Determination of the Mass of Nearby Stars from Astrometric Microlensing Observations

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Abstract: The possibility of determining, with a precision of 10%, the mass of bright, nearby stars by means of astrometric microlensing observations is investigated in the context of the future ground-based and spaceborne high precision astrometric instruments, such as the VLT interferometer, GAIA and SIM.

1 Introduction

When a nearby star passes in front of a distant background star, it acts as a (gravitational) lens, by deflecting the light rays and producing two distinct images of the source (see Fig. 1). Because the angular separation between the two images is really tiny (hundredth to micro arcsec), this is called microlensing. The properties of the background star are then affected in two different ways: photometric variations and astrometric shifts. The former effect has been extensively used by several teams to search for dark compact objects in the disk and the halo of our Galaxy (EROS: Derue *et al.* 1999; MACHO: Alcock *et al.* 2000, OGLE: Paczyński *et al.* 1994). The latter effect has been discussed theoretically by several authors (Hosokawa *et al.* 1993, Miralda-Escudé 1996, Paczyński 1998, Gould 2000 and many others), but no systematic observational campaign has started yet, simply because of the limited astrometric accuracy of current instruments.

The next generation of interferometers and space missions will drastically improve the astrometric performances. With PRIMA starting operations in 2003, the astrometry of pointlike sources with the VLTI will reach a precision ϕ of 10 μarsec, down to $K = 20$ (Delplancke et al. 2000); during its 5 years program starting in 2009, the Space Interferometry Mission (SIM)[1] will yield individual astrometric measurements of stars down to $V = 20$ with $\phi = 7.5$ μarsec; finally, GAIA (launch around 2012) will provide a final astrometry from 3 μarsec at $V = 10$ to 140 μarsec at $V = 20$ (de Boer et al. 2000).

In this paper, we discuss the feasibility and the expected accuracies of stellar mass determinations by astrometric microlensing observations. We concentrate on bright stars (which can

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[1]For more details on SIM, see the URL address: *http://sim.jpl.nasa.gov*

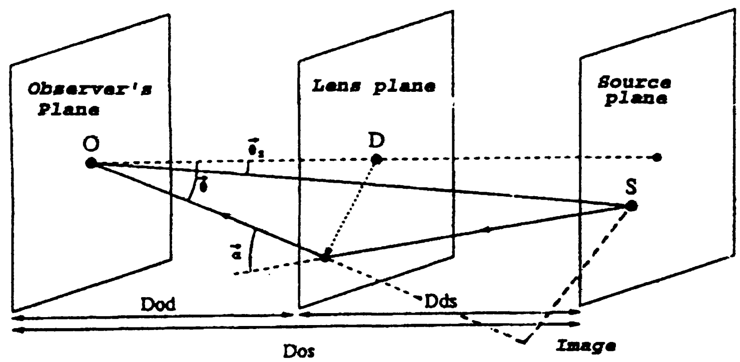

Figure 1: Geometrical configuration of a microlensing system.

be used to phase the interferometer) with important proper motions, so that astrometric shifts of a larger number of lensed background sources can be detected. The latter are assumed to be distant, so that their parallax and proper motion are negligible. We here follow a model independent approach, relying on the absolute astrometry of the bright lens. In Section 2, we present the general principle to determine a stellar mass with astrometric microlensing; in Section 3 we discuss the results and we conclude in the last section.

2 General principle

Figure 1 shows the geometry of gravitational (micro)lensing. The so-called lens equation links the position of the unseen source $\vec{\theta_s}$ to that of the observed image $\vec{\theta}$ and to the deflection angle $\vec{\alpha}(\vec{\theta})$:

$$\vec{\theta_s} = \vec{\theta} - \vec{\alpha}(\vec{\theta})\frac{D_{DS}}{D_{OS}}, \tag{1}$$

where D_{DS} and D_{OS} are the deflector-source and the observer-source distances. A star is considered as a pointlike object with mass M, so that the deflection angle is simply (see e.g. Refsdal and Surdej 1994):

$$\vec{\alpha} = \frac{4GM}{c^2 D_{OD}\theta^2}\vec{\theta}. \tag{2}$$

Figure 2 shows the plane of the sky, where the North and East axes are attached to the first observed position of the lens, L_1. Given Eqs. (1) and (2), the unknown position of the source S $(\theta_{S_x}, \theta_{S_y})$ is related to the $i^{\rm th}$ observed position of the main lensed image A $(\theta_{A_{xi}}, \theta_{A_{yi}})$ and to the corresponding $i^{\rm th}$ observed position of the lens L $(\theta_{\mu_{xi}}, \theta_{\mu_{yi}})$ by the relation:

$$\left\{ \begin{array}{lcl} \theta_{S_x} & = & \theta_{\mu_{xi}} + (1 - \frac{\theta_E^2}{\theta_{A_{xi}}^2 + \theta_{A_{yi}}^2})\theta_{A_{xi}} \\ \theta_{S_y} & = & \theta_{\mu_{yi}} + (1 - \frac{\theta_E^2}{\theta_{A_{xi}}^2 + \theta_{A_{yi}}^2})\theta_{A_{yi}} \end{array} \right. , \tag{3}$$

where $\theta_E = \sqrt{\frac{4GMD_{DS}}{c^2 D_{OD} D_{OS}}}$ represents the Einstein radius of the lens, which simplifies to $\theta_E \simeq \sqrt{\frac{4GM}{c^2 D_{OD}}}$ in the case of a nearby stellar lens and a distant background source.

