Vibration comfort of floors: State-of-the-art

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1 Introduction

The use of increasingly slender and flexible steel and composite structures has sometimes led to vibration problems in bridges (Millennium Bridge, Solferino footbridge) and even in building floors. These problems have been caused by human activity, including walking, as well as rhythmic activities that have been developed strongly in recent years (gyms, dance, etc.). As a consequence, occupants may experience some degree of discomfort.

Eurocode provides few elements for the design of a structure with respect to vibration comfort [1]. The natural frequency limits of the fundamental mode of vibration according to the use of the floor are not always sufficient in practice, especially in the presence of rhythmic activities. International standards, such as ISO 2631 ([2], [3]) and ISO 10137 [4]or national standards such as DIN 4150-2 [5], do however provide elements for assessing the vibration comfort of the occupants of a structure exposed to vibration. These standards are based on an acceleration or velocity approach which will be presented in section 2of this paper.

Following the above-mentioned feedback, guidelines were proposed in the 2000s by AISC/CISC [12]and SCI [11], with the objective of assessing the level of vibration performance of building floors. These guidelines are based on an acceleration approach and refer to the above mentioned ISO standards. The HiVoSS guideline [7], based on a European research project [6], adopts a velocity approach and proposes charts to evaluate the comfort of floors subjected to vertical vibrations caused by a single person walking. The methods included in these three guidelines are presented in section 3and their limitations are also highlighted, in order to illustrate their advantages and disadvantages and to propose some improvements in future guidelines dealing with vibration comfort.

2 Vibration comfort standards

2.1 Introduction

Two international standards are now available to address the comfort of users of a structure exposed to vibration, in order to assess its acceptability for the intended use. ISO 2631, presented in section 2.2, provides values, expressed in terms of acceleration, to assess comfort taking into account the direction of vibration, frequency sensitivity and effects of duration of vibrations. ISO 10137 (see section 2.3) provides acceptability criteria and comfort limits based on the quantities from the previous standard. Finally, DIN 4150-2 (see section 2.4) proposes performance criteria for vibration comfort, no longer based on acceleration, but on velocity.

2.2 ISO 2631 standard

2.2.1 Overview

The international standard ISO 2631 ([2], [3]) provides a detailed procedure for the analytical determination of characteristic parameters for the human-induced vibration response (mainly in acceleration). These parameters are used in the evaluation of the impact of vibrations on the human body (for receivers) with respect to certain criteria (health, motion sickness, perception, comfort). Focus here is on to the criterion of vibration comfort.

The effect of vibrations depends on both the direction of incidence and the position of the human body, which may be standing, sitting or lying. The standard coordinate system is shown in Figure 1 ; human perception is generally more important for vibrations in the x or y direction.

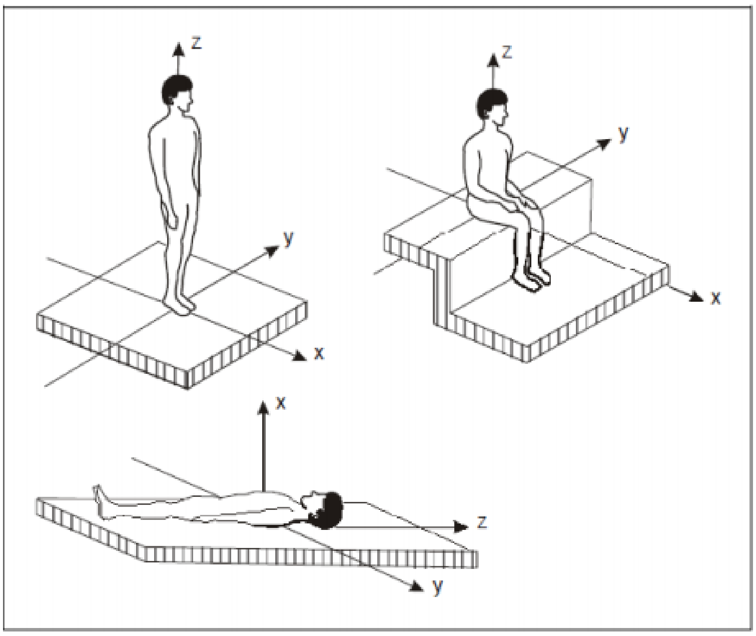
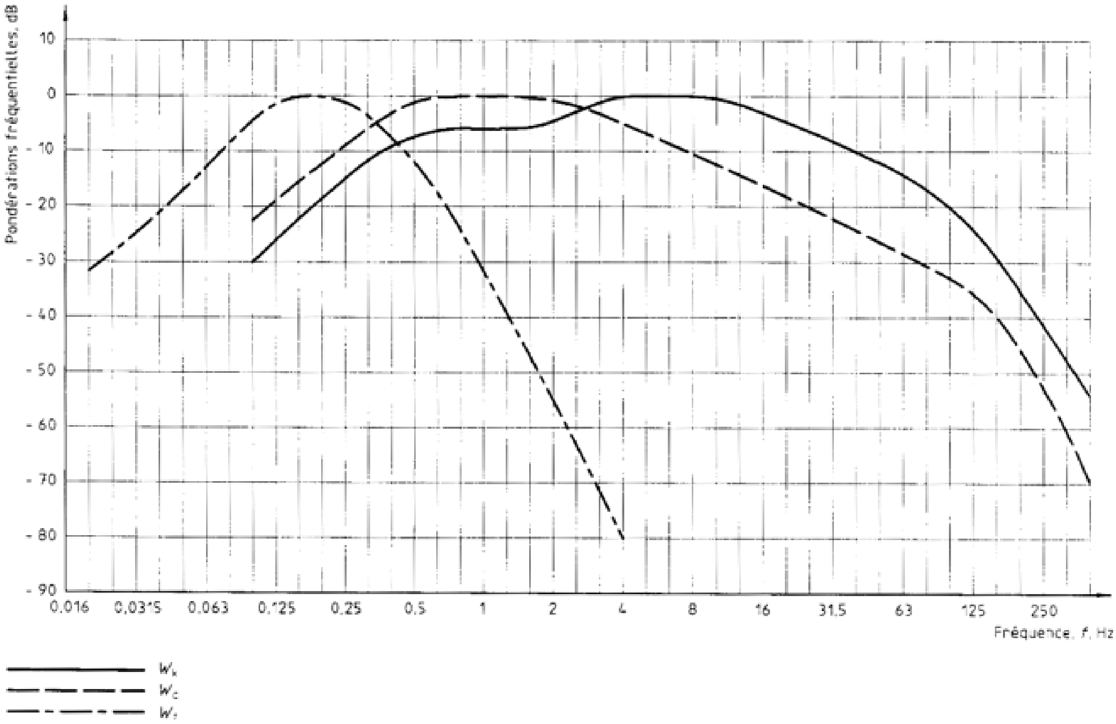


Figure 1: Coordinate system according to human body positions [2]

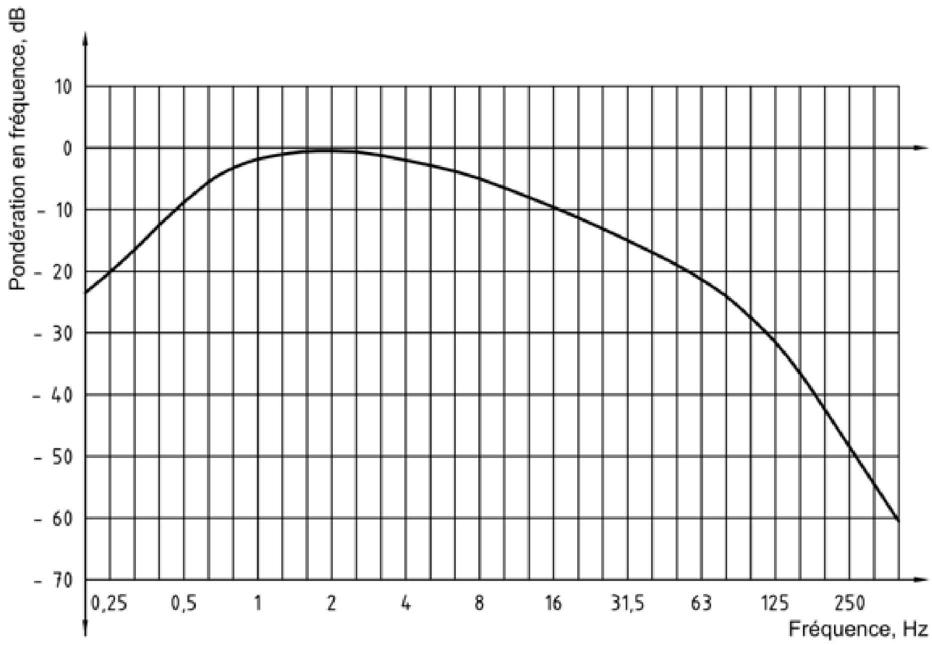
2.2.2 Frequency weighting

The impact of vibration on the occupants of a structure depends on their sensitivity to vibrations, strongly related to its frequency of incidence. In general, there are frequency ranges not very perceptible to humans, where the response is attenuated by means of so-called frequency weighting factors, presented in the form of curves, which are applied to the response terms (see section 2.1.3).

These curves (expressed in dB) are shown in Figure 2. The W_m curve (see Figure 2-b) is obtained by combining the two curves W_d and W_k (see Figure 2-a).



a) W_k, W_d and W_f weights



b) Weighting W_m

Figure 2: Frequency weighting curves [2,3]

According to these curves, the frequency range most perceptible to humans is between 4 and 8Hz for the z-direction (W_k) and between 0.5 and 2Hz for the x- and y-directions (W_d).

The choice of frequency weighting curves is made according to Table 1, depending on the direction of vibration shown in Figure 1.

Axis	Frequency weighting		Multiplying factor
	Known body position	Unknown body position	
X	W_d	W_m	$k_x = 1$
Y	W_d	W_m	$k_y = 1$
Z	W_k	W_m	$k_z = 1$

Table 1: Choice of frequency weighting curves [2,3]

The frequency weighting differs between axes of incidence, since the user perceives vibrations more easily along the *x* and *y* axes than along the *z* axis.

2.2.3 Basic method (weighted rms acceleration)

The basic method is applied in the case of continuous vibrations. It consists of calculating a weighted rms acceleration, which takes into account the reduction of response in frequency ranges of low human perception.

This acceleration is calculated per direction of vibration according to two cases:

- If N_f discrete values of acceleration over time are available, a cumulative acceleration calculation is performed as follows:

$$a_{w,d} = \left[\sum_{i=1}^{N_f} (W_i\, a_i)^2\right]^{\frac{1}{2}} \tag{1}$$

- If a continuous acceleration record is available, an integral is made over the recording time T as follows:

$$a_{w,d} = \left[\frac{1}{T} \int_0^T a^2{}_{w,d}(t)dt\right]^{\frac{1}{2}} \tag{2}$$

The total acceleration can also be obtained as follows:

$$a_v = \sqrt{k_x^2\, a^2{}_{w,x} + k_y^2\, a^2{}_{w,y} + k_z^2\, a^2{}_{w,z}} \tag{3}$$

where:

$a_{w,d}$	is the rms acceleration calculated for each direction d (x, y or z);
W_i	is a frequency weighting factor (see section 2.1.2);
a_i	is the incident acceleration in the direction of the human body;
$a_{w,d}(t)$	is the weighted rms acceleration at each time t;
k_x, k_y, k_z	are multiplying factors equal to 1 for this case (see Table 1).

