Probabilistic capacity assessment for three-phase low-voltage distribution networks

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***Abstract*—The increase of photovoltaic panels and electric vehicles in low-voltage distribution systems leads to over-voltage, under-voltage, and congestion issues. These issues, related to new installations, add a considerable cost to distribution system operators, and therefore to customers. Distribution system operators want to limit these costs by determining the impact of photovoltaic panel production and electrical vehicle charging. This paper presents a probabilistic method enabling operators to evaluate the network capacity, defined as the number of new installations that can be added to the network without adapting it to overcome under- and over-voltage. The method provides multiple probabilistic performance indicators reflecting a large number of possible configurations resulting from new installations added to the low-voltage network. The evaluation of this method is done using a case study based on an existing European network. The method provides tangible results as the maximum number of photovoltaic installations or electric vehicle chargers within a defined confidence level. Results, in the test case, show that the capacity of the network is evaluated as a 45% penetration rate for photovoltaic installations, or 4% for electric vehicle chargers, with a 5% violation of operational indicators.**

***Index Terms*—network capacity, EV, PV, capacity assessment, low-voltage distribution network**

Notation

Sets

$\mathcal{T}$	Set of observation period
$\mathcal{N}$	Set of nodes
$\mathcal{E}$	Set of edges
$\mathcal{P}$	Set of phases
$\mathcal{A}_n$	Set of new possible installations type for node n
$\mathcal{M}_{k,n}$	Set of installed types for node n in configuration k

Variables

Z_e	Impedance of edge $e \in \mathcal{E}$
$\mathbf{S}^b_{n,p}$	Initial base power injection of $n \in \mathcal{N}$ in $p \in \mathcal{P}$
$\mathbf{S}_n$	Power injection of $n \in \mathcal{N}$
$\mathbf{S}_{k,n,m}$	Power injection of installations of type $m \in \mathcal{M}$ at $n \in \mathcal{N}$ for configuration k
$\mathbf{V}_n$	Voltage of $n \in \mathcal{N}$
$\overline{V}$	Over-voltage threshold
$\underline{V}$	Under-voltage threshold
$\mathbf{I}_e$	Edge current of $e \in \mathcal{E}$

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I. Introduction

The European Union directives [1] - [2] encourage increasing penetration rates for both Electric Vehicles (EVs) and Photovoltaic (PV) installations. However, this has become a concern for Distribution System Operators (DSOs). Indeed, adding new installations of these types to the Low-Voltage (LV) network leads to several issues such as over-voltages or under-voltages. To prevent these issues, and to plan future corrective actions, DSOs need to quantify the impact of such installations on their LV networks by evaluating the potential network capacity given likely future new installations in the network. The problem that complicates this quantification is that the decision to install these new devices is not made by DSOs but by the customers themselves according to their financial means and needs. For DSOs, this translates into uncertainty in the type of added installations, their capacity, and the order in which they will be connected to its network. These uncertainties turn the network capacity assessment into a complicated, but equally important problem.

One common method to evaluate this capacity is to check if there exists a possibility of under- or over-voltage, with a power flow for a given set of fixed installations. If it is the case, DSOs consider this set of PV and EV installations as unacceptable when trying to prevent network issues and avoid customers complaints. This conclusion is, however, too conservative with respect to what happens in practice. For instance, when a PV installation produces and the PV inverter detects an over-voltage, it temporarily disconnects the installation from the network. This action of the inverter prevents any significant over-voltage from occurring in the network. Therefore, checking the possibility of over-voltage for a fixed set of installations is not an appropriate method to quantify the network capacity. The real impact for customers and DSOs is that energy not produced due to inverter curtailment resulting from network issues. The same reasoning is applied to EVs as when an under-voltage is detected, charging of EVs is interrupted.

This paper extends the network capacity assessment by (i) determining the number of new PV or EV installations that can be added to the network; (ii) considering the uncertainty related to the position of new installations; (iii) considering time variance by using time-series and (iv) quantifying the capacity in terms of energy. The energy quantifications are stochastic functions of the set of installations obtained using Probability Density Functions (PDF).

