Inverse-problem-based algorithm for sparse reconstruction of Terahertz off-axis holograms

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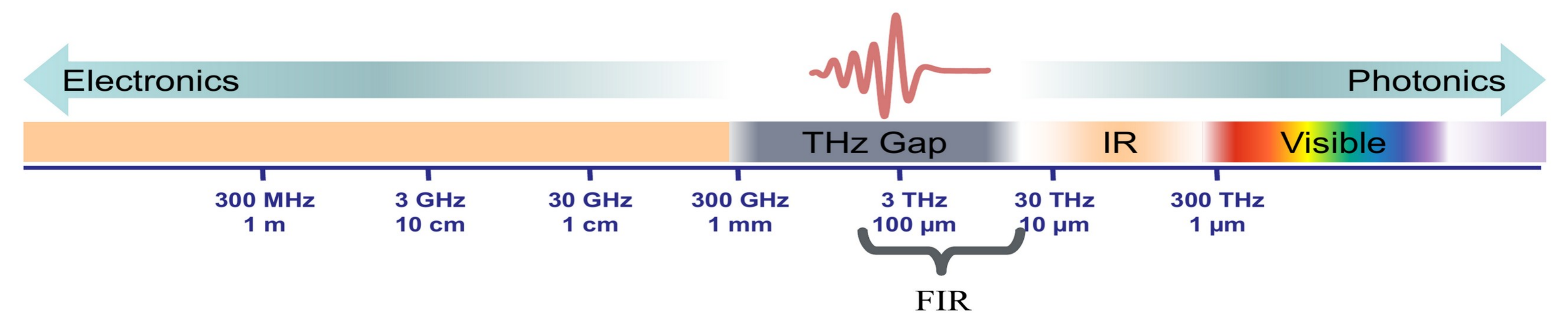
Digital Holography and Three-Dimensional Imaging - July 19 2021

Overview

Terahertz radiation

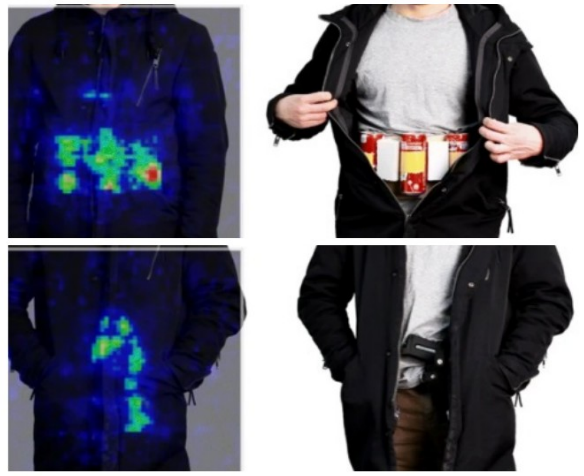
Terahertz (THz) wave range
- Between microwave and infrared
- Frequency: 3-10 THz (1 THz = 1012 Hz)
- Wavelength: 30-1000 µm

Unique properties
- Penetration of non-polar materials
- Ionization free
- Water absorption
- Spectral fingerprint



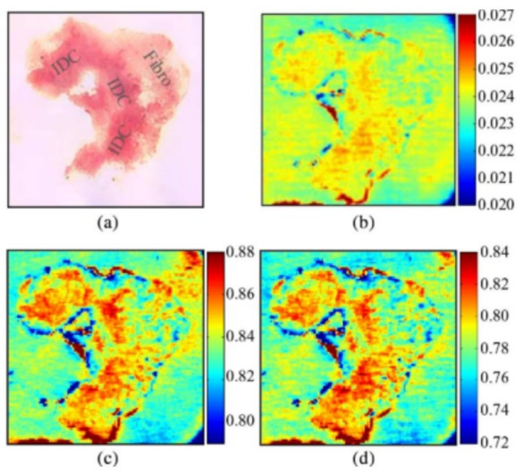
Applications

Security



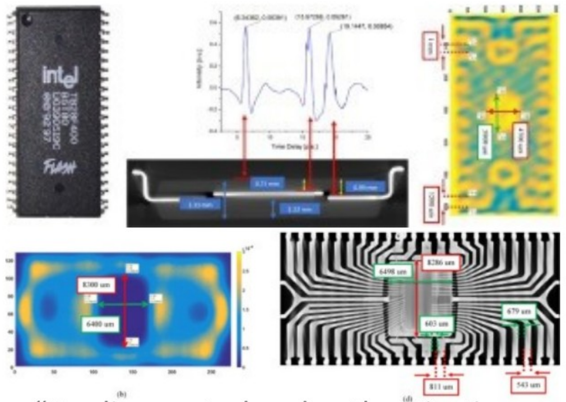
"New Real-Time Sub-Terahertz Security Body Scanner" by Tzydynzhapov, G. *et al.* 2020

Biomedical



"Terahertz Imaging of Excised Breast Tumor Tissue on Paraffin Sections" by T. C. Bowman et al., 2015

