Estimating diagonal second order terms in structural approximations with quasi-Cauchy techniques

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1. Abstract

This paper reports preliminary results obtained when estimating diagonal second order terms to be used in structural approximations with the quasi-Cauchy updates which was recently proposed by Zhu, Nazareth, and Wolkowicz (SIAM J. of Optimization, 9 (4), 1192-1204, 1999). At first, the theory of quasi-Cauchy updates is presented. Main characteristics of the developments that were necessary to use quasi-Cauchy updates in the context of structural optimization are drawn. The available numerical results allow comparing quasi-Cauchy second order term estimations with other estimation procedures.

2. Keywords: Sequential Convex Programming, structural approximations, Quasi-Cauchy updating.

3. Introduction

Since high quality approximation schemes using second order information have been proposed (see for example Ref. [1], [2]), fast estimation procedures of diagonal second order terms have been an interesting research topic. Indeed good quality second order information can substantially improve convergence properties of optimization processes. This means that the number of re-analyses to come to a stationary solution can be reduced while the constraint violations during the optimization process are controlled. In the context of Sequential Convex Programming (SCP), in which dual methods are used to solve the convex sub-problems, the efficiency of the method comes from the separability of the approximations. This means that only diagonal second order terms can be introduced in the approximation. On another hand, second order derivatives are often not available or difficult to calculate. In addition, as the problem size increases, their computation cost becomes very expensive. So it is usual to replace exact values of second order derivatives by estimations. Several approaches have been made to find efficient estimation procedures of the diagonal second order terms. For example in Duysinx et al. [3], a fast estimation procedure of second order terms is derived from Thapa's theory [4] of quasi-Newton update preserving sparse patterns by particularizing it to diagonal structures. However the result is a bit disappointing from a theoretical point of view since one comes to the conclusion that the formula leads to making, more or less, finite differences between the first derivatives at the current and the previous design points. In fact the basic problem stems from the fact that there was no real mathematical (and rigorous) theory to deal with the estimation of a diagonal Hessian matrix. An important break-through was recently realized with the work of Zhu, Nazareth and Wolkowicz [5] and their theory of *quasi-Cauchy diagonal updating*.

4. Quasi-Cauchy updates

Compared to quasi-Newton (QN) updates, which are quite well known in the engineering community, the main characteristics of the quasi-Cauchy theory proposed by Zhu, Nazareth and Wolkowicz [5] are the following:

- The updates satisfy the quasi-Cauchy equation which is a diagonal and weak version of the well-known quasi-Newton relation:

$$\mathbf{s}^T \mathbf{D}_+ \mathbf{s} = \mathbf{s}^T \mathbf{y} \tag{1}$$

  where $\boldsymbol{D}_+$ is the updated approximation of the (true) Hessian matrix we look for, $\mathbf{s} = \mathbf{x}_+ - \mathbf{x}$ is the step between the two design points and $\mathbf{y} = \mathbf{g}_+ - \mathbf{g}$ is the gradient change between these two points.
- The update $\boldsymbol{D}_+$ is required to be *a priori* a diagonal matrix.

According to theoretical results presented in [5], two quasi-Cauchy (QC) updating schemes are available. The first one is based on the update of the matrix $\boldsymbol{D}$ itself and the second one is based on the updating of matrix $\boldsymbol{D}^{1/2}$. In addition, another result, the Oren-Luenberger scaling [7] is also presented, because it is a particular quasi-Cauchy estimation in which every diagonal terms are assumed to be equal.

Updating D. Updating scheme of $\boldsymbol{D}$ is based on the variational problem:

$$\min \ \|\mathbf{D}_+ - \mathbf{D}\| \quad s.t. \quad \mathbf{s}^T \mathbf{D} \mathbf{s} = \mathbf{s}^T \mathbf{y} \tag{2}$$

where $\mathbf{s} \neq 0$, $\mathbf{s}^T \mathbf{y} > 0$, and $\mathbf{D} > 0$. Let $\boldsymbol{D}_+ = \boldsymbol{D} + \boldsymbol{?}$, $a = \boldsymbol{s}^T \boldsymbol{D}\, \boldsymbol{s}$, $b = \boldsymbol{s}^T \boldsymbol{y}$. According to [5], the solution to this problem writes:

$$\boldsymbol{?} = (b-a)/tr(\boldsymbol{E}^2)\ \boldsymbol{E}, \quad \boldsymbol{E} = diag[s_1^2, s_2^2 \ldots s_n^2] \tag{3}$$

When $b<a$, it is interesting to notice that the update matrix $\boldsymbol{D}_+ = \boldsymbol{D} + \boldsymbol{?}$ is not necessarily positive definite. This may be a difficulty if $\boldsymbol{D}$ is used within a metric-based algorithm, but when used in a structural approximation, this is not really an obstacle because one replaces the negative second order terms by a very small positive number to stay with a convex approximation.

