

CLASSIFICATION AND CROSS-SECTION RESISTANCE OF EQUAL-LEG ROLLED ANGLE PROFILES

Marios-Zois Bezas ^{a,b,*}, Jean-François Demonceau ^a, Ioannis Vayas ^b, Jean-Pierre Jaspart ^a

^a *Steel and Composite Construction, UEE Research Unit, Liège University, Belgium*

^b *School of Civil Engineering, Institute of Steel Structures, National Technical University of Athens, Athens, Greece*

Corresponding author at : Université de Liège, Department ArGenCo, Quartier Polytech 1, Allée de la Découverte 9 B52/3, Liège 4000, Belgium.

E-mail addresses: Marios-Zois.Bezas@uliege.be (M.-Z. Bezas), jfdemonceau@uliege.be (J.-F. Demonceau), vastahl@central.ntua.gr (I. Vayas), jean-pierre.jaspart@uliege.be (J.-P. Jaspart).

ABSTARCT

Angle profiles belong to the most common structural steel shapes used in construction. They exhibit specific features that distinguish them from other types of common sections. This paper presents specific rules for the classification of angle profiles as well as formulas for the design cross-section resistances. The rules are based on theoretical-analytical considerations and are subjected to extensive numerical validation. They are written in Eurocode 3 format so as to allow a direct possible inclusion in forthcoming drafts. They cover all cross-section classes and allow a smooth transition between them.

KEYWORDS: CLASSIFICATION -- ANGLE PROFILES -- DESIGN CROSS-SECTION RESISTANCE -- EQUAL-LEG ANGLES -- NUMERICAL SIMULATIONS -- EUROCODE 3

1. Introduction

Angle profiles are among the most common structural steel shapes for construction purposes. Their easy production and transportation, together with an excellent connectivity distinguish them from other profiles. They are available as hot-rolled or cold-formed profiles, as equal or unequal sections depending on the relative length of their legs, in steel grades up to S460, in sizes ranging from small to large, 20 to 300 mm. They are employed either as single or as built-up sections in a back-to-back or star battened configuration. The preferred bolted connection of one leg to gusset plates lead to a most advantageous application as truss or diaphragm members in buildings, bridges or any other structural application and, very importantly, steel towers and masts e.g. for telecommunication or power transmission purposes.

Angles, and especially equal angle profiles considered here, exhibit some properties that distinguish them from other common steel profiles: (i) they are open profiles with very small section constants in both torsion and warping, (ii) they are monosymmetrical sections, (iii) their bending capacity and radius of gyration around the weak axis are substantially lower around strong axis, (iv) their legs are prone to local buckling as external plate elements, (v) their plastic resistances are substantially higher than their elastic ones and (vi) due to the eccentric connection in one leg, they are subjected to some bending in addition to axial force when used as single members.

These features explain that existing design rules for other, mostly doubly symmetric types of sections cannot safely cover angle ones, what inevitably leads to the need for the development of specific design provisions for angle sections. Facing the lack of unified consistent rules for angles, European specifications have adopted a case-by-case approach, embedding individual rules and recommendations in various parts of Eurocode 3. More specifically, EN1993-1-1 [1] provides rules for cross-section classification (classes 1 to 4). EN1993-3-1 [2] presents specific rules for the resistance of angle members used in towers, while EN 1993-1-5 [3] gives rules for buckling resistance of class-4 angle sections prone to local buckling. Another European specification, the CENELEC standard EN 50341-1 [4] provides specific rules for lattice towers used in the field of overhead electrical lines, addressing specific problems linked to such applications; but it also provides specific rules for the verification of the lattice tower and its constituting parts. But for some aspects, the EN 50341-1 design methods for angle sections may diverge from the rules provided in the Eurocodes. In contrast to the European Codes, American Codes have written down in a single document, AISC 2000 [5], all rules concerning angle design.

Extensive research has been carried out to study the behaviour of angle sections. It covers hot-rolled and cold-formed profiles, equal and unequal sections, beams or columns subjected to various types of loading, as well as different connection conditions. Vayas et al. [6] give the inelastic capacity of angle sections to combined axial forces and biaxial bending. Trahair [7] examines angle section beams subjected to uniform eccentric transverse loading and gives the section capacity under combined shear, bending and torsion. Kettler et al. [8] highlight the importance of the end support conditions in their numerical study of rolled angles loaded in compression through bolted connections in one leg and in their comparisons with experimental results and with the provisions

of the European standards. Bezas et al. [9] report on experimental and numerical studies of large rolled angle profiles from high strength steel (HSS) and give an extended literature review on relevant experimental investigations on angles.

In the frame of the European RFCS-funded project ANGELHY [10], existing European specifications on rolled equal angle sections have been reviewed, analytical and numerical studies have been conducted and a complete set of rules covering cross-section classification and cross-section design to individual internal forces and moments, for all ranges of responses (plastic, elastic-plastic or elastic including local buckling) have been developed, duly validated and presented in this paper.

Nomenclature		axis, respectively	
<i>Latin upper-case symbols</i>		<i>Latin lower-case symbols</i>	
A	cross-section area	\bar{b}	appropriate width that is equal to h for angle sections according to EN1993-1-5
A_{eff}	effective area of a cross-section	c	outstand flange width ($c = h - t - r$)
L	length of the member	f_y	yield strength
$M_{el,u}, M_{el,v}$	elastic resistance to bending of the gross cross-section about u-u and v-v axis, respectively	h	width of the cross-section
$M_{pl,u}, M_{pl,v}$	plastic resistance to bending of the gross cross-section about u-u and v-v axis, respectively	i_v	radius of gyration about v-v axis
$M_{ult,u}, M_{ult,v}$	ultimate resistance to bending of the cross-section about u-u and v-v axis, respectively	k_σ	plate buckling coefficient
$M_{u,Rd}, M_{v,Rd}$	design value of the resistance to bending moment about u-u and v-v axis respectively	r	radius of root fillet
$M_{u,Rk}, M_{v,Rk}$	characteristic value of the resistance to bending moment about u-u and v-v axis respectively	t	thickness of the cross-section
$N_{c,Rd}$	design value of the resistance to uniform compression axial force of the cross-section	<i>Greek lower-case symbols</i>	
$N_{c,Rk}$	characteristic value of the resistance to compression axial force of the cross-section	γ_{M0}	partial factor for resistance of cross-sections that equals 1,0 as recommended by EN1993-1-1
N_{pl}	plastic resistance to axial force of the gross cross-section	ε	material parameter depending on f_y , equals $\varepsilon = \sqrt{235/f_y \left[\frac{N}{mm^2} \right]}$
N_{ult}	ultimate resistance to axial force of the gross cross-section	$\bar{\lambda}$	relative slenderness for flexural buckling
$W_{eff,u}, W_{eff,v}$	elastic section modulus of the effective area of a cross-section for bending about u-u and v-v axis, respectively	$\bar{\lambda}_p$	relative slenderness for plate buckling
$W_{el,u}, W_{el,v}$	elastic section modulus for bending about u-u and v-v axis, respectively	$\bar{\lambda}_{LT}$	relative slenderness for lateral torsional buckling
$W_{pl,u}, W_{pl,v}$	plastic section modulus for bending about u-u and v-v	ρ, ρ_u, ρ_v	reduction factors for plate buckling
		σ_1, σ_2	end stresses in a beam
		σ_{cr}	elastic critical plate buckling stress
		ψ	ratio of end moments in a beam

