# Extreme depolarisation for any spin

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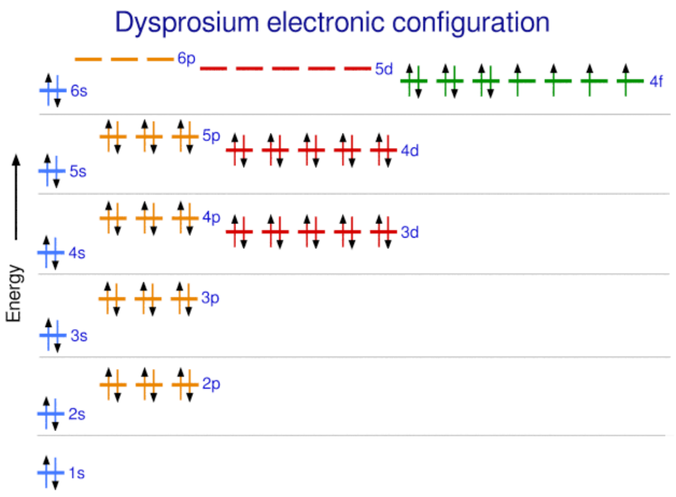
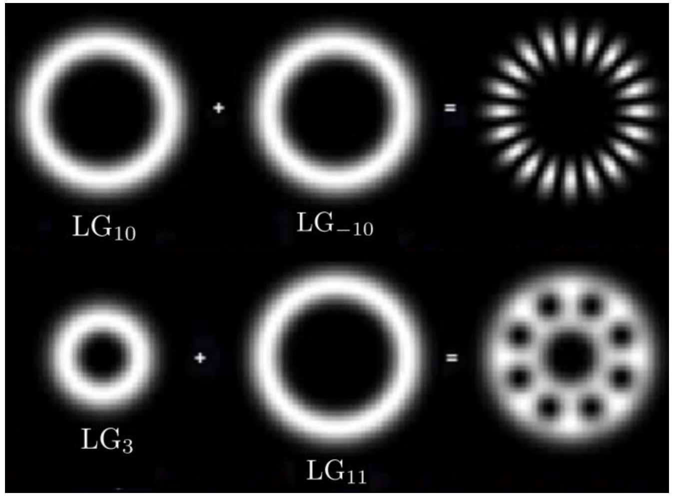
# Motivations

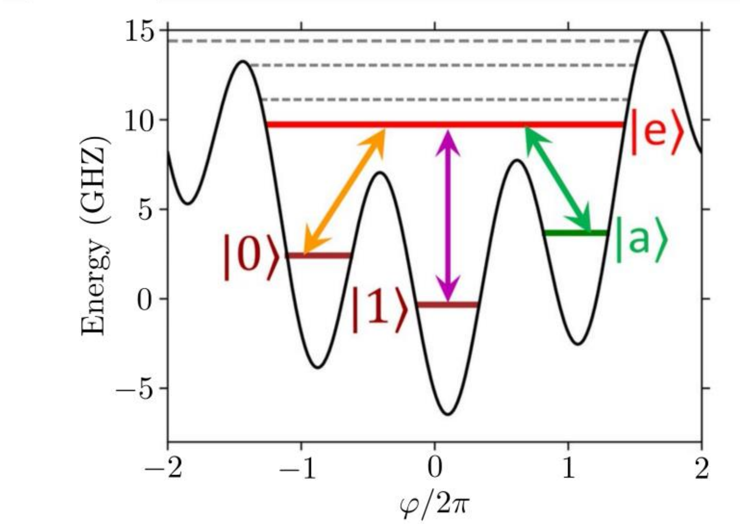
Efforts to use elementary quantum systems with spin number $j > 1/2$ for quantum technologies have been intensified by the availability of such systems in experiments operating in the quantum regime.




Atomic magnetic moment




Light orbital angular momentum




Superconducting qudit

- Which states are more prone to (super)decoherence ?
- How fast does non-classicality of a spin state fade over time ?
- How does decoherence scale with $j$ ?
- How long does it take before a spin state becomes absolutely classical ?