2.2.4 Additional methods

A first method consists in determining a running acceleration (Maximum Transient Vibration Value - *MTVV*), in order to take into account the transient effects of the response, by calculating a rms acceleration noted $a_{w,d}(t_0)$ for small time intervals t_0 along the entire signal as follows:

$$a_{w,d}(t_0) = \left[\frac{1}{\tau} \int_{t_0-\tau}^{t_0} a^2{}_{w,d}(t)dt\right]^{\frac{1}{2}} \tag{4}$$

$a_{w,d}(t)$ being the weighted time-dependant acceleration by vibration direction.

Subsequently, the maximum value of these accelerations is evaluated:

$$MTVV = \max_{t_0}\left[a_{w,d}(t_0)\right]$$

A second method makes it possible to take into account the intermittent nature of the loading, by means of the accumulation of the vibration responses, providing more reasonable results for the evaluation of comfort. This is done by determining a vibration parameter called Vibration Dose Value (*VDV*, in $\mathrm{ms}^{-7/4}$), allowing more perceptible responses, albeit over more limited durations.

It is calculated using the following formula:

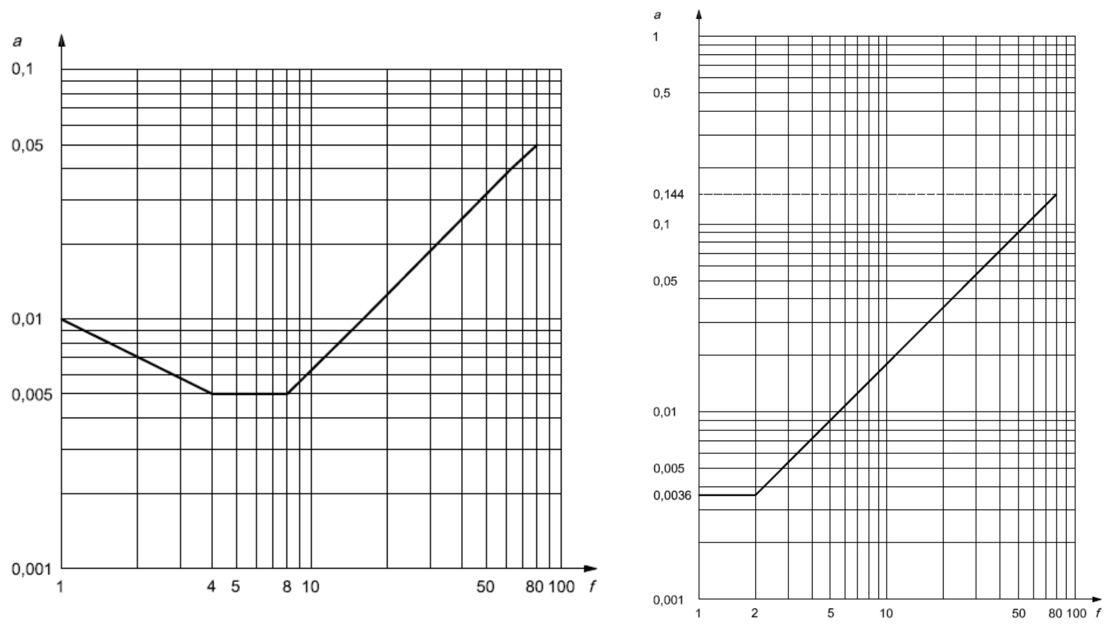
$$VDV = \left[\int_0^T a_{w,d}^4(t)dt \right]^{\frac{1}{4}} \quad (5)$$

2.3 ISO 10137 standard

ISO 10137 standard proposes serviceability acceptance criteria for vibration loading applied to buildings and footbridges, using the characteristic parameters of ISO 2631 (rms acceleration, *VDV*). This standard is limited to the analysis of occupant vibration comfort.

2.3.1 Basic method (rms acceleration)

The basic method is mainly used for continuous vibrations, defined as excitations lasting more than 30 minutes per day. For each excitation frequency, the standard presents basic acceptability curves, showing the acceleration limits beyond which the vibration is perceived by the receiver, depending on the excitation frequency and the direction of vibration shown in Figure 1. These curves are presented in Figure 3.



(a) (b)

Figure 3: Basic acceptability curve: (a) along z axis, (b) along x and y axis [4]

In order to assess the acceptability of vibration, a response factor evaluating the degree to which the vibration perception limit is exceeded, based on the determination of the rms acceleration $a_{w,rms}$ according to ISO 2631 (see section 2.1.3), is calculated as follows:

$$R = \begin{cases} \frac{a_{w,rms}}{\mathrm{a}_z} & \text{for } z \text{ axis} \\ \frac{a_{w,rms}}{\mathrm{a}_{x,y}} & \text{for } x \text{ and } y \text{ axis} \end{cases} \quad (6)$$

where the limits of perception are $a_z = 0.005\mathrm{m/s^2}$ and $\mathrm{a}_{x,y} = 0.00357\mathrm{m/s^2}$.

The response factor R must remain below the values provided in Table 2 to ensure an acceptable level of comfort for the occupants.

Use	**Time**	**Multiplying factors to base curve**	
		Continuous(1) / Intermittent(2) vibration	**Impulsive vibration**
Critical working areas	Day	1	1
	Night	1	1
Residential	Day	2 to 4	30 to 90
	Night	1.4	1,4 to 20
Quiet offices	Day	2	60 to 128
	Night	2	60 to 128
General offices, schools	Day	4	60 to 128
	Night	4	60 to 128
Workshops	Day	8	90 to 128
	Night	8	90 to 128
(1) Continuous vibration: lasts more than 30 minutes per day. (2) Intermittent vibration: occurs more than 10 times per day.			

Table 2: Limiting values of response factor [4]

2.3.2 *VDV* method

For the case of intermittent vibrations (recurring more than 10 times per day), limiting values are also given for the vibration dose value (see section 2.2.4), depending on the duration of exposure and the probability of adverse comments by the occupants. Depending on the level of adverse comments accepted by the client, the *VDV* limit values are provided in Table 3.

Residential buildings (duration)	Adverse comments		
	Low probability	Possible	Probable
16h day	0.2 à 0.4	0.4 à 0.8	0.8 à 1.6
8h night	0.13	0.26	0.51

Table 3: *VDV* limits (in $ms^{-7/4}$) [4]

The major disadvantage of this method is that it is only used on areas of the floors likely to be subject to walking action (notably corridors) which must be known in advance and well defined by the stakeholders.

2.4 DIN 4150-2 standard

2.4.1 General

The German standard DIN 4150-2 proposes a method for assessing the vibration comfort level of building occupants subjected to continuous or non-continuous vibrations with an excitation frequency range between 1 and 80Hz. This standard is not generally used in comfort assessment guidelines ([11], [12]), because it is based on velocity, which is more difficult to measure in practice than acceleration.

2.4.2 Vibration velocity

The velocity characterizes the vibration response of the structure and is directly related to the vibration energy produced over time. It should be measured at the points where the vibrations are most occurring.

The frequency domain vibration velocity $KB(f)$ is determined as follows:

$$KB(f) = \frac{V(f)}{\sqrt{\left(1+\left(\frac{f_0}{f}\right)^2\right)}} \quad (7)$$

where:

$V(f)$ is Fourier transform of the velocity response;

f_0 is a reference frequency equal to 5.6Hz;

f is the frequency of the input signal.

The weighting of $V(f)$ is performed to account for the range of vibration frequencies perceived by humans. By performing the inverse Fourier transform of $KB(f)$, we obtain the time-dependent vibration velocity $KB(t)$ which constitutes the basis of the acceptability assessment parameters described below.

2.4.3 Basic method

This method aims to determine the transient effective vibration amplitude $KB_\tau(t_0)$ obtained as follows:

$$KB_\tau(t_0) = \sqrt{\frac{1}{\tau}\int_{t=0}^{t_0} e^{-\frac{t_0-t}{\tau}}\ KB^2(t)\,dt} \tag{8}$$

where:

τ is the integration time window, taken as 0.125s;

$KB(t)$ is the vibration velocity at time t.

This gives the maximum transient effective velocity:

$$KB_{\mathrm{F_{max}}} = \max_{t_0} KB_\tau(t_0)$$

This value is to be compared with the values noted A_u and A_0 given in Table 4.

Class	Zone	Day		Night	
		A_u	A_0	A_u	A_0
1	Exclusively commercial area	0.4	6	0.3	0.6
2	Mainly commercial area	0.3	6	0.2	0.4
3	Mixed zone	0.2	5	0.15	0.3
4	Residential area	0.15	3	0.1	0.2
5	Protected area	0.1	3	0.1	0.15

Table 4: Limiting values of $KB_{\mathrm{F_{max}}}$ [5]

Three cases are possible:

- If $KB_{\mathrm{F_{max}}} \leq A_u$, the vibration is acceptable.
- If $KB_{\mathrm{F_{max}}} > A_0$, the vibration is not acceptable.
- If $A_u < KB_{\mathrm{F_{max}}} \leq A_0$, then the additional method, described below, must be used.

2.4.4 Additional method

In the case where $A_u < KB_{F_{max}} \leq A_0$, an alternative is to determine a velocity for oscillatory vibration evaluation, denoted $KB_{F_{Tr}}$, which takes into account the accumulation of vibration doses throughout the vibration period. More severe excitation for a shorter period of time can also be allowed.

This velocity is determined by the following equation:

$$KB_{F_{Tr}} = \sqrt{\frac{1}{N}\sum_{i=1}^{N} KB^2{}_{F_{Ti}}} \cdot \sqrt{\frac{T_e}{T_r}} \tag{9}$$

where:

N is the number of 30s cycles during the time of evaluation;

$KB_{F_{Ti}}$ is the maximum transient effective value during a 30s cycle;

T_e is the total time of evaluation (from 6:00 a.m. to 10:00 p.m. during the day and 10:00 p.m. to 6:00 a.m. at night);

T_r is the total time of vibration.

The velocity $KB_{F_{T_r}}$ is then compared to a value noted A_r, given in Table 5.

Class	Zone	Day	Night
		A_r	A_r
1	Exclusively commercial area	0.2	0.15
2	Mainly commercial area	0.15	0.1
3	Mixed zone	0.1	0.07
4	Residential area	0.07	0.05
5	Protected area	0.05	0.05

Table 5: Limiting values of $KB_{F_{T_r}}$ [5]

3 Floor comfort assessment guidelines

3.1 Introduction

In order to address the issue of human induced vibrations in building floors, guidelines and recommendations have been developed over the last decades, aiming at presenting methodologies for the serviceability evaluation of floor vibration performance. These guidelines are the result of European projects, or made by existing technical and scientific centres for steel construction around the world. They are based on the evaluation of comfort, given by the standards described in section 2. The three main guidelines, HiVoSS, SCI P354 and AISC/CISC DG 11 are presented in this section.

Note: A uniform notation for the parameters used by the three guidelines has been adopted in this paper. Therefore, slight differences in notation may be encountered by consulting the official versions of these guidelines.

3.2 HiVoSS Guideline

This guideline stems from the ECSC research project *"Vibrations of Floors"* [6] completed in 2004. It is the result of a dissemination of its outcomes, finalised in 2008, and proposes a spectral method, presented in form of charts, to evaluate the acceptability of human-induced vibrations in a simple way.

3.2.1 Human excitation conditions

This guideline is intended to evaluate the vibration comfort of floors subjected to vertical vibrations caused by the short duration walking of a single person. The persons receiving the vibrations may be standing, sitting or lying on the floor.