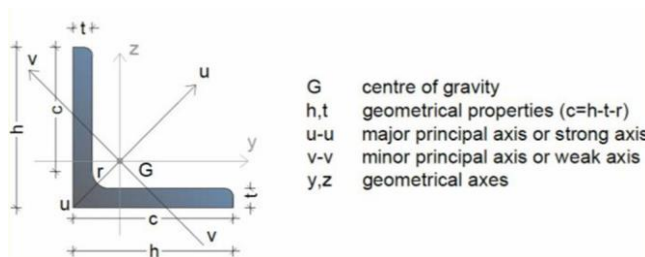
2. Classification of equal-leg angle profiles

Cross-section classification is of importance for the selection of the analysis and design procedures to employ: plastic, elastic or elastic with due account for local buckling. According to EN 1993-1-1, clause 5.5.2 (4) classification should be done for the compression parts of the cross-section that are defined as follows: “Compression parts include every part of the cross-section which is either totally or partially in compression under the load combination considered”. A strict application of this rule requires a separate classification of the cross-section for each combination of applied forces and moments. Since this rule is unpractical for design, a simpler approach is proposed in this article, where the cross-section is classified separately for compression, strong axis and weak axis bending. For the latter, the cross-section class may be different for positive or negative moments due to the monosymmetric shape of the profile, that may lead to different classes when the tip is in compression or in tension. In the following, the limiting width-to-thickness ratios for compression

parts of equal leg angle sections are defined through analytical considerations and numerical calculations, for the above- referred loading conditions.

Fig. 1 illustrates the notations for the geometrical properties as well as for the geometrical and principal axes. It should be also noticed that in the latest available draft of the forthcoming version of EN 1993-1-1 that is available to the authors, namely prEN 1993-1-1 [11], no modification is contemplated regarding the classification of the cross-sections. Therefore, EN 1993-1-1 will be used hereafter when a reference is made.

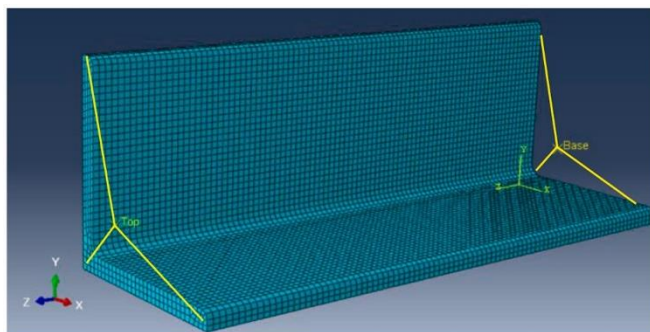
Fig. 1. Notations for geometrical properties and principal axes.



2.1. DESCRIPTION OF THE NUMERICAL MODELS

The numerical models for the parametrical numerical studies were created with ABAQUS non-linear finite element software [12] using volume elements. The samples have been modelled as pin-ended with at least three volume linear elements over the thickness (see Fig. 2). The selection of the elements (linear instead of quadratic) does not influence the results. A denser mesh (i.e four volume elements per thickness) gives better results by 1–2%, but increases substantially the required time of the analysis, that is not desirable in combination with the high number of the performed numerical studies. At the extremities, fictitious end plates have been introduced through a specific constraint, so as to distribute uniformly the external applied loads but also to avoid any local failure at the point of application of the load. The model has been also validated through the experimental tests reported in Ref. [9].

Fig. 2. Sample of the 3-D model used for the numerical analyses.



The finite element analyses were performed considering:

a local leg imperfection equal to $h/100$ (h is the width of the cross- section), based on the tolerances defined in EN 10056–2 [13], with imperfection shape affine to the lower relevant elastic instability

mode obtained through an elastic instability analysis, that has been performed with an axial force or a bending moment for compression or bending cases respectively;

a linear elastic – perfectly plastic material model without strain hardening in accordance with EN 1993-1-14, §4.3.2-(1a) [14].

All nominal dimensions of the cross-sections used in the parametrical studies presented in the following sections, were in mm.

2.2. CLASSIFICATION TO COMPRESSION

The resistance for sections subjected to compression is identical for classes 1, 2 or 3. Accordingly, there is a need to define only the limit between classes 3 and 4. The failure modes on each side of this limit are respectively the yielding of the cross section and the local buckling of the legs. Often, but erroneously, local buckling for class 4 sections is associated to torsional buckling while it is known from Ref. [15] that a pure torsional buckling mode can only be obtained when the load application point is the shear centre, which does not coincide with the centroid in case of angles sections. Accordingly, the relevant failure mode for class 1 to 3 angle section is yielding, while for class 4 sections local plate buckling occurs in the legs.

For the current numerical studies on angles under compression, the cross-sections considered were equal leg angle profiles L 45 × 45 in two thicknesses 3 and 4, L 70 × 70 in two thicknesses 5 and 6, and L 250 × 250 in four thicknesses 17, 20, 22 and 26. For all cross-sections, four steel grades S355, S460, S550 and S690 were considered; steel grade S690 is not available on the market for rolled angle sections, however it has been selected so as to reach higher slenderness ratios. In order to prevent flexural buckling, the length of the samples was selected in such a way that $\bar{\lambda} \leq 0,2$, a slenderness below which the European buckling curves do not provide any strength reduction to buckling; this limitation leads to a length of the samples $L \leq 18,75 \cdot \epsilon \cdot i_v$. Then for the analyses, a value equal to $18,5 \cdot \epsilon \cdot i_v$, has been selected.

Fig. 3. Compression. Ratio between numerical results and plastic resistance vs. h/et ratio.

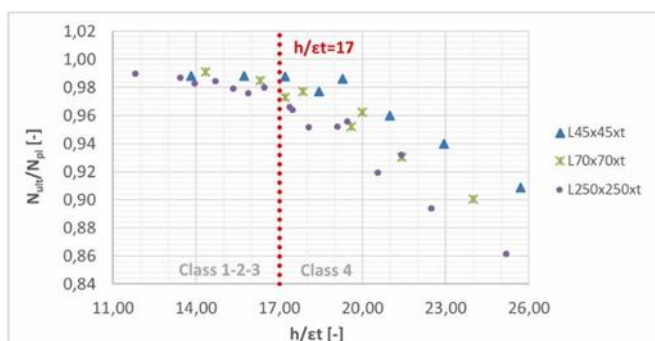


Fig. 4. Compression. Ratio between numerical results and plastic resistance vs. $c/\epsilon t$ ratio.

