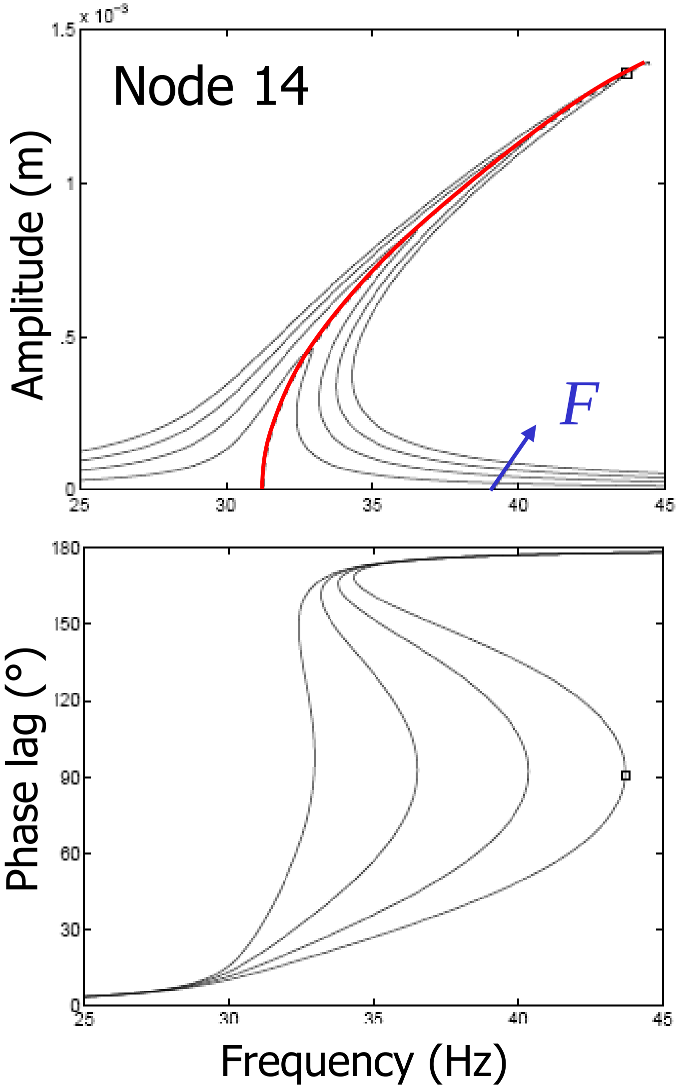
From a practical viewpoint, it is useful to study the quality of imperfect force appropriation consisting of a single-point mono-harmonic excitation, i.e., using a single shaker with no harmonics of the fundamental frequency.

The harmonic force $p(t) = F\sin(\omega t)$ is applied to node 4 of the main beam.



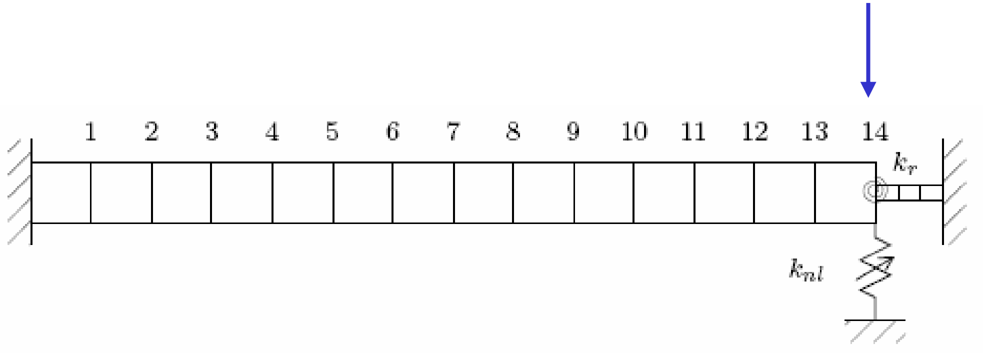
Nonlinear EMA (Illustrative Example)

Nonlinear forced frequency responses to the first resonant frequency



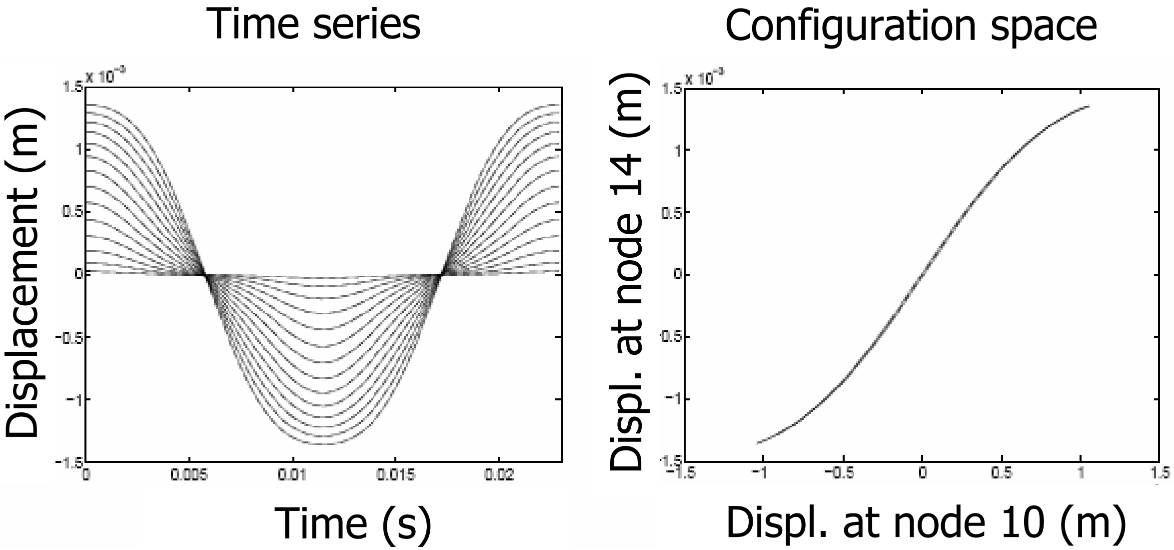
backbone of the first undamped NNM

4 different forcing amplitudes: 1N, 2N, 3N, 4N.



Nonlinear EMA (Illustrative Example)

F = 4N



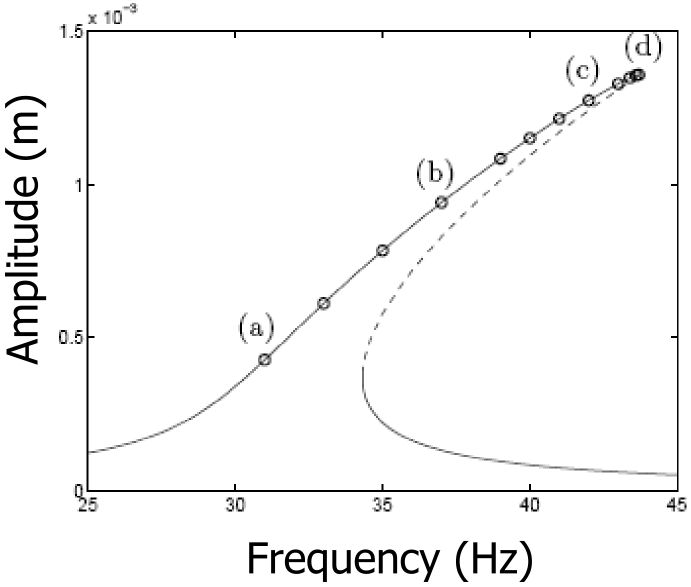
Observations

- The phase lag quadrature criterion is fulfilled close to resonant frequencies.
- Forced responses at resonance occur in the neighbourhood of NNMs.
- Imperfect appropriation can isolate the NNM of interest (the beam has well-separated modes).

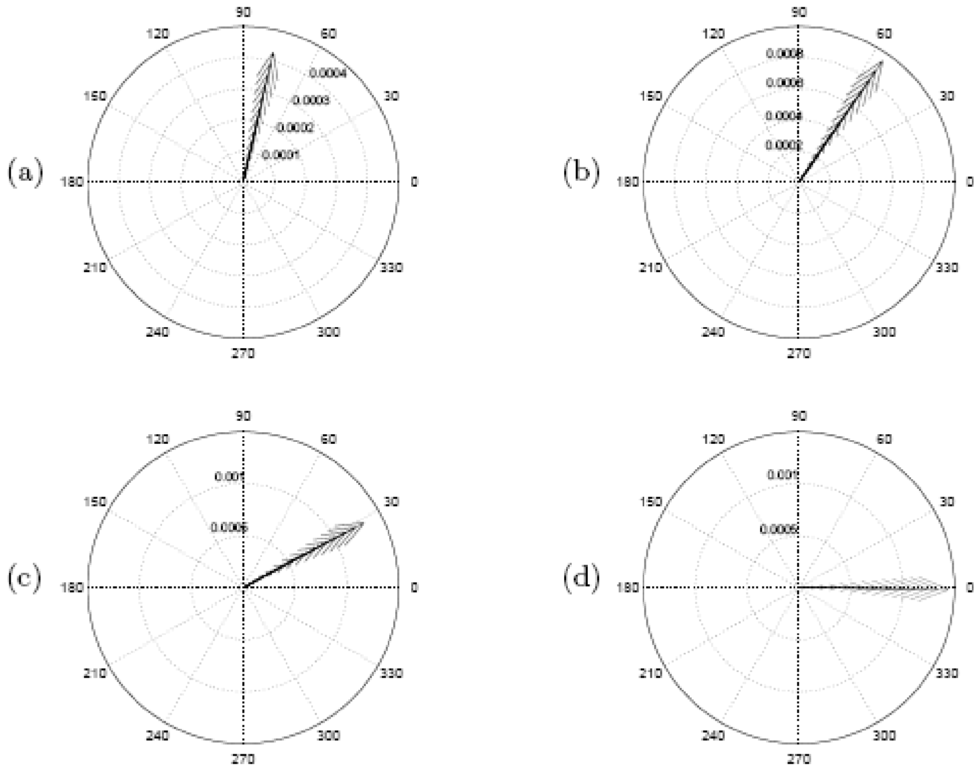
These findings also hold for the second beam NNM.

Nonlinear EMA (Illustrative Example)

Stepped sine excitation procedure for carrying out the NNM force appropriation (F = 4N) of the damped nonlinear beam.