Quality control

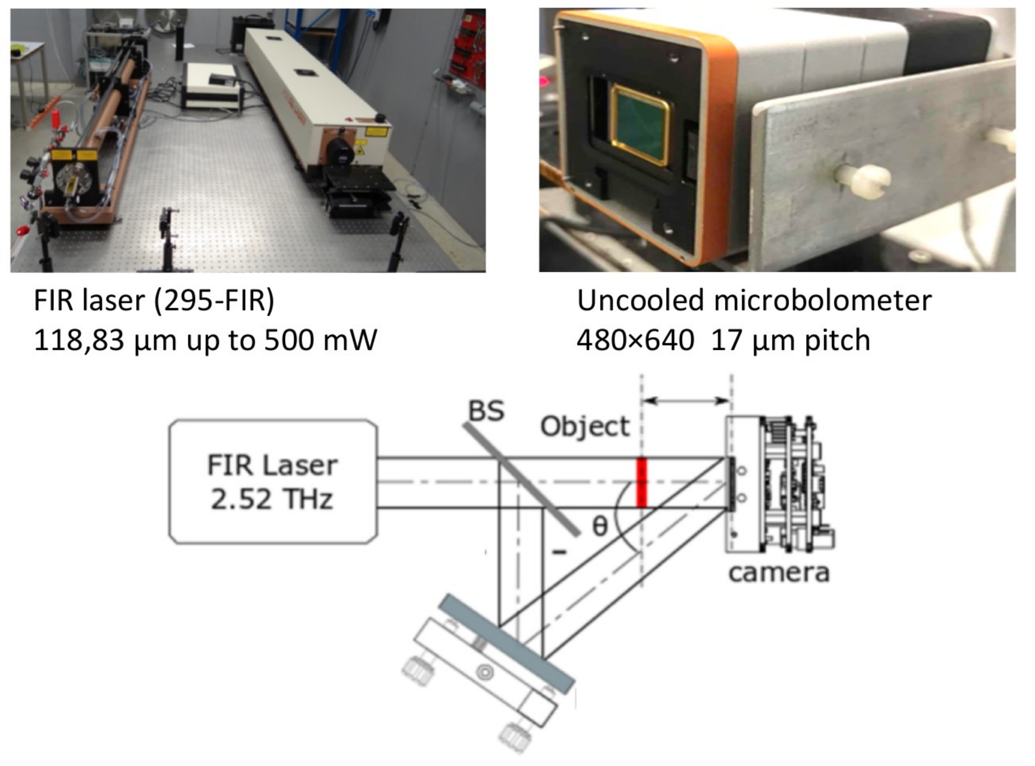


"Quality control and authentication of packaged integrated circuits using enhanced-spatial-resolution terahertz time-domain spectroscopy and imaging" by Kiarash Ahi et al., 2018

Off-axis digital holography in THz

Off-axis digital holography (DH)
- Coherent lensless imaging
- Migration from the visible to the THz band

Acquisition setup ([1])



FIR laser (295-FIR)
118,83 µm up to 500 mW

Uncooled microbolometer
480×640 17 µm pitch

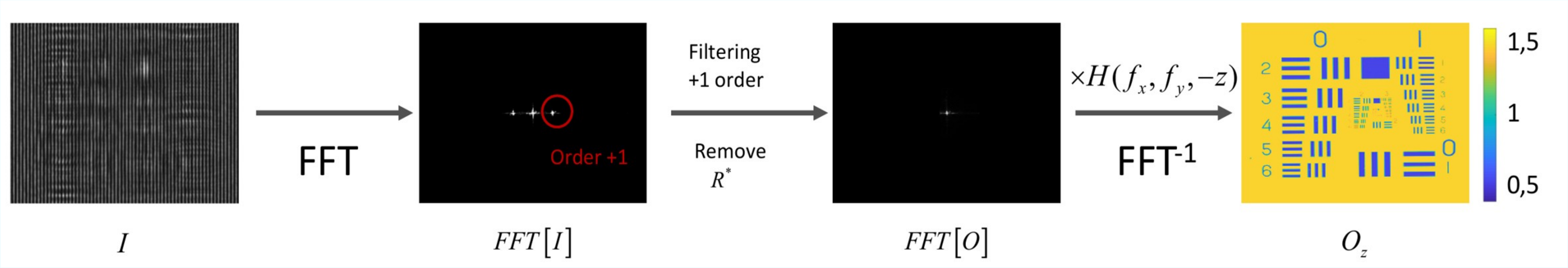
Particular problems
- Low recording distance → unwanted diffraction fringes → reference wave less uniform
- Light field truncation → border effect
- Cameras used for THz: low performance → low resolution

Difficulties of reconstruction

[1] Y. Zhao, M. Kirkove, and M. P. Georges, "Inverse-problem based algorithm for THz off-axis digital holography reconstruction", in Imaging and Applied Optics Congress, The Optical Society (Optical Society of America, 2020), paper HF4G.6.