3.2.2 Calculation of the vibrational response

The response adopted by this guideline is called *OS-RMS*$_{90}$; it is the rms value of the velocity response of a single step corresponding to a single person's walking, covering 90% of people walking normally.

This value is obtained from the charts drawn up in the framework of the "VoF" project [6], based on the principles of DIN 4150-2 [5]. It depends on the natural frequency, the modal mass and the damping of the floor.

3.2.2.1 Basis of *OS-RMS90* charts

The HiVoSS method is based on a normalized step load model that depends on the person's weight Q as well as the person's pacing frequency f_p. This model is reproduced successively during the time of walking.

The ratio of the excitation force (per step) to the weight is written as follows [6]:

$$\begin{cases} \frac{F(t)}{Q} = \sum_{i=1}^{8} k_i \, t^i \text{ if } t \leq t_s \\ F(t) = 0 \text{ if } t > t_s \end{cases} \tag{10}$$

where:

$(k_i)_{1 \leq i \leq 8}$ are factors depending on f_p [6];

t_s is the duration of contact with the floor.

This method was developed by analysing 35 sets of walking frequencies and 20 person weights, resulting in 700 different load models. A probability distribution was considered for each of these parameters.

By varying the dynamic characteristics, a semi-probabilistic calculation based on the 700 load models was used to determine the *OS-RMS90* values which are represented in graphical format.

3.2.2.2 Calculation of *OS-RMS90* value

The modal mass and natural frequency are determined from a Finite Element Model of the floor, or alternatively using the analytical methods presented in appendix (section A.1).

The value of *OS-RMS90* depends also on damping, expressed in terms of damping ratio of the floor, which is determined from Table 6.

Type	Damping ratio
Damping due to the structure ξ_1	
Wood	6%
Concrete	2%
Steel	1%
Steel-concrete	1%
Damping due to the furniture ξ_2	
Traditional office for 1 to 3 persons with separation walls	2%
Paperless office	0%
Open plan office	1%
Library	1%
House	1%
School	0%
Gymnastic venue	0%
Damping due to the finishing ξ_3	
Ceiling under the floor	1%
Free floating floor	0%
Swimming screed	1%
Total damping $\xi = \xi_1 + \xi_2 + \xi_3$	

Table 6: Damping ratios [7]

For the determined modal mass, natural frequency and damping ratio, the $OS\text{-}RMS_{90}$ value can be obtained directly from the appropriate chart (one chart per damping ratio, modal mass on the abscissa, natural frequency on the ordinate).

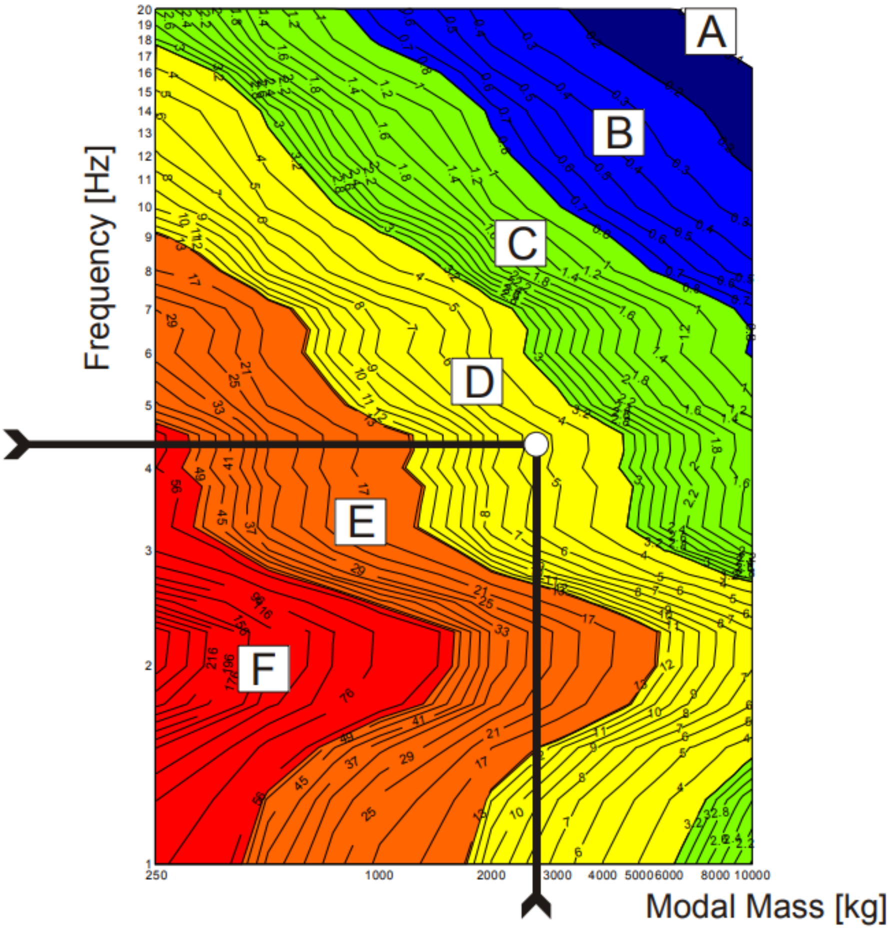


Figure 4: Example of a chart to determine the *OS-RMS₉₀* [7]

For the case of *N* natural modes, the $OS\text{-}RMS_{90}$ value for each mode *i* must be determined.

The final $OS\text{-}RMS_{90}$ value is then approximated by the following equation:

$$OS\text{-}RMS_{90} = \sqrt{\sum_{i=1}^{N}(OS\text{-}RMS_{90,i})^2} \qquad (11)$$

3.2.3 Response acceptability check

The guideline proposes acceptability levels depending on the $OS\text{-}RMS_{90}$ value obtained in the previous step and the type of use of the floor (depending on the building's function). The classes and acceptability levels are summarized in Table 7.

Class	*OS-RMS90*		Function of the building										
	Lower limit	**Upper limit**	**Critical workspace**	**Health**	**Education**	**Residential**	**Office**	**Meeting**	**Retail**	**Hotel**	**Prison**	**Industrial**	**Sport**
A	0	0.1	Green	Green	Green	Green	Green	Green	Green	Green	Green	Green	Green
B	0.1	0.2	Yellow	Green	Green	Green	Green	Green	Green	Green	Green	Green	Green
C	0.2	0.8	Red	Green	Green	Green	Green	Green	Green	Green	Green	Green	Green
D	0.8	3.2	Red	Yellow	Yellow	Green	Green	Green	Green	Green	Green	Green	Green
E	3.2	12.8	Red	Red	Red	Yellow	Yellow	Yellow	Yellow	Yellow	Yellow	Green	Green
F	12.8	51.2	Red	Red	Red	Red	Red	Red	Red	Red	Red	Yellow	Yellow

Caption:
- Green: Recommended
- Yellow: Critical
- Red: Not recommended

Table 7: Acceptability recommendations [7]

N.B.: For floors in the "Sport" category, attention is drawn to the fact that the excitation to evaluate is always caused by the walking of a single person, whereas comfort is evaluated in relation to the occupants in the neighbourhood (doing sport or spectators).

3.2.4 Conclusions on HiVoSS guideline

HiVoSS guideline provides a method for assessing vibration comfort, valid for a wide range of buildings where walking is the predominant vibration activity (residential spaces, offices, hotels, shopping centres, etc.). According to this method, the acceptability of vibration comfort is evaluated on the basis of two parameters, namely the vibration response and the use of the floor. This approach gives a fairly large scope to this method, since it does not consider the position of the walkers, nor the modal shapes, nor some additional aspects, such as the direction of excitation, the type of vibration (permanent or temporary), or the time of day, all of which can have an influence on the evaluation of the vibration comfort of individuals, which has been confirmed by Royvaran et al [8] and Muhammad et al [9]. On the other hand, for places dominated by rhythmic activities or regular machine-induced vibrations, the assessment of vibration comfort needs to be further investigated.

3.3 SCI P354 guideline

SCI P354 guideline was developed by the British Steel Construction Institute. In 1989, a simplified method, based on a brief analysis of the vibration properties of the structure, was proposed in SCI P076 by Wyatt [10]. In order to take into account the important advances in vibration analysis of floors, a new guideline, SCI P354, was published in 2007 by Smith et al. [11] and revised in 2009.

This guideline includes a general method and two simplified methods for the evaluation of the vibration comfort of floors, each focusing on a specific application area.

3.3.1 General method

This method is applied for all types of floors subjected to the action of a single person (walking or rhythmic activities). This method should be privileged when the analysis of the vibration properties of the floor is performed using a finite element model. The response study is carried out first using a basic method and, if necessary, an additional method.

3.3.1.1 Basic method

The vibration response depends on the fundamental natural frequency of the floor studied:

- For low frequency floors, the response of the structure is stationary;
- For high frequency floors, the response of the structure is transient.

The frequency of transition between these two states f_l is presented in Table 8.

Types of floor use	f_l (Hz)
General floors, open plan offices	10
Enclosed spaces (operating theatre, residential)	8
Staircases	12
Floors subject to rhythmic activities	24

Table 8Transition frequency between low/high frequency floors [11]

There are two cases:

- If $f \leq f_l$, both analyses (stationary and transient) must be performed;
- If $f > f_l$, the transient analysis is sufficient.

3.3.1.1.1 Stationary analysis

The resonant or stationary response is due to the continuous nature of the applied loading. For low-frequency floors, a gradual increase in response is perceived until its stabilization (resonance) and until the disappearance of the loading, see Figure 5.

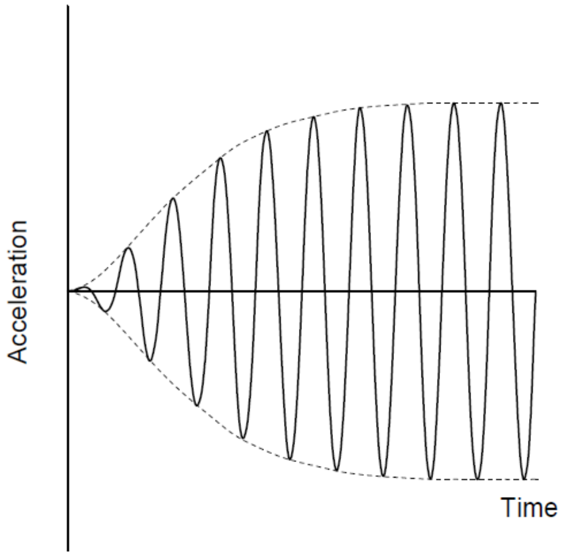


Figure 5: Stationary response [11]

Load models

The equivalent step load is expressed as a Fourier series [6] with four harmonics h:

$$F(t) = Q\left[1 + \sum_{h=1}^{4} \alpha_h \sin\left(2\pi h f_p t + \emptyset_h\right)\right] \tag{12}$$

where:

Q is the average weight of a person, taken as 746N;

α_h is the Fourier coefficient of the h^{th} harmonic;

f_p is the frequency of the human activity;

$\emptyset_h$ is the phase shift of the h^{th} harmonic.

Table 9 provides the values of the loading parameters for walking.

Harmonic h	Frequency range hf_p (Hz)	α_h	$\emptyset_h$
1	1.8 - 2.2	$0.436(hf_p - 0.95)$	0
2	3.6 - 4.4	$0.006(hf_p + 12.3)$	$-\frac{\pi}{2}$
3	5.4 - 6,6	$0.007(hf_p + 5.2)$	π
4	7.2 - 8.8	$0.007(hf_p + 2)$	$\frac{\pi}{2}$

Table 9: Parameters of the equivalent walking load [11]

As for rhythmic activities, the equivalent load is also written as a Fourier series [6], but with three harmonics h:

$$F(t) = Q\left[1 + \sum_{h=1}^{3} \alpha_h \sin\left(2\pi h f_p t + \emptyset_h\right)\right] \tag{13}$$

Each rhythmic activity is characterized by a contact ratio $\alpha_c = \frac{t_s}{T}$, where t_s is the duration of contact with the floor for one excitation and T is the corresponding total time.

