Composite structures design for strength and stiffness with respect to ply thickness and/or fibers orientation

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1. Abstract
This paper presents an optimization approach that allows the design of composite structures under stiffness and strength criteria, with fibers orientations and plies thickness taken into account in the same optimization loop. The approach is based on the sequential convex programming. An industrial application is presented.
2. Keywords: composite structures, sequential convex programming, structural approximations, MMA

3. Introduction
According to their high stiffness to weight ratio and their anisotropic properties, composite materials are widely used in automotive and aerospace applications. The design of fiber reinforced composite structures calls naturally for optimization algorithms. However, the amount of work that has been done for their optimization is weak compared to the effort dedicated to problems involving isotropic materials. This fact is due to the unusual behavior of such materials. For example, structural responses of composites are highly non linear, non monotonous and non convex over fibers orientations. Besides, in problems involving fibers orientations and ply thickness design variables, the structural responses present mixing non monotonous and monotonous behaviors (Figures 1 and 2).

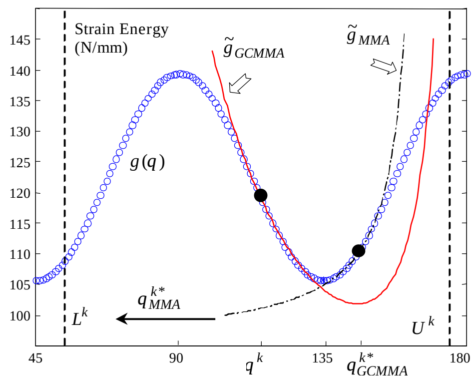

Figure 1. Approximations of the strain energy $g(q)$ for optimal orientation in a laminate. A non monotonous approximation is advised.

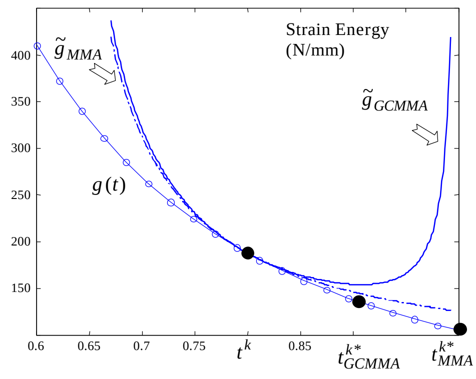

Figure 2. Approximations of the strain energy $g(t)$ for optimal thickness in a laminate. A monotonous approximation is advised.

Because of the solution complexity of this type of problems, research in this field progress slowly and applications to real-life and industrial problems have always been penalized.

This paper focuses on the problem of tailoring an efficient solution procedure to handle composite materials optimization. Applications of this procedure to real-life problems will show that the realized new progress open the door to solve composite problems in a general and robust way, as it is already the case with isotropic materials. Here, the design of composite structures is solved by using the approximation concepts approach [1]. Both fibers orientations and plies thicknesses are the design variables. Improvements of convex approximations of the MMA family [2,3] were already performed in order to deal in a reliable way with the design of structures made of such materials [4]. Later, a generalized approximation scheme was derived and tested on simple laminates [5]. This approximation scheme is able to consider simultaneously monotonous and non monotonous structural behaviors, by generating automatically the best structural approximation of a design function according to each design variable type (Figures 1 and 2). This new solution procedure has been made available in the environment of the commercial optimization software BOSS/Quattro [6] which is linked to the SAMCEF finite elements code [7]. The generality of the optimization approach described in this paper allows the design of composite structures under stiffness and strength criteria, with fibers orientations and plies thickness taken into account in the same optimization loop.