The rest of the paper is presented as follows. Related work is presented in Section II. Section III formalises the network capacity assessment problem. Section IV explains a stochastic approach to solve this problem. Section V presents the results obtained on a case study. Finally, Section VI concludes the paper and presents some discussions

II. Literature review

The hosting capacity assessment problem is a stochastic problem as it involves uncertain parameters. These uncertainties are categorized into aleatory and epistemic [3]. The former refers to power consumption and injection that are unknown variables. The epistemic uncertainties relate to the location or size of future installations in the network.

Several researches, reviewed in [3] and [4], study the low-voltage network capacity assessment problem. All reviewed methods rely on power-flow analyses and can be divided into three categories [3]: (i) time-invariant deterministic, (ii) time-invariant stochastic, (iii) time-variant deterministic. Time-invariant deterministic methods use one typical value for each node as its injection or consumption representing the worst-case scenario. Such consideration leads to a pessimistic assessment that does not represent the real network operation under time-variant production and consumption patterns. Time-invariant deterministic methods discard aleatory uncertainties. Time-invariant stochastic methods capture the stochastic nature of the hosting capacity assessment problem by considering both types of uncertainties. Epistemic uncertainties are only included in some of the stochastic techniques. Deterministic time-series capacity assessment methods ensure the correlation between injections and consumptions are satisfied and entail modelling aleatory uncertainties.

Paper [5] presents a method that addresses epistemic uncertainties for the capacity assessment of EV and PV installations. Epistemic uncertainties are handled with Monte-Carlo simulations, randomly selecting installation sites. To assess the capacity of PV installation, the lowest consumption of the year is taken as a reference consumption and the maximum total PV production is selected for the installations. For EV assessment, the highest consumption values and various nominal charging powers are considered.

The difference in results between [5], [6] and [7] highlights the importance of considering correlated injection/withdrawal time-series. The authors of [7] conclude on the importance of considering high-resolution time-series, but this can only be done at the cost of a shorter study time window that does not truly enable capturing and quantifying the impacts of uncontrolled EV charging.

Most of the papers reviewed in [3] - [4] conclude that the voltage issues are the most important to consider as they occur before any other network issues. Indicators based on these are privileged to quantify the hosting capacity limit. Several papers reviewed in [4] consider certain technical enhancements to increase this limit.

In the scientific papers reviewed, the capacity assessment problem was not addressed with both time-variant and stochastic methodology. This paper presents a methodology that uses time-series to model the aleatory uncertainty and stochastic capacity assessment to consider epistemic uncertainty.

III. Problem Statement

This study considers an unbalanced three-phase LV network. The network topology is represented by a tree graph $\mathcal{G} = (\mathcal{N}, \mathcal{E})$ where $\mathcal{N}$ is the set of nodes in the network and $\mathcal{E}$ is the set of edges linking these nodes. The line impedance of an edge $e \in \mathcal{E}$ is denoted by $Z_e \in \mathbb{C}^3$. The observation period $\mathcal{T}$ of the network is the set of all consecutive time steps denoted t and of length δt in hours. A variable x followed by a subscript t, that is x_t, refers to the value of x at time t while boldface $\mathbf{x}$ refers to the entire time-series. The set $\mathcal{P}$ denotes the set of phases. The initial base power injection time-series of phase $p \in \mathcal{P}$ of node $n \in \mathcal{N}$ is denoted by $\mathbf{S}^b_{n,p} \in \mathbb{C}^{|\mathcal{T}|}$.

A node n of the network can have a set $\mathcal{A}_n$ of new possible installation types, e.g. photovoltaic panels (PV) or electric vehicles (EV). A configuration k is generated by selecting, for each node n, a set of installed types $\mathcal{M}_{k,n} \subset \mathcal{A}_n$ and their corresponding power injections $\mathbf{S}_{k,n,m}$. The set of power injections in node n is:

$$\mathcal{S}_{k,n} = \{\mathbf{S}_{k,n,m} | m \in \mathcal{M}_{k,n}\}. \tag{1}$$

These power injections $\mathbf{S}_{k,n,m} \in \mathbb{C}^{3|\mathcal{T}|}$ modify the power injections of the nodes, such that:

$$\mathbf{S}_{k,n} = \mathbf{S}^b_n + \sum_{\mathbf{s} \in \mathcal{S}_{k,n}} \mathbf{s}. \tag{2}$$

