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DYNAMIC STRESS AND FATIGUE LIFE PREDICTION USING MODAL TESTING AND MODEL CORRELATION

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ABSTRACT

The lifetime prediction of a structure submitted to excessive vibrations under temporary operational conditions is very important in the design process. If the dynamic response of the structure is measured in actual operating conditions, the dynamic stress distribution can be computed by use of the structural model and modal testing results. This paper describes the case history of the lifetime prediction of welded joints used for fixing tubes in a power-plant condenser. During start-up or coast-down, the vacuum containing shell of the condenser is submitted to large vibratory forces caused by steam projection and condensation. A modal testing of the structure is first performed and the results are correlated with finite element model results. Then the lifetime verification of the welded joints is based on the prediction of the dynamic stress distribution by use of FE model analyses.

NOMENCLATURE

[C]	: damping matrix
$[\mathbf{H}(\omega)]$	: transfer matrix
[K]	: stiffness matrix
m	: number of included modes
N	: total number of degrees of freedom
m_r	: modal mass of r^{th} mode
[M]	: mass matrix
{f}	: vector of generalized forces
$\{\mathbf{F}(\omega)\}$	: Fourier transform of generalized forces
$\{\mathbf{P}(\omega)\}$	: Fourier transform of modal forces
{q}	: vector of generalized displacements
$\{\mathbf{Q}(\omega)\}$	: Fourier transform of displacements
r	: current mode number
{x}	: vector of modal coordinates
$\{\mathbf{X}(\omega)\}$	: Fourier transform of modal coordinates
ζ_r	: viscous damping ratio of r^{th} mode
$[\Psi]$	: mode shape matrix
$[\Phi_F(\omega)]$	: PSD matrix of the excitation
$[\Phi_P(\omega)]$	: PSD matrix of the modal excitation
$[\Phi_X(\omega)]$	: PSD matrix of the response
$\Phi_z(\omega)$	: PSD of a response quantity z
ω_r	: natural frequency of r^{th} mode
ω	: frequency
$[\,]^T$ $([\,]^*)$	: transpose (conjugate) of a matrix

1. INTRODUCTION

Structural design using modern techniques usually meets performance and mechanical reliability requirements. However, when unpredictable vibrations are encountered under temporary operational conditions, it becomes important to verify the structural integrity by lifetime calculations. These calculations have to be based on a representative model of the structure and of the excitation.

In the following, the structural model is based on experimental data obtained by modal testing of the structure. The excitation model is built using the model response and observations of the real response to the temporary loading. Thus the reliability analysis is based on the spectral analysis of structures under stationary random excitation. This approach is focused on second-order statistics. It means that the excitation and the response processes are characterized solely in terms of expected values (first moments), variances and covariances (second moments).

2. THEORETICAL BACKGROUND

The general equations of motion of a discretized structure are of the form :

$$[\mathbf{M}]\ \{\ddot{\mathbf{q}}\} + [\mathbf{C}]\ \{\dot{\mathbf{q}}\} + [\mathbf{K}]\ \{\mathbf{q}\} = \{\mathbf{f}\} \tag{1}$$

In the frequency domain, the input-output relation of the structural system takes the form :

$$\{\mathbf{Q}(\omega)\} = [\mathbf{H}(\omega)]\ \{\mathbf{F}(\omega)\} \tag{2}$$

where

$$[\mathbf{H}(\omega)] = \left[-\omega^2\ [\mathbf{M}] + i\ \omega\ [\mathbf{C}] + [\mathbf{K}]\right]^{-1} \tag{3}$$