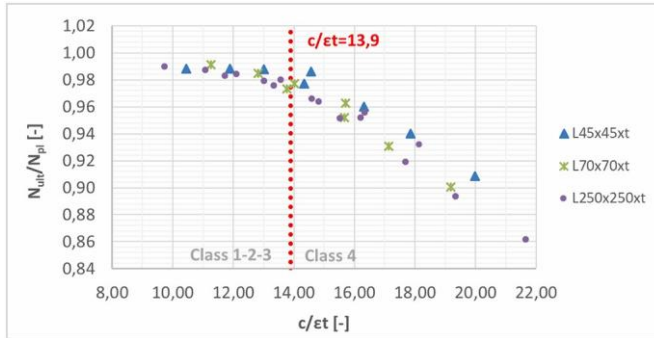


Fig. 3 and Fig. 4 show the ratio between the numerically obtained cross-section (CS) resistance (N_{ult}) and the plastic characteristic resistance ($N_{pl} = A \cdot f_y$), versus the $h/\epsilon t$ and $c/\epsilon t$ ratio respectively. It can be easily observed that the scatter is bigger when the results are correlated with the $h/\epsilon t$ ratio than the $c/\epsilon t$ one, that makes the latter ratio more suitable for classification purposes. The samples that reach their plastic characteristic resistance even with a 3% deviation, that is acceptable due to the selected mesh of the volume elements, can be categorized as class 1 to 3. Therefore, based on the numerical results, the class-3 limit for equal leg angles subjected to compression may be set as $c/t \leq 13,9\epsilon$. In EN1993-1-1, table 5.2 (sheet 3), two conditions are provided to distinguish Class 3 from Class 4 sections for equal leg angles:

$$h/t \leq 11,5\epsilon \text{ and } h/t \leq 15\epsilon \text{ so } h/t \leq 11,5\epsilon \quad (1)$$

For the available hot-rolled equal leg angles, it can be seen that the mean value for parameter c is equal to $0,8 h$. So, in this case, Eq. 1 may be rewritten and becomes $c/t \leq 9,2\epsilon$. This value is seen to be quite conservative.

In the same table 5.2 (sheet 3), of EN1993-1-1, a cross reference to sheet 2 is made for “outstand elements”, from which the class-3 limit is:

$$c/t \leq 14\epsilon \quad (2)$$

This limit may be determined analytically by considering the risk of local plate buckling resistance of the leg. Indeed, the reduction factor for outstand plated elements due to local buckling is given in EN 1993-1-5 as below:

$$\rho = \begin{cases} 1,0 & \text{for } \bar{\lambda}_p \leq 0,748 \\ \frac{\bar{\lambda}_p - 0,188}{\bar{\lambda}_p^2} & \text{for } \bar{\lambda}_p > 0,748 \end{cases} \quad (3)$$

where :

$$\bar{\lambda}_p = \sqrt{\frac{f_y}{\sigma_{cr}}} = \frac{\bar{b}/t}{28,4\epsilon\sqrt{k_\sigma}} \quad (4)$$

In this case, the condition for class 3 to 4 limit is that the resistance of yielding should not be reduced due to local buckling, which may be expressed as $\bar{\lambda}_p \leq 0,748$. In table 4.2 of Ref. [3], the buckling factor for outstand elements in compression ($\psi = 1$) is defined as $k_\sigma = 0,43$. Additionally, for equal leg angles

EN 1993-1-5 defines $b = h$. Introducing the above in the expression for the limit slenderness, the class-3 limit in respect to local buckling may be calculated as:

$$h/t \leq 13,9\epsilon \text{ corresponding approxim. to } c/t \leq 11,1\epsilon \quad (5)$$

But in fact, it appears clearly now that \bar{b} should be selected as equal to c , and not to h . As a conclusion, the class-3 to class-4 limit for equal leg angles subjected to compression may be preferably set equal to:

$$c/t \leq 13,9\epsilon \quad (6)$$

This condition is in line with:

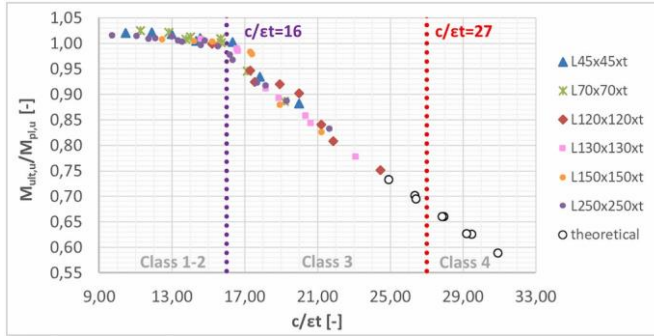
- the numerical studies;
- the current provisions of Eurocode 3 and more specifically with EN 1993-1-1, table 5.2, sheet 2 for class-3 limit of outstand elements ($c/t \leq 14\epsilon$);
- EN 1993-1-5, in which $b = c$ instead of h ($c/t \leq 13,9\epsilon$);
- the standard EN 50341, mainly used in practice in central Europe for the design of lattice towers made of angles ($c/t \leq 13,9\epsilon$);
- the recommendations of EN 1993-3-1 ($c/t \leq 13,9\epsilon$) in which the c/t ratio for angles defined in EN 1993-1-1: table 5.2 may be replaced by the ratio $(h-2t)/t$, the nominator of which is not so far from the exact value $c = h-t-r$.

2.3. CLASSIFICATION TO STRONG AXIS BENDING

The stress distribution for strong axis bending M_u is such that only one leg is under compression and needs classification. The samples considered for these numerical investigations were equal leg angle profiles L 45 × 45 in two thicknesses 3 and 4, L 70 × 70 in two thicknesses 5 and 6, L 120 × 120 in two thicknesses 7 and 8, L 130 × 130 in two thicknesses 8 and 9, L 150 × 150 in two thicknesses 10 and 12 and L 250 × 250 in four thicknesses 17, 20, 22 and 26. For each cross-section, again four steel grades S355, S460, S550 and S690 were considered as for angles in compression, where concerning the grade S690 the same applies as said in section 2.2. The selected parameters lead to $c/\epsilon t$ -ratios below 25. However, in order to extend more the results to higher ratios, additional analyses were carried out for two cross-sections (L 120×120×7 and L 130×130×8) with non-commercially available higher steel grades (S800, S900, S1000 and S1100). In order to prevent lateral torsional buckling, the length of all samples has been each time adapted so as the relative slenderness equals $\lambda_{LT} = 0,4$ which is the limit slenderness under which LTB does not reduce the bending strength.

Fig. 5 shows the ratio between the numerical results for the cross-section resistance ($M_{ult,u}$) and the plastic characteristic resistance ($M_{pl,u} = W_{pl,u} \cdot f_y = 1,5 \cdot W_{el,u} \cdot f_y$), versus the $c/\epsilon t$ ratio. The correlated factor of 1,5 between the elastic and plastic modulus adopted here, has been selected on the safe side and in line with the numerical results, as its mean value for all commercially available angle profiles is 1,58 with a standard deviation of 2%. Details about the exact value of the plastic modulus in respect to the u-u axis can be found in Ref. [16].

Fig. 5. Strong axis bending. Ratio between numerical results and plastic resistance vs. $c/\epsilon t$ ratio.



The samples that reach their plastic characteristic resistance ($M_{pl,u} = 1.5 \cdot M_{el,u}$), even with a 3% deviation, can be categorized as class 1–2, while the class-3 limit can be easily found when $M_{ult,u}$ is equal to $M_{el,u}$ and then $M_{ult,u}/M_{pl,u} = 0.66$. Subsequently, from the numerical results, the class-2 limit for equal leg angles subjected to strong axis bending may be adopted as $c/t \leq 16\epsilon$, while the class-3 limit can be set to $c/t \leq 27\epsilon$.