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# Master equation for depolarisation

**Lindblad master equation for spin *j* :**

$$\dot{\rho}(t) = \frac{i}{\hbar}[\rho(t), H] + \sum_{\alpha=x,y,z} \mathcal{D}_\alpha[\rho(t)]$$

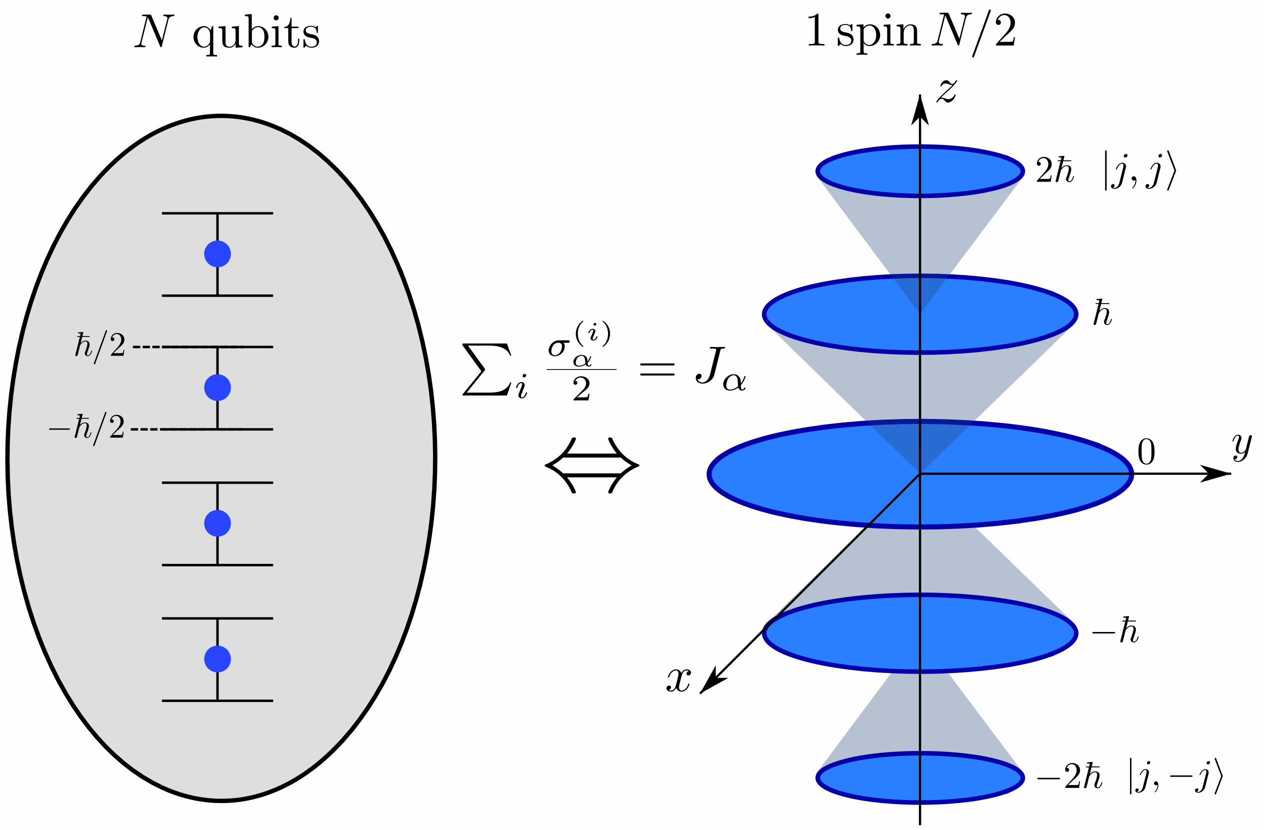
$$H = \hbar\omega J_z \qquad \mathcal{D}_\alpha[\rho] = \gamma_\alpha(2J_\alpha\rho J_\alpha - J_\alpha^2\rho - \rho J_\alpha^2)$$

