Multiscale Modeling of Composites – Piecewise-Uniform Model Order Reduction

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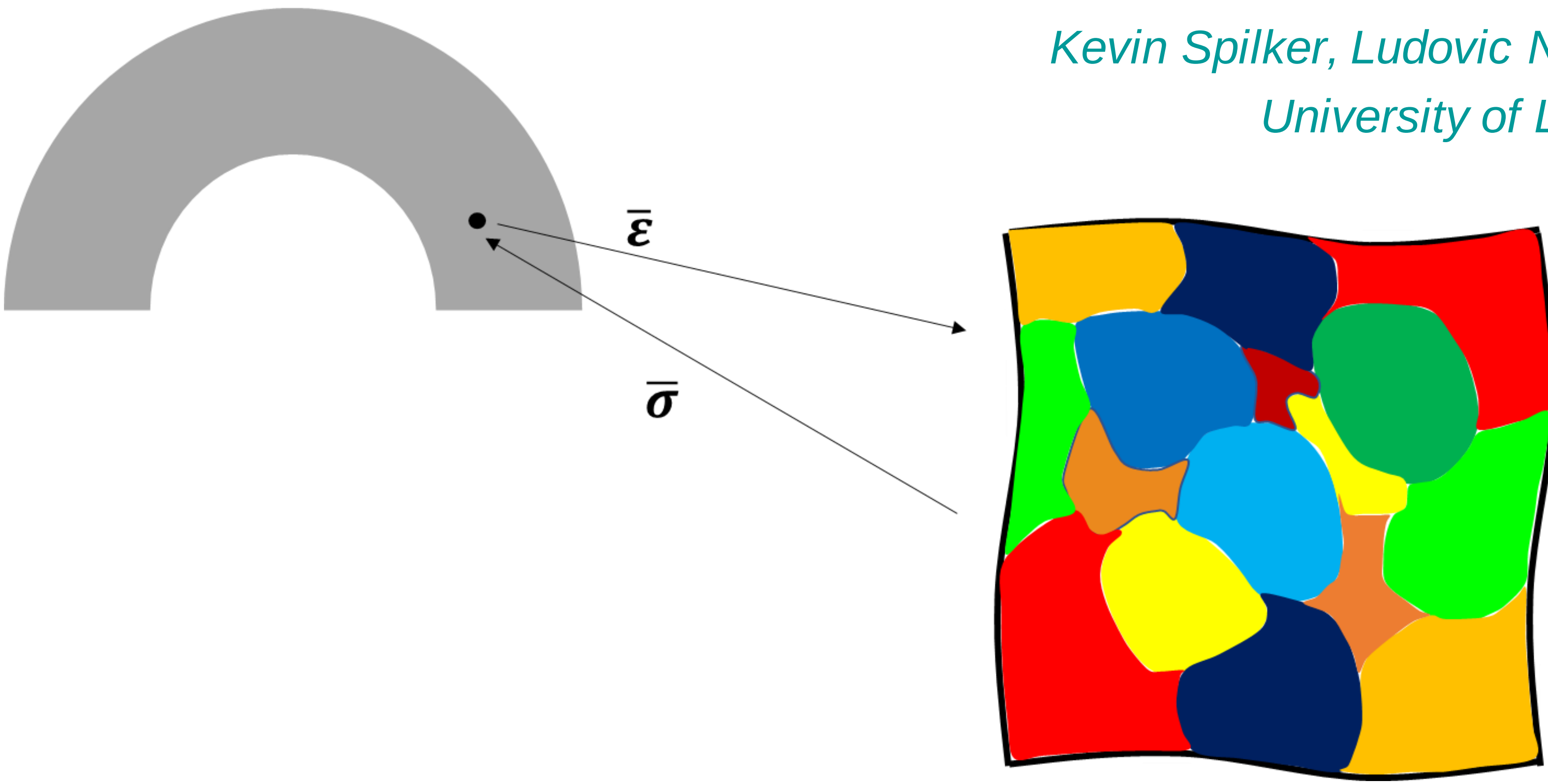