Responses along the branch close to the first resonance

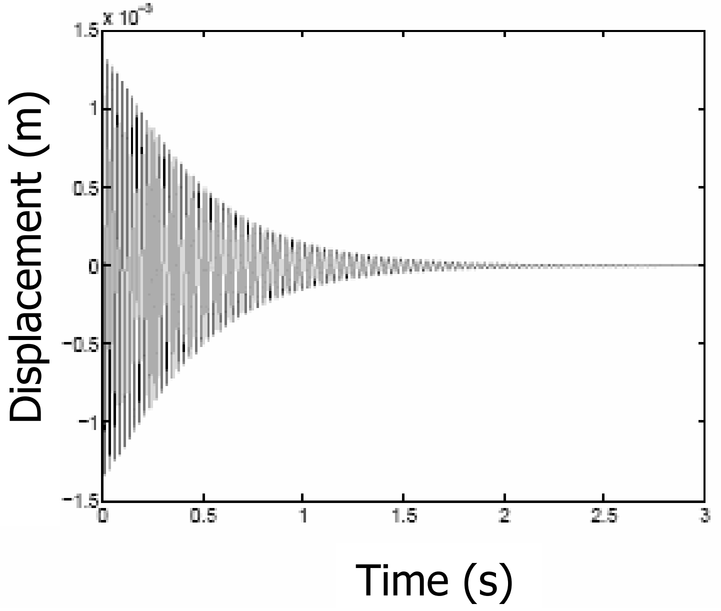


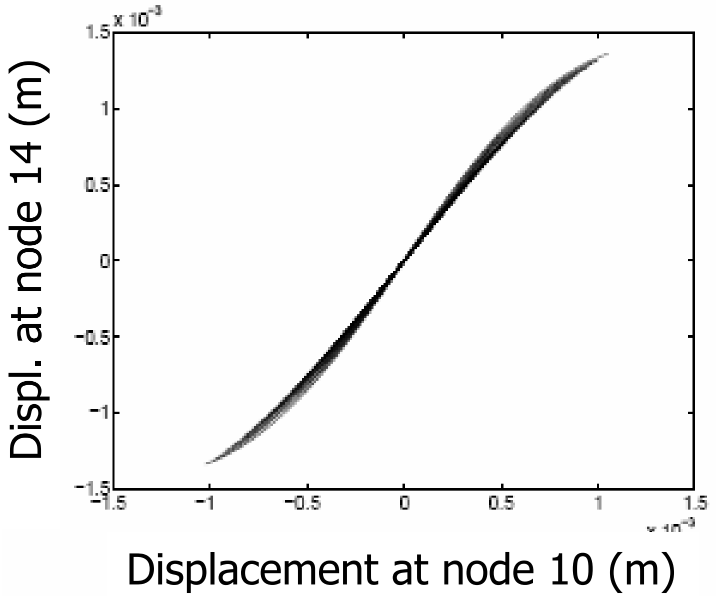
Phase scatter diagrams of the complex Fourier coefficients of the displacements corresponding to the fundamental frequency for the responses (a), (b), (c) and (d).

Nonlinear EMA (Illustrative Example)

NNM free decay identification

Free response of the damped nonlinear beam initiated from the imperfect appropriated forced response





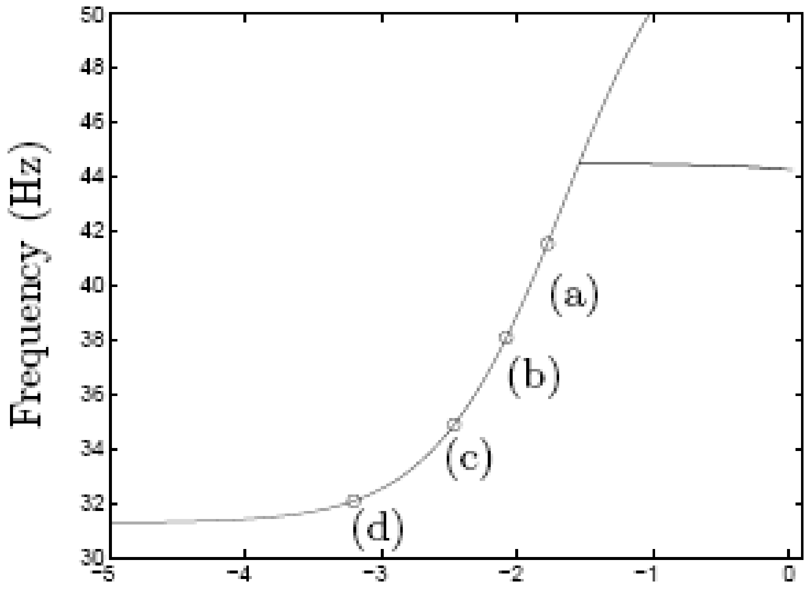
Time series of the displacement at the tip of the main beam (node 14).

Motion in the configuration space composed of the displacements at nodes 10 and 14.

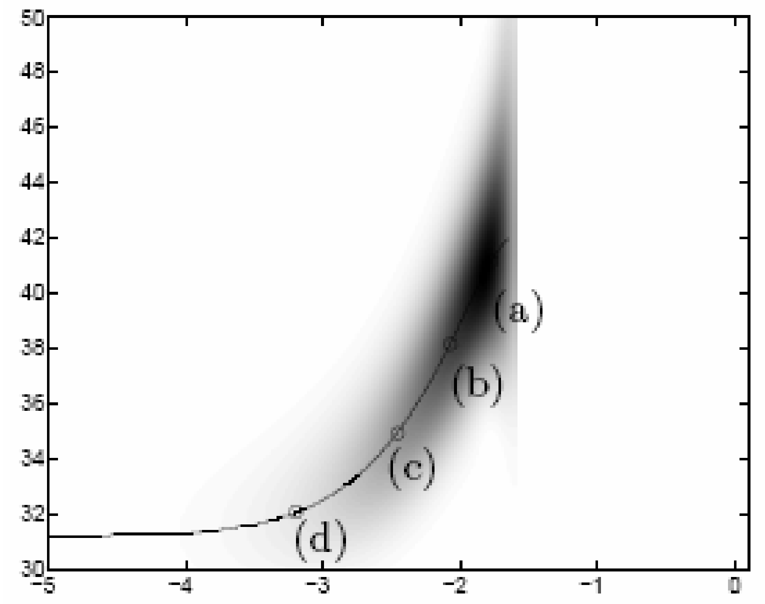
Nonlinear EMA (Illustrative Example)

Frequency-energy plot of the first NNM of the nonlinear beam.

Theoretical FEP



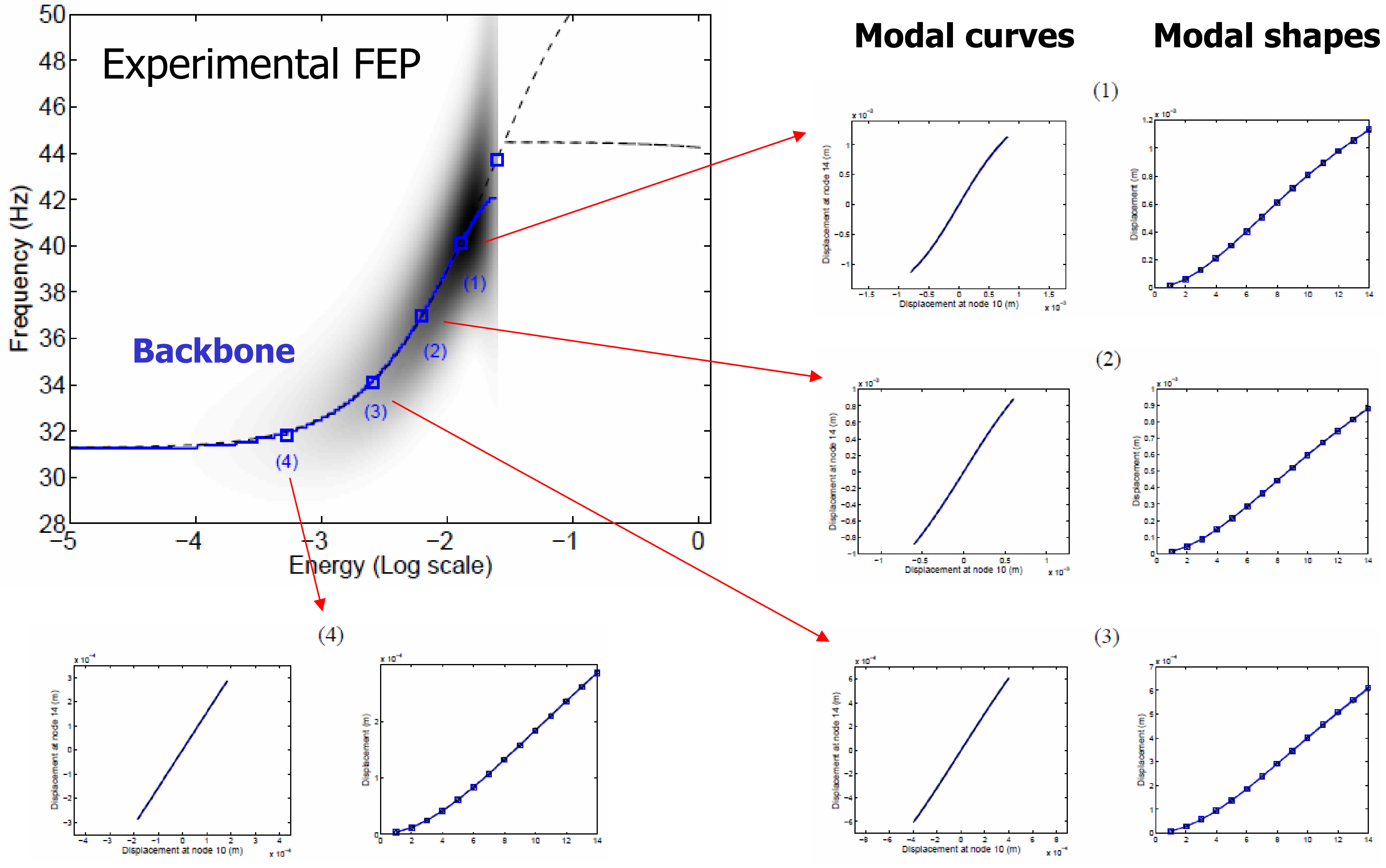
Experimental FEP



This FEP was calculated from the time series of the free damped response using the CWT. The solid line is the ridge of the transform.

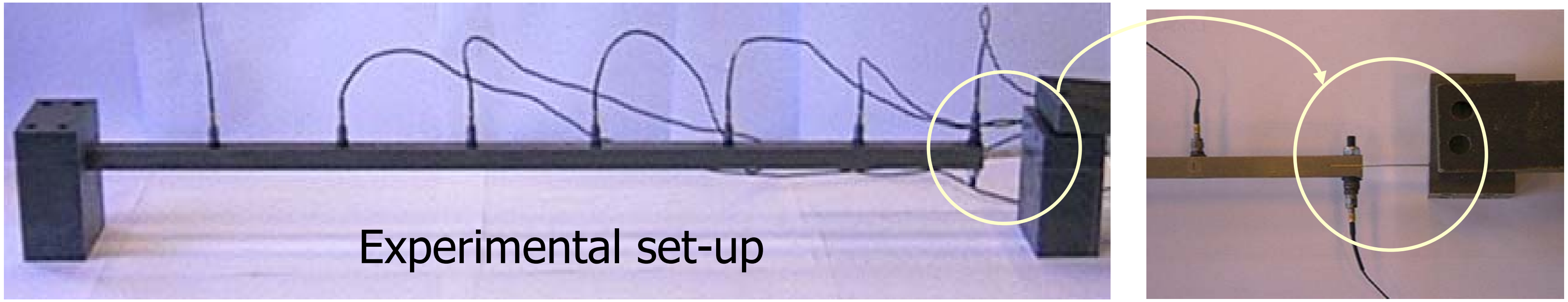
Nonlinear EMA (Illustrative Example)

Modal curves **Modal shapes**



Experimental Demonstration

Benchmark of the European COST Action F3 « Structural Dynamics ».