Purity: $R = \mathrm{Tr}(\rho^2)$ decreases monotonically

Unique stationary state: maximally mixed state $\rho_0$

Examples of spin states: $|\mathrm{DB}\rangle = |j, 0\rangle$

$|\mathrm{GHZ}\rangle = \frac{1}{2}(|j,j\rangle + |j,-j\rangle)$ $\qquad |W\rangle = |j, j-1\rangle$




**Depolarisation rates:**

| | Isotropic depolarisation: | Anisotropic depolarisation: |
|---|---|---|
| | $\gamma_x = \gamma_y = \gamma_z$ | $\gamma_x = \gamma_y \neq \gamma_z$ |
| Possible physical cause: | Noisy Magnetic Field | High temperature bath |

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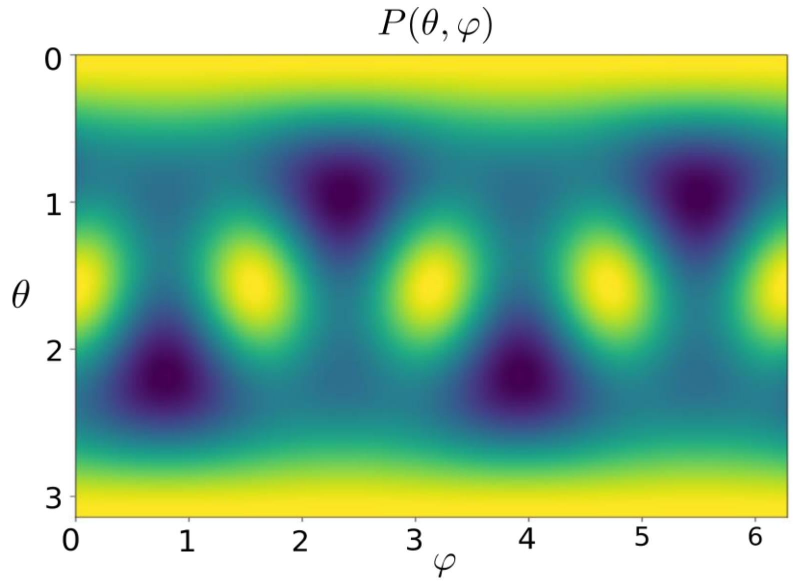
# Anticoherent states and entanglement

**Anticoherent states to order q:** Isotropic states for which the moments of $\mathbf{J}\cdot\mathbf{n}$ up to order q are independent of $\mathbf{n}$ [1]:

$$\mathrm{Tr}[\rho(\boldsymbol{J}\cdot\mathbf{n})^q] \neq f(\mathbf{n})$$

HOAP states: Highest-Order Anticoherent Pure states

$j = 2$: $|\psi\rangle_{HOAP} = \frac{1}{2}\left(|2{,}2\rangle + i\sqrt{2}|2{,}0\rangle + |2,-2\rangle\right)$ $\quad \langle\mathbf{J}\rangle = 0 \quad \Delta J_n^2 \neq f(\mathbf{n})$




[1] J. Zimba, Electr. J. Theor. Phys. **3**, 143 (2006).

**Separability criteria:**

(I) Necessary criterion: $\rho^{T_A} > 0$ (PPT)

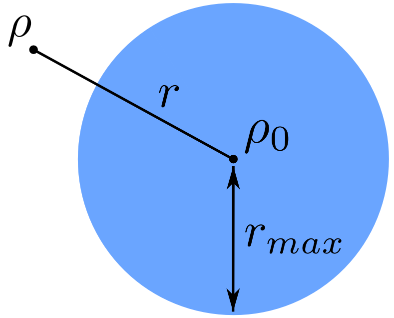
(II) Sufficient criterion: Positive *P* function for all angles $\theta,\varphi$

$$\rho = \frac{2j+1}{4\pi}\int P(\theta,\varphi)|\theta,\varphi\rangle\langle\theta,\varphi| \qquad |\theta,\varphi\rangle \equiv \text{Coherent state}$$

(III) Sufficient criterion: $\rho \in$ ball of absolutely separable states

Absolutely separable state:

$\rho$ such that $\rho' = U\rho U^\dagger$ is separable for any $U$




$$r_{max} \sim j^{-\frac{3}{4}}\, 2^{-(2j+1)}$$

(III) Distance $r = \sqrt{R(\rho) - R(\rho_0)} < r_{max}$

[2] O. Giraud, P. Braun and D. Braun Phys. Rev. A **78**, 042112 (2008).