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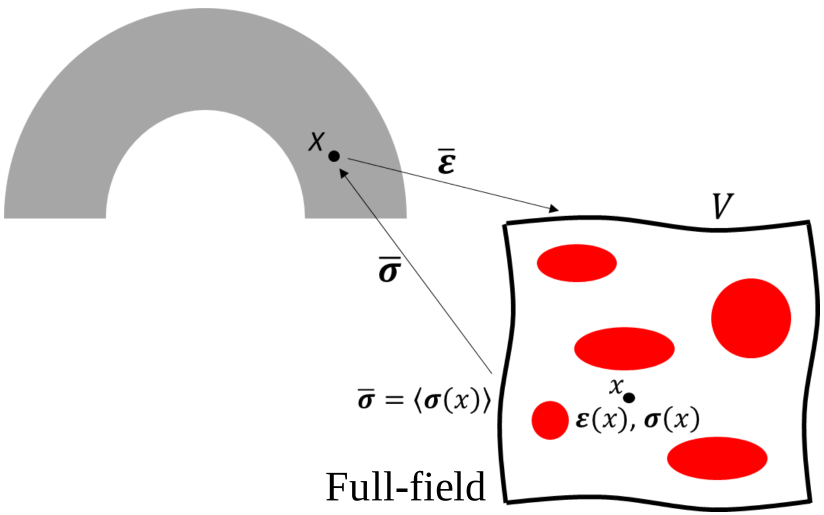
Homogenization of the mechanics of composite materials

- Strategy: finding a representative volume element (RVE)
- RVE represents a macroscopic point X, contains the micro-structure
- Aim: achieving macroscopic/effective RVE response $\bar{\boldsymbol{\varepsilon}} \rightarrow \bar{\boldsymbol{\sigma}}$ (BVP)
- Use of bounds, mean-field homogenization → only need for phase properties and volume fractions
- Two-scale modeling: Linking microscopic and macroscopic scales: $x \leftrightarrow X$



Taking into account micro-structure and microscopic effects

macro-response by averaging over the RVE
→ computational effort is immense

Two-scale reduced order modeling: Uniform fields

Solution strategy

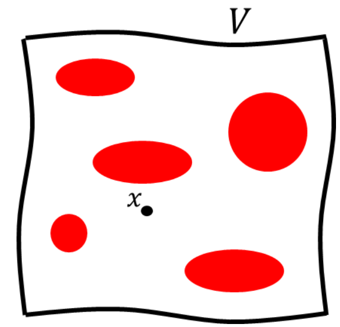
Two-Scale modeling → ROM

Accurate micro-structure consideration

Solve BVP problem

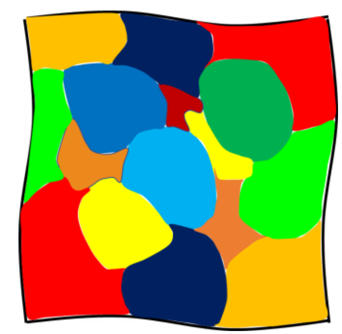
Offline stage:

full-field simulations



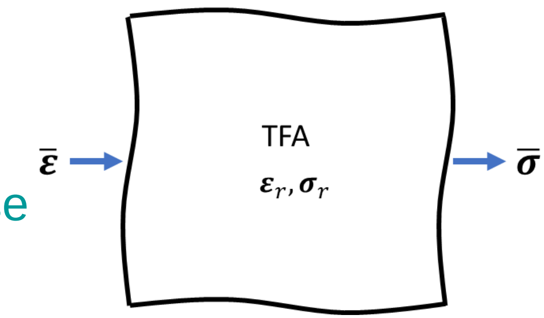
Model order reduction:

spatial subdivision into subdomains



Online solution stage:

solve equations for piecewise uniform fields



Transformation Field Analysis (TFA)

- Two-scale coupling:

eigenstrain $\Delta\boldsymbol{\sigma}_r = \mathbb{C}_r^{\mathrm{el}} : (\Delta\boldsymbol{\varepsilon}_r - \Delta\boldsymbol{\varepsilon}_s^*)$

$\Delta\boldsymbol{\varepsilon}_s^* = \Delta\boldsymbol{\varepsilon}_s^{\mathrm{in}}$

$$\Delta\boldsymbol{\varepsilon}_r = \mathbb{A}_r^{\mathrm{el}} : \Delta\bar{\boldsymbol{\varepsilon}} - \sum_{s=1}^{K} \mathbb{D}_{rs} : \Delta\boldsymbol{\varepsilon}_s^* \coloneqq \mathbb{A}_r^{\mathrm{in}} : \Delta\bar{\boldsymbol{\varepsilon}}$$