In terms of modal parameters, equation (2) can be transformed to :

$$\{\mathbf{X}(\omega)\} = [\mathbf{H}(\omega)]\ \{\mathbf{P}(\omega)\} \tag{4}$$

with

$$\{\mathbf{X}(\omega)\} = [\Psi]^T\ \{\mathbf{Q}(\omega)\} \tag{5}$$

$$\{\mathbf{P}(\omega)\} = [\Psi]^T\ \{\mathbf{F}(\omega)\} \tag{6}$$

and

$$[\mathbf{H}(\omega)] = \left[[\,\check{}\ m_r\ (\omega_r^2 - \omega^2)\,\check{}\,] + i\ \omega\ [\Psi]^T\ [\mathbf{C}]\ [\Psi]\right]^{-1} \tag{7}$$

In the particular case of proportional damping, the modal damping matrix is diagonal and is written :

$$[\Psi]^T\,[\mathbf{C}]\,[\Psi] = 2\,[\,^{\backslash}\zeta_r\; m_r\; \omega_r{}_{\backslash}\,] \tag{8}$$

The response of a linear structure to stationary random vibration is generally written in terms of the power spectral density (PSD) matrices of the response and of the excitation [1]. The PSD matrix of the exciting forces is converted to the PSD matrix of the modal excitation as follows :

$$[\mathbf{\Phi}_P(\omega)] = [\Psi]^T\,[\mathbf{\Phi}_F(\omega)]\,[\Psi] \tag{9}$$

where Hermitian matrices $[\mathbf{\Phi}_F(\omega)]$ and $[\mathbf{\Phi}_P(\omega)]$ are of dimension $(N \times N)$ and $(m \times m)$ respectively. The PSD matrix of the modal responses $[\mathbf{\Phi}_X(\omega)]$ is then given by :

$$[\mathbf{\Phi}_X(\omega)] = [\mathbf{H}(\omega)]\,[\mathbf{\Phi}_P(\omega)]\,[\mathbf{H}(\omega)]^{*T} \tag{10}$$

Thus the PSD $\Phi_z(\omega)$ of any response quantity z (stress, displacement, ...) is given by a relation of the type :

$$\Phi_z(\omega) = \{\mathbf{b}\}^T\,[\mathbf{\Phi}_X(\omega)]\,\{\mathbf{b}\} \tag{11}$$

where $\mathbf{b}$ is a vector of constant that relates the response z to the modal coordinates $\{\mathbf{x}\}$ in the form :

$$z = \{\mathbf{b}\}^T\,\{\mathbf{x}\} \tag{12}$$

From equation (11), the statistics of the response, i.e. the RMS value, the central frequency, the spreading parameter and the peak factor over a period of observation can be estimated. The procedure of calculating the spectral response of a structure submitted to stationary random excitation is summarized in figure 1.