In the plastic domain, the leg is an outstand element subjected to uniform compression and then class-2 limit may be obtained from EN1993-1-1, Table 5.2, sheet 2 as $c/t \leq 10\epsilon$. The background of this value may be found in ESDEP [17] where it is indicated that a class-2 limit can be obtained by defining the value of the reduced plate slenderness $\bar{\lambda}_{p,min}$ by Eq. (7) as equal to 0,6:

$$\bar{\lambda}_p = \frac{c/t}{28,4\epsilon\sqrt{k_\sigma}} \quad (7)$$

The buckling factor is $k_\sigma = 0,43$ for simply supported boundary conditions. So, a c/t value of $11,2\epsilon$ is found (rounded to 10ϵ in Table 5.2, sheet 2). If clamped boundary conditions are now assumed, $k_\sigma = 1,25$ (see Ref. [18] – [19]), this leads to $c/t \leq 19,1\epsilon$. By observing the numerical and analytical results, it can be concluded that the actual class-2

limit is between the above two extreme cases and finally the following limit may be adopted:

$$\frac{c}{t} \leq 16\epsilon \quad (8)$$

which defines the limit between classes 2 and 3. In fact, the leg in tension is seen to bring a torsional restraint to the leg in compression, with an intermediate efficiency between fully pinned and fully fixed boundary conditions.

For elastic behaviour, the compression leg is an outstand element subjected to a stress ratio

$\psi = \frac{\sigma_2}{\sigma_1} = \frac{h-c}{h} \approx \frac{h-0,8h}{h} \approx 0,20$. Based on EN 1993-1-1, Table 5.2, sheet 2, the class-3 limit is equal to $c/t \leq 15,3\epsilon$, which is smaller than the proposed class-2 limit. Again, it is based on an assumption of simply supported boundary condition (i.e $k_\sigma = 0,54$). If, now, clamped boundary conditions are assumed, the corresponding buckling factor [18] – [19] may be taken as equal to $k_\sigma = 1,57$. Then, the class-3 limit may be obtained from the general formula of EN 1993-1-1, Table 5.2, sheet 2:

$$\frac{c}{t} \leq 21\epsilon\sqrt{1,57} = 26,3\epsilon \quad (9)$$

This value is quite close to the numerically obtained one (27ϵ) and is selected here as the proposed class-3 limit.

2.4. CLASSIFICATION TO WEAK AXIS BENDING

When the cross-section is subjected to weak axis bending M_v , the stress conditions for the two legs are identical and then, classification refers to both legs. Two cases are defined and checked afterwards; when the tip is under compression and when it is in tension.

2.4.1. TIP IN COMPRESSION

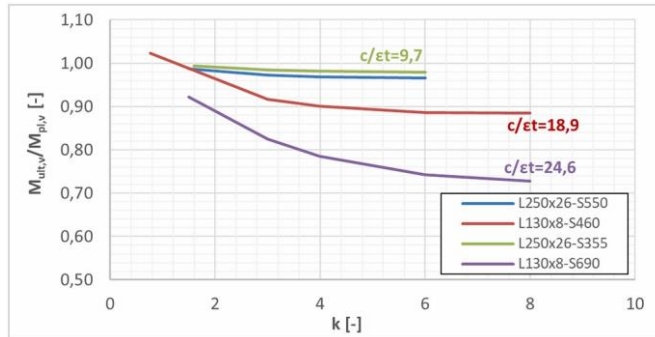
Table 1 presents the cross-sections and the steel grades that have been used for the numerical studies in which the samples are subjected to weak axis bending moment M_v , with the tip in compression. As before, the last 8 analyses, i.e. 15* and 16* in Table 1, are theoretical so as to address slenderer cross-sections.

A number of numerical simulations have been performed to evaluate an optimal length value ($L = k \cdot h$), so that yielding develops freely along the member while the cross-section resistance is independent of the length. It has been found that the value of the sample length $L = 6 h$ [mm] is working quite well (see Fig. 6):

Table 1. Samples for the analyses of the cross-section subjected to weak axis bending M_v .

No	Cross-section	Steel grade
1	L45x45x3	S355 / S460 / S550 / S690
2	L45x45x4	S355 / S460 / S550 / S690
3	L70x70x5	S355 / S460 / S550 / S690
4	L70x70x6	S355 / S460 / S550 / S690
5	L120x120x7	S355 / S460 / S550 / S690
6	L120x120x8	S355 / S460 / S550 / S690
7	L130x130x8	S355 / S460 / S550 / S690
8	L130x130x9	S355 / S460 / S550 / S690
9	L150x150x10	S355 / S460 / S550 / S690
10	L150x150x12	S355 / S460 / S550 / S690
11	L250x250x17	S355 / S460 / S550 / S690
12	L250x250x20	S355 / S460 / S550 / S690
13	L250x250x22	S355 / S460 / S550 / S690
14	L250x250x26	S355 / S460 / S550 / S690
15*	L120x120x7	S700 / S800 / S900 / S950
16*	L130x130x8	S700 / S800 / S900 / S950

Fig. 6. Weak axis bending – tip in compression. Ratio between numerical results and plastic resistance vs. the length parameter k .



- for small c/et ratios which corresponds to class 1 or 2 profiles, the difference between $L = 4h$ and $L = 6h$ is less than 0,5%, what is acceptable.
- for large c/et ratios which corresponds to class 3 and 4 profiles, the difference between $L = 6h$ and $L = 8h$ is less than 1,8%, what is also acceptable.

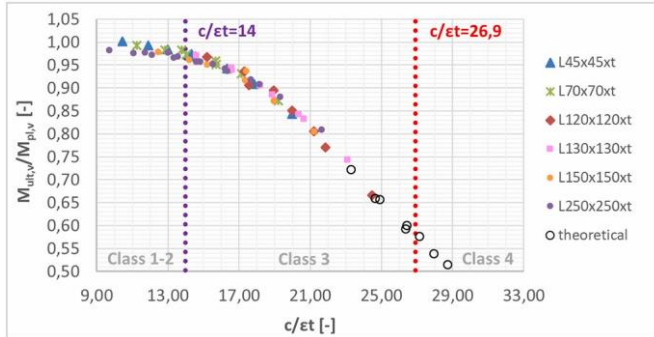
The details and results of this specific parametric study are summarized in Table 2 and Fig. 6.

Fig. 7 shows the ratio between the numerical results for the cross- and the plastic characteristic resistance ($M_{pl,v} = W_{pl,v} \cdot f_y$), versus the c/et ratio. The analytical expressions for the evaluation of $W_{pl,v}$ can be found in Ref. [16]. From the numerical results, one can observe that the class-2 limit, where the samples reach their $M_{pl,v}$ even with a 3% deviation, is $c/t \leq 14\epsilon$, while the class-3 limit (samples that reach their elastic resistance) is $c/t \leq 26,9\epsilon$.

Table 2 Samples of the numerical simulations about the optimal length value.