Image reconstruction in THz off-axis DH: direct methods

Reconstruction by standard direct methods



Limitations

- Border effect
- Intolerant to noise
- Intolerant to sub-sampling
- No consideration of the non-uniformity of the reference beam

Image reconstruction in THz off-axis DH: inverse-problem approach

Inverse-problem-based (IP-based) approach for deconvolution problems

Measures
Equation of the forward model

→ Estimate the original object distribution

Forward model of Thz off-axis DH

$$\boldsymbol{y} = |\boldsymbol{A}_d \boldsymbol{\psi} + \boldsymbol{\alpha} \boldsymbol{r}|^2 + \boldsymbol{n} \qquad (1)$$

where

- $\boldsymbol{y}$ = measures
- $\boldsymbol{\psi}$ = object field (amplitude and phase)
- $\boldsymbol{A}_d$ = propagation function at distance *d*
- $\boldsymbol{n}$ = additive noise
- $\boldsymbol{r}$ = normalized reference beam
- $\boldsymbol{\alpha}$ = relative intensity of the reference beam (non-uniform)

- Unknown = $\boldsymbol{\psi}$
- Non-uniformity of the reference beam → additional unknown = $\boldsymbol{\alpha}$

↓

Estimate $\boldsymbol{\psi}$ and $\boldsymbol{\alpha}$

Advantages

- ✓ Tolerant to noise
- ✓ (Tolerant to sub-sampling)
- ✓ Consideration of the non-uniformity of the reference beam

New IP-based method for image reconstruction in THz off-axis DH

Ill-posed problem

- Indeterminate problem
  # of unknowns ($\boldsymbol{\psi}$ and $\boldsymbol{\alpha}$ in each pixel) > # of measurements ($\boldsymbol{y}$ in each pixel)
- Mathematical properties of $\boldsymbol{A}_d$ → several solutions $\boldsymbol{\psi}$ and $\boldsymbol{\alpha}$ compatible with $\boldsymbol{y}$

Indeterminate problem → difficulties to separate $\boldsymbol{\psi}$ from $\boldsymbol{\alpha}$
Ill-posed problem → regularization required

Regularization

Sparse solutions (as in [2], [3]) → regularization of solutions $\boldsymbol{\psi}$ and $\boldsymbol{\alpha}$ in the wavelet domain
(minimization of the l_1-norm of the wavelet coefficients)

Formulation of the reconstruction problem

Expression of solutions in the wavelet domain

Wavelet coefficients of solutions $\boldsymbol{\psi}$ and $\boldsymbol{\alpha}$: $\boldsymbol{c}_\psi$, $\boldsymbol{c}_\alpha$

Matrices for fast discrete wavelet transforms (DWT) of $\boldsymbol{\psi}$ and $\boldsymbol{\alpha}$: $\boldsymbol{W}_\psi$, $\boldsymbol{W}_\alpha$

Noise-free model:

$$\boldsymbol{m} = |\boldsymbol{A}_d \boldsymbol{\psi} + \boldsymbol{\alpha} \boldsymbol{r}|^2 = |\boldsymbol{A}_d \boldsymbol{W}_\psi^{-1} \boldsymbol{c}_\psi + \boldsymbol{W}_\alpha^{-1} \boldsymbol{c}_\alpha \boldsymbol{r}|^2 \tag{2}$$

[2] S. Bettens, H. Yan, D. Blinder, H. Ottevaere, C. Schretter, and P. Schelkens, “Studies on the sparsifying operator in compressive digital holography,” Opt. Express 25, 18656–18676 (2017)

[3] C. Schretter, D. Blinder, S. Bettens, H. Ottevaere, and P. Schelkens, “Regularized non-convex image reconstruction in digital holographic microscopy,” Opt. Express 25, 16491–16508 (2017)

New IP-based method for image reconstruction in THz off-axis DH

Formulation of the reconstruction problem

Noise-free model:

$$\boldsymbol{m} = |\boldsymbol{A}_d \boldsymbol{\psi} + \boldsymbol{\alpha} \boldsymbol{r}|^2 = |\boldsymbol{A}_d \boldsymbol{W}_\psi^{-1} \boldsymbol{c}_\psi + \boldsymbol{W}_\alpha^{-1} \boldsymbol{c}_\alpha \boldsymbol{r}|^2 \tag{2}$$

Data-fidelity term:

$$D(\boldsymbol{c}_\psi, \boldsymbol{c}_\alpha) = \frac{1}{4} \|\boldsymbol{y} - \boldsymbol{m}\|_2^2 = \frac{1}{4} \|\boldsymbol{y} - |\boldsymbol{A}_d \boldsymbol{W}_\psi^{-1} \boldsymbol{c}_\psi + \boldsymbol{W}_\alpha^{-1} \boldsymbol{c}_\alpha \boldsymbol{r}|^2\|_2^2 \tag{3}$$

Minimization of:

$$D(\boldsymbol{c}_\psi, \boldsymbol{c}_\alpha), \lambda_\psi \|\boldsymbol{c}_\psi\|_1, \lambda_\alpha \|\boldsymbol{c}_\alpha\|_1 \tag{4}$$

Reconstruction problem:

$$(\tilde{\boldsymbol{c}}_\psi, \tilde{\boldsymbol{c}}_\alpha) = \underset{\boldsymbol{c}_\psi, \boldsymbol{c}_\alpha}{\operatorname{argmin}} D(\boldsymbol{c}_\psi, \boldsymbol{c}_\alpha) + \lambda_\psi \|\boldsymbol{c}_\psi\|_1 + \lambda_\alpha \|\boldsymbol{c}_\alpha\|_1 \tag{5}$$