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# Purity loss

**Isotropic depolarisation**

HOAP states are the most superdecoherent states: they lead to the minimal purity at any time

**Superdecoherence:** $\dot{R}(\rho) = -2\gamma R(\rho)N - 2\gamma[R(\rho) - R(\rho_{N-1})]N^2$

Separable state $\Longrightarrow$ $R(\rho) \leq R(\rho_{N-1})$ $\Longrightarrow$ Superdecoherence cannot occur without entanglement

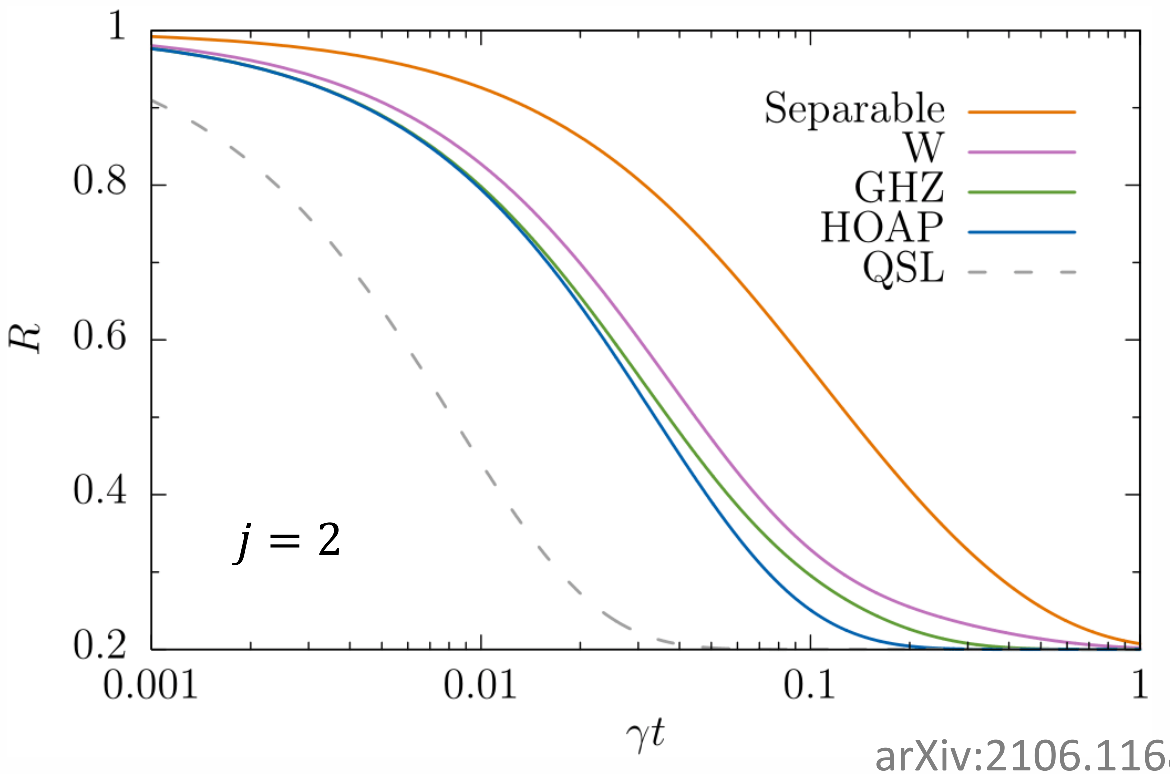
**Depolarisation rate of a pure state:**

$$\dot{R}_{|\psi_0\rangle} = -2\gamma\left(N + \frac{N^2}{2}\mathcal{A}_1\right)$$

with anticoherence measure : $\mathcal{A}_1 = -\frac{1}{j^2}\,|\langle\psi_0|\mathbf{J}|\psi_0\rangle|^2$

**Quantum speed limit (QSL):**

$$R(t) \geq \frac{1}{N+1} + \left(1 - \frac{1}{N+1}\right)e^{-\gamma N(N+1)(N+2)t} \quad [3]$$




[3] R. Uzdin and R. Kosloff, EPL **115**, 40003 (2016).