No	h [mm]	f_y [N/mm ²]	t [mm]	c/et	k	$L = k \cdot h$ [mm]	$M_{ult,v}$ [kNm]	$M_{pl,v}$ [kNm]	$M_{ult,v}/M_{pl,v} [-]$
1	250	355	26	9,7	1,6	400	183,17	184,34	0,99
2	250	355	26	9,7	3	750	181,43	184,34	0,98
3	250	355	26	9,7	3,5	875	181,16	184,34	0,98
4	250	355	26	9,7	4	1000	180,94	184,34	0,98
5	250	355	26	9,7	6	1500	180,57	184,34	0,98
6	250	550	26	11,7	1,5	375	281,90	285,6	0,99
7	250	550	26	11,7	3	750	277,57	285,6	0,97
8	250	550	26	11,7	4	1000	276,54	285,6	0,97
9	250	550	26	11,7	6	1500	275,98	285,6	0,97
10	130	460	8	18,9	0,77	100	21,42	20,94	1,02
11	130	460	8	18,9	3	390	19,19	20,94	0,92
12	130	460	8	18,9	4	520	18,86	20,94	0,90
13	130	460	8	18,9	6	780	18,54	20,94	0,89
14	130	460	8	18,9	8	1040	18,53	20,94	0,88
15	130	690	8	24,6	1,5	195	28,96	31,42	0,92
16	130	690	8	24,6	3	390	25,92	31,42	0,82
17	130	690	8	24,6	4	520	24,65	31,42	0,78
18	130	690	8	24,6	6	780	23,32	31,42	0,74
19	130	690	8	24,6	8	1040	22,88	31,42	0,73

Fig. 7. Weak axis bending – tip in compression. Ratio between numerical results and plastic resistance vs. c/et ratio. section resistance ($M_{ult,v}$)



The stress ratio for elastic stress distribution (see Fig. 8) is given by $\psi = \frac{\sigma_2}{\sigma_1} = -\frac{e-(h-c)}{h-e}$. As it is shown in Fig. 9, the stress ratio $\psi \approx -0.1$ for usual angle sections. Therefore, the buckling factor may be conservatively set equal to $k_\sigma = 0.57$ (valid for $\psi = 0$) and according to EN 1993- 1-1, Table 5.2, sheet 2, the class-3 limit is:

$$c/t \leq 15.9\epsilon \quad (10)$$

Fig. 8. Stress distribution (compression taken as positive) for weak axis bending (M_v) – tip in compression.

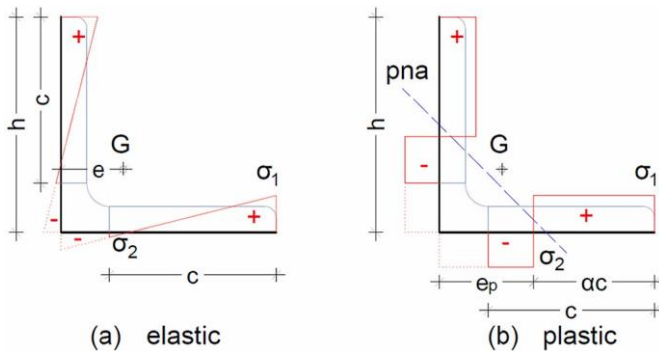
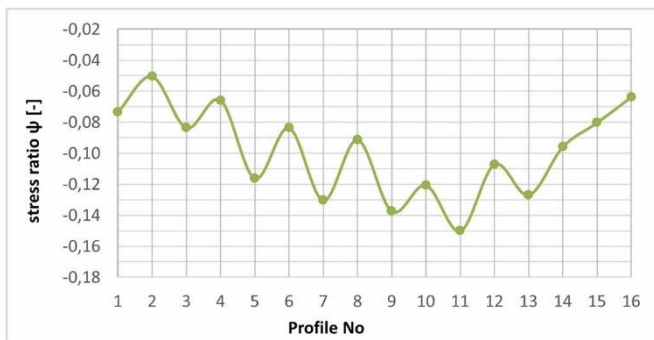


Fig. 9. Stress ratio ψ for elastic stress distribution – tip in compression (angles from 70 to 300).



If now clamped boundary conditions are considered, as have been for strong axis bending too, the buckling factor is equal to $k_\sigma = 1,65$ (based on ref. [18] for stress ratio $\psi \approx -0,1$). Then, the class-3 limit may be obtained again from the general formula of EN 1993-1-1, Table 5.2, sheet 2 as follows:

$$\frac{c}{t} \leq 21\varepsilon\sqrt{k_\sigma} = 21\varepsilon\sqrt{1,65} = 26,9\varepsilon \quad (11)$$

The class-3 limit may so be defined accordingly as it is in agreement with the numerical results.

For the plastic stress distribution, the proportion of the leg in compression is $\alpha = 1 - \frac{e_p - t - r}{h - t - r}$. By considering all the available angle sections, it can be seen that the value of α is ranging between 0,50 to 0,62 with a mean value of approximately 0,58 and a standard deviation of 2%, and therefore, a value of 0,60 is adopted. The class 2 limit may be accordingly obtained from the general formula of EN 1993-1-1, Table 5.2, sheet 3, from

$$\frac{c}{t} \leq \frac{10\varepsilon}{\alpha} = \frac{10\varepsilon}{0,6} = 16,6\varepsilon \quad (12)$$

The classification limits proposed currently by Eurocode 3, i.e. Eq. 10 and Eq. 12, are not at all consistent as the c/t -ratio for class 3 ($c/t \leq 16\varepsilon$) is lower than the one obtained for class 2 ($c/t \leq 16,6\varepsilon$). The reason is that the mechanical model for class 2 sections in Eurocode 3, when the tip is in compression, is not correct because the outstand elements partially in compression are treated as elements fully in compression with a reduced width αc . This means that the hinge support is introduced exactly at the position where the compression starts. This is questionable, since tension is beneficial to local buckling. For that reason, it is proposed here to keep the c/t limit for class 2 equal to:

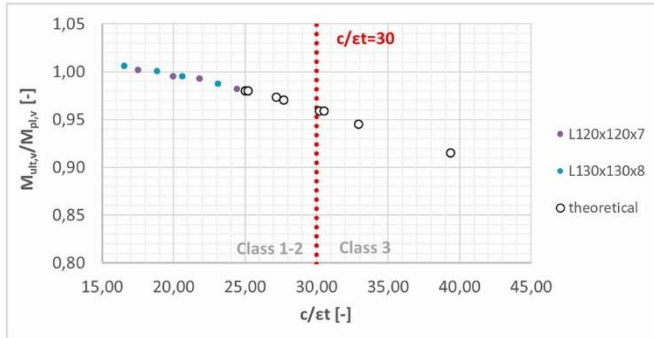
$$\frac{c}{t} \leq 14\varepsilon \quad (13)$$

which is in good agreement with the numerical results.

2.4.2. TIP IN TENSION

The samples for the numerical simulations were equal leg angle profiles L 120 × 120 in two thicknesses 7 and 8 and L 130 × 130 in two thicknesses 8 and 9. For each cross-section, four steel grades S355, S460, S550 and S690 were considered. In order to extend the investigations in a range of $c/\varepsilon t$ ratios higher than 25, some additional analyses have been performed. For these ones, two cross-sections (L 120×120×7 and L 130×130×8) were considered with theoretical steel grades ranging from S720 to S2000. The value $L = 6h$ [mm] for the length has been adopted in this case too, as explained in section 2.4.1. Fig. 10 shows the ratio between the numerically-obtained values of the cross-section resistance ($M_{ult,v}$) and the plastic resistance, according to the $c/\varepsilon t$ ratio. From the numerical results, one can observe that the class-2 limit for equal leg angles subjected to weak axis bending when the tip is in tension, equals $c/t \leq 30\varepsilon$.

Fig. 10. Weak axis bending – tip in tension. Ratio between numerical results and plastic resistance vs. $c/\epsilon t$ ratio.