For each configuration k, the power injections of new installations change the edge currents and the node voltages. The updated voltages and currents can be obtained solving the power flow equations G:

$$G_{\mathcal{G}}(\{\mathbf{S}_{k,n} | n \in \mathcal{N}\}) = \{\mathbf{V}_{k,n} | n \in \mathcal{N}\}, \{\mathbf{I}_{k,e} | e \in \mathcal{E}\}. \tag{3}$$

Each configuration of installations is evaluated using a set $\mathcal{I}$ of indicators, such as the maximum power flowing through the main substation transformer. These indicators are defined in Section IV. Each indicator $i \in \mathcal{I}$ is calculated as a function $h_i(\mathbf{V}, \mathbf{I}) : \mathbb{C}^{|\mathcal{N}||\mathcal{T}|} \times \mathbb{C}^{|\mathcal{E}||\mathcal{T}|} \to \mathbb{R}$. The inputs of h_i are the voltages for each node in $\mathcal{N}$ over each time step in $\mathcal{T}$ and currents over the same time period of each edge of $\mathcal{E}$. DSO can define a threshold rate $\overline{h}_i$ over which the configuration is not acceptable.

Defining these rates $\{\overline{h}_i | i \in \mathcal{I}\}$ enables one to determine the number of installations the network can support. For a fixed number of installations r to distribute on $\#\mathcal{N}$ nodes, $\frac{\#\mathcal{N}!}{(\#\mathcal{N}-r)! \cdot r!}$ combinations exists. The set of configurations $\mathcal{C}_r$ contains all possible combinations with r installations. The probability distribution over the configurations within $\mathcal{C}_r$ is assumed to be uniform. Values of indicator i, obtained for this set, are used as the support of a probability density function

$f_{i,r} : \mathbb{R} \to \mathbb{R}^+$ defined from $\{h_i(\mathbf{V}_k, \mathbf{I}_k) | k \in \mathcal{C}_r\}$. The network capacity, i.e. the number of installations r that can be accommodated, depends on the risk that the DSO can tolerate. Denoting this risk tolerance $\overline{F}_i \in [0, 1]$, the tolerated number of installations r_i for an indicator i is such that:

$$\int_{\overline{h}_i}^{+\infty} f_{i,r_i}(h_i) dh_i \leq \overline{F}_i \tag{4}$$

where the integral of the PDF of indicator i, from the maximum threshold rate $\overline{h}_i$ to $+\infty$, gives the probability of having a configuration with h_i over this threshold. The total capacity of the network is defined as the minimum of r_i values obtained for a set $\mathcal{I}$ of indicators:

$$R = \min\{r_i | i \in \mathcal{I}\}. \tag{5}$$

R presents the number of installations that can be accommodated, for which none of the indicators exceeds the defined thresholds $\overline{h}_i$, with a probability equal to or less than the DSO risk tolerance $\overline{F}_i$.

Require: K, $G_{\mathcal{G}}(\mathbf{S})$, $\{\mathcal{A}_n | n \in \mathcal{N}\}$
1: $r \leftarrow 0$
2: **while** $r < \sum_{n \in \mathcal{N}} |\mathcal{A}_n|$ **do**
3: $\quad \mathcal{K} \leftarrow$ sample K configurations with r installations
4: $\quad \{\mathbf{V}_k, \mathbf{I}_k | k \in \mathcal{K}\} \leftarrow \{G_{\mathcal{G}}(\mathbf{S}_k) | k \in \mathcal{K}\}$
5: $\quad$ **for each** $i \in \mathcal{I}$ **do**
6: $\quad\quad F_{i,r} \leftarrow \sum_{k \in \mathcal{K}: h_i(\mathbf{V}_k, \mathbf{I}_k) > \overline{h}_i} 1 \;/\; |\mathcal{K}|$
7: $\quad\quad$ **if** $F_{i,r} > \overline{F}_i$ **then**
8: $\quad\quad\quad R \leftarrow r - 1$
9: $\quad\quad\quad$ **return** R
10: $\quad\quad$ **end if**
11: $\quad$ **end for**
12: **end while**
13: $R \leftarrow \sum_{n \in \mathcal{N}} |\mathcal{A}_n|$
14: **return** R

Algorithm 1: Network capacity assessment algorithm.