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# Entanglement dynamics

**Relevant times:**

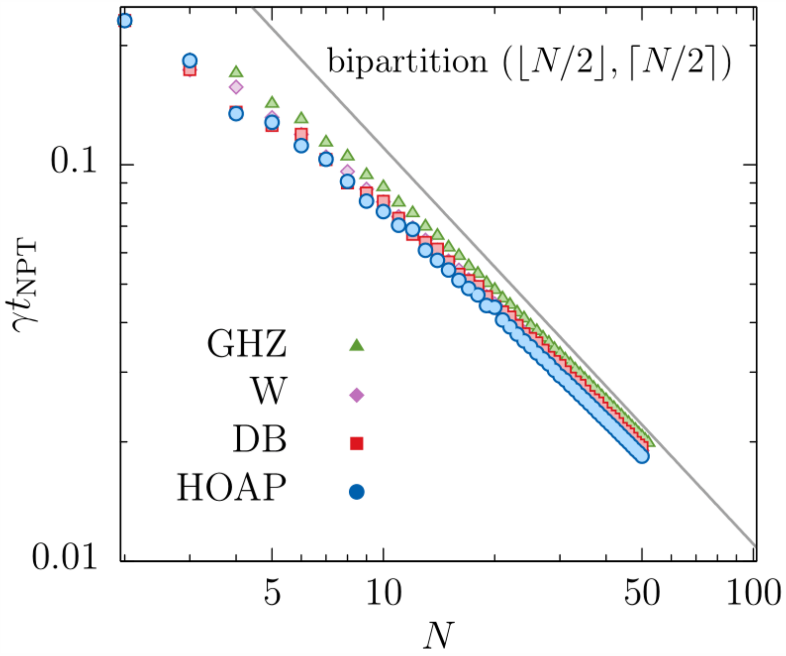
$t_{EST}$: time at which the state becomes separable

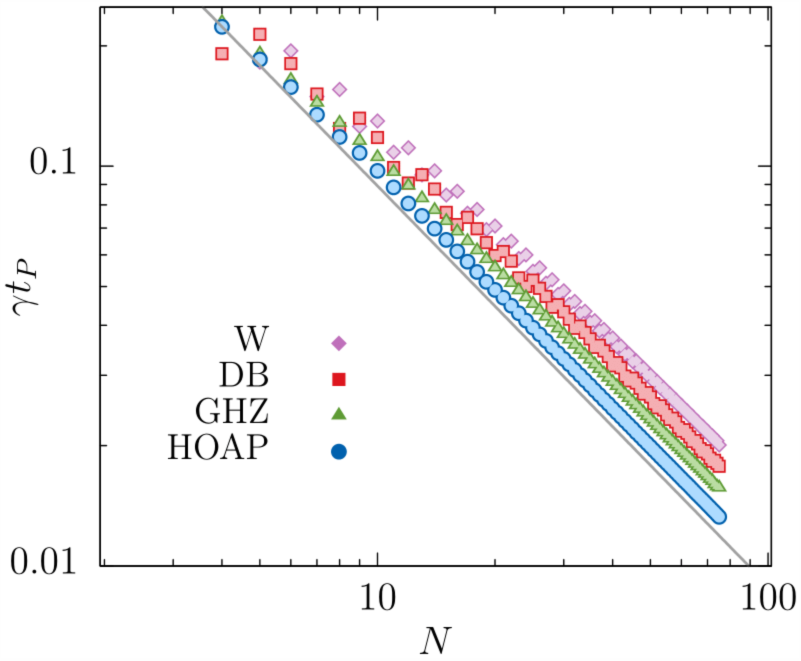
$t_{NPT}/t_P/\, t_{r_{max}}$ : times at which the separability criteria (I)/(II)/(III) are fullfilled