Table 10 provides the values of the loading parameters for rhythmic activities depending on α_c.

$\alpha_c = \frac{t_s}{T}$	**Type of activity**	**Parameter**	***h*=1**	***h*=2**	***h*=3**
$\frac{2}{3}$	Aerobics (low impact)	α_h	$\frac{9}{7}$	$\frac{9}{55}$	$\frac{2}{15}$
		$\emptyset_h$	$-\frac{\pi}{6}$	$-\frac{5\pi}{6}$	$-\frac{\pi}{2}$
$\frac{1}{2}$	Aerobics (high impact)	α_h	$\frac{\pi}{2}$	$\frac{2}{3}$	0
		$\emptyset_h$	0	$-\frac{\pi}{2}$	0
$\frac{1}{3}$	Normal jumping	α_h	$\frac{9}{5}$	$\frac{9}{7}$	$\frac{2}{3}$
		$\emptyset_h$	$\frac{\pi}{6}$	$-\frac{\pi}{6}$	$-\frac{\pi}{2}$

Table 10: Parameters of rhythmic activity loads [11]

Weighted rms acceleration

The weighted rms acceleration $a_{w,rms,e,r,h,n}$ for a natural mode n depends on the excitation point e, the response point r and the excitation harmonic h, and is expressed as:

$$a_{w,rms,e,r,h,n} = \mu_{e,n}\, \mu_{r,n}\, \frac{F_h}{M_n\sqrt{2}}\, D_{n,h}\, W_h \qquad (14)$$

where:

$\mu_{e,n}$ is the amplitude of the normalized deformed shape of the excitation point e;

$\mu_{r,n}$ is the amplitude of the normalized deformed shape of the response point r;

$F_h = \alpha_h Q$ is the exciting force of the h^{th} harmonic (depending on the type of activity);

M_n is the modal mass of the n^{th} mode;

$D_{n,h}$ is the dynamic magnification factor applied to the acceleration response (with $\beta_n = \frac{f_p}{f_n}$):

$$D_{n,h} = \frac{(h\beta_n)^2}{\sqrt{(1-(h\beta_n)^2)^2+(2\xi h\beta_n)^2}}$$

W_h is the frequency weighting factor corresponding to hf_p.

In the case where amplitudes of deformed shapes are unknown, $\mu_{e,n} = \mu_{r,n} = 1$ are conservatively taken.

The damping ratio ξ is shown in Table 11.

Type of floor finishes	Damping ratio
Fully welded steel structures (staircases)	0.5%
Completely bare floors or floors when only a small amount of furnishings is present	1.1%
Fully fitted-out and furnished floors in normal use	3%
Floors with partitions interrupt the relevant mode(s) of vibration	4.5%

Table 11: Damping ratio [11]

The frequency weighting W_h is obtained from one of the weighting curves in BS 6841 according to Table 12.

Type of use	Vibration axis	Category	Weighting curve
Critical working areas (hospital operating theatres, precision laboratories)	z	Vision - hand control	W_g
	x-y	Perception	W_d
Residential, offices, wards, general laboratories, consulting rooms	z	Discomfort	W_b
	x-y	Discomfort	W_d
Workshops and circulation spaces	z	Discomfort	W_b
	x-y	Discomfort	W_d

Table 12: Choice of frequency weighting curves [11]

The associated weighting curves are shown in Figure 6.

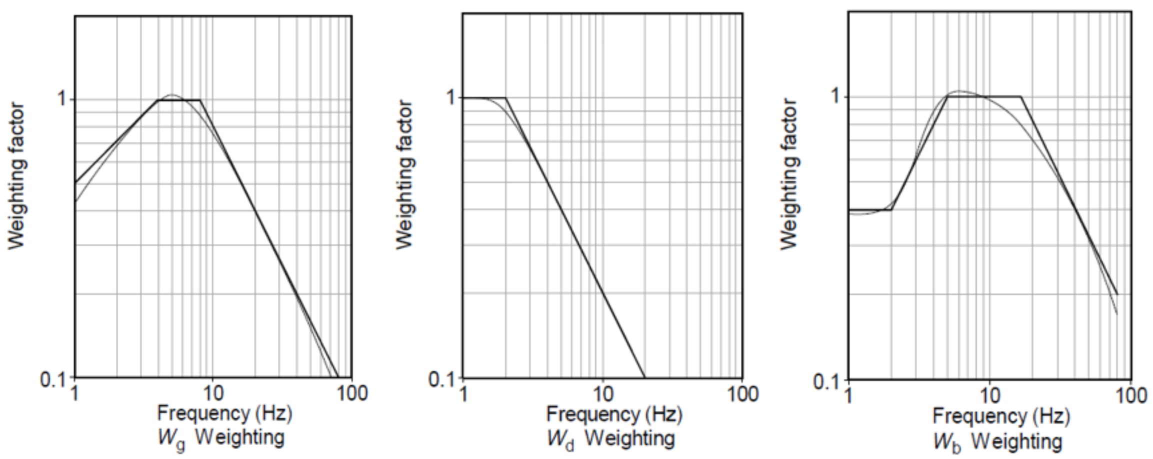


Figure 6Frequency weighting curves [11]

For walking, when the length of the walking path is limited, the time of excitation is limited. As a result, the floor does not fully reach the resonant state during walking, which leads to a reduction in the stationary response.

This reduction is taken into account by multiplying the acceleration $a_{w,rms,e,r,h,n}$ by a resonance build-up factor ρ given by equation (15):

$$\rho = 1 - e^{\left(\frac{-2\pi\xi L_p f_p}{v}\right)} \qquad (15)$$

where:

L_p is the length of the walking path;

f_p is the pacing frequency;

v is the walking velocity, determined for $1.7\text{Hz} \leq f_p \leq 2.4\text{Hz}$ by:

$$v = 1.67{f_p}^2 - 4.83 f_p + 4.5 \qquad (16)$$

The above calculation shall be made with respect to the dominant modes of vibration, the natural frequency of which is lower than the transition frequency (given in Table 8) increased by 2Hz.

The weighted rms acceleration for all dominant natural modes is then calculated by combining the responses of the ***H*** harmonic responses according to the ***N*** dominant modes of vibration as follows:

$$a_{w,rms,e,r} = \sqrt{\sum_{h=1}^{H}\left[\sum_{n=1}^{N}\left(a_{w,rms,e,r,h,n}\right)\right]^2} \qquad (17)$$

3.3.1.1.2 Transient analysis (only for walking)

This analysis reflects the intermittent nature of the load, in which the high frequency floor behaves as if the load were composed of a series of consecutive pulses, see Figure 7.

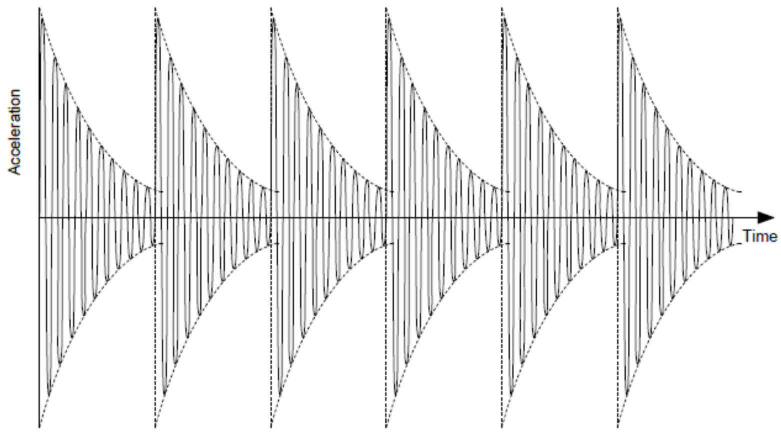


Figure 7: Transient response [11]

In this case, the impact induced by each step is modelled by an impulsive loading expressed as follows:

$$F_I = 60\,\frac{f_p^{1.43}}{f_n^{1.3}}\,\frac{Q}{700} \qquad (18)$$

where:

f_p is the pacing frequency;

f_n is the natural frequency of the n^{th} mode;

Q is the average weight of a person, taken as 746N.

The maximum acceleration for the studied natural mode n is determined by equation (19):

$$a_{w,max,e,r,n} = 2\pi f_n \sqrt{1-\xi^2}\, \mu_{e,n}\, \mu_{r,n}\, \frac{F_I}{M_n}\, W_n \tag{19}$$

where:

F_I is the impulsive exciting force;

W_n is the appropriate weighting factor with respect to f_n.

The other terms are given by equation (14).

In this case, the dominant natural modes are those whose natural frequency does not exceed the double of the fundamental natural frequency.

The total acceleration of all these modes will then be:

$$a_{w,e,r}(t) = \sum_{n=1}^{N} a_{w,max,e,r,n}\, e^{-\xi 2\pi f_n t} \sin(2\pi f_n \sqrt{1-\xi^2}\, t) \tag{20}$$

The associated rms acceleration $a_{w,rms,e,r}$ is deduced as follows:

$$a_{w,rms,e,r} = \sqrt{\frac{1}{T}\int_0^T a^2{}_{w,e,r}(t)} \tag{21}$$

where $T = \frac{1}{f_p}$.

3.3.1.1.3 Acceptability check

In accordance with BS 6472, which is very close to ISO 10137, a response factor R is determined with respect to the perception base curve along the vibration axis using equation (6), recalled below:

$$R = \begin{cases} \dfrac{a_{w,rms}}{\mathrm{a}_z} & \text{for } z \text{ axis} \\ \dfrac{a_{w,rms}}{\mathrm{a}_{x,y}} & \text{for } x \text{ and } y \text{ axis} \end{cases}$$

$a_{w,rms}$ being the weighted rms acceleration calculated for the stationary or transient response, with $a_z = 0.005\text{m/s}^2$ and $a_{x,y} = 0.00357\text{m/s}^2$.

The response factor R is to be compared with the multiplying factors related to the perception limit.

BS 6472 defines multiplying factors for common (non-hospital) occupancies shown in Table 13.

Location	Time	Multiplying factor	
		Continuous vibration(1)	Impulsive vibration(2)
Critical working areas	Day	1	1
	Night	1	1
Residential	Day	2 to 4	60 to 90
	Night	1.4	20
Offices	Day	4	128
	Night	4	128
Workshops	Day	8	128
	Night	8	128
(1) Time: 16 hours during the day and 8 hours at night (2) For more than 3 occurrences			

Table 13: Limiting values of factor R (general locations) [11]

For offices, contrary to the value of 4 given in Table 13, the guideline recommends a multiplying factor equal to 8.

For hospital environment, the multiplying factors are taken from HTM 08-01 and presented in Table 14.

Type of room	Multiplying factor
Operating theatres, precision laboratories, audiometric testing booth	1
Wards	2
General laboratories, treatment areas	4
Offices, consulting rooms	8

Table 14: Limiting values of factor R (hospitals) [11]

For car parks, a multiplying factor $R = 65$ is recommended (with a damping ratio of 1.1%).