instantaneous strain concentration tensor

Elastic strain concentration tensor

eigenstrain-strain interaction tensor

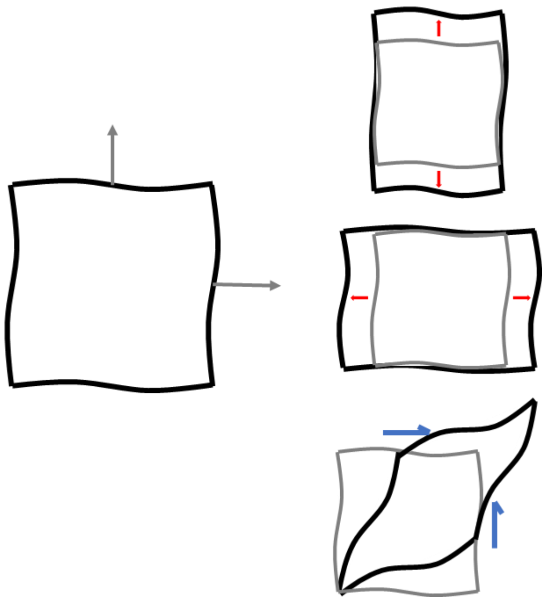
$$\Delta\bar{\boldsymbol{\varepsilon}} = \sum_{r=1}^{K} v_r \Delta\boldsymbol{\varepsilon}_r$$

→ Determination of $\mathbb{A}_r^{\mathrm{el}}$ for each subdomain and $\mathbb{D}_{rs}$ between all subdomains in the offline stage

Offline stage

- Microscopic elastic strain concentration tensors

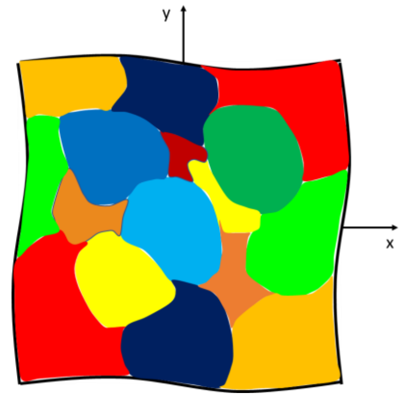
$$\boldsymbol{\varepsilon}(\boldsymbol{x}) = \mathbb{A}^{\mathrm{el}}(\boldsymbol{x}) : \bar{\boldsymbol{\varepsilon}}$$



apply boundary conditions $\bar{\boldsymbol{\varepsilon}}$

compute local strains $\boldsymbol{\varepsilon}(\boldsymbol{x})$

determine entries of $\mathbb{A}^{\mathrm{el}}(\boldsymbol{x})$

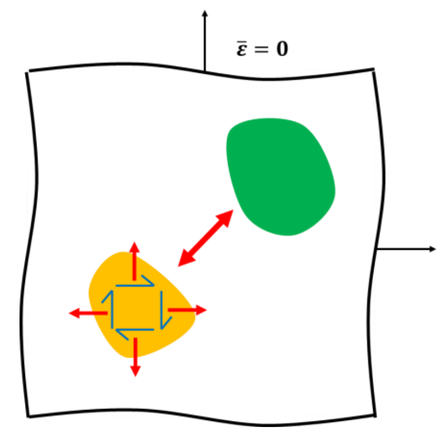


Spatial decomposition based on $\mathbb{A}^{\mathrm{el}}(\boldsymbol{x})$

→ $\mathbb{A}_r^{\mathrm{el}}$

- Eigenstrain – strain interaction tensors
  - With $\bar{\boldsymbol{\varepsilon}} = \mathbf{0}$ and eigenstrain in one subdomain

$$\boldsymbol{\varepsilon}_r = \mathbb{D}_{rs} : \boldsymbol{\varepsilon}_s^*$$



apply eigenstrain $\boldsymbol{\varepsilon}_s^*$

compute $\boldsymbol{\varepsilon}_r$

determine entries of $\mathbb{D}_{rs}$

Results

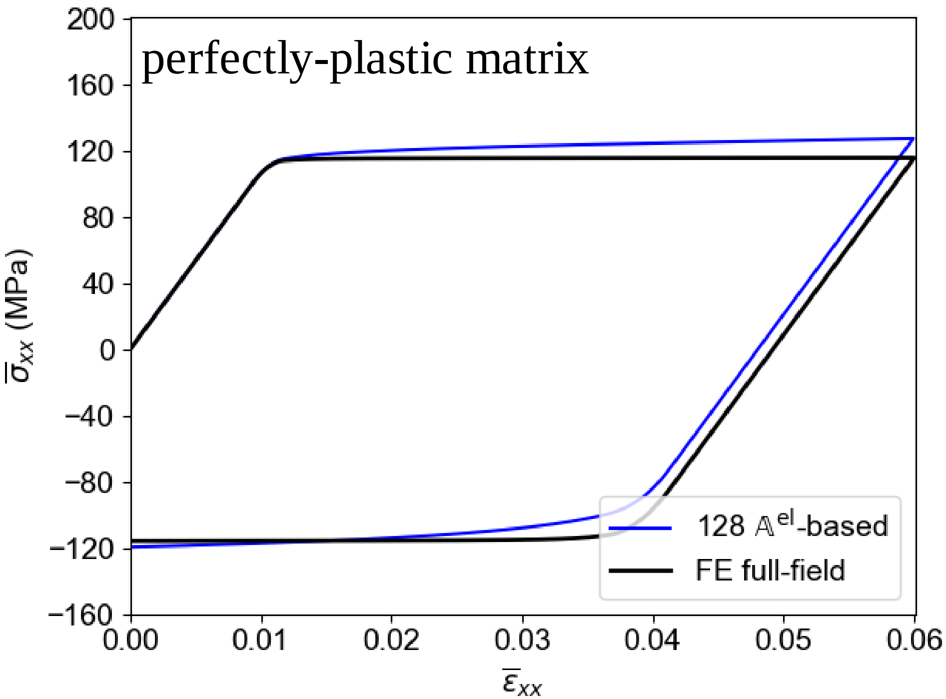
- TFA issue: not well-captured inelastic fields using elasticity-based subdomains
- Under-represented inelastic fields ⟷ under-represented interaction effects