In the plastic domain, the proportion of the leg subjected to compression is $\alpha = \frac{e_p - t - r}{h - t - r}$ and taking $r = t$ as an approximation, it may be shown (Fig. 11), that for usual angle sections it is $\alpha \approx 0.4$. Then, the class-2 limit may be obtained from the general formula of EN 1993-1-1, Table 5.2, sheet 2:

$$\frac{c}{t} \leq \frac{10\epsilon}{\alpha\sqrt{\alpha}} = \frac{10\epsilon}{0.4\sqrt{0.4}} = 39.5\epsilon \quad (14)$$

At the end, the class-2 limit may be kept as provided by the numerical results ($c/t \leq 30\epsilon$), which is on the safe side concerning the normative approach. In any case, both limits are far from the highest $c/\epsilon t$ ratios obtained for available angles and steel grades. Therefore, all angle sections may practically always develop their plastic moment for weak axis bending when the tip is in tension.

2.5. SUMMARY OF CLASSIFICATION FOR EQUAL LEG ANGLE SECTIONS

The complete set of the proposed duly validated classification criteria is summarized in Table 3. It may be seen that, unlike in the current Eurocodes, the same geometric parameters, c and t , are used for all cross-section loading situations.

3. Design resistances of angle cross-sections

In the following paragraphs, formulae for the evaluation of the cross-section design resistance of equal leg angles are proposed and validated. The notations for the material properties, safety factors and other properties follow those given in the nomenclature, and therefore no further definitions are given here, unless it is necessary. Finally, based on the results obtained in the course of classification, a linear transition between plastic and elastic bending resistances is adopted. This smooth transition has been proposed and validated for double symmetric cross-sections in the SEMI-COMP European funded project [20] and will be adopted for these sections in the forthcoming new version of Eurocode 3 Part 1-1 (prEN1993-1-1).

Fig. 11. Ratios α for tip in tension (angles from 70 to 300).

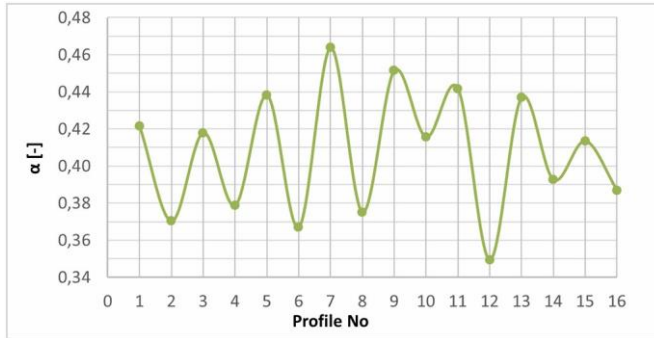


Table 3 Maximum width-to-thickness ratios for compression parts of equal leg angle sections.

	Section in compression	Section in strong axis bending M_u	Section in weak axis bending M_v – tip in compression	Section in weak axis bending M_v – tip in tension
Class 1-2	-	$\frac{c}{t} \leq 16\varepsilon$	$\frac{c}{t} \leq 14\varepsilon$	$\frac{c}{t} \leq 30\varepsilon$
Class 3	$\frac{c}{t} \leq 13,9\varepsilon$	$\frac{c}{t} \leq 26,3\varepsilon$	$\frac{c}{t} \leq 26,9\varepsilon$	-

where $\varepsilon = \sqrt{235/f_y \left[\frac{N}{mm^2} \right]}$

3.1. CROSS-SECTION RESISTANCE TO COMPRESSION

The cross-section design resistance for axial compression may be determined from:

$$N_{c,Rd} = \begin{cases} \frac{A f_y}{\gamma_{M0}} & \text{for class 1, 2 and 3 profiles} \\ \frac{A_{eff} f_y}{\gamma_{M0}} & \text{for class 4 profiles} \end{cases} \quad (15)$$

where,

A_{eff} is the area of the effective cross-section defined as:

$$A_{eff} = A - 2ct(1 - \rho) \quad (16)$$

ρ is the reduction factor for plate buckling, evaluated through Eq. 3, considering a reduced plate slenderness of the legs equal to:

$$\bar{\lambda}_p = \sqrt{\frac{f_y}{\sigma_{cr}}} = \frac{c/t}{28,4\varepsilon\sqrt{0,43}} = \frac{c/t}{18,6\varepsilon} \quad (17)$$

$\gamma_{M0} = 1,0$ (as recommended by EN1993-1-1) is the partial safety factor for resistance.

The corresponding characteristic resistance $N_{c,Rk}$ is given by Eq. 15, without consideration of γ_{M0} . It is equal to $N_{c,Rd}$ in the specific case when $\gamma_{M0} = 1,0$.

In order to be in line with the classification limits as derived in Section 2.2, the geometric property b must be defined differently than in the current Eurocode provisions. Consequently, the statement of EN 1993-1-5, §4.4(2) that $b = h$ for equal leg angles should be replaced for this type of section by $b = c$. This constitutes the only difference between the current proposal and the existing Eurocode 3 provisions.

Fig. 12 shows the ratio between the numerically determined cross-section resistance (N_{ult}), and the characteristic resistance ($N_{c,Rk}$), versus the c/et ratio. It may be seen that the response is not influenced by the cross-section size. Furthermore, it may be seen that, for stocky class 1 to 3 legs, there is a small overestimation of resistance by the proposed formulae. However, this is largely counterbalanced by strain hardening that is not considered in analysis. For class-4 sections the proposed rules are largely on the safe side.

Fig. 12. Cross-section resistance to compression. Ratio between numerical results and characteristic compression resistance vs. c/et ratio

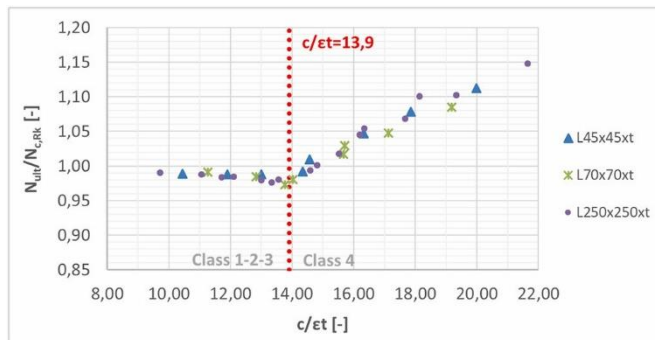
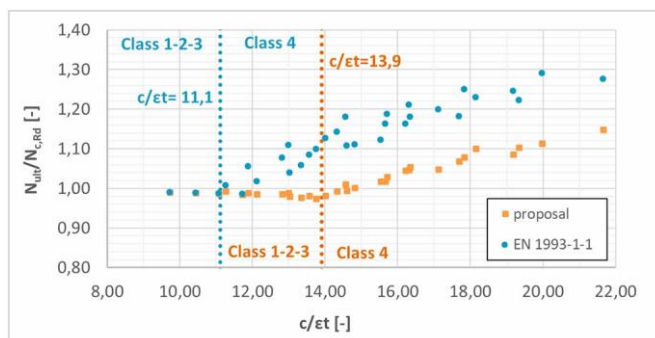


Fig. 13 illustrates the ratio between the numerically determined cross-section resistance (N_{ult}), and the design resistance ($N_{c,Rd}$), versus the c/et ratio. The design resistances have been evaluated firstly with the current proposal and then based on the existing provisions of EN1993-1-1 in combination with EN1993-1-5 for class-4 sections. The vertical dot lines represent the class-3 limit as suggested in the paper ($c/t \leq 13,9\epsilon$) and the one calculated by Eurocode 3 provisions ($c/t \leq 11,1\epsilon$ - Eq. 5). It can be easily seen that the proposed model is less conservative for class-4 profiles.