4 Optimization procedure
The approximation concepts approach is used to solve the general optimization problem (1) where $g_0(X)$ is the objective function and $g_j(X)$ are the structural restrictions.

$$\begin{aligned} &\min g_0(X) \\ &g_j(X) \le g_j^{\max} \qquad j = 1,\dots,m \\ &\underline{X_i} \le X_i \le \overline{X_i} \qquad i = 1,\dots,n \end{aligned} \tag{1}$$

Here, the design variables X are the plies thicknesses and the fibers orientations of a laminate. In the approximation concepts approach, the primary optimization problem (1) is replaced by the solution of a sequence of explicit approximated sub-problems (2) generated trough first or second order Taylor series expansion of the structural functions in terms of specific linearization variables.

$$\begin{aligned} &\min\ \tilde{g}_0(X) \\ &\tilde{g}_j(X) \le g_j^{\max} \qquad j = 1,\ldots,m \\ &\underline{X}_i^k \le X_i \le \overline{X}_i^k \qquad i = 1,\ldots,n \end{aligned} \tag{2}$$

In this work, the generalized first order approximation scheme (3) derived in [5] is used to approximate each structural response entering the optimization problem (1).

$$\begin{aligned} \tilde{g}_j(X) = g_j(X^k) &+ \sum_{i\in A} p_{ij}^k \left(\frac{1}{U_i^k - X_i} - \frac{1}{U_i^k - X_i^k} \right) + \sum_{i\in A} q_{ij}^k \left(\frac{1}{X_i - L_i^k} - \frac{1}{X_i^k - L_i^k} \right) \\ &+ \sum_{+,i\in B} p_{ij}^k \left(\frac{1}{U_i^k - X_i} - \frac{1}{U_i^k - X_i^k} \right) + \sum_{-,i\in B} q_{ij}^k \left(\frac{1}{X_i - L_i^k} - \frac{1}{X_i^k - L_i^k} \right) \end{aligned} \tag{3}$$

This scheme, called GBMMA-MMA, is based on the non monotonous GBMMA approximations [4] using the gradients from previous iteration to improve the quality of the approximation and to speed up the convergence process, and on the monotonous MMA [2]. At a given stage k of the iterative optimization process, the choice of the approximation, that is monotonous or not, is based on the tests (4), (5) and (6) computed for given structural response $g_j(X)$ and design variable X_i.

$$\frac{\partial g_j(X^k)}{\partial X_i} \times \frac{\partial g_j(X^{k-1})}{\partial X_i} > 0 \qquad \Rightarrow \qquad \text{MMA} \tag{4}$$

$$\frac{\partial g_j(X^k)}{\partial X_i} \times \frac{\partial g_j(X^{k-1})}{\partial X_i} < 0 \qquad \Rightarrow \qquad \text{GBMMA} \tag{5}$$

$$\frac{\partial g_j(X^k)}{\partial X_i} - \frac{\partial g_j(X^{k-1})}{\partial X_i} = 0 \qquad \Rightarrow \qquad \text{linear expansion} \tag{6}$$

Using the gradients values at two successive iterations of the optimization, it is determined if a structural response is monotonous (4) or not (5) for a design variable variation, and the most reliable scheme is then chosen. A and B in (3) are the sets of design variables leading to a non monotonous and a monotonous behavior, respectively, in the considered structural response $g_j(X)$. Test (6) detects the linear structural behaviors: the approximation (3) is then locally reduced to a linear expansion.

As explained in [5], tests (4) to (6) are performed on a given number of iterations *ICHECK* for avoiding the use of a monotonous approximation to approximate a non monotonous structural behavior (Figure 3).

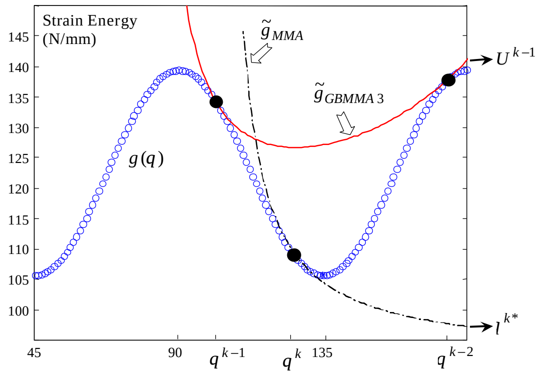

Figure 3. Approximations of the strain energy $g(q)$ for optimal orientation in a laminate. At the iteration k, a monotonous approximation is generated although the structural response is non monotonous.