  → Incorrect strain distributions over the subvolumes

  → Especially: overestimated strain accumulation in stiff material phase

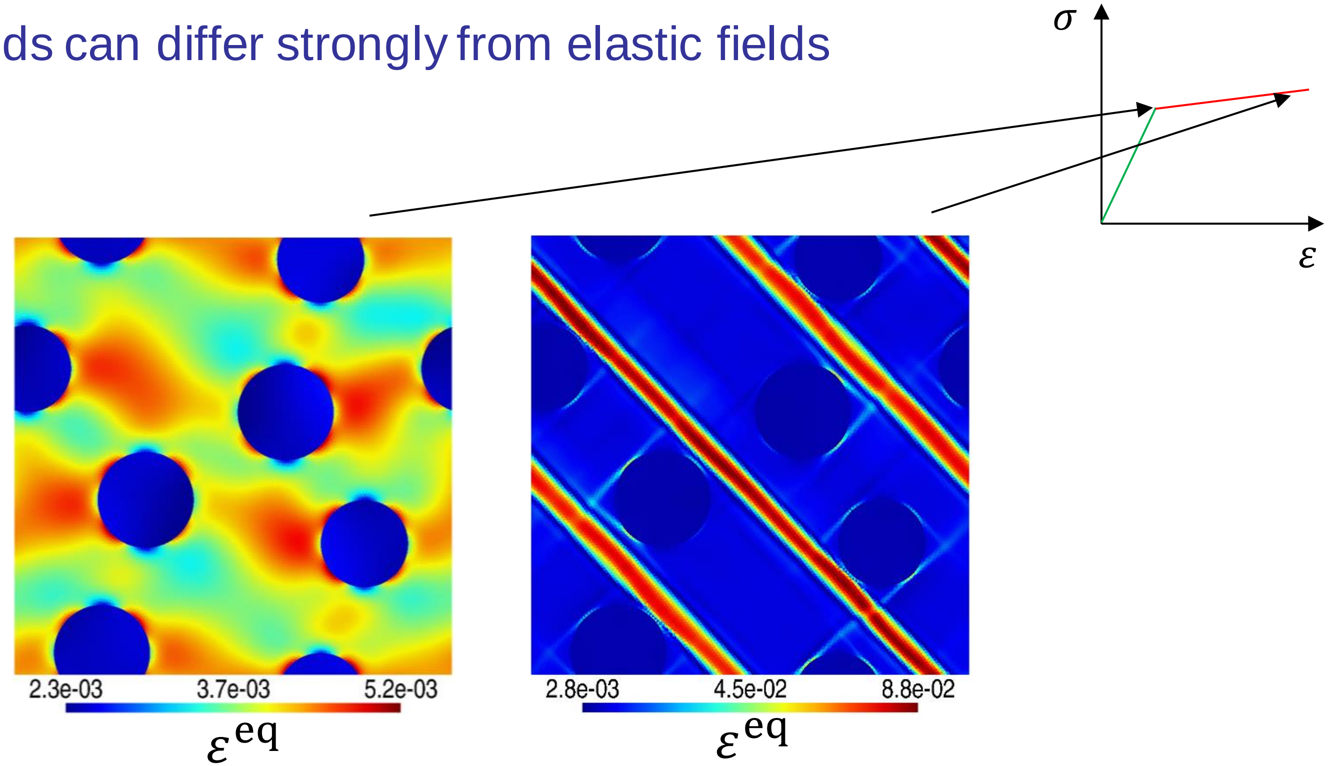
→ Over-stiff composite overall response

Particularly in cases of
highly-localized plasticity or
highly heterogeneous plastic fields



TFA: Integration of inelasticity

- Inelastic fields can differ strongly from elastic fields



→ Change of plan: cluster decomposition based on inelastic micro-fields to respect inelastic micromechanical patterns