IV. Methodology

This section presents the methodology used to assess the network capacity as the number of new installations the network can sustain. The methodology takes as inputs a graph $\mathcal{G}$ and its impedances $Z_e \;\forall e \in \mathcal{E}$, initial base power injection time-series $\mathbf{S}^b_{n,p} \;\forall (n, p) \in \mathcal{N} \times \mathcal{P}$ and sets $\mathcal{A}_n \;\forall n \in \mathcal{N}$ of new possible installation types that can be added to node n. In addition to these inputs, the number of configurations evaluated was limited to a number $K \in \mathbb{R}$.

The algorithm, whose pseudo code can be found in Alg. 1, iterates on the number of new installations r that is limited by the total number of possible new installations. For each r, K configurations are generated with r new installations and for each of these installations the currents and voltages are determined using a power flow. These currents and voltages are used to compute indicators for each configuration. Line 6 computes the ratio of generated configurations that have values for indicators higher than their accepted limit. These ratios are the probabilities defined in Eq. (4) for a configuration to have indicators exceeding their limit. The probabilities are compared to the corresponding risk tolerance of their indicator $\overline{F}_i$, the first indicator ratio to exceed its limit has the smallest r and is set as the total capacity of the network R.

The Key Performance Indicators (KPI) are computed to evaluate the impact of installations on the network. To a given configuration k corresponds a set of indicators $\mathcal{H}_k = \{h_{i,k}(\mathbf{V}_k, \mathbf{I}_k) | i \in \mathcal{I}\}$. This study considers three indicators $\{\alpha, \beta, \gamma \in \mathcal{I}\}$.

The power flowing through the transformer, α, is used as an indicator to assess the total impact of installations on the network.

$$h_{\alpha,k} = \max_{(p,t) \in \mathcal{P} \times T} |S_{k,0,p,t}| \tag{6}$$

where 0 designates the main substation node of the network.

The remaining two indicators quantify the impact of curtailing installations. Indeed, installations such as PV panels are temporarily disconnected from the network by the converter, thus, not producing energy when an over-voltage occurs. Simulating this behaviour and generalising it to all production installations leads to an indicator of the energy that was not produced by the installations when the node voltage exceeds $\overline{\mathbf{V}}$. This indicator, referred to as energy spilled and denoted as β, is defined as, for a configuration k:

$$h_{\beta,k} = \sum_{(n,p,t) \in \mathcal{N} \times \mathcal{P} \times T : \mathbf{V}_{k,n,p,t} > \overline{\mathbf{V}}} |S_{k,n,p,t}| \cdot \delta t \tag{7}$$

In other words, this indicator calculates the sum of the energy spilled over the study interval, because of PVs disconnected due to the over-voltage limit.

To avoid under-voltage on the network, EV could also be disconnected using power electronic control. Therefore, an indicator on the energy not served to be consumed is defined as the corollary to the energy spilled for consumption. The energy unserved, γ, quantifies the energy not consumed when under-voltage occurs and is defined as:

$$h_{\gamma,c} = \sum_{n,p,t \in \mathcal{N} \times \mathcal{P} \times T : \mathbf{V}_{c,n,p,t} < \underline{\mathbf{V}}} |S_{c,n,p,t}| \cdot \delta t \tag{8}$$

This indicator calculates the sum of the energy not served to EVs over the study interval, due to the under-voltage limit.

V. Case study and results

This section presents the results of the proposed method applied to a case study based on an existing European LV network. The network has 128 customers and 256 nodes. For each penetration rate, the number of studied configurations, K, is limited to 500. This parameter reduces the factorial growth of the number of configurations depending on the number of possible installation nodes available. The value of K is set considering a trade-off between having a large number to depict as many configurations as possible, and a small enough value to be computationally feasible. The number of new installations depends on the penetration rate and their

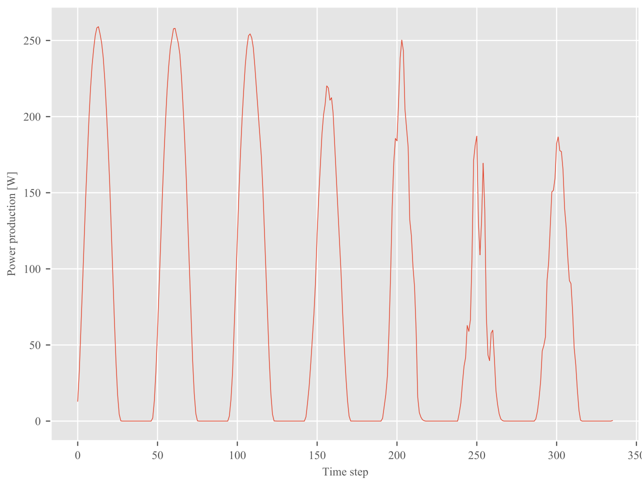


Fig. 1: Reference PV production over a week for the network case study.