3.3.1.2 Additional method

This method is used when the comfort limits are exceeded according to the basic method and is based on the principles of ISO 10137 (see section 2.3.2), with a vibration dose value VDV (in $\mathrm{ms}^{-\frac{7}{4}}$) given by the following equation:

$$VDV = 0.68\, a_{w,rms} \sqrt[4]{n_a T_a} \tag{22}$$

where:

$a_{w,rms}$ is the weighted rms acceleration in the stationary (see equation (17)) or transient (see equation (21)) case;

n_a is the number of occurrences of the activity for one exposure time slot (16h during the day, or 8h at night);

T_a is the duration of human activity ($T_a = \frac{L_p}{v}$).

The obtained value shall be compared with values given below.

In the general case, BS 6472 defines *VDV* limits according to Table 15.

Residential buildings (exposure time slot)	**Adverse comments**		
	Low probability	**Possible**	**Probable**
16h day	0.2 à 0.4	0.4 à 0.8	0.8 à 1.6
8h night	0.13	0.26	0.51

Table 15: *VDV* limits (buildings) [11]

It should be noted that the additional method is not applicable for precision work (1st row of Table 14). For other usages, the acceptability limits in Table 16, taken from HTM 08-01 are used.

Type of room	**VDV limit ($m/s^{1.75}$)**
Wards, residential (day)	0.2
General laboratories, offices	0.4
Workshops	0.8

Table 16: *VDV* limits (other usages) [11]

In case where the number n_a is unknown, another alternative to this method is to use equation (23) to determine a maximum number of occurrences on the floor during the considered exposure time:

$$n_{a,max} = \frac{1}{T_a}\left[\frac{VDV_{max}}{0.68\, a_{w,rms}}\right]^4 \qquad (23)$$

3.3.2 1st simplified method

This simplified method is applied for ordinary floors subjected to normal walking activity and covers floors where a general response estimate is desired, or where the modal properties of the floor are not determined numerically. Analytical formulae are proposed for this purpose for the natural frequency and the modal mass of the fundamental mode and are presented in appendix (section A.2).

3.3.2.1 Basic method

The basic method is only valid for floors with a natural fundamental frequency higher than 3Hz and is also developed for stationary or transient responses (see section 3.3.1).

3.3.2.1.1 Stationary analysis (3 Hz ≤ f_1 ≤ 10 Hz)

The rms acceleration for the stationary analysis is written as follows:

$$a_{w,rms} = \mu_e\, \mu_r\, \frac{0.1Q}{2\sqrt{2}\, M_1\, \xi}\, W_1\, \rho \qquad (24)$$

where:

μ_e is the amplitude of the normalized deformed shape of the excitation point e;

μ_r is the amplitude of the normalized deformed shape of the response study point r;

Q is the average weight of a person, taken as 746N;

M_1 is the modal mass of the fundamental mode;

ξ is the damping ratio of the floor;

W_1 is the appropriate weighting factor with respect to f_1;

f_1 is the natural frequency of the fundamental mode;

ρ is the resonance build-up factor, obtained from equation (15).

In the case where amplitudes of deformed shapes are unknown, $\mu_e = \mu_r = 1$ are conservatively taken.

3.3.2.1.2 Transient analysis ($f_1 > 10$ Hz)

For a transient analysis, the rms acceleration becomes:

$$a_{w,rms} = 2\pi \, \mu_e \, \mu_r \, \frac{185}{M_1 \, f_1^{0.3}} \, \frac{Q}{700} \, \frac{W_1}{\sqrt{2}} \qquad (25)$$

In the case of an unknown direction of excitation, W_1 can be determined from Figure 8.

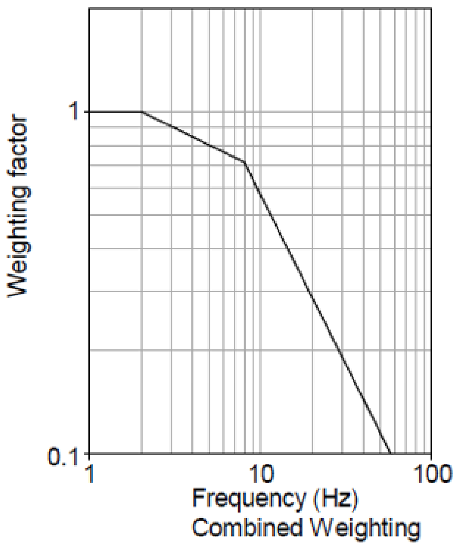


Figure 8: Frequency weighting curve for an unknown direction of excitation [11]

For the acceptability check, the response factor is used (see section 3.3.1.1.3):

$$R = \frac{a_{w,rms}}{0.005} \qquad (26)$$

3.3.2.2 Additional method

The number of crossing occurrences on a floor during the exposure time (16h during the day and 8h at night) is compared with the maximum number of occurrences, obtained by equation (23).

The values of VDV_{max} are similar to those in Section 3.3.1.2.

A second alternative is to determine this number as a function of the response factor and the length of the corridor from Figure 9.

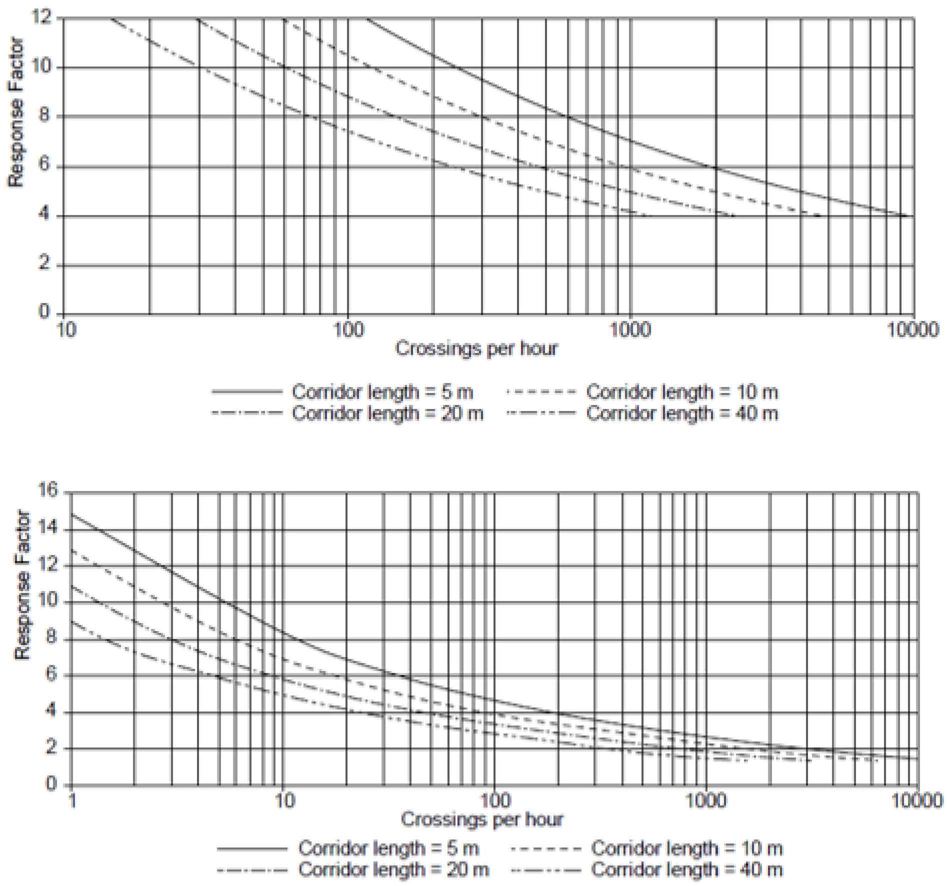


Figure 9: Maximum number of occurrences for vibrations in the z-axis direction for 16h during the day (top) and in the x and y-axis direction for 8h at night (bottom) [11]

3.3.3 2nd simplified method

This method covers floors subject to the same conditions as the 1st simplified method, but is limited to lightweight floors. Lightweight floors covered by this method have joists composed of cold-formed, Z or C-shaped profiles.

3.3.3.1 Stiffness check

Two conditions must be met for these types of floors, namely:

- A fundamental natural frequency exceeding 8Hz in the general case and 10Hz in the case of corridors, under permanent loads plus a load equal to $0.3kN/m^2$, to move away from the resonant state;
- A limited static deflection under point load of 1kN to ensure sufficient stiffness of the floor. It must be less than the limits given in Table 17.

Span (m)	3.5	3.8	4.2	4.6	5.3	6.2
Limit (mm)	1.7	1.6	1.5	1.4	1.3	1.2

Table 17: Deflection limits in function of floor span [11]

3.3.3.2 Evaluation of the vibration comfort

This evaluation is to be performed in the case where the determination of vibration response is required by the stakeholders. Section A.2 of the appendix provides analytical expressions for determining the natural frequency and the modal mass of the fundamental mode in this particular case.

3.3.3.2.1 Basic method

Since a high natural frequency is required for these floors, only the transient response is considered. The weighted rms acceleration is expressed as in the case of the 1st simplified method (see equation (25)).

The response factor R calculated with equation (6) must be less than 16 for all types of floor's usage.

3.3.3.2.2 Additional method

The number of people crossing the floor during the time of exposure (16h during the day and 8h at night) is compared with the maximum number, obtained by equation (23).

The values for VDV_{max} (for a low probability of reporting discomfort) are given below:

- Residential building (16h day): 1.6;
- Residential building (8h night): 0.51.

3.3.4 Conclusions on the SCI P354 guideline

The SCI P354 guideline, based on the principles of ISO 2631 and ISO 10137 standards, is the most comprehensive document dealing with vibration comfort in steel and composite structures nowadays. A wide range of structures, from office buildings, to hospitals and car parks, as well as human loads, such as walking and rhythmic activities, are covered. The general method, which requires the use of a finite element model to extract the vibration properties of the structure, is relatively accurate but will be quite time consuming to implement. The 1st simplified method considers only the fundamental natural mode of vibration and uses a Fourier coefficient equal to 0.1, resulting in a fairly safe estimate of acceleration responses. Through the analysis of more than 50 floors, it has been shown by Royvaran et al [8]that this simplified method is quite conservative.

3.4 AISC/CISC DG 11 guideline

This guideline, like the SCI P354 guideline, is applicable to floor vibrations under the action of walking or rhythmic activities of a single person. It was initially established in collaboration between the American Institute of Steel Construction and the Canadian Institute of Steel Construction in 1997, and revised in 2003.

Modal finite element analysis is also favoured in this guideline, while analytical expressions for determining the properties of the fundamental vibrational mode when needed are presented in appendix (section A.3).

3.4.1 Walking activity

The method described below is applied for floors with a fundamental frequency above 3Hz, which are subject to walking action. As with the SCI P354 guideline, the evaluation of comfort depends on the fundamental frequency of the floor under consideration.

The equivalent walking load is decomposed into a 4-harmonic Fourier series as follows:

$$F(t) = Q\left[1 + \sum_{h=1}^{4} \alpha_h \cos\left(2\pi h f_p t\right)\right] \tag{27}$$

where:

Q is the average weight of a person, taken as 700N;

α_h is the Fourier coefficient of the h^{th} harmonic;

f_p is the pacing frequency.

Table 18 provides the parameters of this decomposition.

Harmonic	f_p **(Hz)**	α_h
1	1.6-2.2	0.5
2	3.2-4.4	0.2
3	4.8-6.6	0.1
4	6.4-8.8	0.05

Table 18: Walking load parameters [12]

3.4.1.1 Response study

The maximum floor acceleration, as a proportion of the acceleration of gravity g, is determined by equation (28):

$$\frac{a_p}{g} = \frac{P_0 \exp(-0.35 f_n)}{\xi W_n} \tag{28}$$

where:

$P_0 = 0.29$kN for buildings;

f_n is the natural frequency of the considered mode;

ξ is the damping ratio of the floor;

W_n is the modal weight of the floor (related to the modal mass of the considered mode).