Algorithm

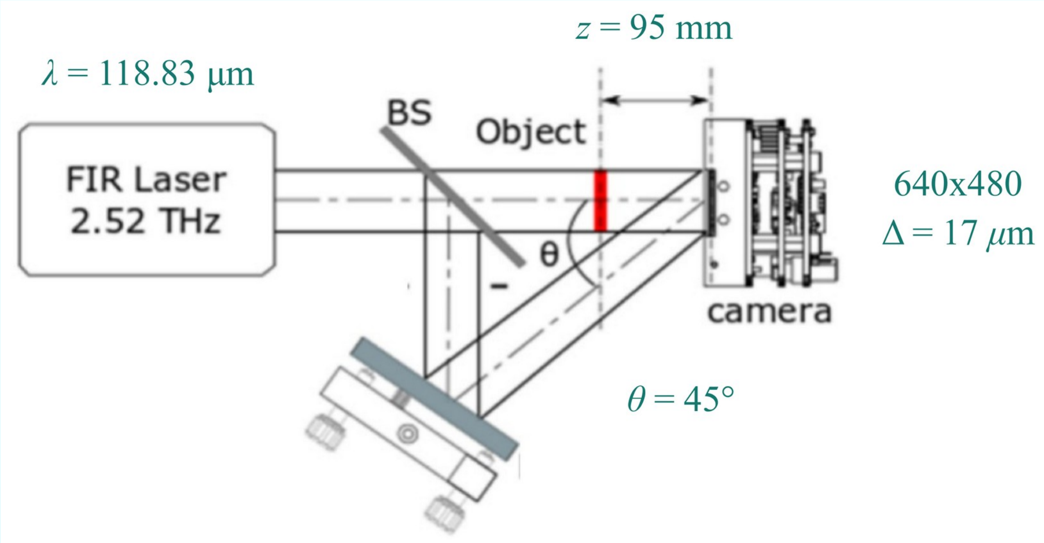
- Wavelet filter: (CDF) 9/7 wavelet
- Based on an alternating direction method of multipliers (ADMM) based framework
- Using 2 projection operators and 2 soft thresholding operators

Parameters

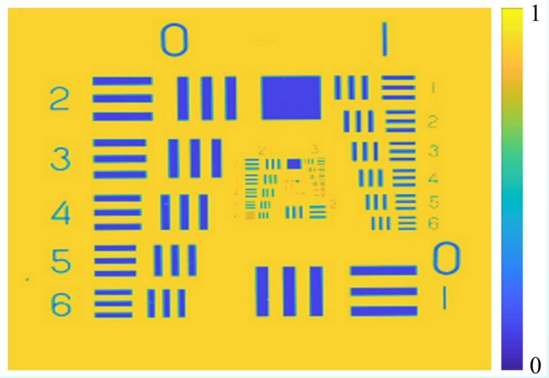
- λ_ψ and λ_α: scalar parameters for regularizations strengths
- σ: standard deviation of the noise
- l_ψ and l_α: number of wavelet decomposition levels
- n_i: number of iterations

Synthetic data

Acquisition parameters

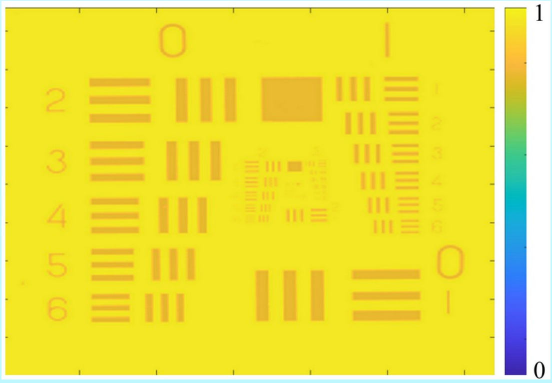


Object field

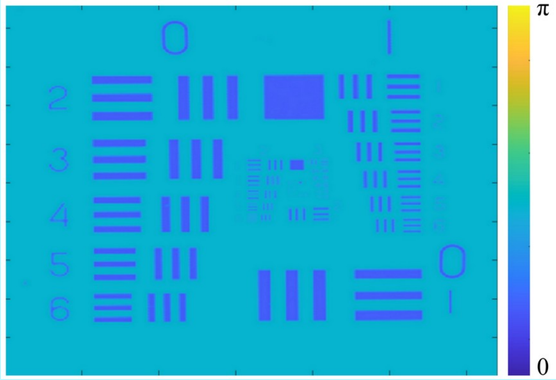


Highly transparent phase object
Amplitude: 0.8-1

Object field (ψ, amplitude)



Object field (ψ, phase)