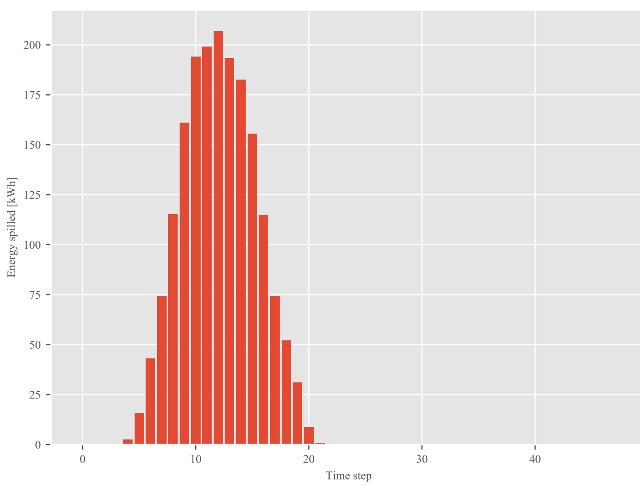
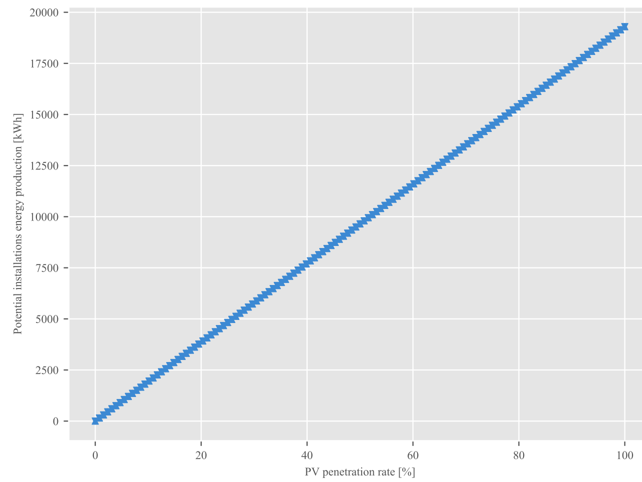
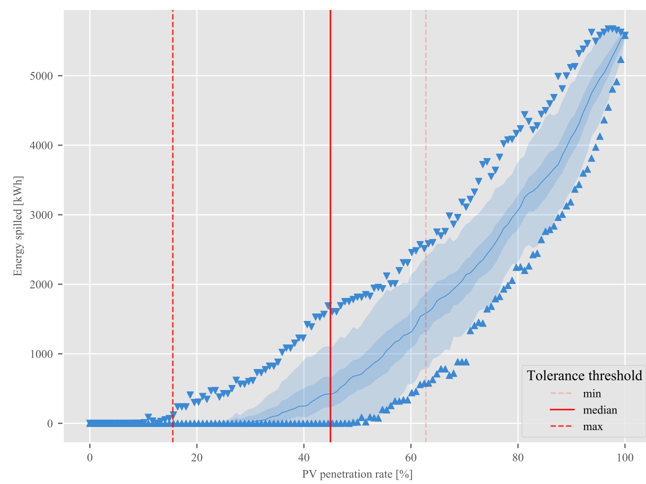


Fig. 2: Energy spilled by day over 500 configurations and 336 time steps with 85 installations.



(a) Potential energy production



(b) Energy spilled

Fig. 3: Distributions, over 500 configurations of energy spilled and potential energy consumption.

positions are defined randomly for each configuration. For each configuration, the power flow is performed over one week with 30-minute time steps to enable KPI computation.