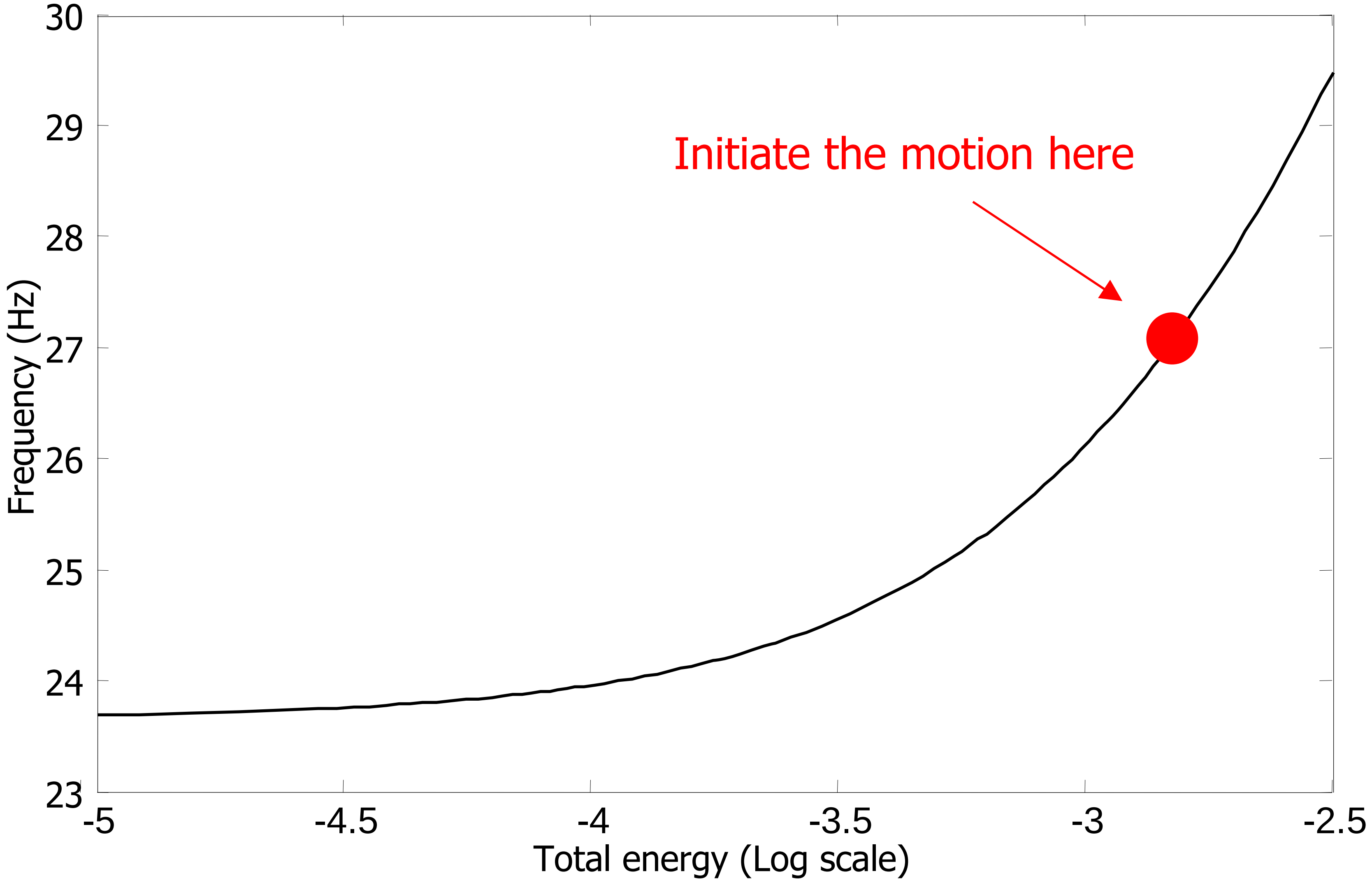

Experimental set-up

Test conditions

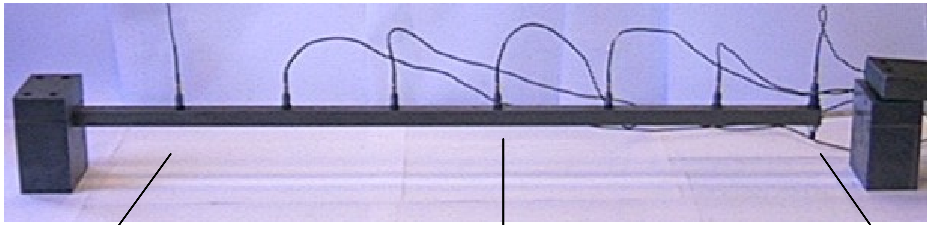
- Harmonic excitation of the nonlinear beam.
- Response measured using seven accelerometers.
- Very preliminary results.