This approximation scheme and its dual solver are available in the BOSS/Quattro optimization toolbox [6]. Although it includes a multi objective formulation, mono objective optimization problems are only considered here.

5. Discussion about the parameterization and the character of the optimum

It is well known that when fibers orientations and ply thicknesses are the design variables, many local optimum may exist. Besides, gradient based methods are generally not able to find the global optimum of the problem. Usually, the designer contents himself with a local solution, or anyway with a feasible one, if strength constraints are included in the design problem.

6. Academic application

This application concerns the design of a composite membrane divided in 20 regions of constant fibers orientations and made of one ply laminates. The initial configuration includes laminates with 45° or –45° layers, with equal thicknesses of 1mm. The base material is glass reinforced polyester (Table 1) and the applied load *P* is equal to 5000N (Figure 4).

E_X	E_Y	G_{XY}	n_{XY}	X_T	X_C	Y_T	Y_C	S	d
38050	8140	3130	0.285	820	491	21	107	40	1860

Table 1. Mechanical properties of the glass reinforced polyester.
Tensile, compressive and shear strengths. Values in MPa. Density d in kg/m^3.

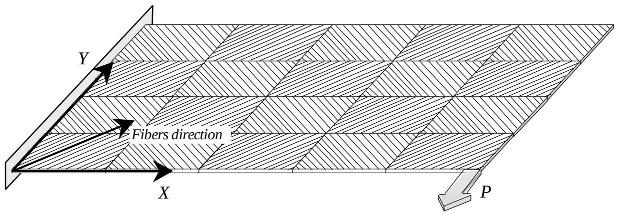

Figure 4. Composite membrane.

20×16 quadrangular membrane multi layer finite elements are used [7]. There are 40 design variables, that is 20 ply thicknesses and 20 fibers orientations. In the design problem (7) the strain energy is the objective function because it is expressed in terms of both kinds of design variables. The strength constraint is related to the Tsai-Hill (*T.H.*) safety margin (*MS*). When this margin is positive, the applied stress can increase by a factor 1+*MS*; when *MS* is negative, the applied stress has exceeded the strength by a factor 1/(1+*MS*). The strength restriction that the optimum has to fulfill is an overall minimum safety margin of 0.15. Considering such a value allows to take into account the numerous uncertainties that surround the design of composite structures. Such a definition of the safety factor is to be used for a proportional loading. A restriction on the structural weight is also imposed. Plies thicknesses are allowed to vary continuously between 0.01 and 5 mm.

$$\begin{gathered}\min_{q,t} Compliance \\ weight \le 1.5kg \\ \min\left(\frac{1}{\sqrt{T.H.}}-1\right) \ge 0.15\end{gathered} \qquad (7)$$

The obtained solution is illustrated in Figures 5, 6 and 7, and the iteration history is plotted in Figure 8. Note that the initial design is not feasible. The stopping criteria is such that the relative variation of the objective function at two successive iterations be lower than 0.01%.

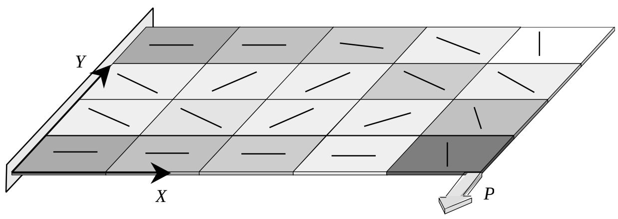

Figure 5. Illustration of the solution.

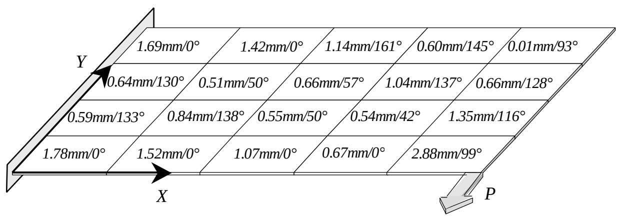

Figure 6. Solution for the composite membrane.