A. *Capacity assessment for photovoltaic panels*

The power time-series for PV panel production were extracted from ELIA [8]. The number of panels added when adding an installation to a customer is set to 13. This number corresponds to the number of panels needed to fulfil the average household energy consumption in Belgium as described in [9]. The watt peak of the panels is the common watt peak of Belgium panel production and is set at 290W [9]. The reference power production over the studied week is shown in Fig. 1. Figure 2 shows the energy spilled computed using Eq. (7) and time aggregated by taking the mean over the 500 configurations considered for each time step. The peak of energy spilled occurs around 11am and 1:30pm which coincides with both the PV production peak resulting from the sun cycle and lower consumption habits of households during these hours.

Figures 3a and 3b show PDFs by penetration rates. The maximum and minimum values of these are represented by the lower and higher arrows respectively, the thick line in the middle is the median, and in-between values are the different percentiles. Figure 3a shows the potential energy production by penetration rate. The potential energy production is the sum of all added PV installations' energy production. The same reference PV panel and the same number of panels by installation was used for each new installation, resulting in a one-value PDF by penetration rate for potential energy production. Figure 3b depicts the energy spilled PDFs approximations by penetration rate. These approximations are based on the randomly sampled configurations. The missing values between

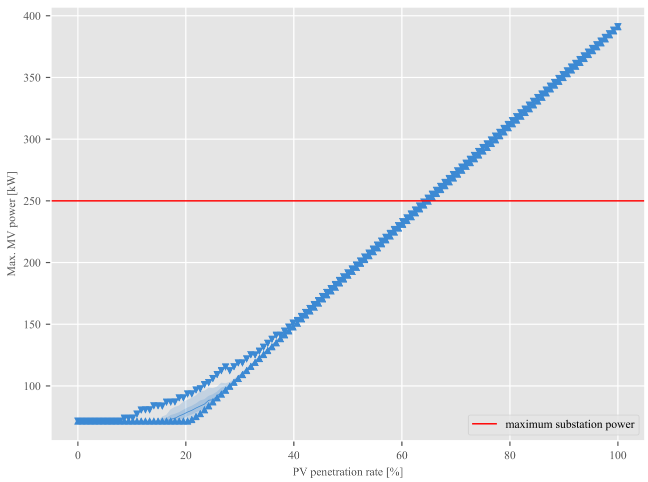


Fig. 4: Medium voltage substation maximum power distribution by PV penetration rate.

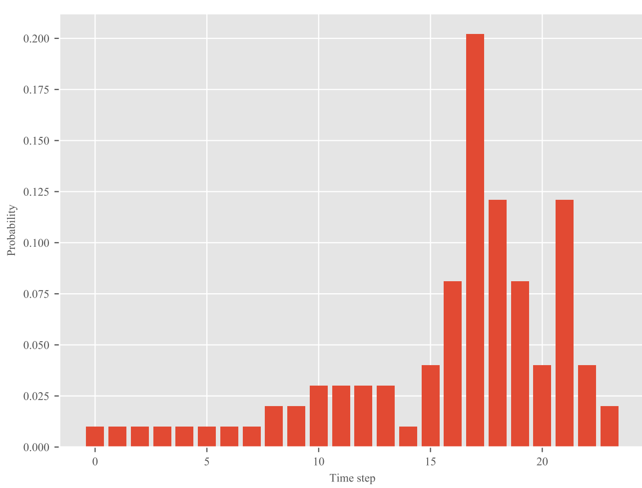


Fig. 5: Daily probability distribution of EV charging for the network case study.

the minimum and maximum of the distribution which comes from the lack of configurations. For the maximum penetration rate, where all customers add a PV installation, the energy spilled represents 30% of the potential energy produced. If the threshold of energy spilled $\overline{h}_\beta$ is set to 5%, a strict pessimistic risk tolerance $\overline{F}_\beta$ of 0 enables a 15% penetration rate while a 0.5 value, represented by the median, enables a 45% penetration rate, as shown by the vertical line in Fig. 3b.