The damping ratio ξ to be used is taken from Table 19.

Type of floor finishes	ξ
Floors with few non-structural components (offices, residences, shopping malls, places of worship)	0.02
Floors with non-structural components and small demountable partitions (modular offices)	0.03
Floors with full-height partitions	0.05

Table 19: Damping ratio [12]

This acceleration is to be compared with the acceleration limit $\frac{a_0}{g}$, obtained on the basis of the acceptability curves of ISO 2631-2 (version 1989), shown in Figure 10.

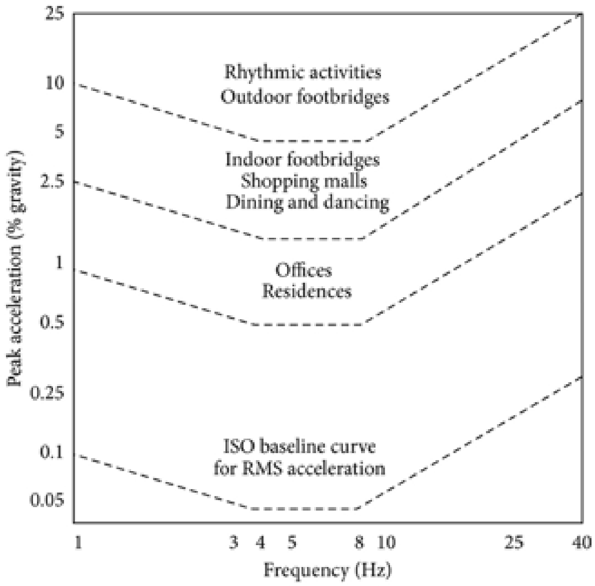


Figure 10Vibration acceptability curves [12]

The values of $\frac{a_0}{g}$ are taken conservatively by extending the horizontal line of each of the above curves for all frequencies, resulting in the following values:

- For offices, residences and places of worship: $\frac{a_0}{g} = 0.5\%$;
- For shopping centres: $\frac{a_0}{g} = 1.5\%$.

3.4.1.2 Case of high frequency floors

For floors with a fundamental natural frequency higher than 9Hz, in addition to the above check, the risk related to the effects of the transient response which may be induced by the harmonics of walking must be eliminated.

Hence, the static deflection of the floor calculated under a point load of 1kN must not exceed 1mm.

3.4.2 Rhythmic activities

For the case of rhythmic activities, the associated load is decomposed into a 3-harmonic Fourier series as follows:

$$F(t) = Q\left[1 + \sum_{h=1}^{3} \alpha_h \cos\left(2\pi h f_p t\right)\right] \tag{29}$$

where:

Q is the average weight of a person, taken as 700N;

α_h is the Fourier coefficient of the h^{th} harmonic;

f_p is the excitation frequency of the considered rhythmic activity.

Table 20 provides the parameters for this loading.

Harmonic	Jumping (aerobics)		Dancing		Sports event or lively concert	
	$f_p\ (Hz)$	α_h	$f_p\ (Hz)$	α_h	$f_p\ (Hz)$	α_h
1	2-2.75	1.5	1.5-3	0.5	1.5-3	0.25
2	4-5.5	0.6	-	-	3-5	0.05
3	6-8.25	0.1	-	-	-	-

Table 20: Parameters of rhythmic activity loads [12]

3.4.2.1 Natural frequency limitation

For each harmonic excitation, it must be checked that the maximum acceleration remains below $\frac{a_0}{g}$, which is expressed as a condition on the natural frequency f_1 of the fundamental mode, presented in equation (30):

$$f_1 \geq f_p \sqrt{1 + \frac{k}{\frac{a_0}{g}} \frac{\alpha_h w_p}{w_t}} \tag{30}$$

where:

$k = 2;\ \frac{a_0}{g} = 5\%$ for jumping and aerobics;

$k = 1.3;\ \frac{a_0}{g} = 2\%$ for dancing;

$k = 1.7;\ \frac{a_0}{g} = 5\%$ for sports event or lively concert;

w_p is the maximum surface weight of individuals (related to their occupying surface), according to the h^{th} harmonic;

w_t is the surface weight of the floor including the weight of people.

The excitation frequency f_p and weight w_p are given in Table 21.

Harmonic	Jumping (aerobics)		Dancing		Sports events or concerts	
	f_p (Hz)	w_p (kPa)	f_p (Hz)	w_p (kPa)	f_p (Hz)	w_p (kPa)
1	2.75	0.2	3	0.6	3	1.5
2	5.5	0.2	-	-	5	1.5
3	8.25	0.2	-	-	-	-

Table 21: Parameters f_p and w_p for rhythmic activities [12]

3.4.2.2 Response study

As the frequency limitation given in 3.4.2.1 can be quite penalizing, a response study can be considered, in particular, when the criterion of the previous method is not verified for some harmonics. For a natural mode n (frequently the fundamental one), three possible cases are to be considered for each harmonic (with excitation frequency f_p):

- If $0.83\ f_p \ \leq f_n \leq 1{,}2\ f_p$, the response is resonant and the maximum acceleration is:

$$\frac{a_p}{g} = \frac{k}{2\xi}\frac{\alpha_h\, w_p}{w_t} \tag{31}$$

- If $f_n > 1{,}2\ f_p$, the response is transient and the maximum acceleration is:

$$\frac{a_p}{g} = \frac{k}{\left(\frac{f_n}{f_p}\right)^2 - 1}\ \frac{\alpha_h\, w_p}{w_t} \tag{32}$$

- If $f_n < 0.83\ f_p$, the following equation must be applied:

$$\frac{a_p}{g} = \frac{k\frac{\alpha_h\, w_p}{w_t}}{\sqrt{\left[\left(\frac{f_n}{f_p}\right)^2 - 1\right]^2 + \left[2\xi\frac{f_n}{f_p}\right]^2}} \tag{33}$$

The damping ratio ξ is taken equal to 0.06.

The set of accelerations determined for H harmonics is then combined to give the total acceleration by equation (34):

$$\frac{a_m}{g} = \left[\sum_{p=1}^{H}\left(\frac{a_p}{g}\right)^{1.5}\right]^{\frac{1}{1.5}} \tag{34}$$

This acceleration must be less than the following limits:

- Dancing: 1.5%;
- Jumping - sport event or lively concert: 5%.

3.4.3 Conclusions on the AISC/CISC DG 11 guideline

The walking method of this guideline neglects the properties of excitation harmonics in the maximum response calculation, and the rhythmic activity method is limited to the fundamental mode of vibration in most cases, which may result in an underestimation of the acceleration response in general. On the other hand, in some cases, the calculation of the peak acceleration can result in a too severe judgment of the vibration comfort (case of local *maxima*). The analysis of Royvaran et al [8]**Erreur ! Source du renvoi introuvable.**on the feedback from the usage of offices and the application of this method to more than 50 floors in the United States shows nevertheless that it is quite matching with the users' experience. This is also confirmed by the comparative analysis findings of Muhammad et al [9].

3.5 Common limitations of guidelines

The three comfort evaluation guidelines presented in this paper, although widely used and efficient in many cases of floor vibration, have theoretical and usage limitations, both specific to each guideline (as described above) and common to all of them.

The first common limitation is that all guidelines consider human loading as a deterministic periodic action, with at most a semi-probabilistic characterization, which is far from being the case in reality. Indeed, each person in movement produces a load different from the others in terms of amplitude and frequency (intervariability) and the same person cannot also reproduce exactly the same excitation during his movement (intravariability). Muhammad et al. [9] highlight this problem in the analysed guidelines and advocate the need to initiate vibration analysis in a probabilistic framework for the development of more robust load models in future standards.

The second limitation is the assumption that the floor is loaded by a single person in service conditions, which is only the case for a limited category of floors (certain offices and residential spaces). The case of crowd loading is not explicitly studied by the guidelines (reference to equivalent static loads in the SCI P354 guideline), although it is very frequent in many structures (commercial and sports venues, railway stations, etc.). Vijayan et al [13] confirm the impact of the group effect (especially for slender floors) and invite to take it into account in the design of floors. In practice, the SCI P354 guideline proposes Fourier coefficients for the case of a group performing "normal jumping" and appendix A of the ISO 10137 standard provides coefficients that take into account the effect of reduction of the total force of a crowd with respect to the sum of the forces of the individuals, for the case of walking as well as for rhythmic activities. These coefficients can be adopted in the load models of SCI and AISC guidelines to have a first estimate of the floor's acceleration due to a crowd action, without being able to use them in HiVoSS guideline based on a spectral approach with a fixed load model.

Given that these shortcomings can lead to unsafe (uncomfortable, as a matter of fact) results, further consideration of the randomness of human loading, as well as the effect of a group of people exciting floors (with a synchronization study), would be key elements to promote in the development of future methods for evaluating the vibration comfort of floors.

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Appendix: Simplified calculation of modal properties

This appendix presents analytical methods proposed by the three comfort evaluation guidelines to determine the natural frequency and modal mass of the fundamental mode of vibration, when finite element analysis of the floor cannot be performed.

The reader is referred to the recommendations of Eurocode 4 for the calculation of the composite geometrical and inertial parameters mentioned through analytical methods.

A.1 HiVoSS guideline

A.1.1 General considerations

A mass per unit area should be taken into account for the floor, which is the mass corresponding to the floor's self-weight plus other permanent loads (furniture, finishes). When the designer can be sure of the existence of the latter loads, semi-permanent loads should be taken into account up to a limit of 10% of the nominal permanent loads. A fraction of 10 to 20% of the mass due to live loads is also added, with a minimum representative mass of a person having 30kg weight for very light floors. In addition, an elastic modulus of concrete increased by 10% over the static secant modulus must be used.

A.1.2 Natural frequency

For the natural frequency, three methods are presented for composite floors: the direct method, the Dunkerley method and the arrow method.

A.1.2.1 Direct method

The geometrical characteristics of the composite floor are shown in Figure 11.

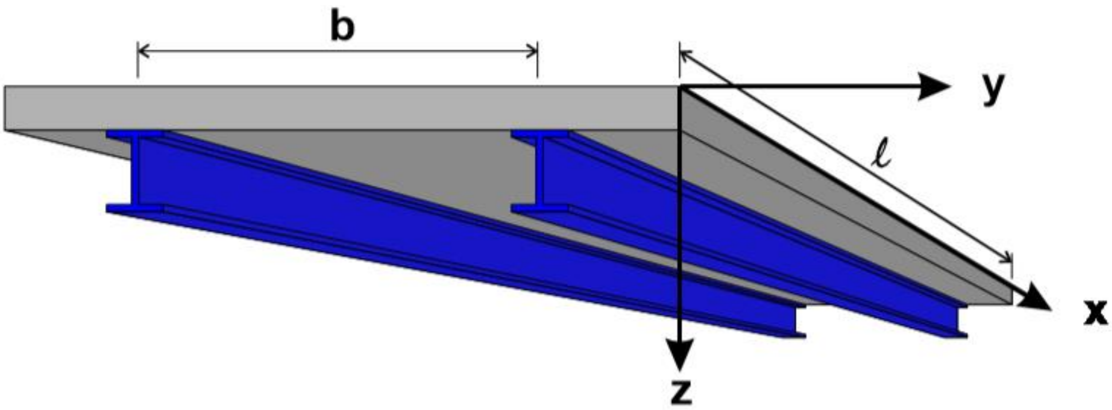


Figure 11: Geometry of the composite floor [7]

The natural frequency is obtained as follows:

$$f_1 = \frac{\pi}{2}\sqrt{\frac{E_y\,I_y}{\mu_l l^4}}\sqrt{1+\left[2\left(\frac{b}{l}\right)^2+\left(\frac{b}{l}\right)^4\right]\frac{E_x\,I_x}{E_y\,I_y}} \tag{35}$$

where:

$E_y\, I_y$ is the bending stiffness of the composite floor in the *y* direction;

$E_x\, I_x$ is the bending stiffness of the composite floor in the *x* direction;

μ_l is the linear mass of the composite beam (with slab centre distance width).