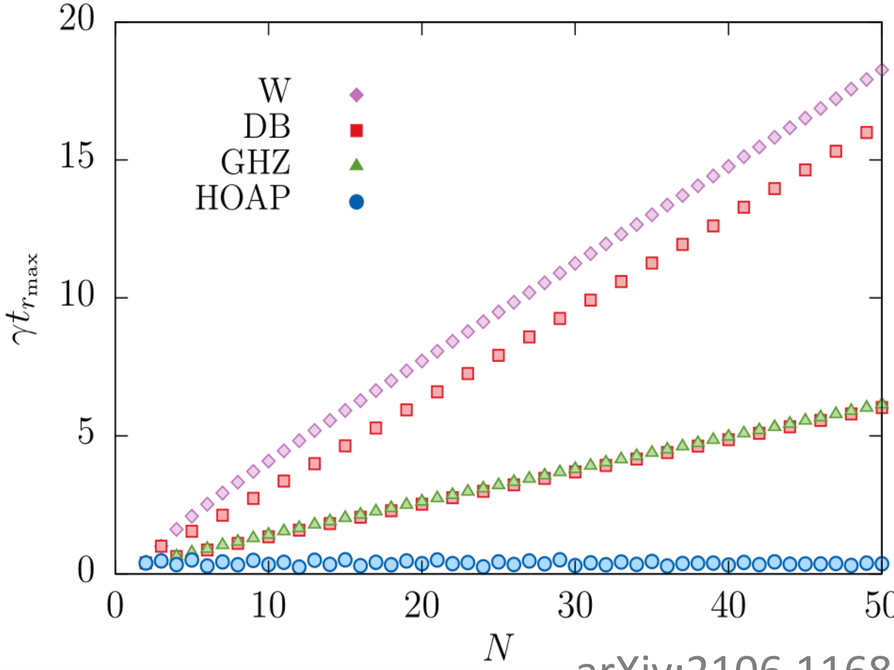
**Results:**

$$t_{NPT} \leq t_{EST} \leq t_P \qquad \left.\begin{array}{l} t_{NPT} \sim 1/N \\ t_P \sim 1/N \end{array}\right\} \implies t_{EST} \sim 1/N$$

$t_{r_{max}}$ is independent of the number of qubits for HOAP states

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# Anisotropic depolarisation

## Anisotropic depolarisation

$$\gamma_{\perp z} = \gamma_x = \gamma_y \neq \gamma_z$$

HOAP states do no longer minimize purity at any time

## Initial purity loss rate: $\dot{R}_{|\psi_0\rangle} = -4\left[\gamma_z\, \Delta J_z{}^2 + \gamma_{\perp z}\left(\Delta J_x{}^2 + \Delta J_y{}^2\right)\right]$

Extremal states at short times :