The maximum power flowing through the substation KPI is computed using Eq. (6) and is shown in Figure 4. For penetration rates above 40%, the maximum power passing through the substation, h_α, does not drastically change for the same penetration rate. This leads to a nearly monotonous tolerance risk. The maximum power capacity of the substation is 250kVA, represented by the red line, the value with a zero penetration rate represents 30% of it. Using the maximum va-

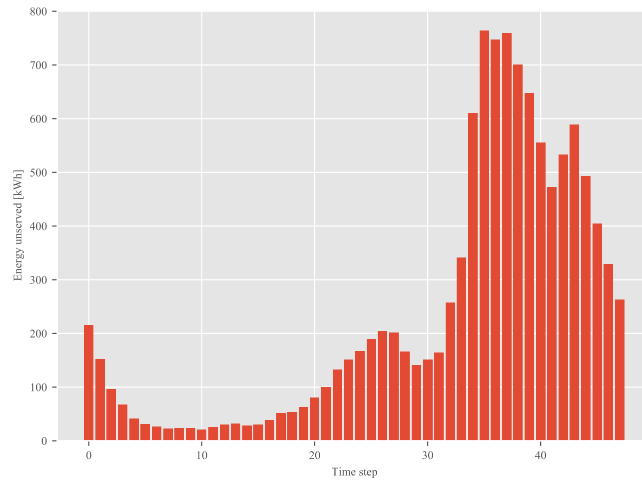
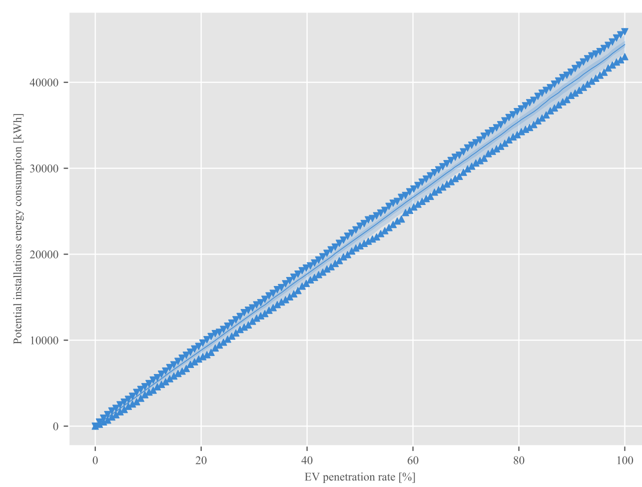
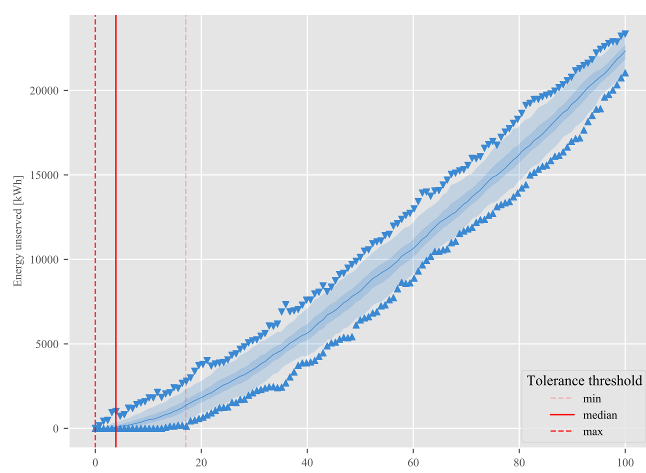


Fig. 6: Energy unserved by day over 500 configurations and 336 time steps with 79 installations.



(a) Potential energy consumption



(b) Energy unserved

Fig. 7: Distributions, over 500 configurations, of energy unserved and potential energy consumption.

lue as the KPI limit, the penetration rate is 64%. Following (5), the total PV penetration rate for this case study network is determined by the energy spilled.

B. Capacity assessment for electric vehicles

The EV charging time-series' profiles were generated assuming a 40kWh battery capacity that is a common value for available EVs, and the charging power was limited to 3.6kW; both values were derived from [10]. The EV charging consumption time series are randomly generated using a probability distribution shown in Fig. 5. This distribution was modelled on drivers commuting regularly to work during peak hours. Figure 6 depicts the energy unserved computed using Eq. 8 for all the sampled configurations, and aggregated by taking the mean of all configuration for each time step. The energy unserved follows the EV charging probability.