→ include more information of the real physics into the two-scale modeling approach

TFA: Integration of inelasticity

- Evident: well-chosen boundary conditions in order to capture different possible shear bands which can occur in the online stage

- Selection based on low-fidelity FE simulations:

  compare plastic fields under various complex loading conditions and under several proportional loadings

- Chosen four monotonic strain boundary conditions

(1) $\bar{\varepsilon} = \frac{1}{2} E \, (e_1 \otimes e_1 - e_2 \otimes e_2)$ bi-axial isochoric

(2) $\bar{\varepsilon} = \frac{1}{2} E \, (e_1 \otimes e_2 + e_2 \otimes e_1)$ pure shear

(3) $\bar{\varepsilon} = (1) + \frac{1}{2} \, (2)$ mixed isochoric

(4) $\bar{\varepsilon} = \frac{1}{2} (1) + (2)$ mixed isochoric

→ Ability to detect different kinds of evolving shear bands

A numerical application

- Material system: elastic inclusions in a J2-plastic matrix

  $\sigma^{Y} = \sigma^{Y0} + R$ yield stress

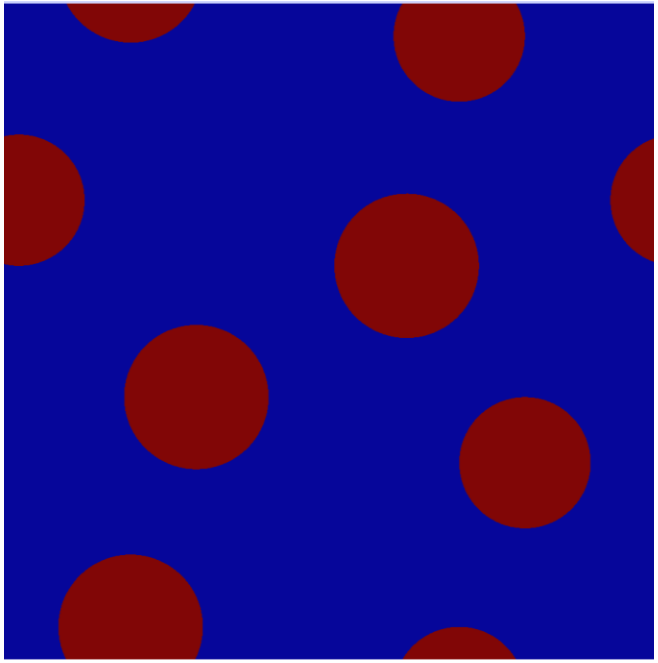
  $R = Hp^{m}$ hardening stress

Perfect plasticity

	G (GPa)	***K*** (GPa)	***υ*** (-)	***H*** (MPa)	*m* (-)
Matrix	3	10	0.36	~ 0	1
Inclusions	6	20	0.36	-	-