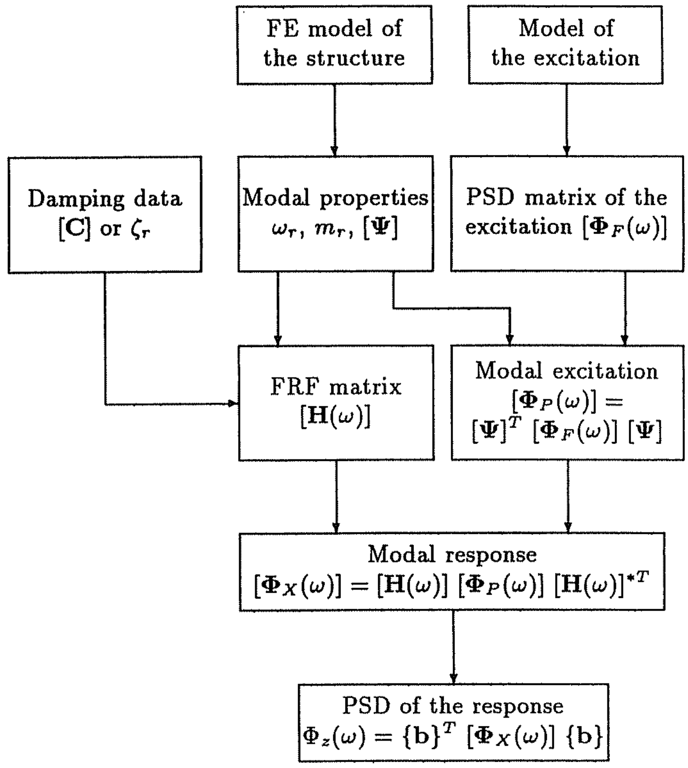

Figure 1: Spectral analysis of the stationary random response

3. DESCRIPTION OF THE PROBLEM

To illustrate this concept, we consider the case history of the lifetime assessment of welded joints used for fixing tubes in a condenser attached to the low-pressure exhaust of a steam turbine. The equipment arrangement of the combined cycle power-plant considered here is shown in figure 2. The plant consists of an installation of two open-cycle gas turbines (1) arranged to exhaust to steam generators (2). The steam produced is then supplied to a steam turbine (3) which exhausts downward into the condenser (4). During starting and stopping of the turbine units, it is imperative to dispose of large quantities of steam and the condenser is used as a dumping place for this steam. During this operation, the turbine is by-passed and the steam flow is directly fed through inlet pipes into the vacuum containing shell of the condenser. The steam ejection and condensation induce large vibratory forces so that the structural members of the condenser have to accomodate the high-energy steam without damage.

The objective of this study is to decide whether or not the design is adequately reliable. For this purpose, the time-varying pressure induced by the steam on the condenser shell is considered as a stationary random process. As the stochastic model for the specific loading considered here is not known, it is idealized as a band-limited white noise. Thus the procedure consists in constructing a FE model of the condenser wall that correlates with results of modal testing. Then the lifetime verification of the welded joints is based on the prediction of the static and dynamic stress distributions using FE model analyses.

4. MODAL TESTING

4.1. Vibration Severity Measurements

The root-mean-square (RMS) value of the acceleration amplitude was measured at equally-spaced coordinates on the condenser envelope in the frequency range from 0 to 128 Hz. The measurements were done during normal operation conditions of the power-plant and also during a by-pass operation of the turbine. The critical zone of the condenser wall (i.e. where the vibration severity is the highest) was found to be located in front of the steam inlet tubes as shown in figure 3. PSD spectra were measured during the by-pass operation of the turbine at 90 equally-spaced measurement points on the critical zone (figure 4). A typical example of a measured PSD spectrum is shown in figure 5.

4.2. Modal Identification

A modal identification of the structure was performed *in situ* using impact testing. The modal testing was made with the system out of operation but with the vacuum maintained into the condenser. The frequency response function (FRF) at each measurement coordinates of the critical zone was mea-