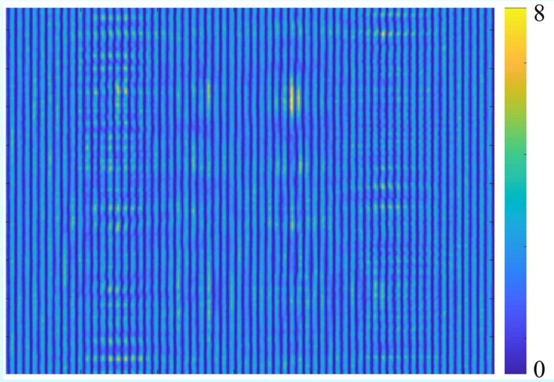


Synthetic data

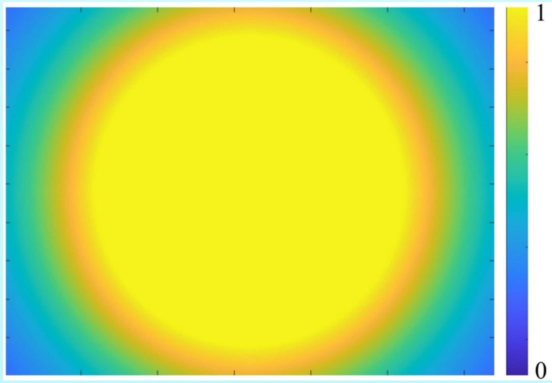
Reference field amplitude (α)

Constant: 0.9505

Measures (y)

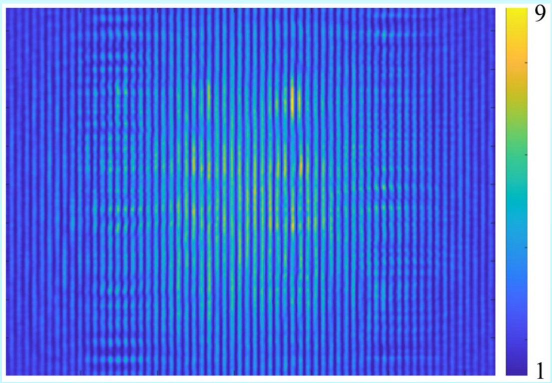


Reference field amplitude (α)



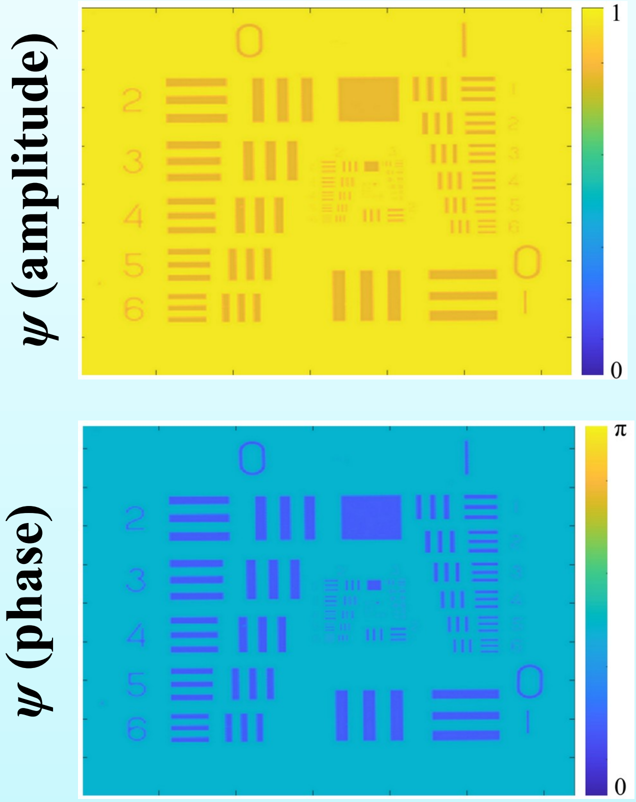
Variable: Gaussian function (mean ~ 0.9505)

Measures (y)