- 2D periodic RVE micro-structure: random distribution of circular inclusions

- Inclusion volume fraction: 21 %

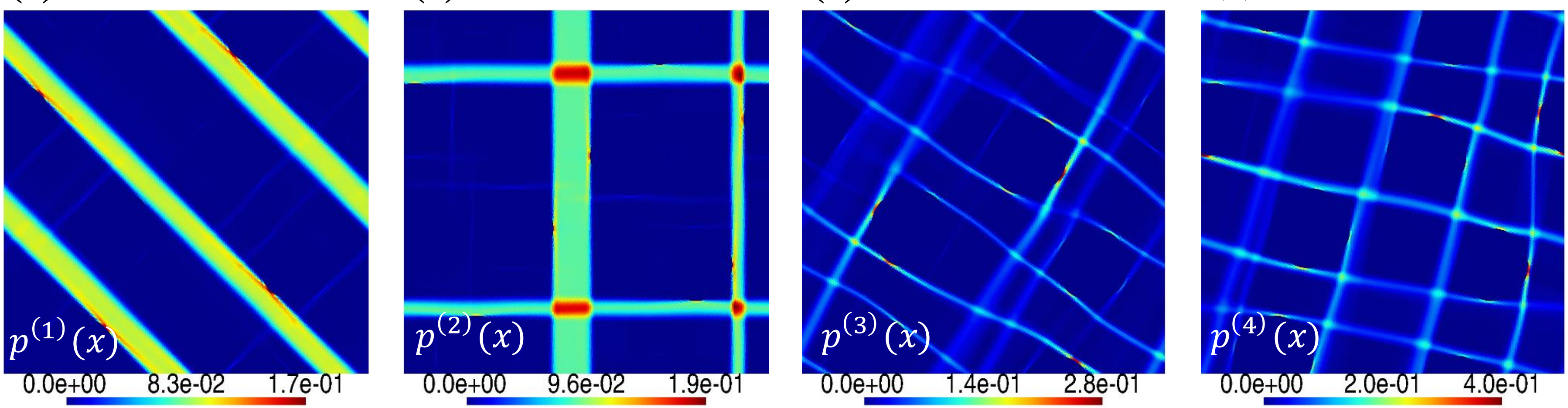


Offline stage: Application

- perfectly-plastic matrix behavior

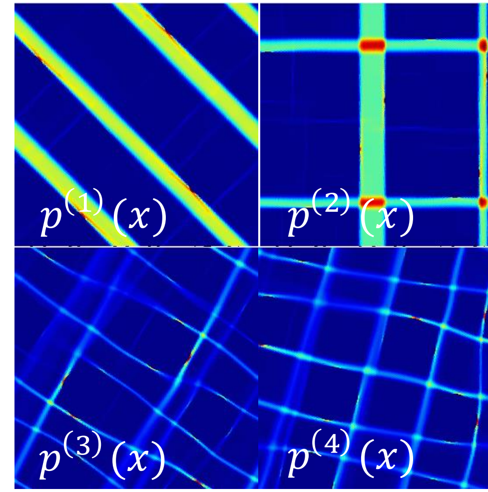
(1) $\bar{\varepsilon} = \frac{1}{2} E \left(e_1 \otimes e_1 - e_2 \otimes e_2 \right)$	bi- axial isochoric
(2) $\bar{\varepsilon} = \frac{1}{2} E \left(e_1 \otimes e_2 + e_2 \otimes e_1 \right)$	pure shear
(3) $\bar{\varepsilon} = (1) + \frac{1}{2}\ (2)$	mixed isochoric
(4) $\bar{\varepsilon} = \frac{1}{2}(1) +\ (2)$	mixed isochoric

(1) (2) (3) (4)



Offline stage: Application

- Weighting factors: $w^{(1)} = 10, w^{(2)} = 10, w^{(3)} = 1, w^{(4)} = 1$



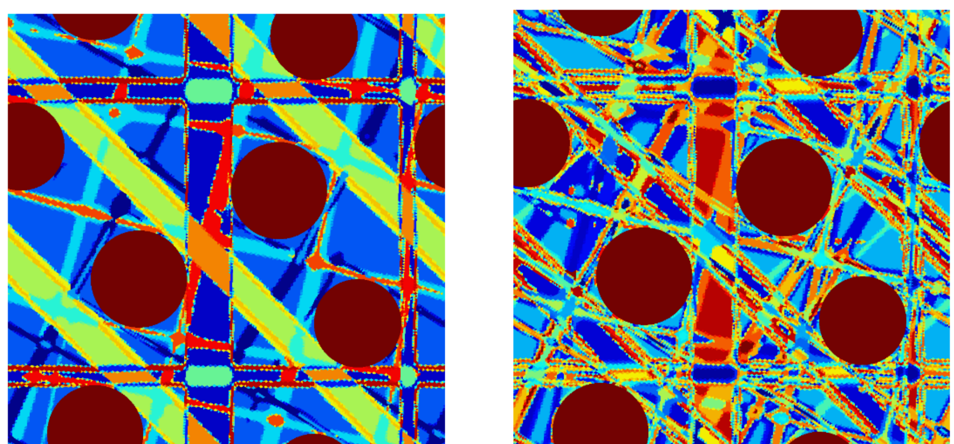
- Normalized, weighted plastic fields

$$q^{(l)}(x) = w^{(l)} \frac{p^{(l)}(x)}{\bar{p}^{(l)}} \quad l \in 1, \dots, 4$$

$$\rightarrow \boldsymbol{q}(x) = (q^{(1)}(x), q^{(2)}(x), q^{(3)}(x), q^{(4)}(x))^T$$