Experimental Demonstration

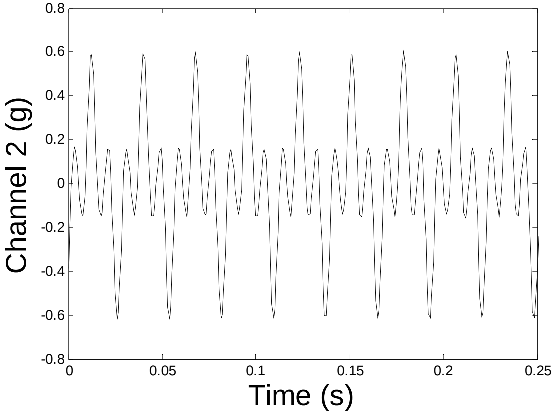
Excitation of the 1st NNM of the beam

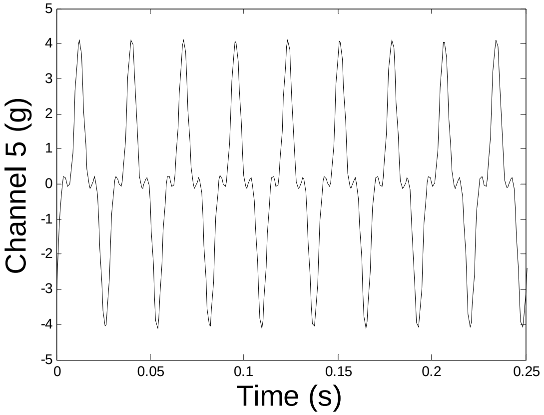


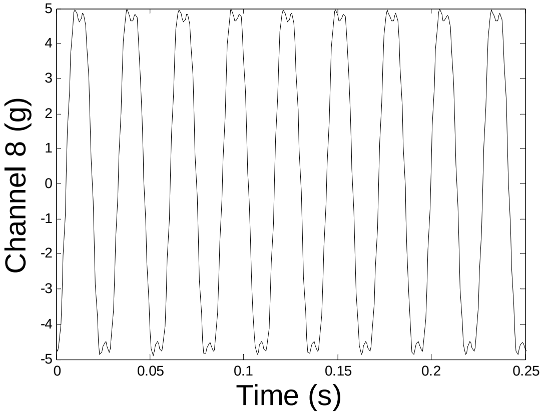
Experimental Demonstration

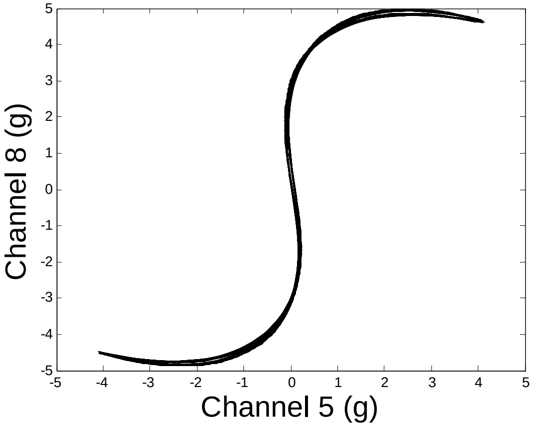


Sustained harmonic excitation



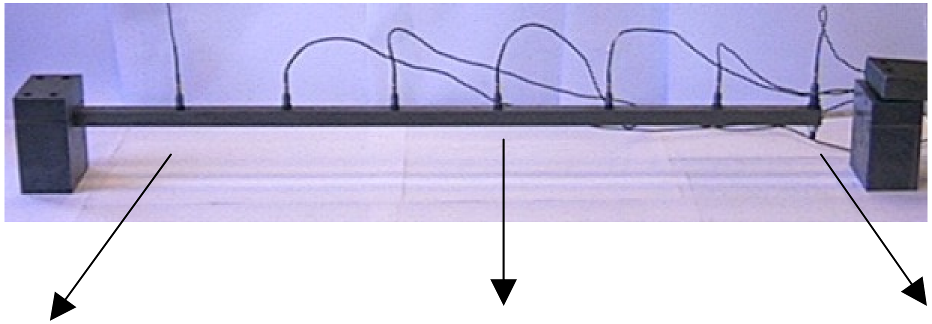




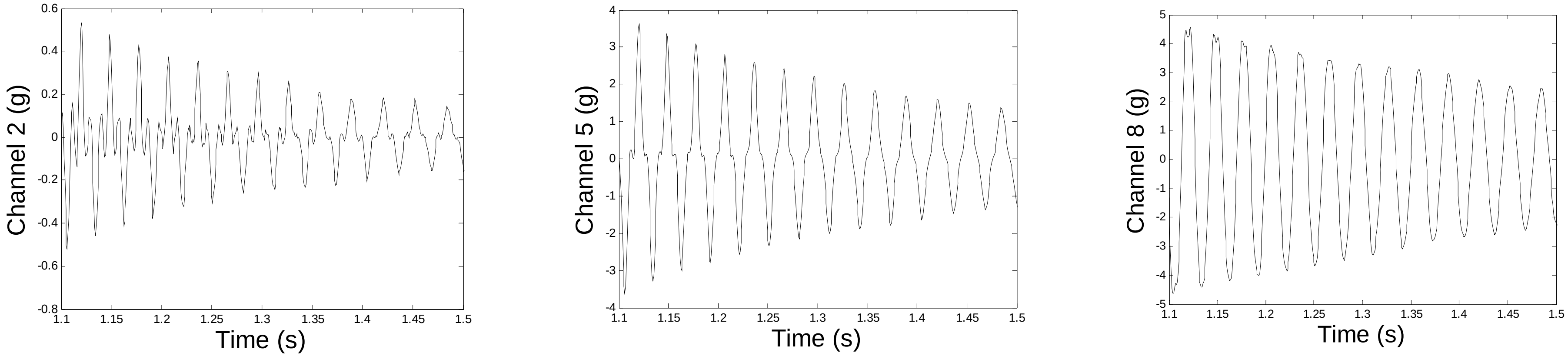


Experimental Demonstration

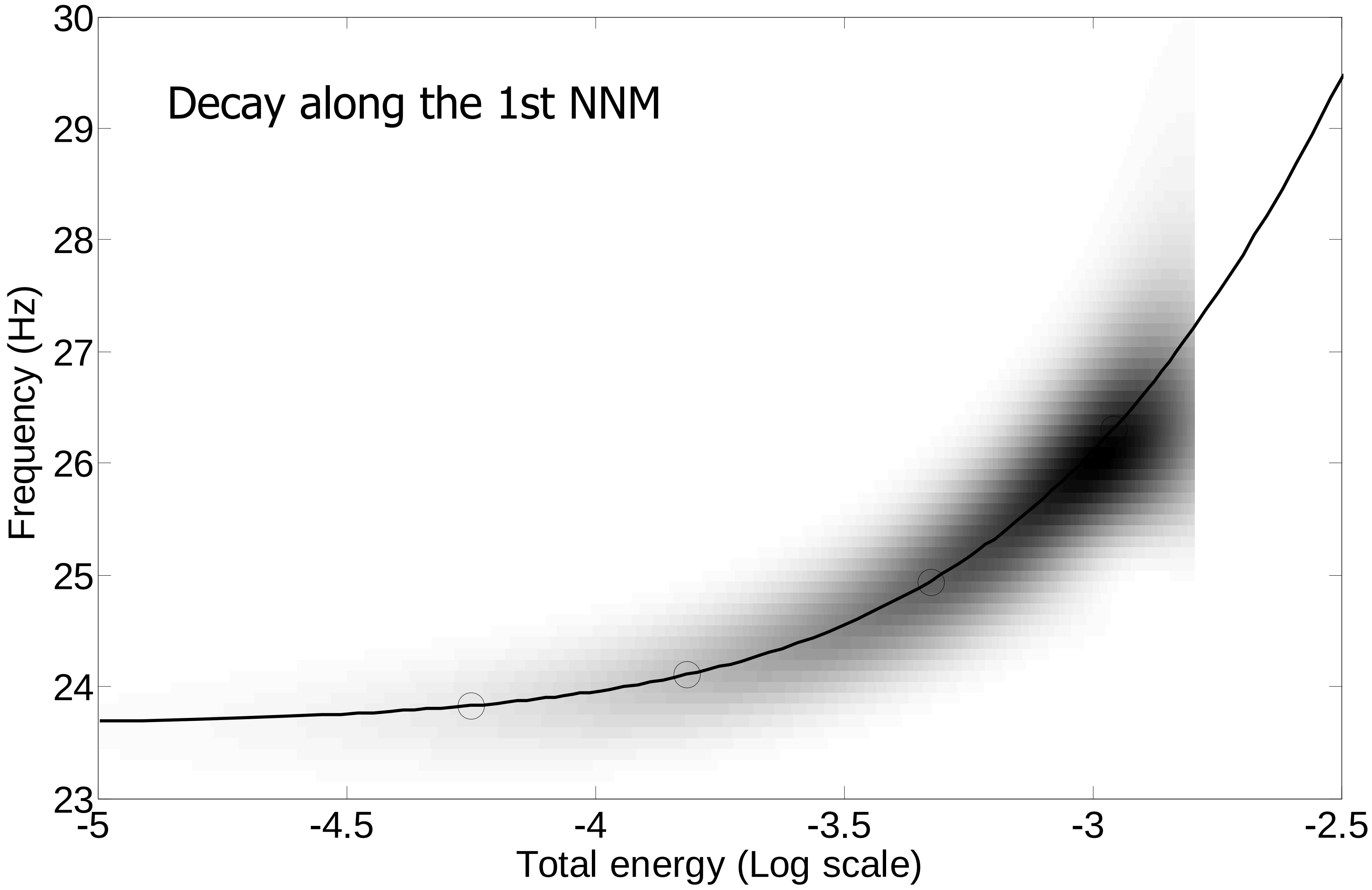
Turn Off the Shaker



Burst sine

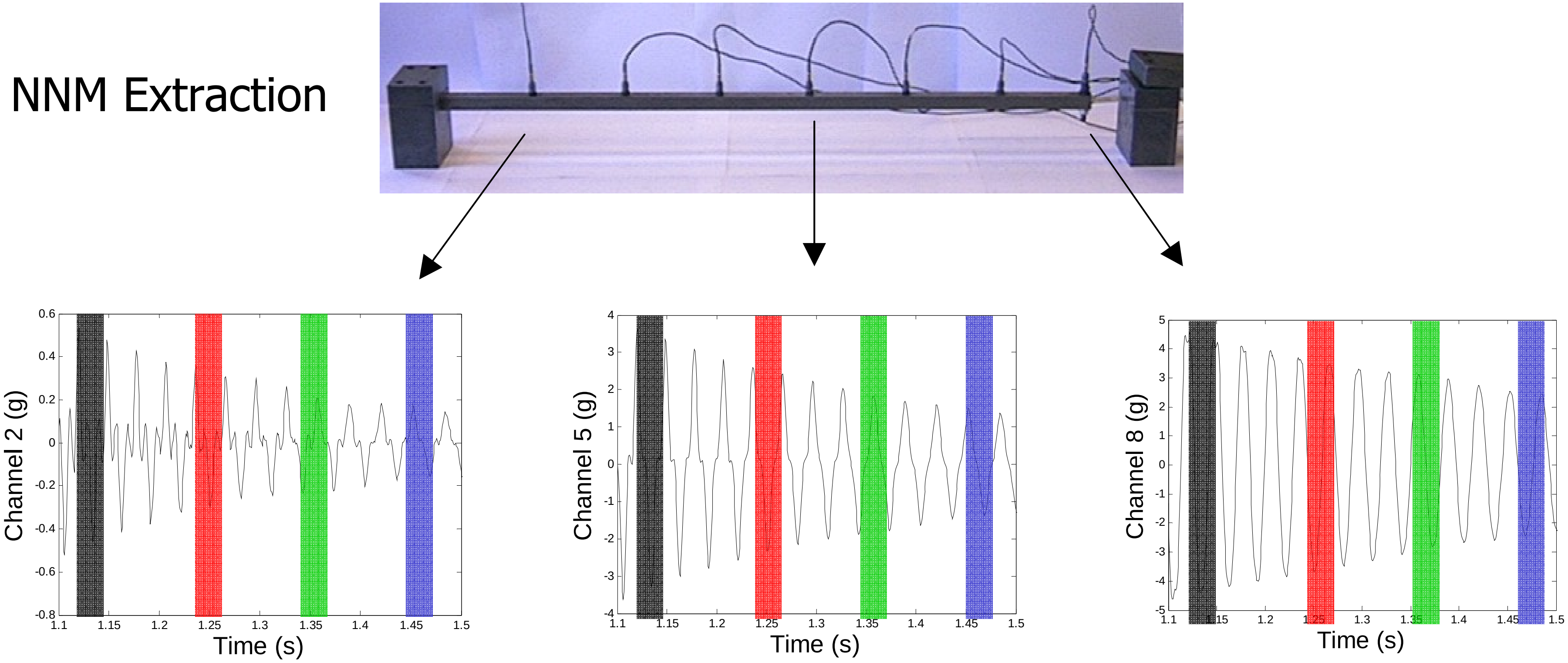


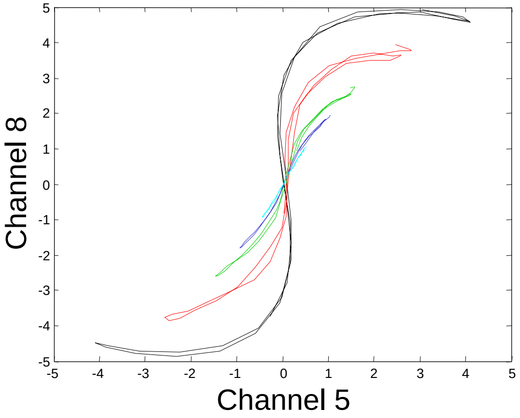
Experimental Demonstration



Experimental Demonstration

NNM Extraction





1st NNM of the beam at different energy levels

Outline

1. Introduction

2. Theoretical Modal Analysis of Nonlinear Systems

3. Nonlinear Experimental Modal Analysis

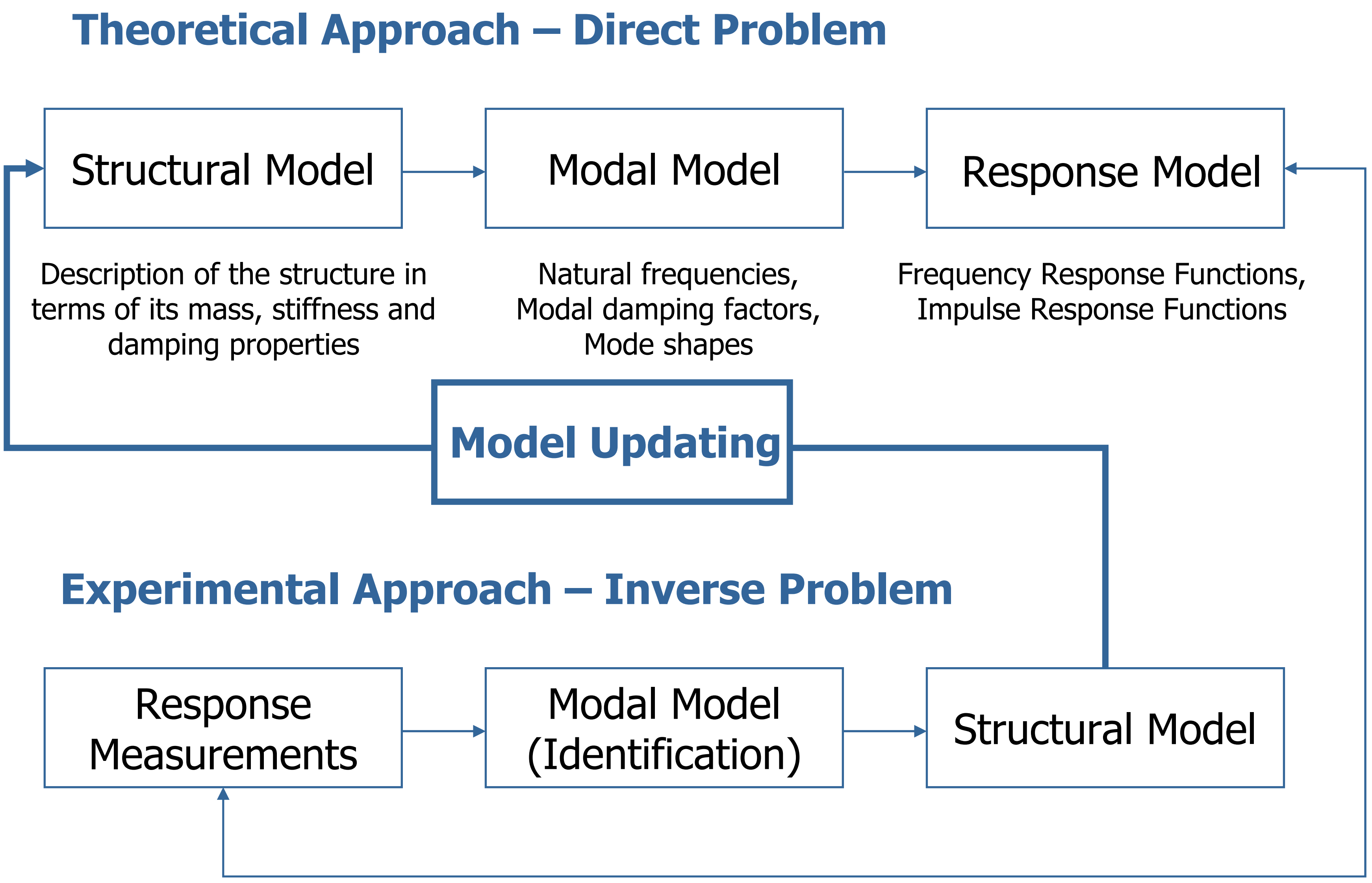
4. Model Parameter Estimation Techniques

   - Parameter Estimation Using POD

   - Parameter Estimation Using Nonlinear EMA

5. Concluding Remarks

Model Updating



Theoretical Approach – Direct Problem

Structural Model

Modal Model

Response Model

Description of the structure in terms of its mass, stiffness and damping properties

Natural frequencies,
Modal damping factors,
Mode shapes

Frequency Response Functions,
Impulse Response Functions

Model Updating

Experimental Approach – Inverse Problem

Response Measurements

Modal Model (Identification)

Structural Model

Model Updating

Assumption

- The linear counterpart of the structure is known (updated).