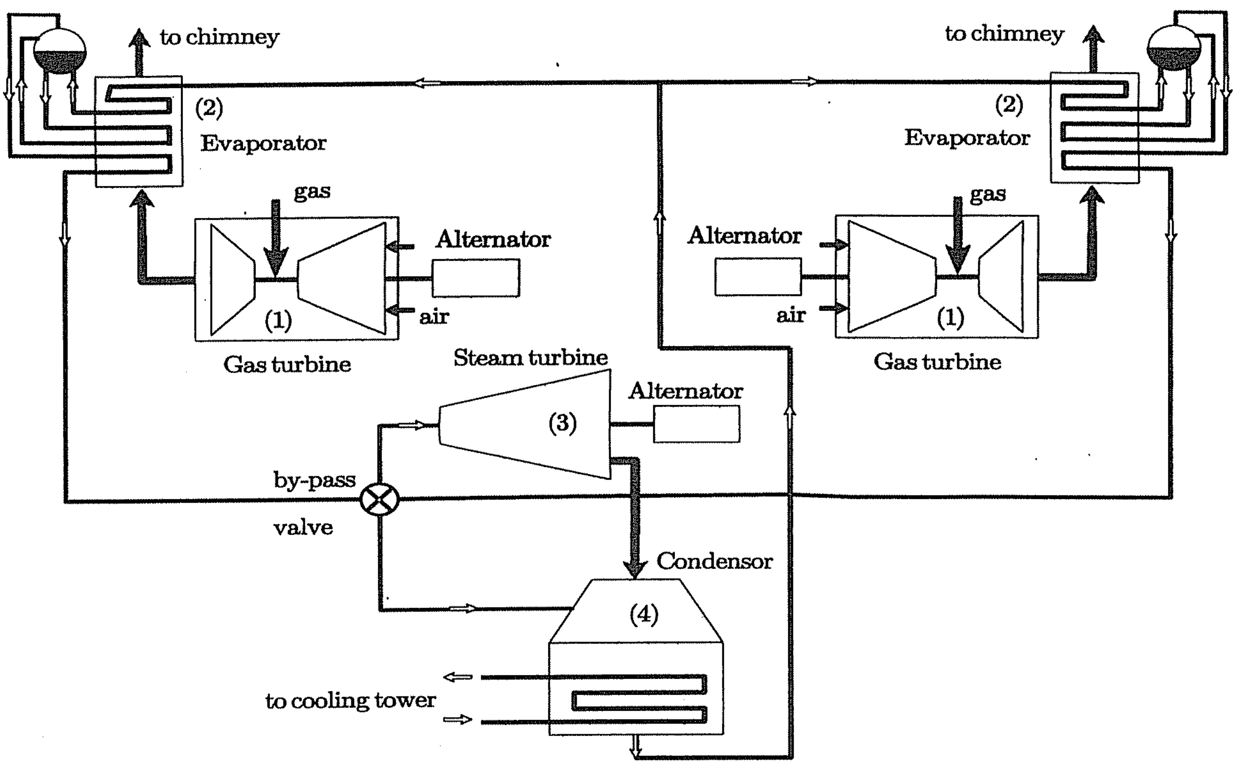

Figure 2: Combined cycle power-plant

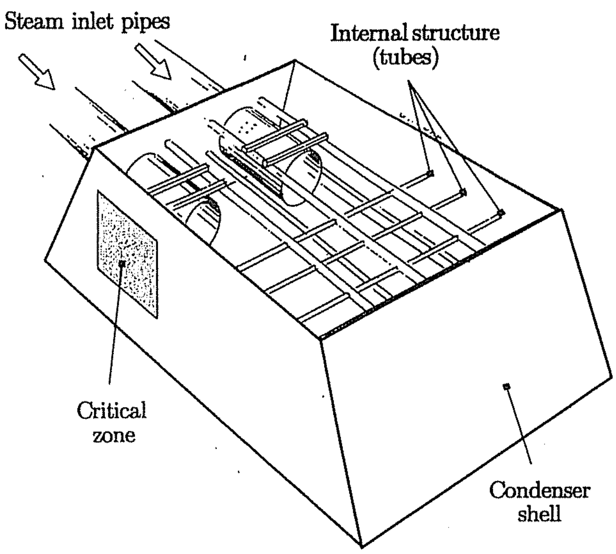

Figure 3: The steam condensor

sured at 512 frequencies for a baseband frequency range of $0-128$ Hz. An example of a measured FRF is shown in figure 6. The modal parameter extraction was performed using the modal analysis software LMS Cada-X [2].

5. MODAL ANALYSIS

5.1. Structural Model Description

The structural model of the condenser vertical wall is made using the FE software SAMCEF [3]. The FE discretization of the wall resulting in a total of 808 isoparametric shell and beam elements and 7 865 degrees of freedom (DOFs) is shown in figure 4. The wall perimeter is supposed to be fixed in the normal direction to the wall plane and additional boundary conditions were added to prevent rigid body modes. The horizontal tubes supporting the wall are modelled by beam elements. Linking constraints between the tube ends were added to take into account the own weight of the internal structure. The boundary conditions were corrected to achieve a good correlation between the FE and the modal testing results.