Fig. 13. Cross-section resistance to compression. Ratio between numerical results and design compression resistance obtained from the current proposal and Eurocode 3 vs. c/et ratio.



3.2. CROSS-SECTION RESISTANCE TO STRONG AXIS BENDING

The cross-section design resistance to strong axis bending M_u may be determined from:

$$M_{u,Rd} = W_u \frac{f_y}{\gamma_{M0}} \quad (18)$$

where,

W_u is the section modulus about u axis that equals :

$$W_u = \alpha_{i,u} W_{el,u}, i = 2,3,4 \quad (19)$$

where,

$$\alpha_{2,u} = 1,5 \text{ for class 1 or 2} \quad (20)$$

$$\alpha_{3,u} = \left[1 + \left(\frac{26,3\varepsilon - c/t}{26,3\varepsilon - 16\varepsilon} \right) (1,5 - 1) \right] \text{ for class 3} \quad (21)$$

$$\alpha_{4,u} = \frac{W_{eff,u}}{W_{el,u}} = \rho_u^2 \text{ for class 4} \quad (22)$$

ρ_u is the reduction factor for plate buckling, determined from Eq. 3 under consideration of a plate slenderness of legs expressed by:

$$\bar{\lambda}_p = \sqrt{\frac{f_y}{\sigma_{c,r}}} = \frac{c/t}{28,4\varepsilon\sqrt{1,57}} = \frac{c/t}{35,6\varepsilon} \quad (23)$$

$\gamma_{M0} = 1,0$ (as recommended by EN1993-1-1) is the partial safety factor for resistance.

For strong axis bending of class-4 sections, the effective cross-section becomes non-symmetric due to the fact that only one leg is in compression. This changes the position of the centroid, the directions of the principal axes and all cross-section properties. In order to avoid such laborious calculation, an approximate solution for the effective section modulus is envisaged. This may be achieved by reducing equally the other leg too. The comparison of the ratio between the two cross-sections is shown in Fig. 14. It may be seen that the proposed approach, applied to for a large number of cross-sections considered, is on the safe side.

Fig. 14. Ratio of the strong axis moduli between the initial and the effective cross-section.

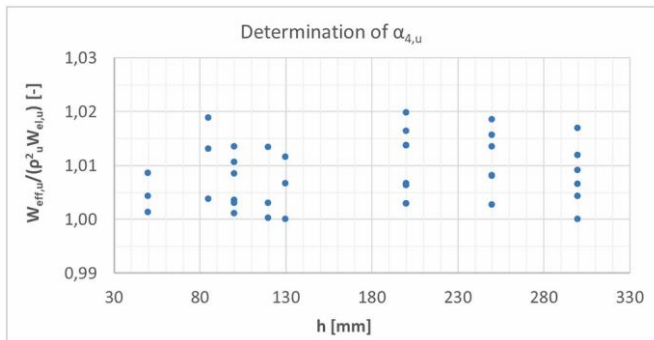


Fig. 15 shows the ratio between the numerically determined cross- section resistance ($M_{ult,u}$), and the characteristic resistance ($M_{u,Rk}$), versus the $c/\varepsilon t$ ratio. It may be seen that the response is not influenced by the cross-section size. For stocky class 1 and 2 legs, the proposed formulae predict almost exactly the resistance. For class-3 sections there is a small overestimation of resistance in a

very limited range of $c/\epsilon t$ ratios, while for class-4 sections the proposed rules are always on the safe side.

Fig. 15. Cross-section resistance to strong axis bending. Ratio between numerical results and characteristic moment resistance vs. $c/\epsilon t$ ratio.

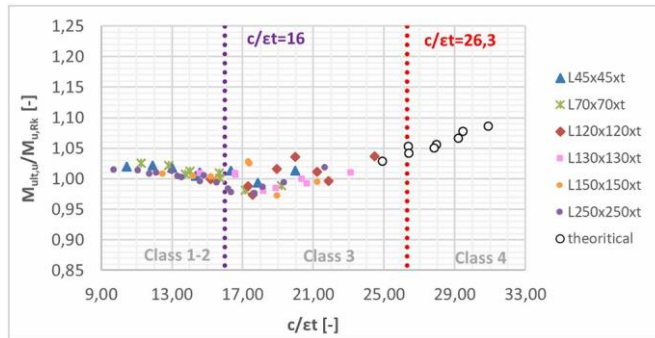
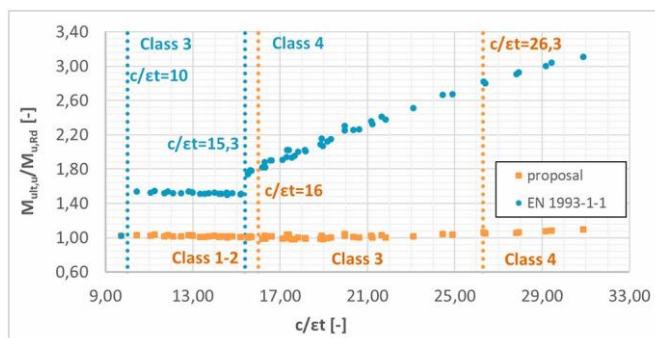


Fig. 16 illustrates the ratio between the numerically determined cross-section resistance ($M_{ult,u}$), and the design resistance ($M_{u,Rd}$), versus the $c/\epsilon t$ ratio. The design resistances have been evaluated both with the current proposal and the existing provisions of EN1993-1-1. The vertical dot lines represent the class limits as suggested in the paper (i.e. $c/t \leq 16\epsilon$ for class 2 to 3 and $c/t \leq 26,3\epsilon$ for class 3 to 4) and as calculated by EN1993-1-1 provisions (i.e. $c/t \leq 10\epsilon$ for class 2 to 3 and $c/t \leq 15,3\epsilon$ for class 3 to 4). The benefits and the improvements brought from the proposals (classification and cross-section resistance) on the design of the cross-sections may be observed.

Fig. 16. Cross-section resistance to strong axis bending. Ratio between numerical results and design compression resistance obtained from the current proposal and Eurocode 3 vs. $c/\epsilon t$ ratio.