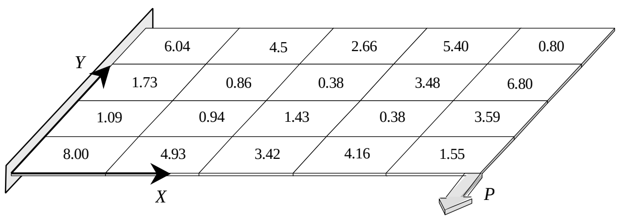

Figure 7. Minimum Tsai-Hill safety margins in the different plies.

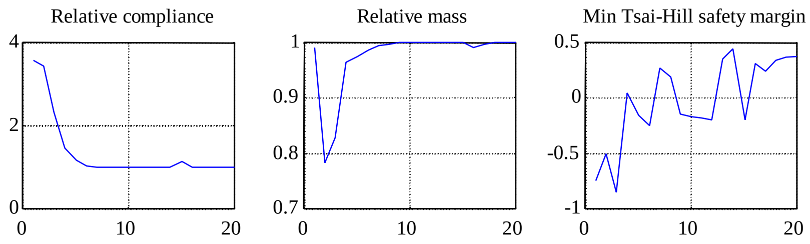

Figure 8. Iteration history for the composite membrane optimization using GBMMA-MMA (*ICHECK = 5*).
The maximum allowable weight is of 1.5 kg.

In Figure 8, the non dimensional compliance is plotted with regards to its optimal value and the mass is normalized according to its upper allowable bound. At the iteration 13, the design is feasible and the relative variation of the objective function is of 0.3%. It is seen from Figures 7 and 8 that the overall minimum Tsai-Hill safety margin at the solution is larger than 0.15. This means that the weight still can be decreased. If the problem is restarted with an upper bound of 1.25 kg for the structural weight, the obtained solution is quite similar to the preceding one in terms of fibers orientations. The corresponding iteration history is plotted in Figure 9. A feasible design is found after 6 iterations, while 8 iterations are needed for stabilizing the objective function value with a relative variation of 0.1%.

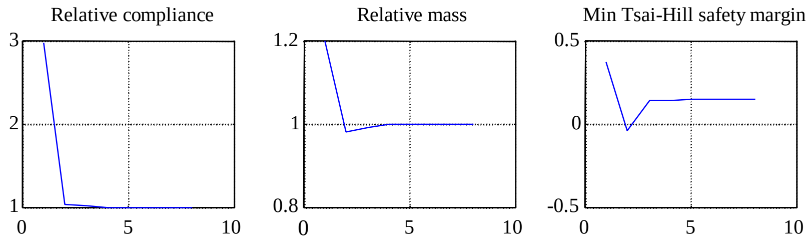

Figure 9. Iteration history for the composite membrane optimization using GBMMA-MMA (*ICHECK = 5*).
The maximum allowable weight is of 1.25 kg.

It is to be noted than when only plies thicknesses are considered in problem (7), a solution seems to be very difficult to be obtained. Although GCMMA [3] is globally convergent, it is not able to find any optimal design in a reasonable time. After 100 iterations, the overall minimum Tsai-Hill safety margin is of -0.125 and the restriction on the mass is slightly violated. This put into the light the limitation of the "only thickness" formulation and the key role of the fibers orientations in the design of composite structures [8]. In Figure 10, the non dimensional compliance is plotted with regards to its value at iteration 100. The mass is normalized according to its upper allowable value.