Methodology

- Estimation of nonlinear parameters only (which will be based on FE updating techniques).

Parameters for Model Updating (Crucial step!)

The number of parameters :

- should be kept small to avoid problems of ill-conditioning,
- should be chosen with the aim of correcting recognised features in the model.

→ requires physical insight → leads to **knowledge-based** models.

Model Parameter Estimation Techniques

Mathematical Background

Consider the general equation governing the dynamics of a structure

$$\mathbf{M}\,\ddot{\mathbf{x}} + \mathbf{C}\,\dot{\mathbf{x}} + \mathbf{K}\,\mathbf{x} + \underbrace{\mathbf{f}_{NL}}_{\text{Vector of nonlinear forces}}(\mathbf{x}, \dot{\mathbf{x}}) = \mathbf{g}(t)$$

Step 1: definition of a penalty function involving modal features of the system (residual between analytical and measured dynamic behaviour)

The measured quantities may be assembled into a measurement vector $\mathbf{z}$.

Model Parameter Estimation Techniques

The vector of modal features $\mathbf{z}$ depends on parameters $\mathbf{p}$

$$\mathbf{z} = \mathbf{z}(\mathbf{p})$$

The choice of parameters is a crucial step in model updating. For nonlinear identification purposes, we will assume that a *knowledge-based* model exists (the physically meaningful model and the associated parameters are supposed to be known).

Penalty function methods are based on the Taylor series expansion of the modal data in terms of the unknown parameters

$$\mathbf{z} = \mathbf{z}(p_0) + \left[\frac{\partial \mathbf{z}}{\partial \mathbf{p}}\right]_{\mathbf{p}=\mathbf{p}_0} (\mathbf{p} - \mathbf{p}_0) + O(\mathbf{p}^2)$$

Sensitivity matrix

Initial estimation of the parameters

This expansion is often limited to the first two terms.

Model Parameter Estimation Techniques

Define the weighted penalty function

$$J = \boldsymbol{\varepsilon}^{\mathbf{T}} \ \mathbf{W} \ \boldsymbol{\varepsilon}$$

Positive definite weighting matrix

where $\boldsymbol{\varepsilon} = \Delta\mathbf{z} - \mathbf{S}\ \Delta\mathbf{p}$ is the error in the predicted measurements.

$$\mathbf{S} = \left[\frac{\partial \mathbf{z}}{\partial \mathbf{p}}\right]$$ is the sensitivity matrix.

Minimising J with respect to $\Delta\mathbf{p}$ leads to

$$\Delta\mathbf{p} = \left(\mathbf{S}^T \ \mathbf{W}\ \mathbf{S}\right)^{-1} \mathbf{S}^T \ \mathbf{W}\ \Delta\mathbf{z}$$

With the assumption that the number of measurements is larger than the number of parameters, the matrix $\mathbf{S}^T \ \mathbf{W}\ \mathbf{S}$ is square and hopefully full rank.

Model Parameter Estimation Techniques

Definition of the measurement vector **z** containing the modal features.

- In the case of **linear** systems

i **th mode shape vector**

$$\mathbf{z}^T = \left(\omega_1, \mathbf{\Phi}_1^T, \ldots, \omega_i, \mathbf{\Phi}_i^T, \ldots, \omega_r, \mathbf{\Phi}_r^T\right)^T$$

i **th eigenvalue**

- In the case of **nonlinear** systems
  - ☐ Proper Orthogonal Decomposition
  - ☐ Nonlinear Modal Analysis

Parameter Estimation Using POD

Linear Systems

$$\mathbf{M}\,\ddot{\mathbf{x}} + \mathbf{C}\,\dot{\mathbf{x}} + \mathbf{K}\,\mathbf{x} = \mathbf{p}(t)$$

Deterministic approach

Eigenvalue problem:

$$(\mathbf{K} - \omega^2\,\mathbf{M})\,\boldsymbol{\Phi} = \mathbf{0}$$

Response:

Spatial information

$$\mathbf{x}(t) = \sum_{i=1}^{n} \eta_i(t)\,\boldsymbol{\Phi}_{(i)}$$

Natural frequencies

$$\eta_i = A_i\cos(\omega_i\,t) + B_i\sin(\omega_i\,t)$$

Nonlinear Systems

$$\mathbf{M}\,\ddot{\mathbf{x}} + \mathbf{C}\,\dot{\mathbf{x}} + \mathbf{K}\,\mathbf{x} + \mathbf{f}_{NL}(\mathbf{x},\dot{\mathbf{x}}) = \mathbf{p}(t)$$

Statistical approach

Proper Orthogonal Decomposition:

$$\mathbf{X} = \mathbf{U}\,\boldsymbol{\Sigma}\,\mathbf{V}^{\mathbf{T}}$$

Response:

Spatial information

$$\mathbf{x}(t) = \sum_{j=1}^{n} a_j(t)\,\mathbf{u}_{(j)}$$

Time information

→ **Instantaneous frequencies**

Parameter Estimation Using POD

Principle of the method :

Minimise the residuals between the bi-orthogonal decompositions of the measured and simulated data.

$$\mathbf{X} = \mathbf{U}\,\boldsymbol{\Sigma}\,\mathbf{V}^T$$

POM (Spatial information)

Associated energy (Mode participation)

Time information

Wavelet Transform
Instantaneous frequencies

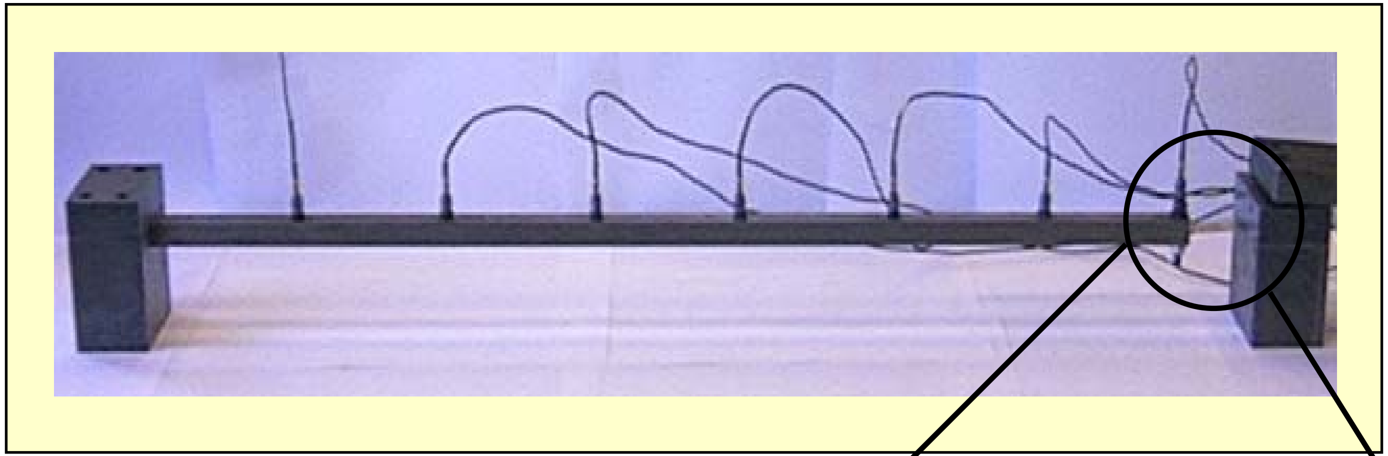
Penalty function :

$$J = \sum_i \sum_j (\Delta U_{ij})^2 + \sum_j (\Delta \Sigma_{jj})^2 + \sum_j \sum_k (\Delta V_{jk})^2$$

→ Selection of the POMs with the highest POV

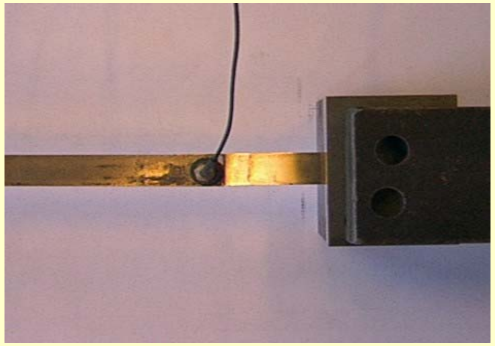
Parameter Estimation Using POD

Benchmark of the European COST Action F3 « Structural Dynamics »

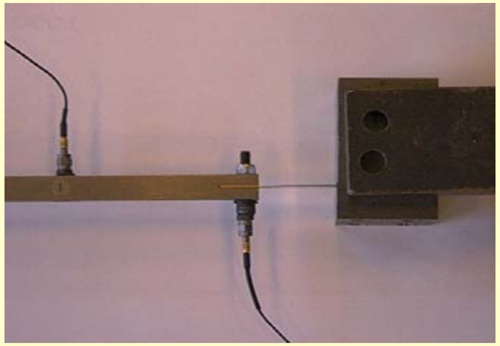


Experimental set-up

Horizontal view

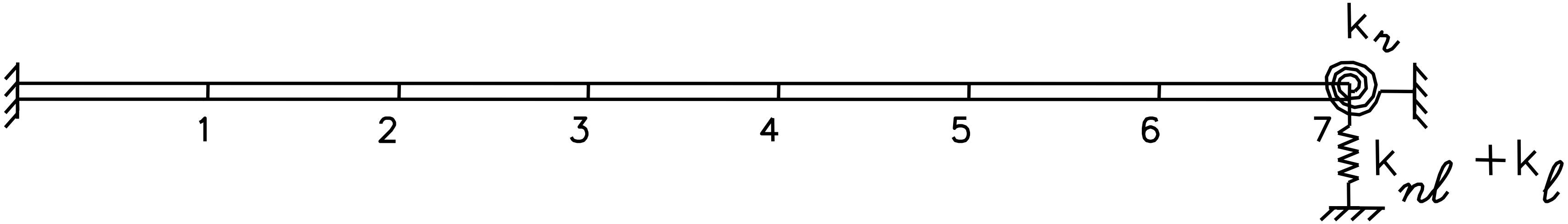


Vertical view



Parameter Estimation Using POD

Finite Element model



The nonlinear stiffening effect of the thin beam is modelled by a nonlinear function in displacement of the form:

$$f_{nl} = A\left|x\right|^{\alpha}\ sign(x)$$

where A and α are nonlinear parameters to be identified.