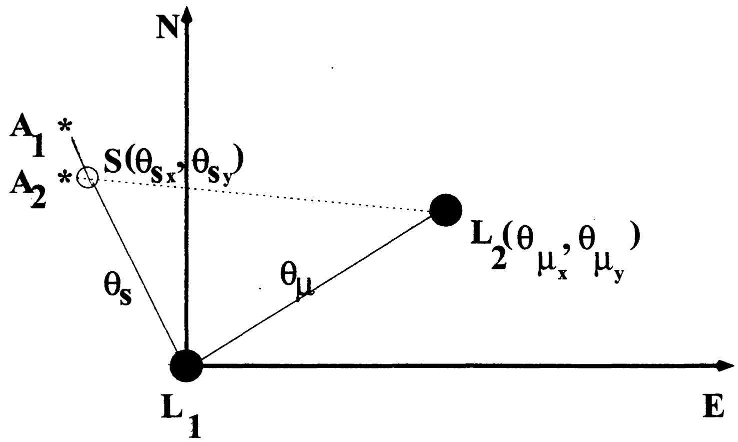

Figure 2: Projection on the plane of the sky of the source S, the lens L at two different positions and the two corresponding (main) lensed images A_1 and A_2.

So, observing at least twice the lens and one background stellar image yields four equations for the three unknowns $(\theta_{S_x}, \theta_{S_y})$ and θ_E; the mass of the lens is derived from the latter, provided the lens distance is known.

3 Results and discussion

The precision on the mass determination will not only depend on the astrometric precision ϕ of the observations but also on the number of observations, the total angular shift of the lens during the observations, the angular distance between the lens and the lensed background source, and, of course, on the number of background sources available in the angular vicinity of the lens. In order to test the ability to retrieve the lens mass, we simulated astrometric datasets for one background star according to different instrumental precisions (see Section 1). We then minimized the chi squared function defined as:

$$\chi^2 = \sum_{i=1}^{n_{obs}} \left([\theta_{S_x}(\theta_{A_{xi}}, \theta_{A_{yi}}, \theta_{\mu_{xi}}, \theta_{\mu_{yi}}, \overline{\theta_E}) - \overline{\theta_{S_x}}]^2 + [\theta_{S_y}(\theta_{A_{xi}}, \theta_{A_{yi}}, \theta_{\mu_{xi}}, \theta_{\mu_{yi}}, \overline{\theta_E}) - \overline{\theta_{S_y}}]^2 \right) , \qquad (4)$$

where the upper bars refer to the unknown parameters and the functions θ_{S_x} and θ_{S_y} are given by Eq. (3).

All our simulations were performed for the case of a 1.5 $M_\odot$ lens located at 10 pc, thus corresponding to $\theta_E = 0.035''$. The uncertainty on the lens distance is not taken into account. The results displayed in Fig. 3 show the fractional error on the mass determination as a function of the angular separation between a unique background source and the lens at first observation. The proper motion of the lens is assumed to be perpendicular to the projected direction along which the source is seen from L_1 (see Fig. 2); this is the worst situation since the shift of the lensed image is then minimal.

In the case of SIM ($\phi \sim 8$ μarcsec), the lower group of curves in Fig. 3A shows the expected accuracy when the total proper motion of the lens is $3''$, while the upper group of curves illustrates the situation when the lens does only move by $0.5''$. It appears that an important proper motion of the lens during the mission is crucial. On the other hand, even