5.2. Linear Static Analysis

A FE linear static computation was performed to simulate the 34 mbars vacuum condition inside the condenser shell. This load case corresponding to a depressure of 978 mbars was superposed to the own weight of the internal structure supported by the wall. The computed reaction forces between the tubes and the wall allow to evaluate the static stress levels inside the welded joints.

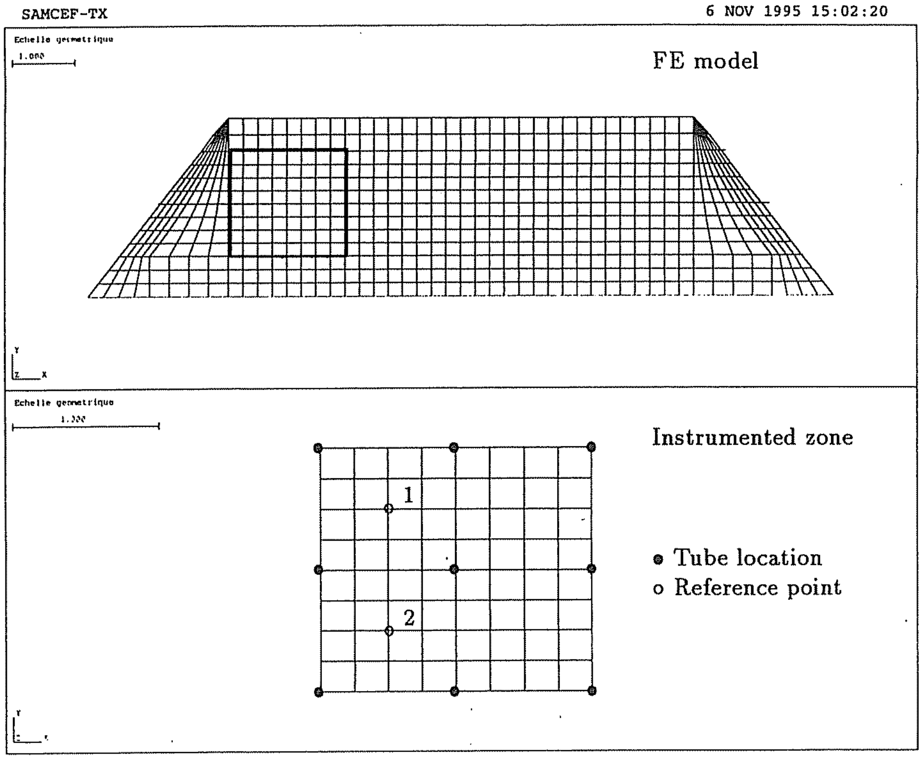

Figure 4: FE model of the steam condensor shell

5.3. Dynamic Analysis

A FE dynamic computation was performed using the corrected stiffness matrix of the linear static computation in order to take into account the influence of the static loading on the dynamic behaviour of the structure. The results show a high modal density (an amount of about 30 eigenmodes is found on a frequency interval of 60 Hz). Some of the most significant eigenmodes are shown in figure 7.

5.3.1. Correlation of FE and Experimental Results

The degree of correlation between the FE and the experimental models was investigated by visual inspection of mode shapes and by calculation of the mode shape correlation coefficients. The correlation between the FE and the experimental models was improved by a FE model updating using the boundary conditions as parameters. However, the high modal density of the structure compared to the frequency resolution (0.25 Hz) used for impact testing does not allow us to hope for a high degree of correlation between FE and experimental results. Unfortunately, the test conditions (impact hammer, vibration noise due to vacuum maintenance and *in situ* measurements, limited testing time due to operational constraints) could not be improved. Furthermore, the Modal Assurance Criterion (MAC) matrix shows that the FE eigen modes that do not correlate with experimental mode shapes do not give any contribution to the energy deformation of the instrumented zone of the wall. For these reasons, the degree of correlation between the first 6 experimental mode shapes and 6 FE eigen modes was found to be satisfactory for use in a spectral analysis.