$\gamma_z > \gamma_{\perp z}$ GHZ

$\gamma_z < \gamma_{\perp z}$ Dicke balanced

An appropriate squeezing can reduce decoherence on short time scales

## Optimal purity loss at any time:

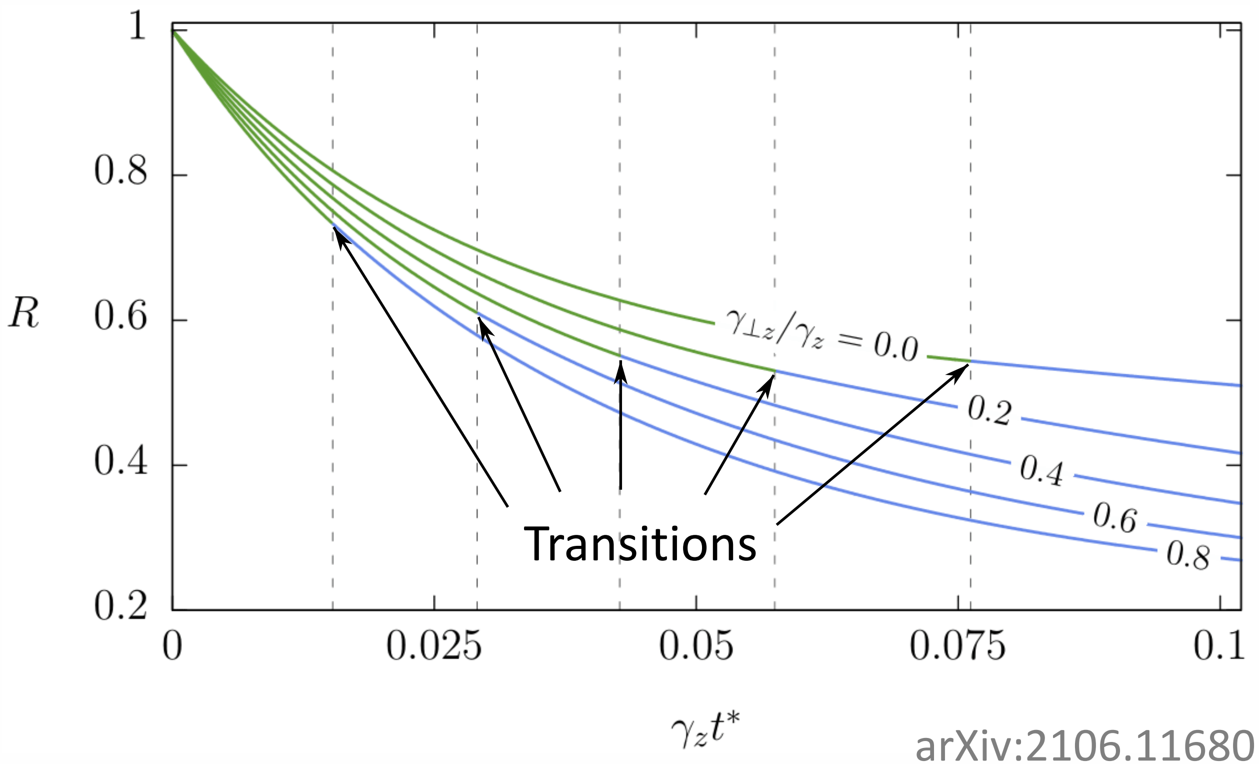
N= 2: $\gamma_z > \gamma_{\perp z}$ GHZ

$\gamma_z < \gamma_{\perp z}$ Dicke balanced

N= 4: Short times : GHZ or Dicke balanced

Long times : $|\mu\rangle = \mathcal{N}\left(|2{,}2\rangle + \mu|2{,}0\rangle + |2,-2\rangle\right)$

with $\mu = i\sqrt{\frac{7\left(e^{8(4\gamma_{\perp z}-3\gamma_z)t^*} - e^{8\gamma_z t^*}\right)}{4\, e^{4(7\gamma_{\perp z}+2\gamma_z)t^*} - 7\, e^{24\gamma_{\perp z}t^*} + 3\, e^{8\gamma_z t^*}} + 2}$

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# Conclusion and perspectives

**Conclusion :**

- Entanglement is a necessary condition for (isotropic) superdecoherence
- Direct link between isotropic depolarization rate and anticoherence measure for pure states
- The time at which entanglement is lost is inversely proportional to the quantum spin number
- Squeezing can be used to reduce decoherence of a spin system

**Perspectives :**

- Condition of superdecoherence in anisotropic depolarization ?
- Experimental investigation of large spin depolarization (e.g. with Dysprosium [4])
- Usefulness of anticoherent states for quantum sensing [5] or cryptography ?

[4] T. Satoor, A. Fabre, J.-B. Bouhiron, A. Evrard,R. Lopes, and S. Nascimbene, arXiv:2104.14389.

[5] F. Gebbia *et al.*, Phys. Rev. A **101**, 032112 (2020).

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