A.1.2.2 Dunkerley's method

This method superimposes two natural modes: an isotropic slab's mode (s - *slab*) and a composite beam's mode (b - *beam*). The total natural frequency is written as follows:

$$\frac{1}{{f_1}^2} = \frac{1}{{f_s}^2} + \frac{1}{{f_b}^2} \tag{36}$$

Keeping the notations in Figure 11, the frequencies f_s and f_b are obtained by the following equations:

$$f_s = \frac{\alpha}{2\pi l^2}\sqrt{\frac{E_c\, h^3}{12\,\mu\,(1-\nu^2)}} \tag{37}$$

$$f_b = \frac{\beta}{\pi}\sqrt{\frac{3E_a I_b}{\gamma \mu_l l^4}} \tag{38}$$

where:

h is the thickness of the slab;

E_c is the elastic modulus of concrete;

E_a is the elastic modulus of steel;

I_b is the moment of inertia of the composite beam;

ν is the Poisson's ratio of concrete (about 0.2);

μ is the mass per unit area of the slab (in kg/m^2);

μ_l is the linear mass of the beam (in kg/m, with slab centre distance width).

The values of parameters α, β and γ depend on the boundary conditions of slabs and beams, according to Tables 22 and 23.

Boundary conditions	$\boldsymbol{\alpha}$ **(with** $\boldsymbol{\lambda = \frac{l}{b}}$**)**
Simply supported on 4 edges	$\alpha = 1.57\,(1 + \lambda^2)$
Clamped on 2 opposite edges	$\alpha = 1.57\sqrt{1 + 2.5\,\lambda^2 + 5.14\,\lambda^4}$
Clamped on 3 edges	$\alpha = 1.57\sqrt{5.14 + 2.92\,\lambda^2 + 2.44\,\lambda^4}$
Clamped on 1 edge	$\alpha = 1.57\sqrt{1 + 2.33\,\lambda^2 + 2.44\,\lambda^4}$
Clamped on 2 perpendicular edges	$\alpha = 1.57\sqrt{2.44 + 2.72\,\lambda^2 + 2.44\,\lambda^4}$
Clamped on 4 edges	$\alpha = 1.57\sqrt{5.14 + 3.13\,\lambda^2 + 5.14\,\lambda^4}$

Table 22: Parameter α [7]

Boundary conditions	β	γ
Clamped (both supports)	4	0.37
Clamped-simply supported	2	0.2
Simply supported (both supports)	2	0.49
Cantilever	0.5	0.24

Table 23: Parameters β and γ [7]

A.1.2.3 Self-weight method

The natural frequency of a floor can be approximated as follows:

$$f_1 = \frac{18}{\sqrt{\delta_{max}}} \tag{39}$$

Where δ_{max} (in mm) is equal to the sum of the deflection of the composite beam and that of the slab, the latter is determined by assuming zero deflection for the beam.

A.1.3 Modal mass

The modal mass is based on the determination of the deflections of the slab and the composite beam, according to the following equation:

$$M_1 = M_{tot} \left[\frac{\delta_x^2 + \delta_y^2}{2\delta^2} + \frac{8}{\pi^2} \frac{\delta_x \, \delta_y}{\delta^2}\right] \tag{40}$$

where:

M_{tot} is the total mass of the floor;

δ_x is the deflection of the composite beam;

δ_y is the deflection of the slab assuming zero deflection for the beam;

δ is the total deflection: $\delta = \delta_x + \delta_y$.

When the lateral stiffness of the slab is negligible compared to that of the composite beam, the calculation can be limited to the modal mass of the composite beam alone, determined by one of the equations presented in Table 24 (μ_l being its linear mass).

Boundary conditions	Modal mass
Clamped (both supports)	$M_1 = 0.41 \, \mu_l \, l$
Clamped-simply supported	$M_1 = 0.45 \, \mu_l \, l$
Simply supported (both supports)	$M_1 = 0.5 \, \mu_l \, l$
Cantilever	$M_1 = 0.64 \, \mu_l \, l$

Table 24: Modal mass of a beam [7]

A.2 SCI P354 guideline

The simplified equations presented in this section are never used in the general method presented in section 3.3.1.

A.2.1 General considerations

The following loads must be taken into account for calculating the mass per unit area when determining the dynamic characteristics:

- Self-weight of the floor;
- Weight of non-structural elements (ceilings and equipment);
- Semi-permanent loads (only if the previous weight is present, excluding rhythmic activity floors);
- 10% of live loads.

Table 25 shows the recommended values for the dynamic modulus of concrete.

Type of concrete	Dry weight (kN/m³)	Dynamic modulus (GPa)
Ordinary concrete	23.5	38
Lightweight concrete	18	22

Table 25: Concrete properties [11]

A.2.2 Natural frequency

The methods presented in this section apply to the simplified methods in the SCI P354 guideline (see sections 3.3.2 and 3.3.3).

The natural frequency of the fundamental mode depends on two deformation modes:

- Mode of secondary beams, resulting in a deflection δ_b (in mm);
- Mode of primary beams, resulting in a deflection δ_p (in mm).

The natural frequency is obtained according to the following expression:

$$f_1 = Min\left(\frac{18}{\sqrt{\delta_b}}, \frac{18}{\sqrt{\delta_p}}\right) \tag{41}$$

To determine these deflections, there are two cases:

- The spans are independent and the displacements depend on the floor configurations shown in Table 26.

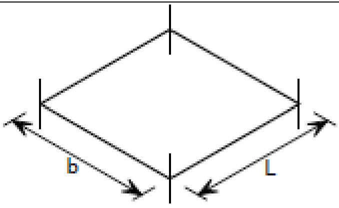
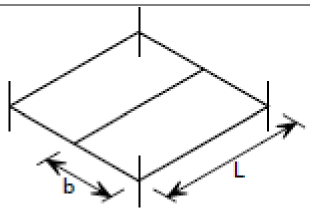
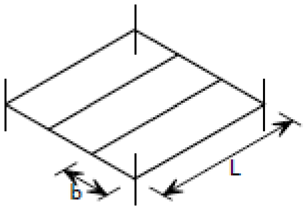
Configuration	δ_b	δ_p
	$\frac{mgb}{384\,E}\left(\frac{5L^4}{I_b}+\frac{b^3}{I_s}\right)$	-
	$\frac{mgb}{384\,E}\left(\frac{5L^4}{I_b}+\frac{b^3}{I_s}\right)$	$\frac{mgb}{384\,E}\left(\frac{64\,b^3L}{I_p}+\frac{L^4}{I_b}+\frac{b^3}{I_s}\right)$
	$\frac{mgb}{384\,E}\left(\frac{5L^4}{I_b}+\frac{b^3}{I_s}\right)$	$\frac{mgb}{384\,E}\left(\frac{368\,b^3L}{I_p}+\frac{L^4}{I_b}+\frac{b^3}{I_s}\right)$

Table 26: Deflections for primary and secondary beams [11]

where:

m is the mass per unit area of the floor;

g is the acceleration of gravity;

E is the Young's modulus of the steel;

I_b is the moment of inertia of the composite secondary beam;

I_p is the moment of inertia of the composite primary beam;

I_s is the moment of inertia of the slab (per meter).

- The spans are continuous and a correction factor, presented in Table 27, is applied to the displacements obtained for the case of independent spans.

Number of continuous spans	**Correction factor**
2	$\dfrac{0.4+\frac{I_M L_S}{I_S L_M}\left(1+0.6\,\frac{L_S^2}{L_M^2}\right)}{1+\frac{I_M L_S}{I_S L_M}}$
3	$\dfrac{0.6+2\,\frac{I_M L_S}{I_S L_M}\left(1+1.2\,\frac{L_S^2}{L_M^2}\right)}{3+2\,\frac{I_M L_S}{I_S L_M}}$

Table 27: Correction factor for continuous spans [11]

In Table 27, I and L are respectively the moment of inertia and the span of the studied span. M is the index of the longest span and S is the index of the shortest span.

For the case of the 2nd simplified method, lightweight floors are generally much more flexible in the direction of the secondary beams. Therefore, the natural frequency is calculated only with respect to this mode:

$$f_1 = \frac{18}{\sqrt{\delta_b}} \tag{42}$$

A.2.3 Modal mass

The modal mass is obtained according to the following expression:

$$M_1 = m\, L_{eff}\, S \tag{43}$$

where:

m is the total mass per unit area of the floor;

L_{eff} is the effective length of the floor;

S is the effective width of the floor.

The effective length and width depend on the configuration of the slab and beam, as well as the type of floor, whether it is ordinary (using the 1st simplified method) or lightweight (using the 2nd simplified method).

A.2.3.1st simplified method

In this method, two configurations are considered:

- 1st configuration: the slab overcomes the beams (case of common floors);
- 2nd configuration: the slab is located between the beams flanges (case of thin floors).

The calculation of dimensions L_{eff} and S for these two configurations is detailed below.

A.2.3.1.1st configuration: slab over beams

In this configuration, the slab is cast on the top flanges of beams, resulting in the geometrical characteristics illustrated in Figure 12.

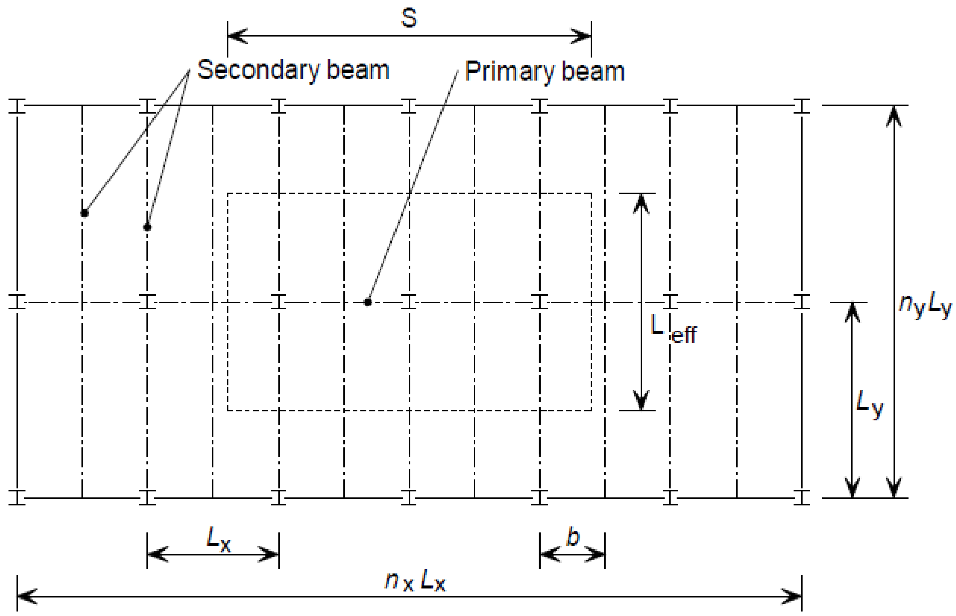


Figure 12: Geometry of the 1st configuration [11]

The effective length and width are expressed as follows (when $n_x \leq 4$ and $n_y \leq 4$):

$$L_{eff} = 1.09 \times (1{,}1)^{n_y - 1} \left[\frac{(EI)_b}{mbf_1^2}\right]^{\frac{1}{4}} \tag{44}$$

$$S = \eta\ (1.15)^{n_x - 1} \left[\frac{(EI)_s}{mf_1^2}\right]^{\frac{1}{4}} \tag{45}$$

where:

m is the total mass per unit area of the floor;

f_1 is the fundamental natural frequency of the floor;

$(EI)_b$ is the bending stiffness of the composite secondary beam;

$(EI)_s$ is the bending stiffness of the slab (along the strong axis);

η is a factor taking into account the influence of the natural frequency on the response of the slab, obtained from Table 28.