- K-means clustering based on similarity of local field $\boldsymbol{q}(x)$

resulting subdomains



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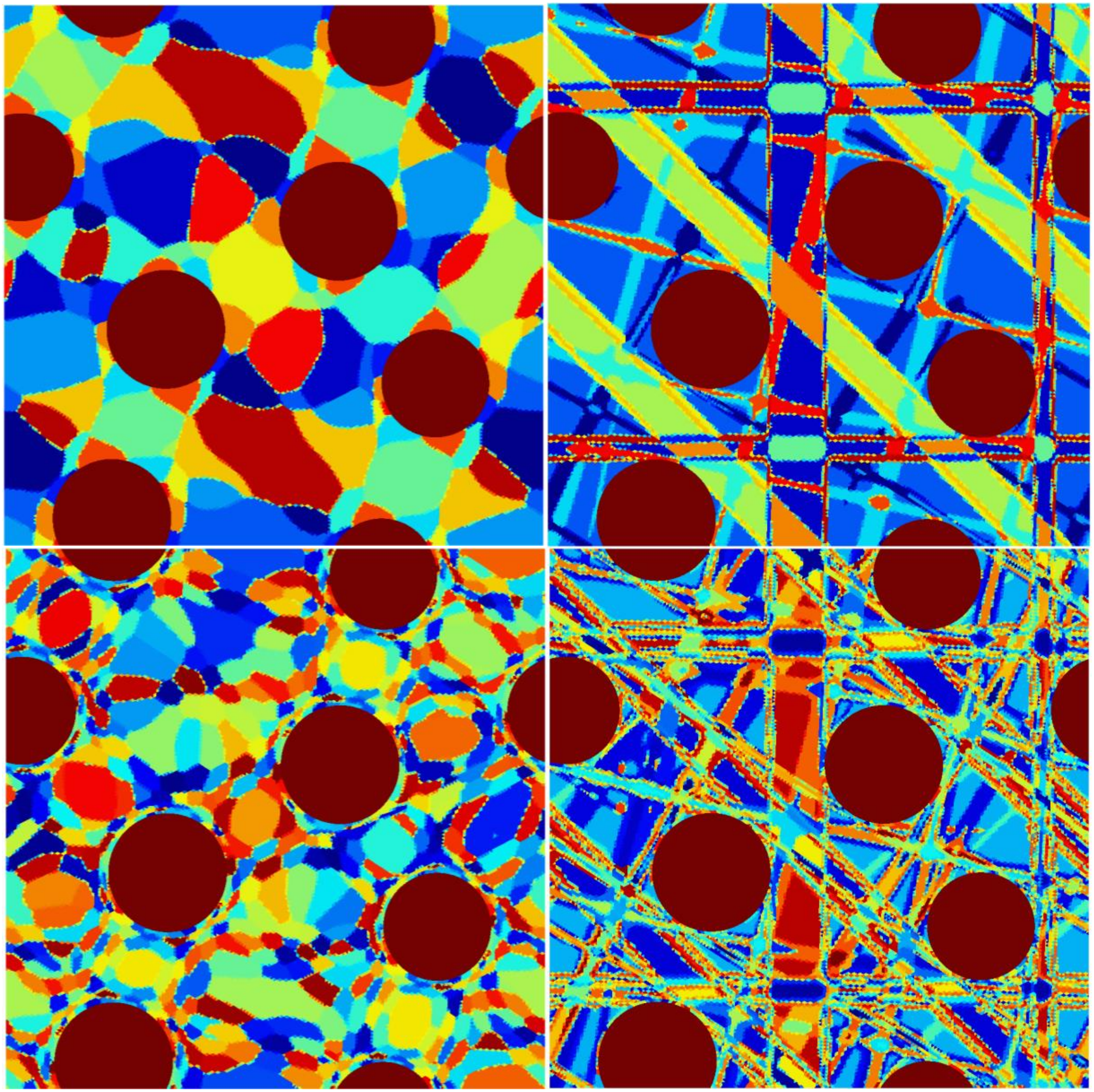
Offline stage: Application