Parameter Estimation Using POD

Simulated results

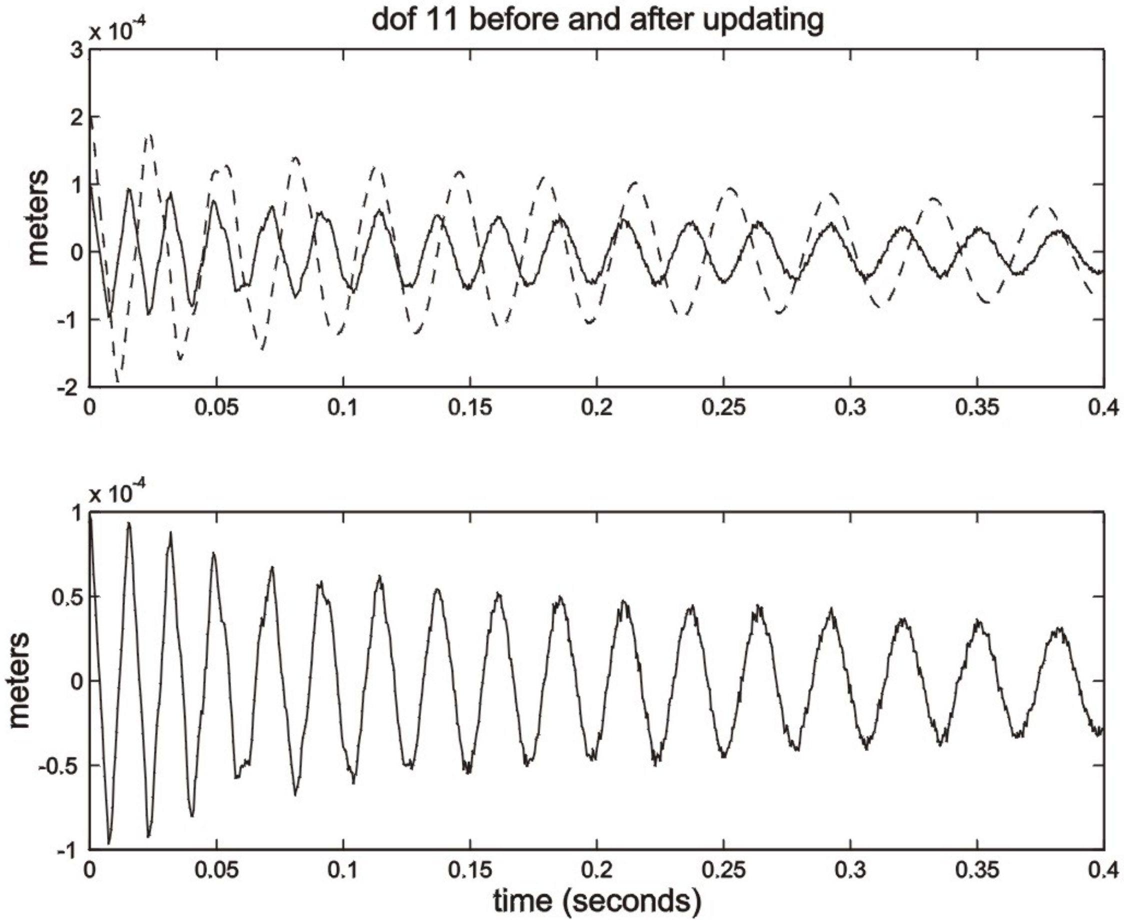
Identification of linear and nonlinear parameters

- 2 parameters : nonlinear stiffness + Young’s modulus
- Penalty function in terms of the first POM
- Simulation time = 0.4 sec
- Gaussian white noise of 1 %
- Nonlinear parameter correction < 10 %
- Linear parameter correction < 50 %

Parameter Estimation Using POD

Simulated results

Comparison between the original (–) and the reconstructed (--) signals



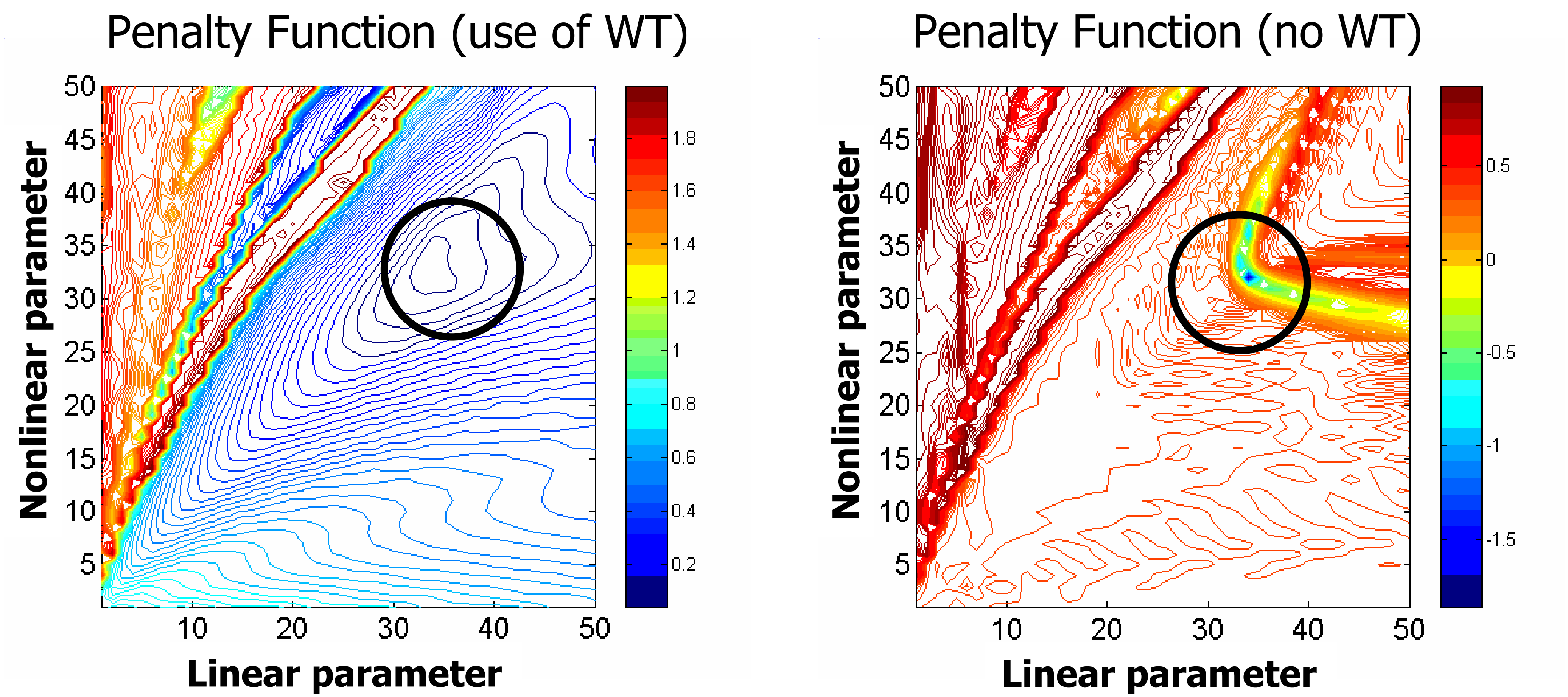
Before updating

After updating

Parameter Estimation Using POD

Simulated results

Contour Plot



Penalty Function (use of WT)

Penalty Function (no WT)

Well-conditioning

Ill-conditioning

Parameter Estimation Using POD

Experimental results (Vertical set-up)

Model of the nonlinear stiffness

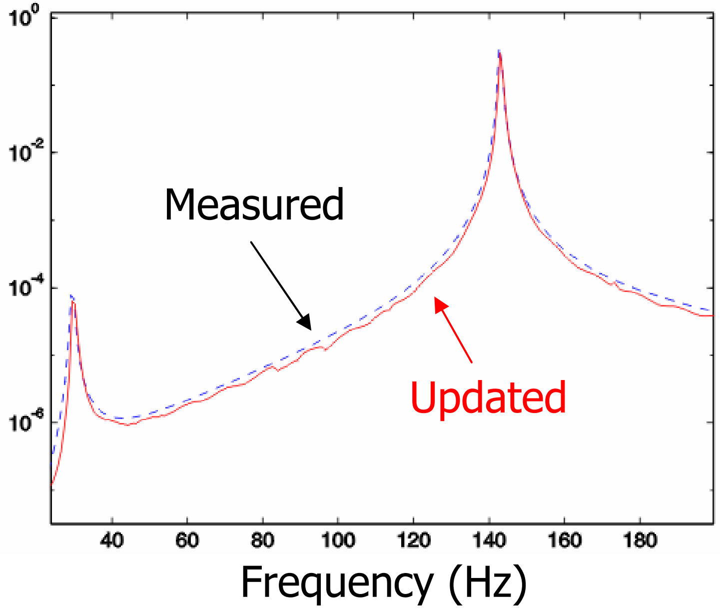
$$f_{nl}(x) = A|x|^{\alpha} sign(x)$$

Results of the identification of the nonlinear parameters based on the model updating method:

$$\alpha = 2.8$$

$$A = 1.65\ 10^9\ \text{N/m}^{2.8}$$

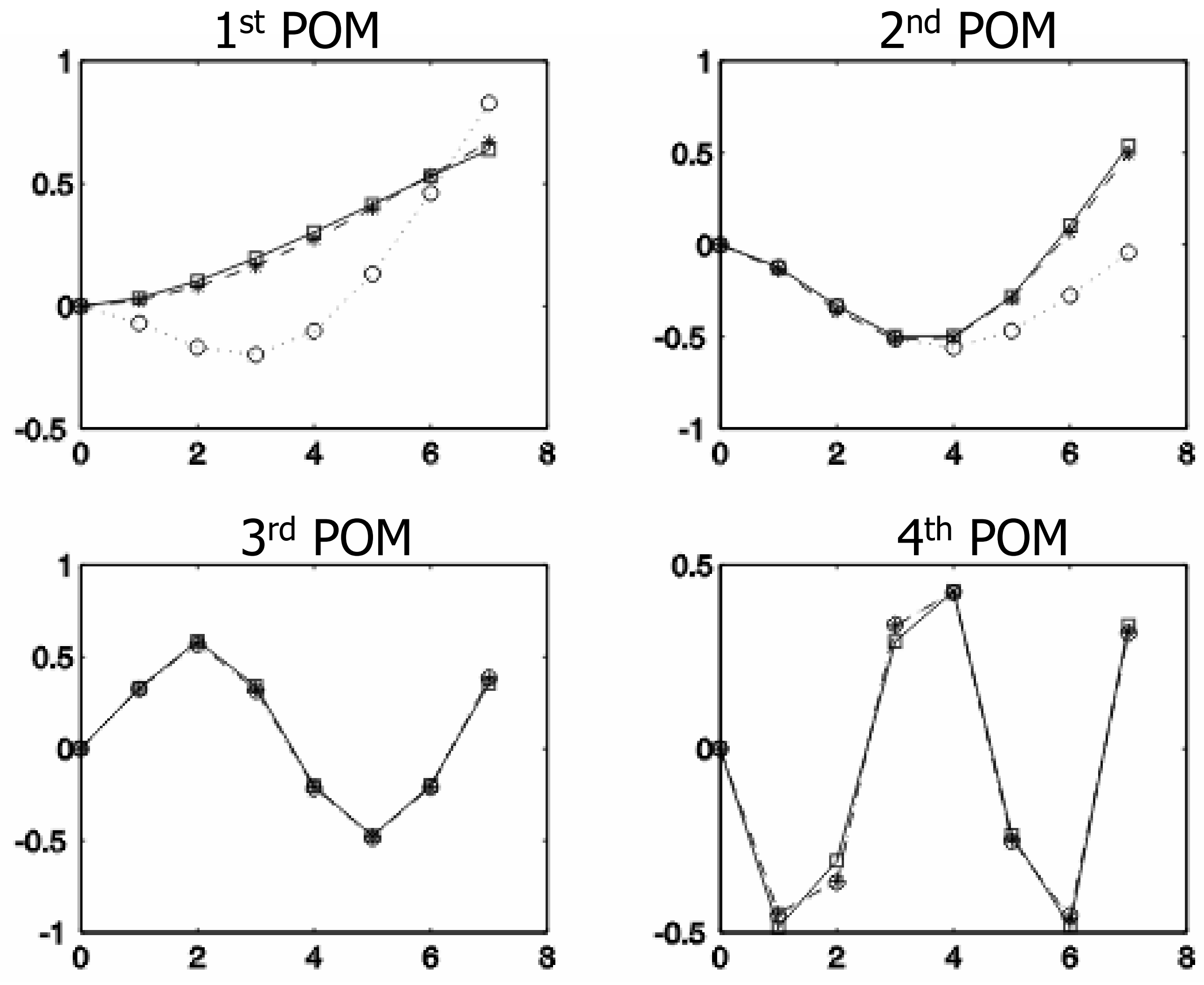
PSD of the time evolution of the 1st POM



Parameter Estimation Using POD

Experimental results (Vertical set-up)

Comparison of the POM



1st POM

2nd POM

3rd POM

4th POM

□ experimental

* nonlinear model
(after updating)

o linear model
(before updating)

Parameter Estimation Using Nonlinear EMA

Nonlinear MDOF systems

$$\mathbf{M}\,\ddot{\mathbf{x}} + \mathbf{K}\,\mathbf{x} + \mathbf{f}_{NL}\left(\mathbf{x},\dot{\mathbf{x}}\right) = 0$$

The concept of Nonlinear Normal Modes (NNMs) is a rigorous extension of the concept of eigenmodes to nonlinear systems.

Caution: the solution is energy-dependent !

Parameter Estimation Using Nonlinear EMA

Vector of modal features: $\mathbf{z}^T = \left(\mathbf{z}_1^T, \ldots, \mathbf{z}_i^T, \ldots, \mathbf{z}_r^T\right)$

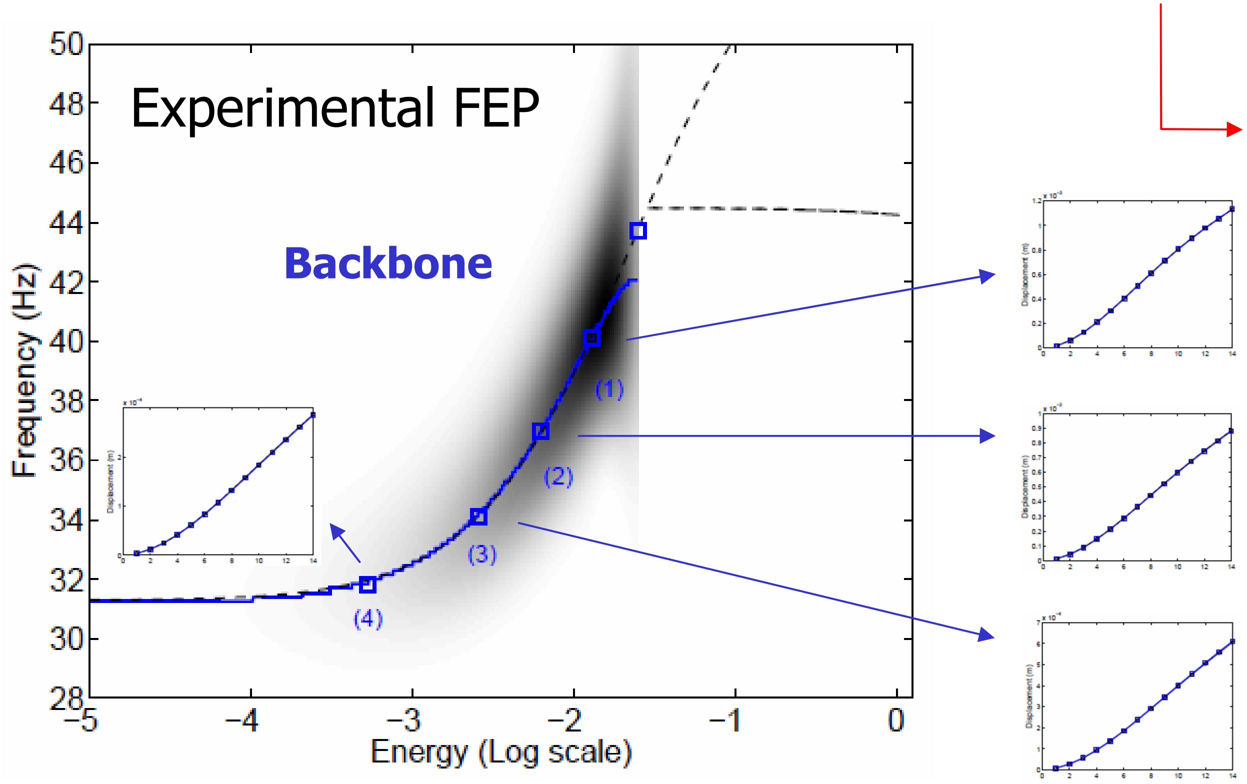
i th **backbone**

energy level

$$\mathbf{z}_i^T = \left(\omega_i^{(1)}, \mathbf{\Phi}_i^{(1)T}, \omega_i^{(2)}, \mathbf{\Phi}_i^{(2)T}, \omega_i^{(3)}, \mathbf{\Phi}_i^{(3)T}, \omega_i^{(4)}, \mathbf{\Phi}_i^{(4)T}, \ldots\right)^T$$