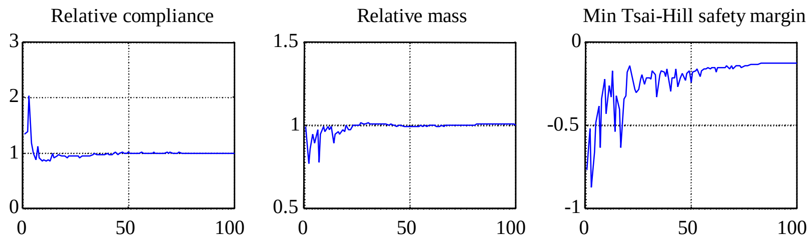

Figure 10. Iteration history for the composite membrane optimization using GCMMA.
Plies thicknesses are the only design variables.

7. Industrial application

The optimization procedure is applied to the design of a light rail vehicle (LRV) made of composite material (Figure 11).
The sandwich panels configuration used in the LRV is symmetric and comprises skin layers of E-glass reinforced epoxy resin while the foam core is manufactured from polyurethane (Table 2).

	E_X	E_Y	G_{XY}	n_{XY}
FOAM	21.5	21.5	7.7	0.39
GLASS-EPOXY	43640	9760	3810	0.274

Table 2. Mechanical properties of the glass reinforced epoxy and the foam material (in MPa).

The initial configuration of the panel includes a 24mm thick foam core with 8×0.06 mm face sheets building up with 8 plies and a lay-up of $[0°/45°/-45°/90°]_S$. The loading condition is derived from the British Railway Board standard [9]: it implies a compressive force of 1500 kN at the buffer position.

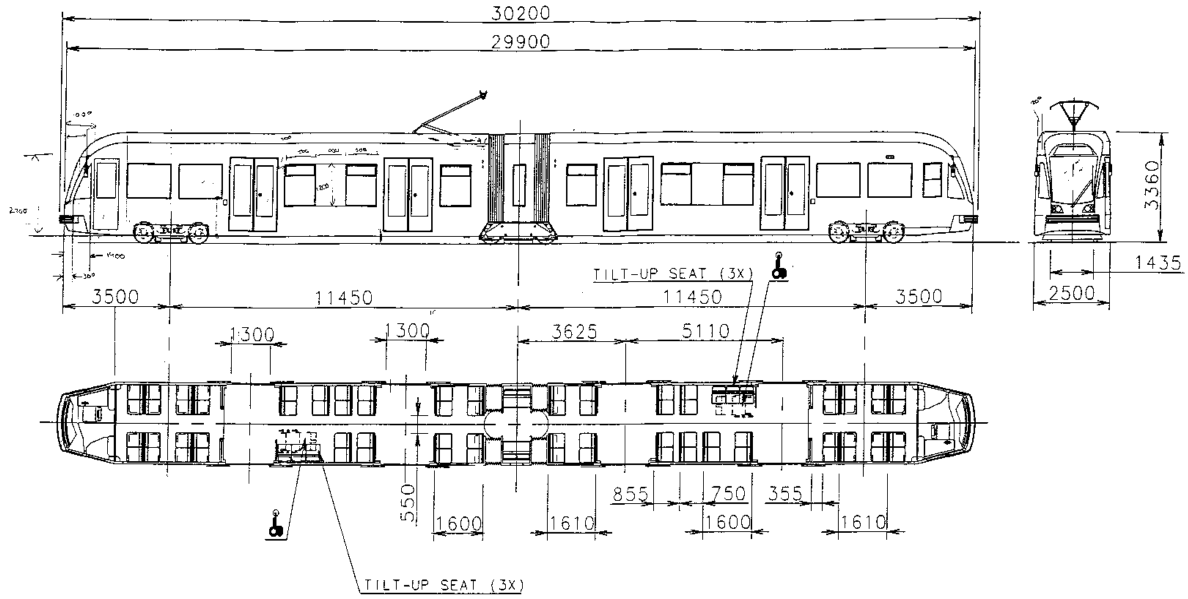

Figure 11. Geometry and dimensions of the railway vehicle.