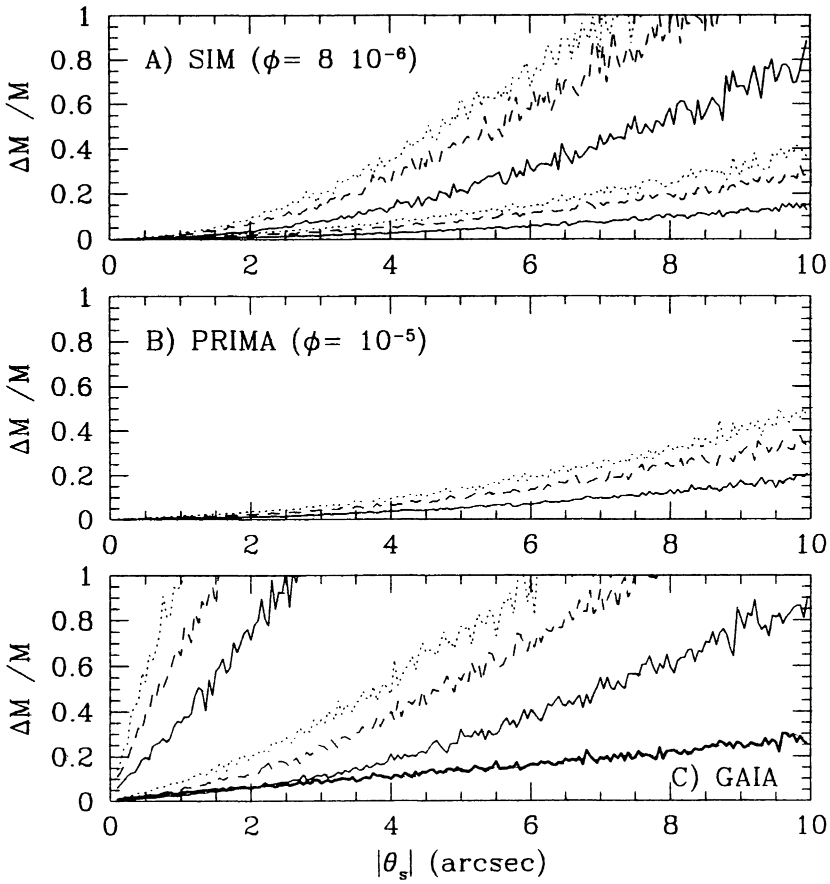

Figure 3: Expected precision on the mass determination of a 1.5 $M_\odot$ star located at a distance of 10 pc, by astrometric microlensing with SIM (top), PRIMA (center) and GAIA (bottom). Dotted, dashed and full lines correspond to 2, 10 and 50 observations of one single star, respectively (see text for details).

if two observations are theoretically sufficient to determine the mass of the lens, repeated observations are necessary to increase the signal to noise. The case of PRIMA at the VLTI ($\phi \sim 10$ μarcsec) is comparable to that of SIM (see Fig. 3B), except that PRIMA works in the infrared. Finally, the case of GAIA is less favourable. We estimated the precision on the *individual* absolute astrometry of the bright lens to be 25 μarcsec while that on the relative astrometry of the faint source is only 1 milli arcsec at $V = 20$ and 70 μarcsec at $V = 15$. The upper-left (resp. central) group of curves in Fig. 3C is computed for a background star with $V = 20$ (resp. $V = 15$). The thick curve corresponds to an extreme proper motion of the lens of 25 arcsec instead of 3 arcsec, and a background star with $V = 15$.

If the proper motion of the lens is 3″, 50 observations made with PRIMA (resp. SIM), can be used to determine the stellar mass with an uncertainty smaller than 10% if one star is located within 7″ (resp. 8″) from the lens. With GAIA, this is possible only if one $V = 20$ star is closer than 0.25″ from the lens, or if one $V = 15$ star is closer than 3″. Alternatively, 40 stars should be observed at 8″ from the lens to reach the same accuracy. Can these requirements be met in practice?

According to the so-called Besançon galactic model (Robin *et al.* 1995) the expected star counts range between 100 000 and 500 000 per square degree at $V = 20$ and between 8000 and

12000 at $V = 15$ in the galactic plane, depending on the longitude. Therefore the probability to find a background star around a bright lens becomes substantial only within an angular vicinity typically larger than 5″ (resp. 20″) at $V = 20$ (resp. $V = 15$). Thus, it is clear that for a given bright lens, SIM will more "systematically" be able to provide a good determination of its mass than GAIA, which will rely on chance alignment with background stars even for a lens with a large proper motion.

On the other hand, IR star counts culminate to $N(K < 20) \sim 1.5\,10^8$ per square degree toward the galactic center (Mamon, private communication), yielding more than 1500 suitable stars to determine the lens mass with PRIMA!...

We may also wonder how many bright stars with large proper motions are known. To answer this question, we looked into the Tycho-2 catalog (Høg et al. 2000) for stars with $\mu \geq 0.4''$/year, located at low galactic latitudes ($|b| \leq 10°$) in order to maximize the number of background stars. The selection resulted in 110 stars with V mag typically brighter than 10; 22 of those stars have a proper motion larger than 1″ per year.

4 Conclusion

Determining the mass of a stellar lens with a precision of 10% is theoretically possible with future high precision astrometric instruments such as PRIMA at the VLTI, GAIA and SIM. However, the radius of the angular zone around the lens, in which a background star must lie to detect its angular shift induced by microlensing, is a function of the astrometric accuracy. It is of the order of 7″ for SIM and PRIMA (V or $K = 20$), but for GAIA, according to our estimates of its accuracy on individual measurements, the angular zone is only 0.25″ at $V = 20$ and 3″ at $V = 15$. According to predicted starcounts in the galactic disk, the *a priori* probability to find a suitable background star in the vicinity of a given bright lens is much closer to 1 for SIM and PRIMA than for GAIA. The expected accuracy on the mass determination of fast lenses with GAIA is closer to $\sim 60\%$.

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