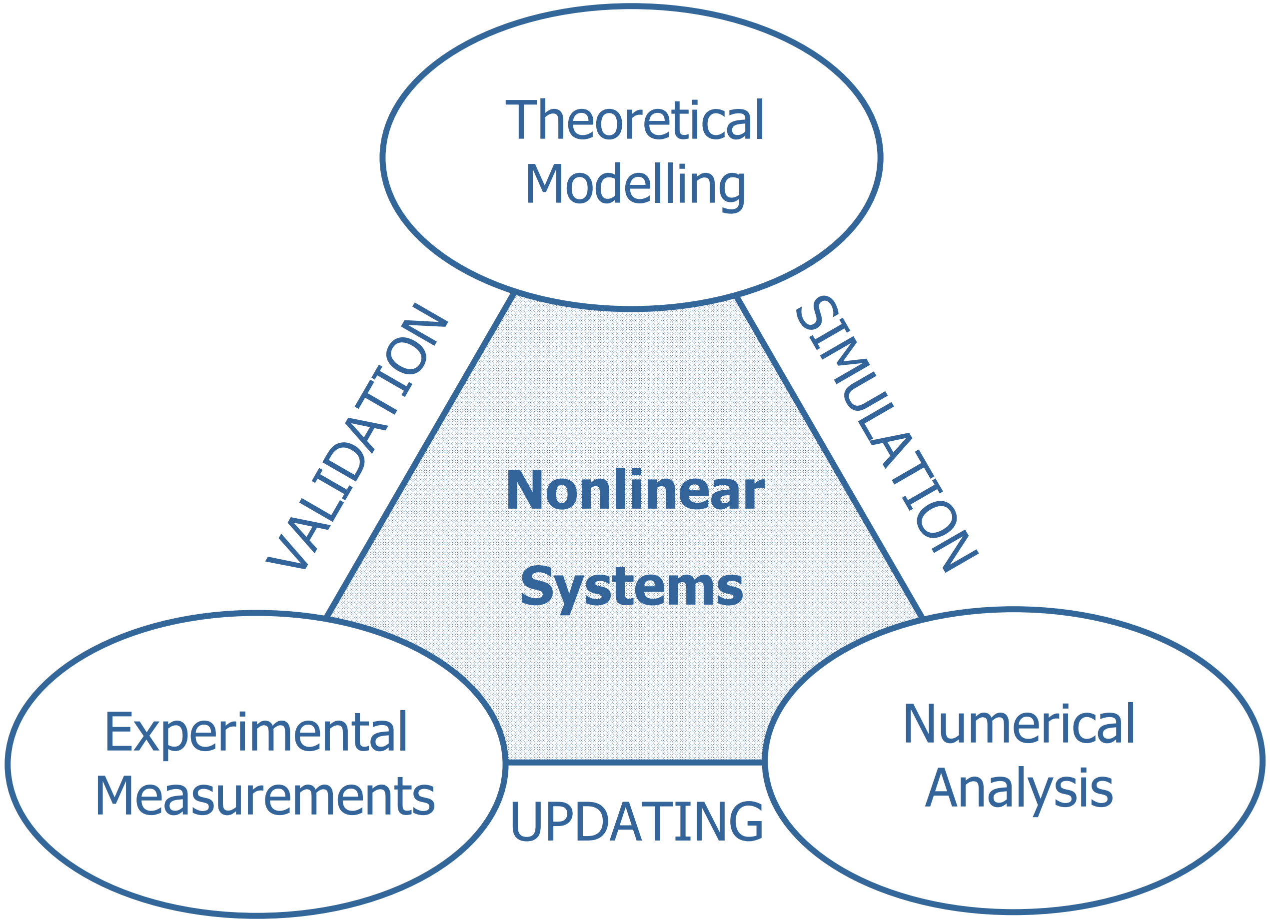
modal shape

frequency



Conclusion

The Structural Dynamicist’s Toolkit



Theoretical Modelling

VALIDATION

SIMULATION

Nonlinear Systems

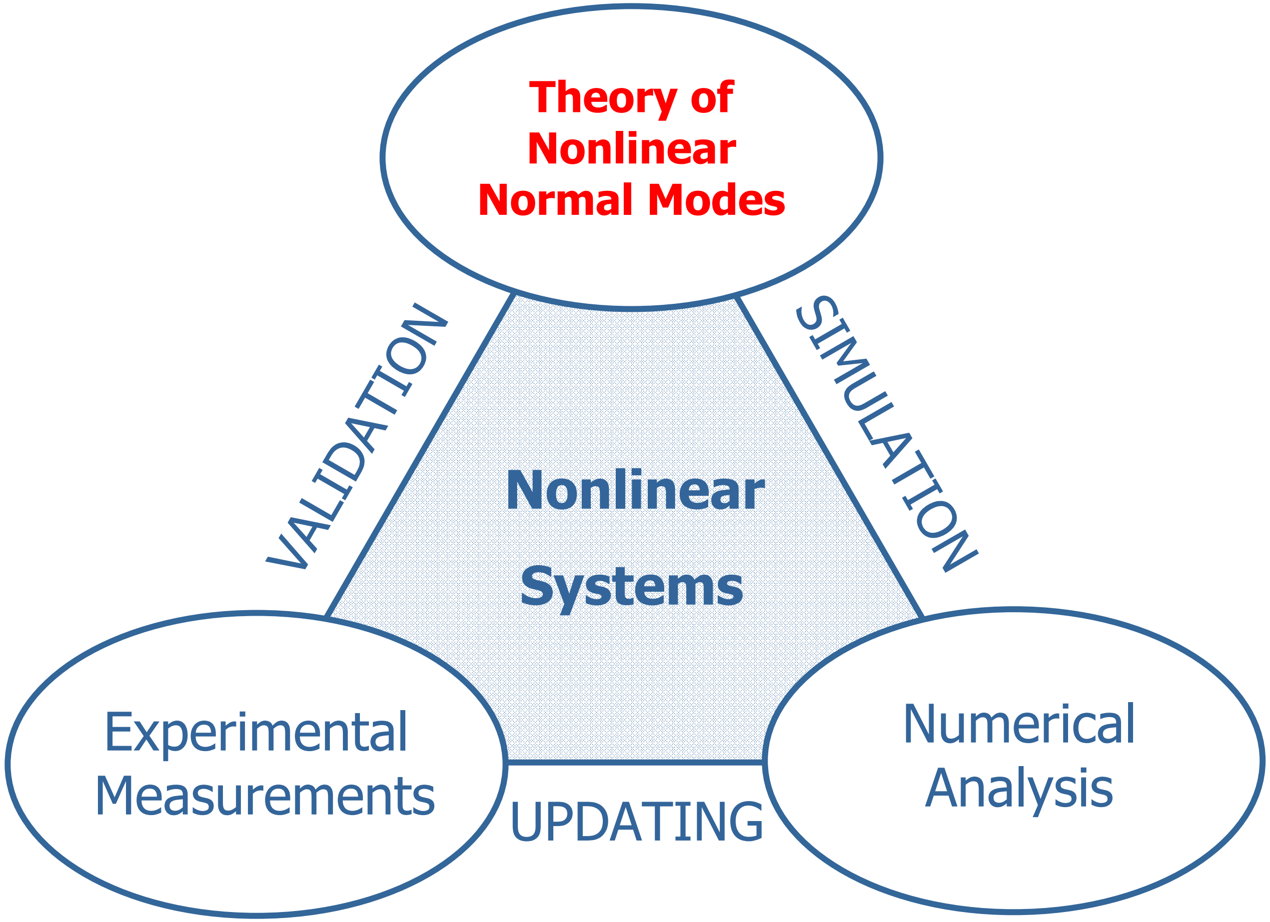
Experimental Measurements

UPDATING

Numerical Analysis

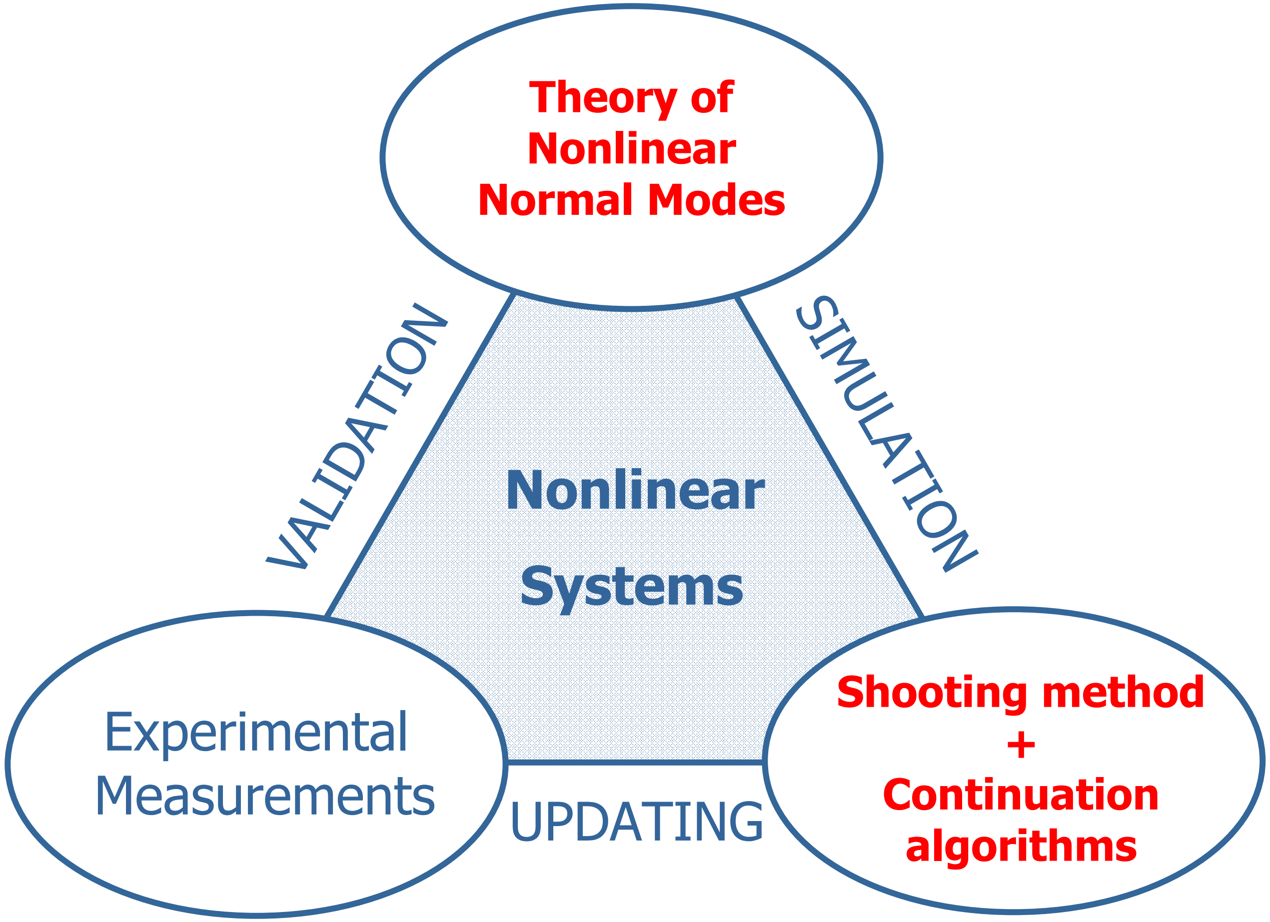
Conclusion

The Structural Dynamicist's Toolkit



Conclusion

The Structural Dynamicist's Toolkit



Theory of Nonlinear Normal Modes

VALIDATION

SIMULATION

Nonlinear Systems

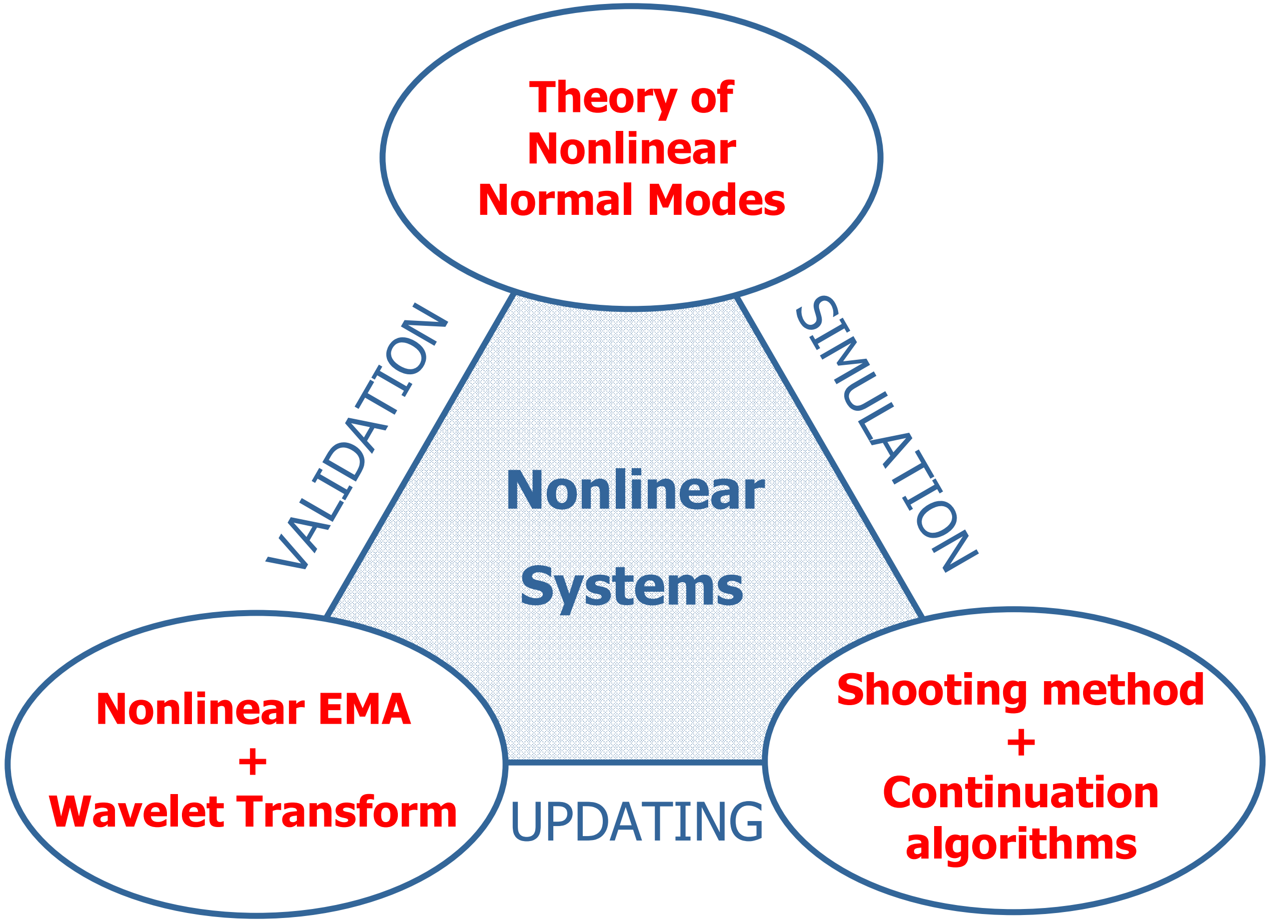
Experimental Measurements

UPDATING

Shooting method + Continuation algorithms

Conclusion

The Structural Dynamicist's Toolkit



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Thank you for your attention.