Results on synthetic data

Simulation



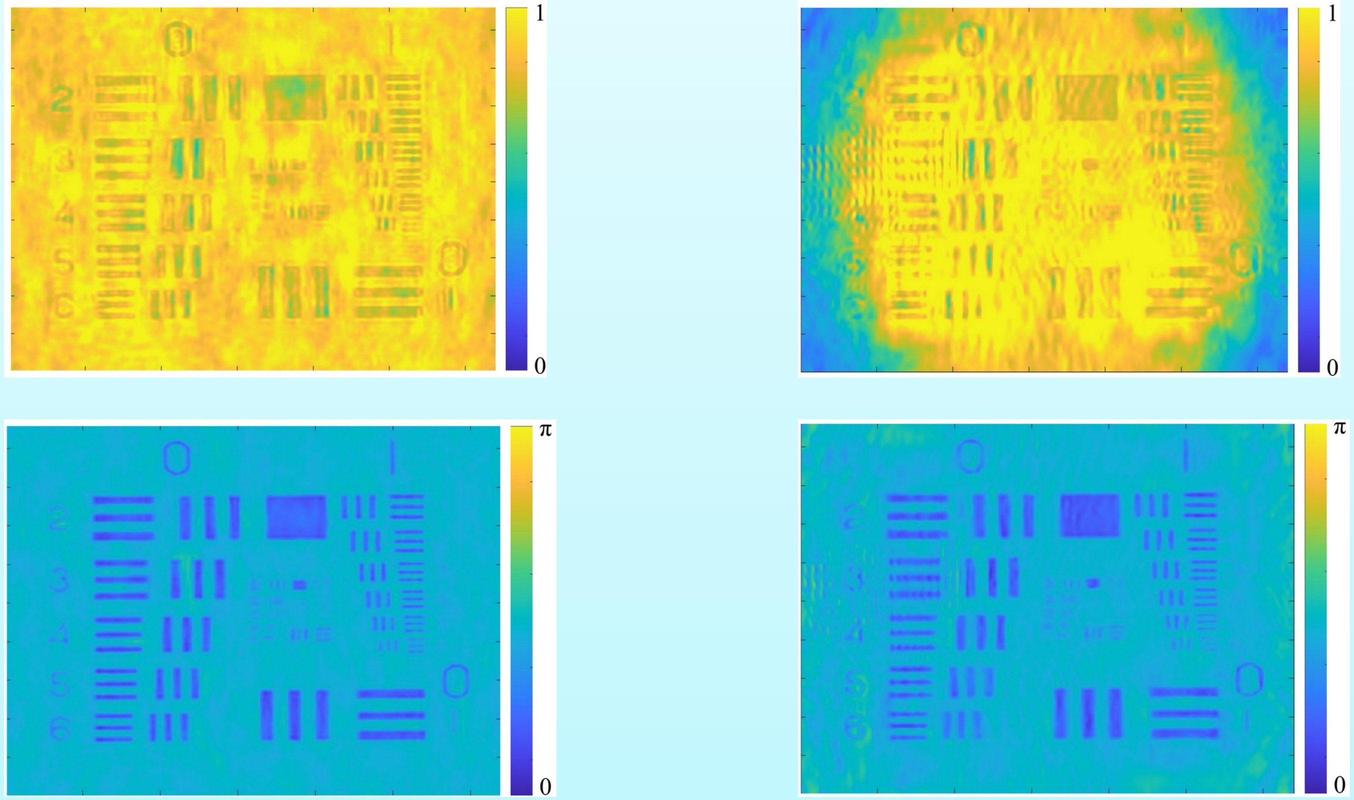
Processing parameters

$\lambda_\psi = 0.12$, $\lambda_\alpha = 0.8$, $l_\psi = 5$, $l_\alpha = 5$
$\sigma = 0.025$, $n_i = 180$

$\lambda_\psi = 0.125$, $\lambda_\alpha = 0.25$, $l_\psi = 5$, $l_\alpha = 4$
$\sigma = 0.075$, $n_i = 255$

Reconstruction

Original α: constant **Original α: variable**



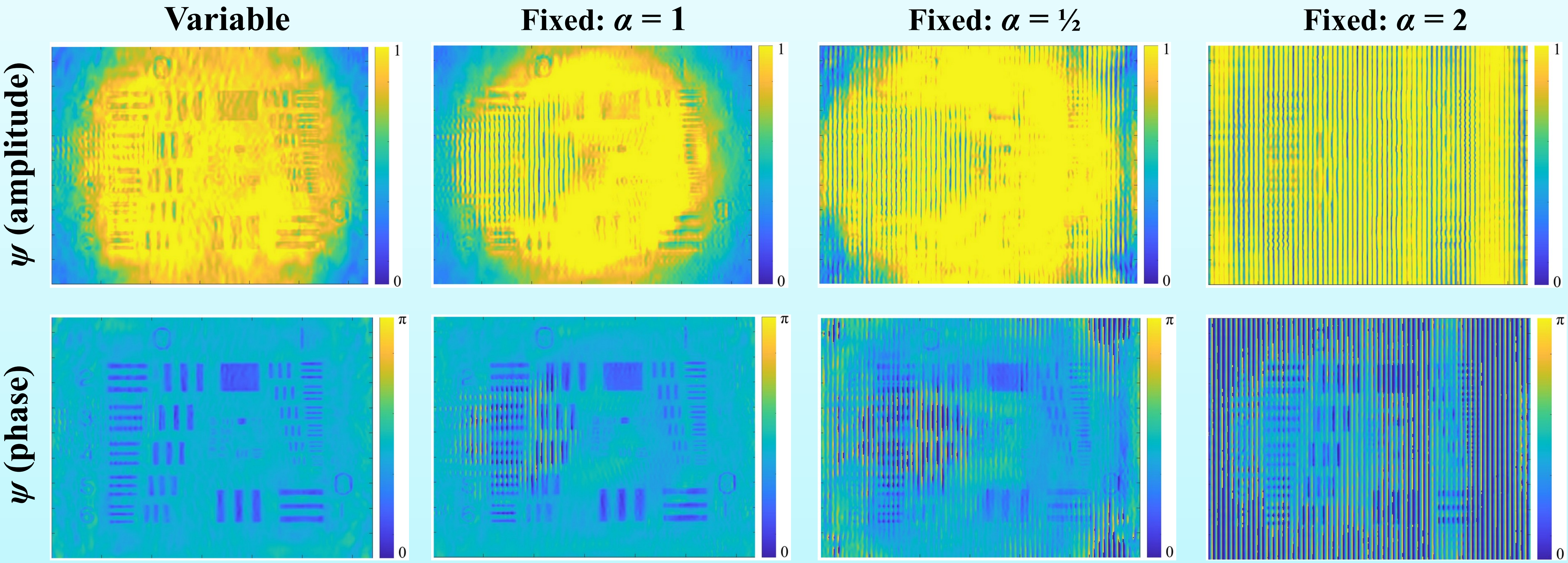
Signal-to-noise ratios (SNRs)