3.3. CROSS-SECTION RESISTANCE TO WEAK AXIS BENDING

3.3.1. TIP IN COMPRESSION

The design characteristic resistance of angle cross-sections to weak axis bending M_v –when the tip is in compression – may be determined from:

$$M_{v,Rd} = W_v \frac{f_y}{\gamma_{M0}} \quad (24)$$

where,

W_v is the section modulus about v axis that equals :

$$W_v = \alpha_{i,v} W_{el,v}, i = 2,3,4 \quad (25)$$

where,

$$\alpha_{2,v} = W_{pl,v}/W_{el,v} \text{ for class 1 or 2} \quad (26)$$

$$\alpha_{3,v} = \left[1 + \left(\frac{26,9\varepsilon - c/t}{26,9\varepsilon - 14\varepsilon} \right) (\alpha_{2,v} - 1) \right] \text{ for class 3} \quad (27)$$

$$\alpha_{4,v} = \frac{W_{eff,v}}{W_{el,v}} = 0,94 \cdot \rho_v^2 \text{ for class 4} \quad (28)$$

ρ_v is the reduction factor for plate buckling, determined from Eq. 3 under consideration of a plate slenderness of legs expressed by:

$$\bar{\lambda}_p = \sqrt{\frac{f_y}{\sigma_{cr}}} = \frac{c/t}{28,4\varepsilon\sqrt{1,65}} = \frac{c/t}{36,5\varepsilon} \quad (29)$$

$\gamma_{M0} = 1,0$ (as recommended by EN1993-1-1) is the partial safety factor for resistance.

It should be noted that, for weak axis, $W_{pl,v} \neq 1,50 \cdot \min(W_{el,v}^{tip}, W_{el,v}^{toe})$, in contrast with the case of strong axis bending. However, based on the numerical tested samples it appears that $W_{pl,v} = (1,65 - 1,95) \cdot \min(W_{el,v}^{tip}, W_{el,v}^{toe})$ and so, a value $\alpha_{2,v} = 1,75$ could be possibly adopted as a rough estimation only for a preliminary design.

For class 4 cross-sections, a similar procedure than for strong axis bending is followed. Fig. 17 shows that the modulus of the effective cross section is approximately equal to the modulus of the initial cross-section multiplied with the factor $0,94 \cdot \rho_v^2$. Therefore, $\alpha_{4,v}$ is fixed accordingly.

Fig. 17. Ratio of the weak axis section moduli between the full and the effective cross-section.

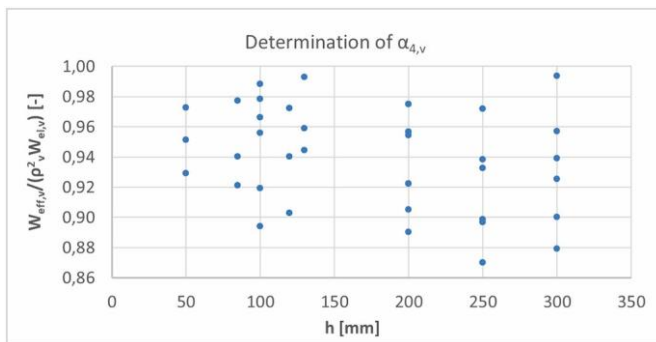


Fig. 18 shows the ratio between the numerically determined cross-section resistance ($M_{ult,v}$), and the characteristic resistance ($M_{v,Rk}$ using the exact value for $W_{pl,v}$), versus the $c/\varepsilon t$ -ratio. It may be seen that the response is not influenced by the cross-section size; therefore, the unconservative character of the selection of a mean value for $\alpha_{4,v}$ can be justified. For stocky legs of class 1 and 2, entering even in class-3, there is a small overestimation of resistance. This may be counterbalanced by strain hardening not considered here. This is also observed for large $c/\varepsilon t$ -ratios in the border between class 3 and 4. However, such ratios are not applicable to existing hot-rolled angle profiles.

Fig. 18. Cross-section resistance to weak axis bending – tip in compression. Ratio between numerical results and characteristic moment resistance vs. c/et ratio Tip in tension.

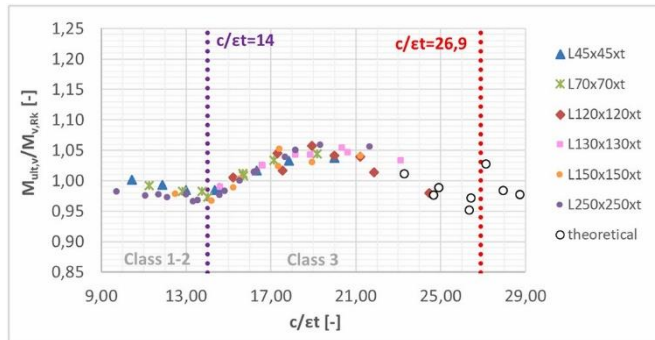
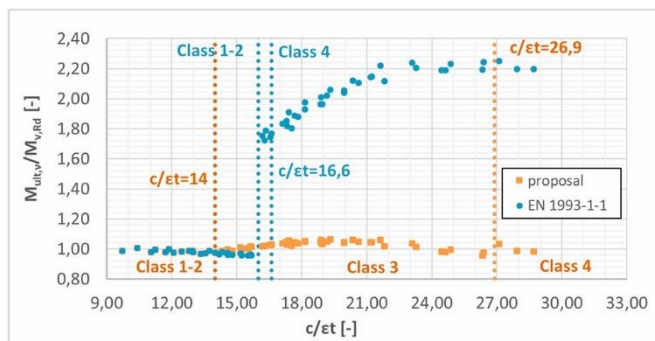


Fig. 19 illustrates the ratio between the numerically determined cross-section resistance ($M_{ult,v}$), and the design resistance ($M_{v,Rd}$), versus the c/et ratio. The design resistances have been evaluated both with the current proposal and the existing provisions of EN1993-1-1. The vertical dot lines represent the class limits as suggested in the paper (i.e. $c/t \leq 14\epsilon$ for class 2 to 3 and $c/t \leq 26,9\epsilon$ for class 3 to 4) and as calculated by EN1993-1-1 provisions (i.e. $c/t \leq 16,6\epsilon$ for class 2 to 3 and $c/t \leq 15,9\epsilon$ for class 3 to 4 as explained in Section 2.4.1-the profiles in between 26 and 16,6 are treated as class 4 sections for the calculations). The benefits and the improvements brought from the current proposals (classification and cross-section resistance) on the design of the sections is again observed.

Fig. 19. Cross-section resistance to weak axis bending– tip in compression. Ratio between numerical results and design compression resistance obtained from the current proposal and Eurocode 3 vs. c/et ratio.



3.3.2. TIP IN TENSION

The design resistance of angle cross-sections to weak axis bending M_v – tip in tension – may be determined from:

$$M_{v,Rd} = W_{pl,v} \frac{f_y}{\gamma_{M0}} \quad (30)$$

As it is shown in Fig. 10 for c/et ratio less than 30, the analytical approach for the cross-section resistance subjected to weak axis bending, when the tip is in tension, is in good agreement with the numerical results.

4. Summary and conclusions

Angle profiles belong to the most common structural steel shapes used in construction. As monosymmetric sections with negligible torsion and warping stiffness, they distinguish themselves from other common steel shapes and require specific rules for their design. Current European regulations provide rules of rather limited application, scattered in various codes and often exhibit significant lack of consistency. This paper presents design rules for rolled equal leg angle profiles, and more specifically deals with cross-section classification and resistance. The main features of the proposed design rules are as follows:

- They are based on theoretical-analytical considerations and are duly validated through extensive numerical analysis.
- They are written in the format of the existing Eurocode 3 specifications that allows a direct adoption in forthcoming drafts.
- They are simple to apply.
- They cover all cross-section classes and remove inconsistencies of existing specifications.
- They include all important loading conditions such as compression, weak and strong axis bending.
- They allow a smooth transition between cross-section classes, removing any artificial stepwise prediction of resistance.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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