5.4. Spectral Analysis

During starting and stopping of the turbine units, the produced steam is injected in the vacuum containing shell of the condenser through two multiple-orifice inlet pipes. The steam ejection and condensation induce a fluctuating pressure that results in large vibratory responses of the condenser wall. The excitation pressure could not be measured and was assimilated to a band-limited white noise of constant amplitude

$$[\Phi_F(\omega)] = \Phi_0 \qquad (13)$$

in the considered frequency range $0 - 128$ Hz. The PSD of the excitation reduces to a single component as the spatial correlation of the excitation is assumed to be perfect on the critical zone of the condenser wall.

The identified modal dampings obtained from modal testing of the condenser wall are not precise enough to be used in

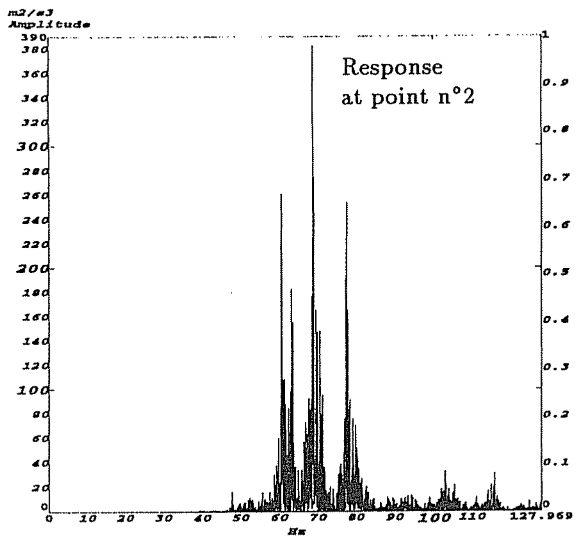

Figure 5: Measured PSD spectrum

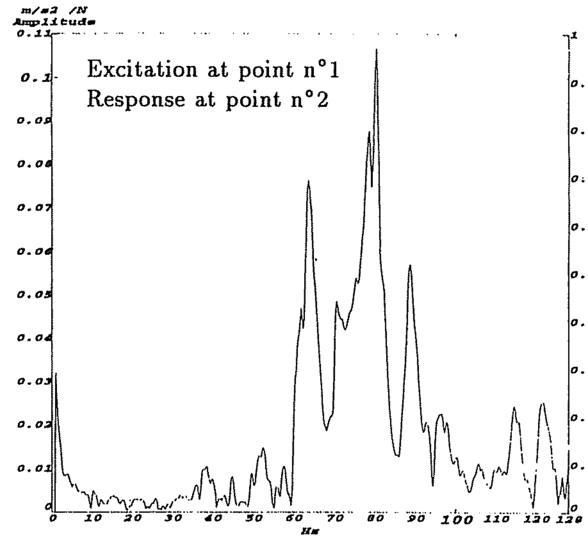

Figure 6: Example of measured FRF

calculations. It is a consequence of *in situ* test conditions as mentioned earlier. For this reason, the modal damping associated to each eigenmodes in the frequency interval $0-128$ Hz is fixed to 0.5 % which is a common value for steel. However, the modal damping of the two first modes had to be lowered in order to enhance the correlation between the predicted and measured PSD of the responses. Thus the spectral response of the structural model was computed using the FE software SAMCEF. The PSD amplitude of the excitation pressure was calibrated in order to minimize the differences between RMS values of the model responses and of the measured responses respectively. For example, the predicted PSD spectrum for reference point 2 shown in figure 8 can be directly compared with the corresponding measured PSD spectrum given in figure 5.