$SNR_{\psi\_amp} = 22.3$, $SNR_{\psi\_ph} = 24.8$
$SNR_\alpha = 21.8$

$SNR_{\psi\_amp} = 11.2$, $SNR_{\psi\_ph} = 21.5$
$SNR_\alpha = 11.6$

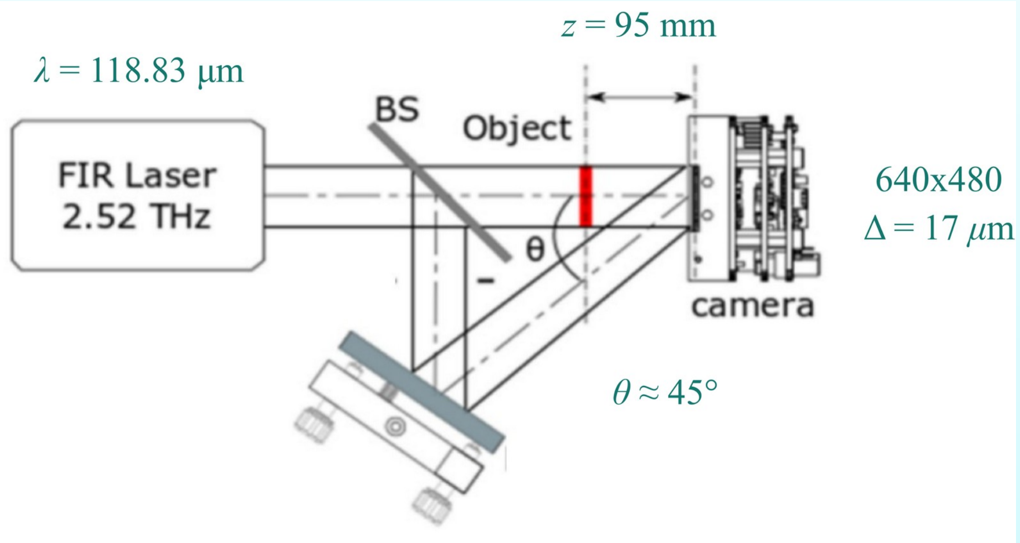
Results on synthetic data

Algorithm: variability of α

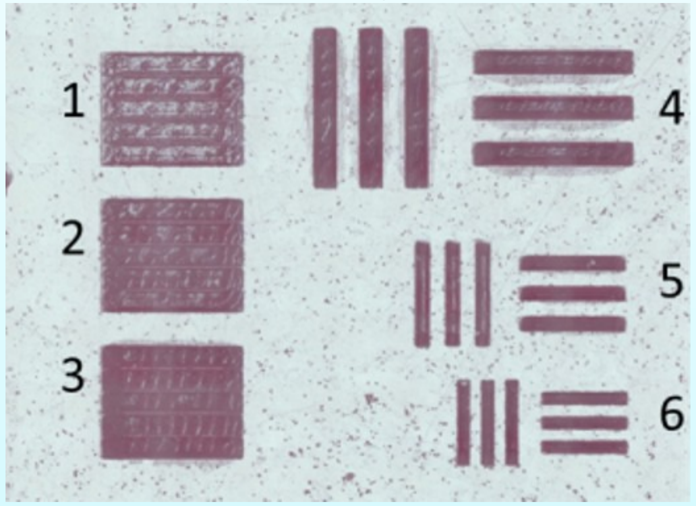


Experimental data

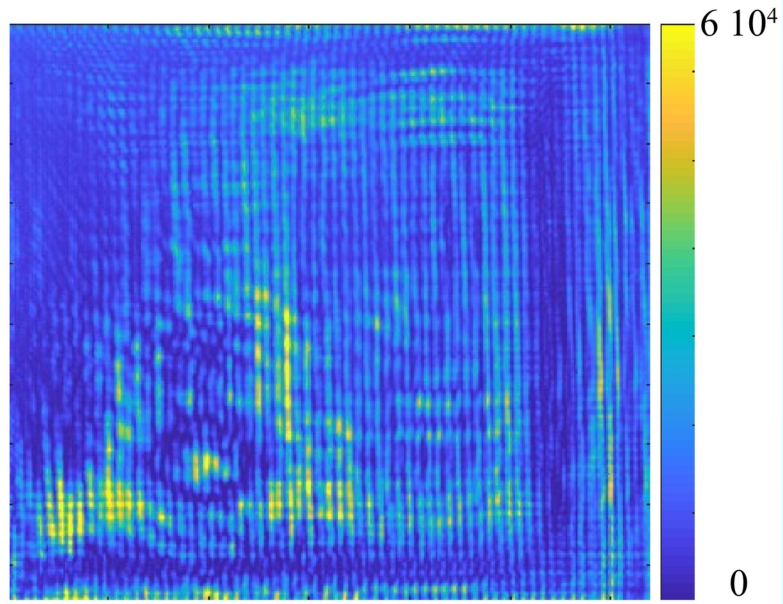
Acquisition setup



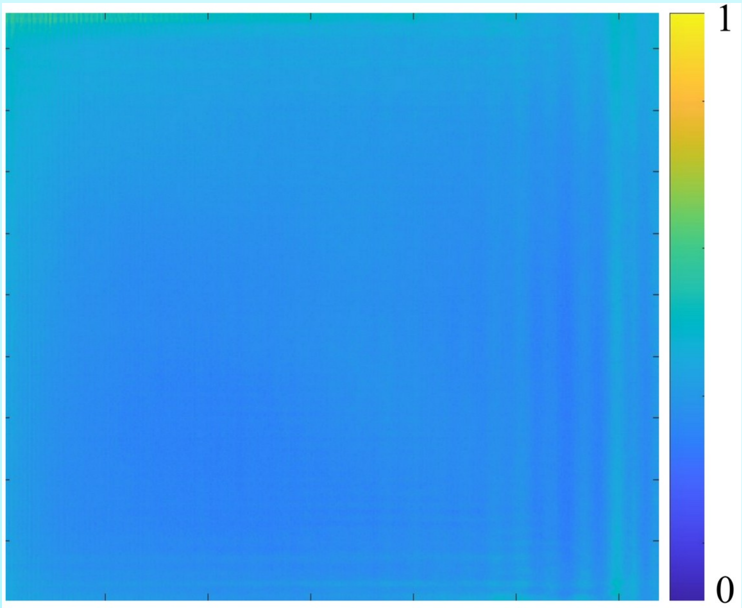
Sample (polypropylene, $n = 1.49$)



Hologram (y)



Reference amplitude



Results on experimental data

Processing parameters

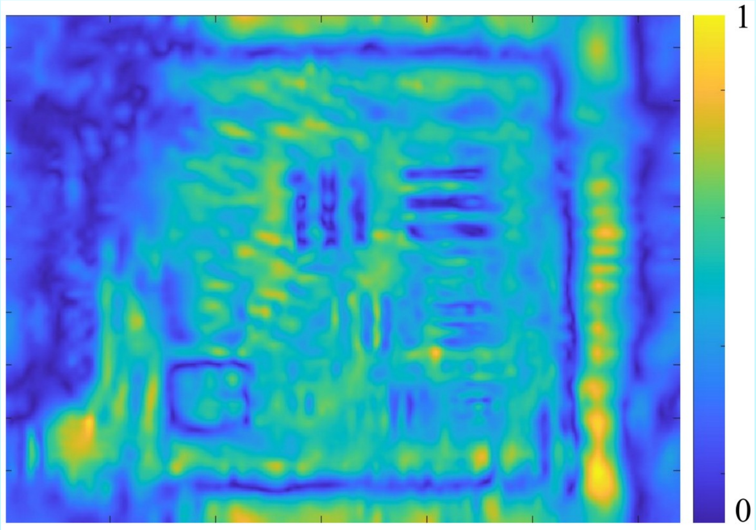
$\lambda_\psi = 0.45, \lambda_\alpha = 0.125$

$l_\psi = 5, l_\alpha = 4$

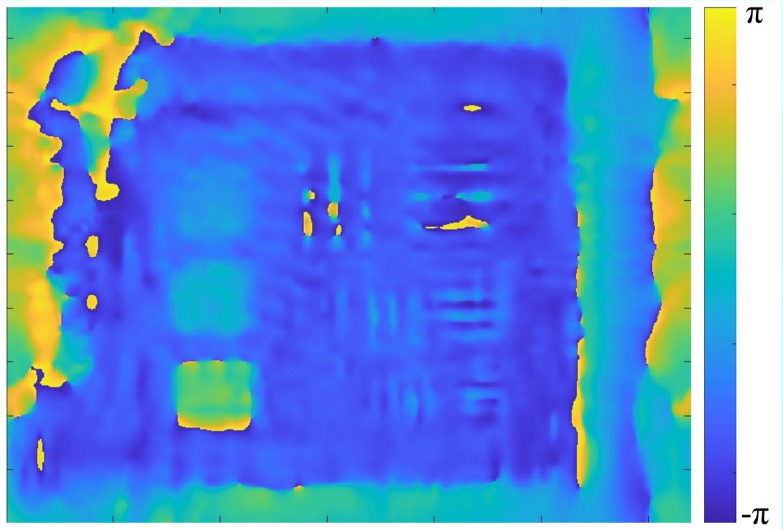
$\sigma = 0.08875$

$n_i = 500$

ψ (amplitude)



ψ (phase)



Conclusions of current work

Advantages of the method

- Some benefits of the IP approach
  - ✓ No border effect
  - ✓ Tolerance to noise
  - ✓ Consideration of the non-uniformity of the reference beam
- Benefit due to wavelet-based regularization
  - ✓ Regularization adapted with respect to the awaited resolution
- Additional advantages
  - ✓ Acceptable convergence
  - ✓ Parameter tuning reliable

Limitations of the method

- ✗ Difficulties to separate the solutions $\boldsymbol{\psi}$ and $\boldsymbol{\alpha}$
- ✗ Not tolerant to down-sampling

Future work

Steps of future work

- Assessment of the performances of the method
- Consideration of down-sampling
- Improvement of separation between the solutions ψ and α by consideration of additional measures
- Study of the potential of the method in other spectral bands

Thank you for your attention!

Questions: M.Kirkove@uliege.be