Natural frequency	$\boldsymbol{\eta}$
$f_1 < 5\text{Hz}$	0.5
$5\text{Hz} \leq f_1 \leq 6\text{Hz}$	$0.21 f_1 - 0.55$
$f_1 > 6\text{Hz}$	0.71

Table 28: Factor η [11]

A.2.3.1.2 2nd configuration: slab between beams flanges - thin floors

In this configuration, the slab is placed on the lower flanges of beams, resulting in the geometrical characteristics shown in Figure 13.

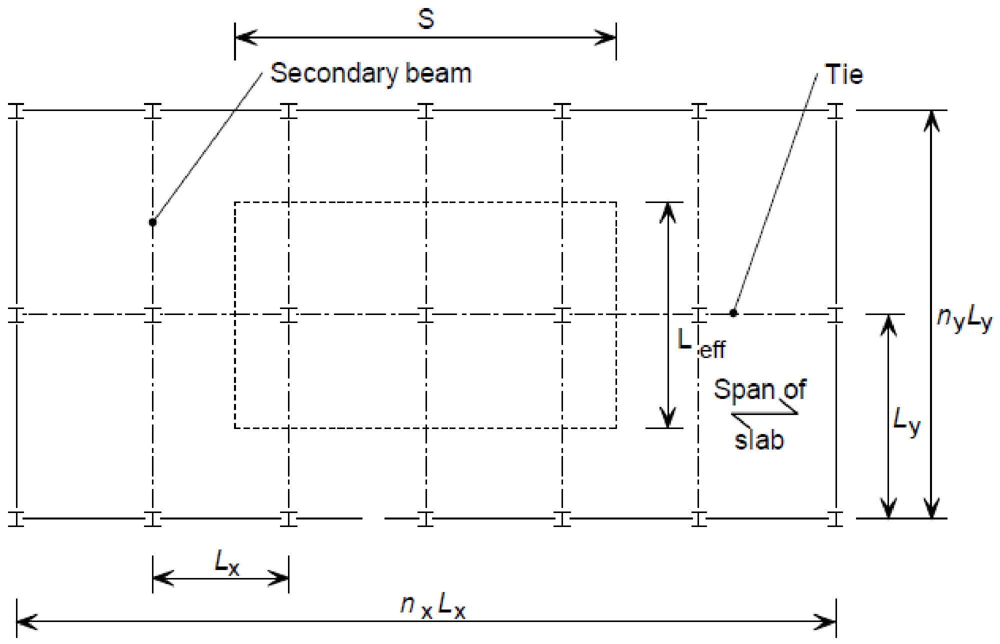


Figure 13: Geometry of the 2nd configuration [11]

The effective length and width are expressed as follows (when $n_x \leq 4$ and $n_y \leq 4$):

$$L_{eff} = 1.09 \left[\frac{(EI)_b}{mL_x f_1^2}\right]^{\frac{1}{4}} \tag{46}$$

$$S = 2.25 \left[\frac{(EI)_S}{m f_1^2}\right]^{\frac{1}{4}} \tag{47}$$

where the description of the variables is the same as for equations (44) and (45).

A.2.3.2 2nd simplified method

The geometrical characteristics of a lightweight floor are shown in Figure 14.

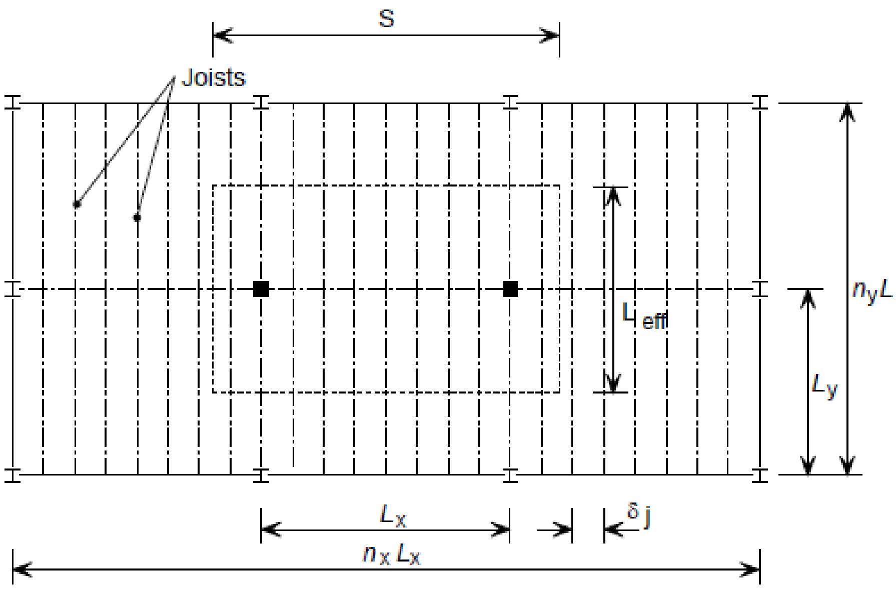


Figure 14: Geometry of a lightweight floor [11]

The effective length and width are expressed as follows (when $n_x \leq 4$ and $n_y \leq 4$):

$$L_{eff} = n_y \left[0.2\, L_y^2 - 2.1 L_y + 7.5\right] \sqrt{\frac{I_b}{5.3\times10^{-6}}} \tag{48}$$

$$S = 0.75\,(L_x + 1)\sqrt{\frac{I_b}{5.3\times10^{-6}}} + 5.9\,(0.6 - \delta_j) \tag{49}$$

I_b being the moment of inertia per meter of the composite secondary beam.

A.3 AISC/CISC DG 11 guideline

A.3.1 General considerations

The surface weights to be taken into account in calculation are:

- Self-weight per unit area of the floor;
- Permanent surface loads of non-structural elements (ceilings and equipment);
- A live load of 0.25kN/m^2 for residential floors and 0.5kN/m^2 for office floors.

In addition, the dynamic modulus of concrete $E_{c,dyn}$ is obtained by increasing the secant static modulus by 35%.

When evaluating the inertia of composite beams, the effective width of the slab is obtained from the following expression:

$$b_{eff} = Min\,(e_b, 0.4\,L) \tag{50}$$

L being the length of the floor and e_b the centre distance between beams.

A.3.2 Natural frequency

The Dunkerley equation is used to calculate the natural frequency:

$$\frac{1}{f_1{}^2} = \frac{1}{f_j{}^2} + \frac{1}{f_g{}^2} \quad (51)$$

This natural frequency is commonly expressed as a function of deflections of joists and girders:

$$f_1 = 0.18\sqrt{\frac{g}{\Delta_j+\Delta_g}} \quad (52)$$

where:

g is the acceleration of gravity;

Δ_j is the static deflection of a joist;

Δ_g is the static deflection of a girder.

For high-rise buildings (with more than 6 floors), the axial displacement of columns Δ_c should be added:

$$f_1 = 0.18\sqrt{\frac{g}{\Delta_c+\Delta_j+\Delta_g}} \quad (53)$$

For independent spans, there are two cases concerning the calculation of deflection for the considered beam (joist or girder):

- Simply supported beam:

$$\Delta_i = \frac{5 w_i L_i^4}{384\, E_s I_i} \quad (54)$$

- Cantilever beam:

$$\Delta_i = \frac{w_i L_i^4}{8\, E_s I_i} \quad (55)$$

where:

Δ_i is the deflection of the considered beam;

L_i is the span of the beam;

w_i is the surface weight supported by the beam;

$E_s I_i$ is the bending stiffness of the composite beam.

For the case of continuous spans, this deflection is multiplied by the same correction factors of the SCI P354 guideline, given in Table 27.

A.3.3 Modal weight

The modal weight is used in the walking method, presented in section 3.4.1. The weight of the two modes of joists and girders are combined as follows:

$$W_1 = \frac{W_j\, \Delta_j + W_g\, \Delta_g}{\Delta_j+\Delta_g} \quad (56)$$

where:

W_j is the modal weight supported by a joist;

W_g is the modal weight supported by a girder.

The modal weight of each beam *i* (joist or girder) is:

$$W_i = w_i \, L_i \, B_i \tag{57}$$

where:

W_i is the modal weight supported by the beam;

w_i is the surface weight supported by the beam;

L_i is the span of the beam;

B_i is the effective width of the beam, the calculation of which will be detailed below.

A.3.3.1 Effective joist width

The effective width of joists is obtained according to the following expression:

$$B_j = Min\left(C_j \left(\frac{D_s}{D_j}\right)^{\frac{1}{4}} L_j, \frac{2}{3} L_f\right) \tag{58}$$

where:

L_f is the length of the studied floor;

L_j is the span of a joist;

C_j is a coefficient equal to 2 for interior joists and 1 for edge joists;

D_s is the equivalent moment of inertia of the slab, obtained by: $D_s = \frac{1}{n} \frac{d_e^3}{12}$, such that:

- $d_e = h_{slab} + \frac{h_{flooring}}{2}$ is effective depth;
- $n = \frac{E_s}{E_{c,dyn}}$ is the steel/concrete equivalence factor;

D_j is the moment of inertia of a joist divided by the centre distance between joists S.

A.3.3.2 Effective girder width

The effective girder width is expressed as follows:

$$B_g = Min\left(C_g \left(\frac{D_j}{D_g}\right)^{\frac{1}{4}} L_g, \frac{2}{3} l_f\right) \tag{59}$$

where:

l_f is the width of the floor studied;

L_g is the span of a girder;

C_g is a coefficient depending on the connection between joists and girders, equal to:

- 1.6 if this connection is at the level of the upper flanges of girders;
- 1.8 if this connection is at the level of the webs of girders;

D_j is the moment of inertia of a joist divided by the centre distance between joists S;

D_g is the moment of inertia of a girder divided by the average joist span L_j.

For a girder located at the edge, the width is written as: $B_g = \frac{2}{3} L_j$.

A.3.3.3 Special cases

If the floor is formed from continuous beams (joists or girders), with a span ratio between adjacent spans greater than 0.7, their modal weight is increased by 50%.

Furthermore, if $L_j \leq \frac{L_g}{2}$, both the joist mode and the combined mode are verified separately (each with its natural frequency and modal weight).

Finally, if $0.5 \leq \frac{L_g}{B_j} \leq 1$, a correction is assigned to Δ_g, f_1 and W_1 as follows:

$$\Delta_g' = \frac{L_g}{B_j} \Delta_g \tag{60}$$

$$f_1 = 0.18 \sqrt{\frac{g}{\Delta_j + \Delta_g'}} \tag{61}$$

$$W_1 = \frac{W_j \Delta_j + W_g \Delta_g'}{\Delta_j + \Delta_g'} \tag{62}$$