6. LIFETIME PREDICTION

The results show that the reaction in the welded joints is a narrow-band process. Due to the dominance of the central frequency in the process, the local behavior resembles a harmonic oscillation. For this reason, the classical rules used for lifetime prediction of welded joints are assumed to be applicable in this case. The reliability of the design is made on the basis of the comparison between a comparison stress (combined real stresses) and an allowable stress. Refering to the procedures recommended in [4], [5] and [6], the comparison stress σ_C is defined as

$$\sigma_C = \sqrt{\sigma_{\parallel}^2 + \frac{1}{\alpha^2}\left(\sigma_{\perp}^2 + \lambda\,(\tau_{\parallel}^2 + \tau_{\perp}^2)\right)} \qquad (14)$$

$$\text{with} \quad \alpha = 0.8\,(1 + \frac{1}{a}) \quad \text{and} \quad \lambda = 1.8$$

where $\sigma_{\parallel}$, $\sigma_{\perp}$, $\tau_{\parallel}$ and $\tau_{\perp}$ are defined in figure 9. For static loading, the comparison stress has to be compared to the tensile limit stress ($160\ 10^6 N/m^2$ for ST37 steel). In case of dynamic loading, the comparison stress has to be compared to the fatigue strength ($36\ 10^6 N/m^2$ for ST37 steel). In the case-study presented here, the safety factors were found to be :

1.12	for static loading
2.88	for dynamic loading

For a by-pass operation of the steam turbine unit of thirty minutes, the peak factor of the stress leads to a safety factor of 1.15 (for a comparison stress of $235\ 10^6 N/m^2$).

7. CONCLUDING REMARKS

Traditionally, structural design relies on the perfect knowledge of the loads. However, during its lifetime, the structure may be submitted to unexpected loads that might affect its safety. In this case, the reliability of the real structure becomes a matter of concern. An synergistic approach, using both modal testing, *in situ* measurements and FE modelling, was outlined to verify the integrity of welded joints submitted to large amplitude vibrations. Modal testing associated to structural modelling provide the necessary tools to perform reliability calculations and to control structural safety because it accounts for the observed performance of real structures.

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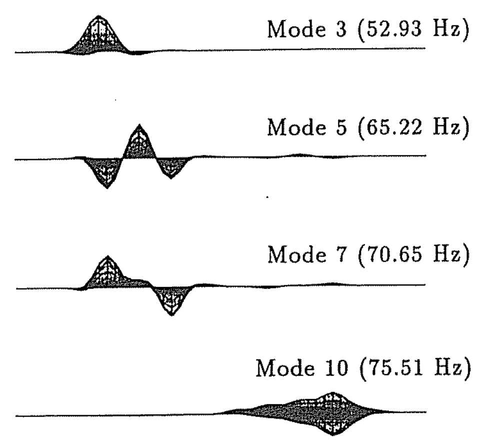

Figure 7: Typical mode shapes of the condenser wall

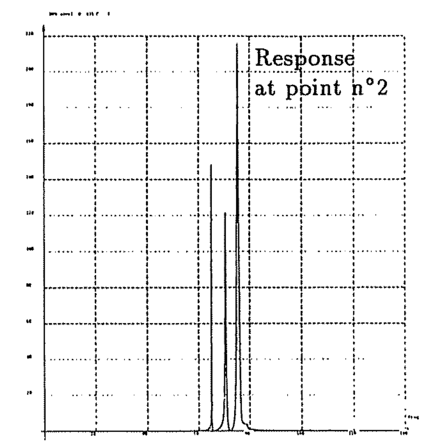

Figure 8: Predicted PSD spectrum

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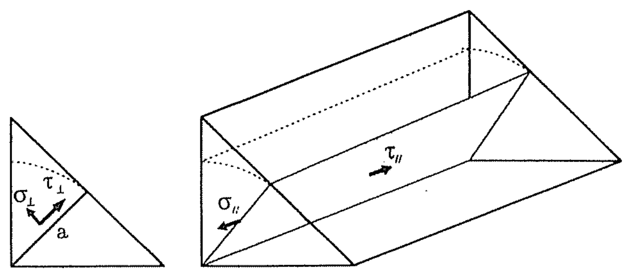

Figure 9: Welding joint geometry