The mesh of one quarter of the wagon is presented in Figure 12. It is made of 2012 multi layer 8-noded Mindlin shell finite elements. The design domain is the dark panel represented in Figure 12. It is divided in three different design zones. Ply thicknesses and fibers orientations are considered as design variables. Plies thicknesses are allowed to vary continuously between 0.01 and 5 mm.

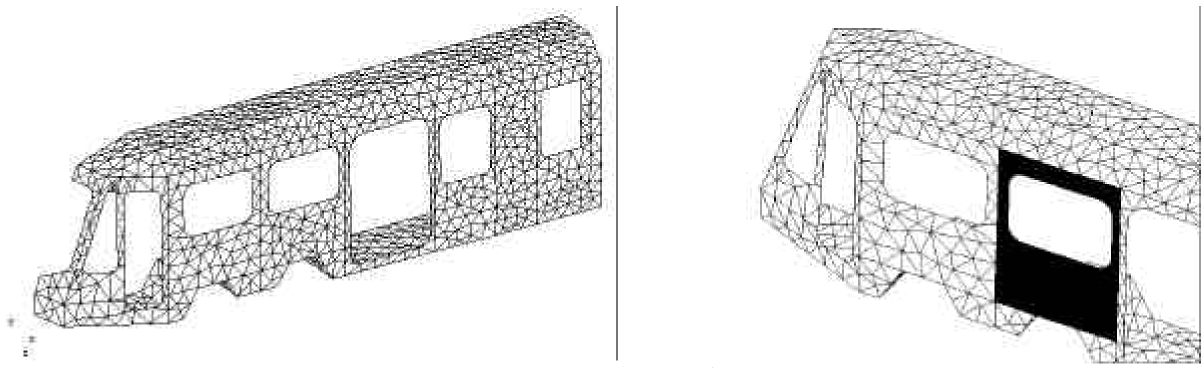

Figure 12. Finite element model of the railway vehicle.
The dark panel on the rigth is the part to be designed.

The goal of the optimization is to increase the strutural stiffness while maintening the panel mass at 30 kg. Strength constraints are imposed on the upper and lower faces of each ply via a Tsai-Hill safety margin of 0.05. The problem has 24 design variables (12 orientations and 12 thicknesses) and 97 structural constraints.

Preliminary results are reported in Figure 13 where the number of violations at each design step is plotted with the evolutions of the compliance (values related to the optimum) and the normalized mass.

The initial design is non admissible: 68 strength restrictions are violated. It is seen in Figure 13 that 11 iterations are needed to find an admissible design. At this stage, the corresponding relative variation of the objective function is of 0.5% and the solution is supposed to be reached.

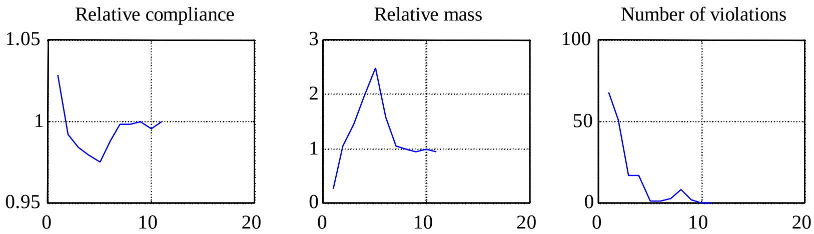

Figure 13. Iteration history for the optimization of the LRV using GBMMA-MMA (*ICHECK = 2*).

8. Conclusions

Numerical applications showed that the proposed solution procedure allows the design of composite structures under stiffness and strength criteria, over fibers orientations and plies thicknesses. Those two kinds of structural responses have to be taken into account in real-life applications. Besides, great savings in structural performances can be achieved by considering fibers orientations in the design of composite structures. Due to the lack of convexity resulting from the orientation/thickness formulation, the solution is not the global optimum. This is the price to pay for designing the structure for strength [10].

9. Acknowledgments

The main part of this work was performed under the contract BRPR-CT96-0332 of the European project BRITE-EURAM (COMPOPT).

10. References

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