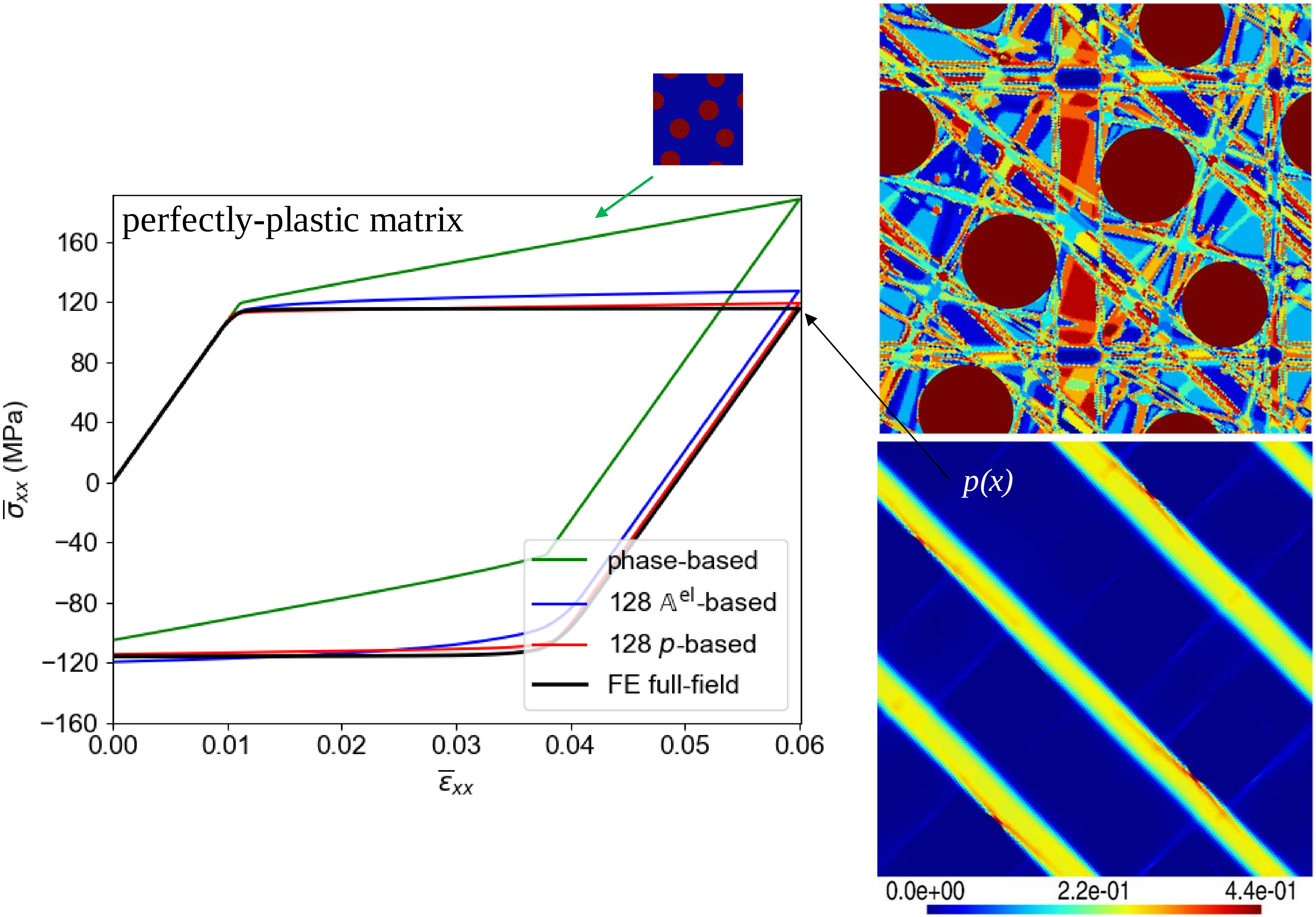
16

128



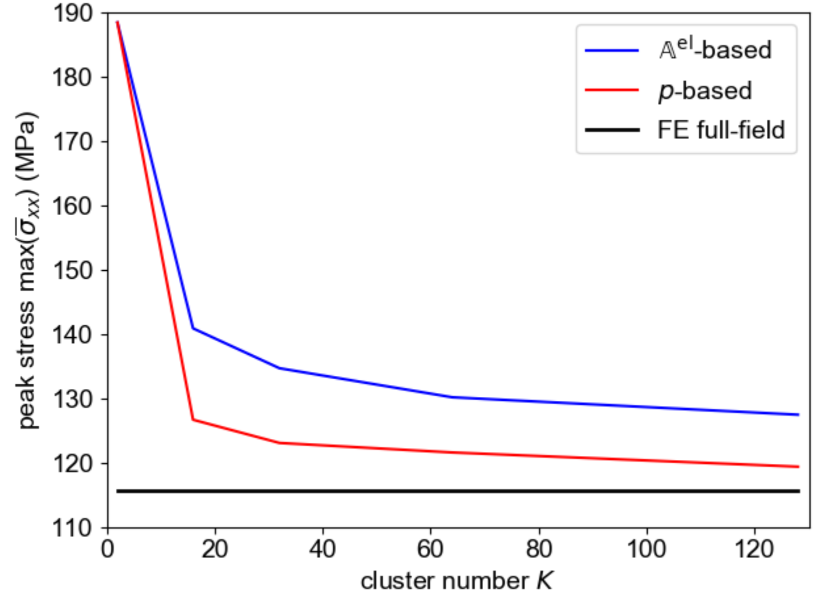
Elasticity Plasticity

Results: Uniaxial tension



Results

- Plasticity-based clustering allows more accurate modeling than elasticity-based clustering



- Clearly faster convergence to full-field result

- Drawbacks:
  - Inelastic simulations with fine mesh required in offline stage
  - Integration of many more loading cases in offline stage → interferencing patterns

References

- Transformation Field Analysis:

George J. Dvorak (1992), Transformation Field Analysis of Inelastic Composite Materials, *Proceedings: Mathematical and Physical Sciences*, 437: 311-327

Pierre Suquet (1997), Continuum Micromechanics, Springer