The energy unserved distribution aggregated by the penetration rate is shown in Figure 7b, while the potential energy consumption distribution is shown in Fig. 7a. Figure 7b shows that using distributions enables one to avoid worst-case scenarios. For the maximum penetration rate, the energy unserved represents more than 50%. If the threshold $\overline{h}_\gamma$ is set to 5% of energy unserved allowed, the worst-case scenario $\overline{F}_\gamma$ set to 0 would not even allow a 1% penetration rate. A more optimist approach, with a risk tolerance of 0.5 as shown by the median in the figure, allows a 4% penetration rate. As the same conclusions can be drawn for the maximum power flowing through the substation KPI for EV as for PV, i.e. the energy unserved is more restrictive, the obtained PDFs for this KPI are not presented for lack of space.

VI. Conclusion

This paper presents a new method to assess the network capacity considering uncertainty on installation positions and production/consumption time-series. Two types of installations are considered: PV and EV. The impact of adding PV installations to a network is quantified by the energy not produced due to the curtailment by the inverter when detecting an over-voltage. For EVs, a symmetric reasoning is applied; the capacity is defined as the unserved energy due to under-voltage limit curtailment. The proposed method is able to consider the uncertainties in installation location and production/consumption time series, providing stochastic results as a set of KPIs. Results obtained with a full-size European-based test case show a 45% penetration rate for photovoltaic installations or 4% for electric vehicle chargers, with 5% tolerance for energy spilled and energy unserved respectively and 0.5 risk tolerance. The method is applicable on any distribution system, it has low data requirements, and it provides tangible results which can be of help for DSOs in their real-world practical cases.

This work can be extended along several lines. Improvements should first be focused on optimizing the computation time to enable considering both EV and PV installations simultaneously and computing more granular network capacity. Two options are possible to minimise this computation time: decreasing the number of evaluated configurations or the number of time steps simulated. The current number of configurations could be reduced using more-complex scenario-selection methods such as sampling techniques. Selecting a subset of days representing most days of a year would significantly decrease the number of time steps evaluated by the method while providing an overview of the network capacity on the entire year. Finally, another extension of this work could be simultaneous capacity assessment for different types of installations. However, this will increase the connection possibilities, and hence the computation time.

References

[1] European Parliament, Council of the European Union, *Directive (EU) no 2009/33/ec*, https://eur-lex.europa.eu/legal-content/EN/TXT/HTML/?uri=CELEX:32009L0033&from=EN, 2009.

[2] Council of European Union, *Directive (EU) no 2019/1161*, https://eur-lex.europa.eu/legal-content/EN/TXT/HTML/?uri=CELEX:32019L1161&from=EN, 2019.

[3] E. Mulenga, M. H. Bollen, and N. Etherden, "A review of hosting capacity quantification methods for photovoltaics in low-voltage distribution grids," *International Journal of Electrical Power & Energy Systems*, vol. 115, p. 105 445, 2020.

[4] S. M. Ismael, S. H. A. Aleem, A. Y. Abdelaziz, and A. F. Zobaa, "State-of-the-art of hosting capacity in modern power systems with distributed generation," *Renewable energy*, vol. 130, pp. 1002–1020, 2019.

[5] M. H. Bollen and S. K. Rönnberg, "Hosting capacity of the power grid for renewable electricity production and new large consumption equipment," *Energies*, vol. 10, no. 9, p. 1325, 2017.

[6] S. Cundeva, A. K. Mateska, and M. H. Bollen, "Hosting capacity of lv residential grid for uncoordinated ev charging," in *2018 18th International Conference on Harmonics and Quality of Power (ICHQP)*, IEEE, 2018, pp. 1–5.

[7] J. Quirós-Tortós, L. F. Ochoa, A. Navarro-Espinosa, M. Gillie, and R. Hartshorn, "Probabilistic impact assessment of electric vehicle charging on residential uk lv networks," in *CIRED*, 2015, pp. 1–5.

[8] Elia, *Production photovoltaïque*, https://www.elia.be/fr/donnees-de-reseau/production/production-photovoltaique, 2020.

[9] Gysel, Pieter, *Combien de panneaux solaires me faut-il et que vont-ils me coûter ?* https://blog.eneco.be/fr/soleil/combien-panneaux-solaires/, 2018.

[10] C. Gaete-Morales, H. Kramer, W.-P. Schill, and A. Zerrahn, "An open tool for creating battery-electric vehicle time series from empirical data: Emobpy," *arXiv preprint arXiv:2005.02765*, 2020.