Updating $D^{1/2}$**.** An alternative approach is similar to the principle used to derive the BFGS update in the quasi-Newton setting. It consists in updating the square root or Cholesky factor $\boldsymbol{D}^{1/2}$ to give the corresponding $\boldsymbol{D}_+^{1/2}$. The update is defined by $\mathrm{D}_+^{1/2} = \mathrm{D}^{1/2} + \mathrm{O}$. The $\boldsymbol{O}$ update is calculated via the solution of the minimization problem:

$$\min \quad \|\mathrm{O}\| \quad s.t. \quad \mathrm{s}^T (\mathrm{D}^{1/2} + \mathrm{O})^2\, \mathrm{s} = \mathrm{s}^T \mathrm{y} > 0 \tag{4}$$

Let $\mathbf{D} > 0$, and $\mathbf{s} \neq 0$. There is a unique solution to the minimization problem (4) (see theorem 2.2.1 of Ref. [5]) and it is given by:

$$\mathbf{O} = \begin{cases} 0 & if\ b = a \\ -m^* \mathbf{E}(\mathbf{I} + \mu^* \mathbf{E})^{-1} \mathbf{D}^{1/2} & if\ b \neq a \end{cases} \tag{5}$$

where μ^* is the largest solution of the non linear equation $F(\mu) = b$ with

$$F(\mu) = \mathbf{s}^T (\mathbf{D}(\mathbf{I} + \mu \mathbf{E})^{-2}) s = \sum_{\{i: s_i \neq 0\}} \frac{d_i s_i^2}{(1 + \mu s_i^2)^2}. \tag{6}$$

One can demonstrate that the solution $\boldsymbol{D}_+$ is always definite positive. However this updating scheme requires the solution of a one-dimensional non-linear equation.
The solution of the one-dimensional equation $F(\mu) = b$ is not too difficult to realize numerically for example with a bisection iteration scheme. In addition, the efficiency of the solution can be improved when studying the function $F(\mu)$. The function has poles at ($-1/s_i^2$), i=1,…n. If the largest pole is ($-1/s_j^2$), the function $F(\mu)$ is positive and strictly decreasing from $+\infty$ to 0 on the interval $]-1/s_j^2, +\infty[$. Thus, the largest solution m^* to $F(\mu) = b = \mathbf{s}^T \mathbf{y} > 0$ can be searched in this interval. Moreover, one can even evaluate $F(0)$. If $F(0) > b$, then the search interval can be further restricted to $[0, +\infty[$ whereas if $F(0) < b$, the search interval is $]-1/s_j^2, 0]$. Nevertheless the one-dimensional equation can become quite ill-conditioned when the algorithm comes to an accumulation point, because the steps become very small, in which case the numerical values of μ^* and $1/s_i^2$ become very large. In this case a special care must be taken to the numerical implementation. Finally, one may notice that the solution of the one-dimensional equation goes into trouble when the function is non convex during the step, i.e. $b = \mathbf{s}^T \mathbf{y} < 0$. Indeed $F(\mu)$ is strictly positive on the search interval. It comes that the QC update can not be applied anymore and the update procedure is not possible. In this case, our parade is to restart the update procedure.

Oren-Luenberger scaling. The simplest relation derived from QC relation (1) is known from a long time [7]. The Oren-Luenberger scaling matrix is the unique matrix that is obtained from QC relation with the further restriction that the diagonal matrix is a scalar multiple of identity matrix $\boldsymbol{I}$, that is $\mathbf{D}_+ = d\,\mathbf{I}$. It follows that $\mathbf{s}^T \mathbf{D}_+ \mathbf{s} = d\, \mathbf{s}^T \mathbf{s}$. The QC relation allows identifying d, which leads to the result:

$$\mathbf{D}_+ = \left(\mathbf{s}^T \mathbf{y} / \mathbf{s}^T \mathbf{s}\right) \mathbf{I} \tag{7}$$

Properties of Quasi-Cauchy updatings. Both quasi-Cauchy updating schemes present the following major advantages:

- They require very little storage (O(n)) so that they are very well adapted to the solution of large-scale problems such as topology problems in structural optimization.
- The update provides a rigorous way to avoid off-diagonal second order terms and to replace them by a weighted effect over the diagonal.

Nonetheless, the weakness of the theory up to now is that, to the authors' knowledge, there are no demonstrated convergence properties for quasi-Cauchy techniques conversely to quasi-Newton techniques. Our numerical experiments showed that the convergence speed towards the true diagonal second order terms can be slow.

5. Estimating second order terms of structural approximations

The research work is devoted to adapt the general theory of quasi-Cauchy updating to tailor an efficient procedure in the framework of structural optimization.

Approximation schemes.

First of all, we remind the reader with the expression of the two approximation schemes, which serve to realize our numerical applications. The estimated second order terms are used in 2 high quality approximation schemes, namely the Generalized Method of Moving Asymptotes (GMMA) [1] and the quadratic separable scheme [2].

The *quadratic separable approximation* (QUA) is a second order Taylor's expansion of the structural response, in which the second order coupling terms (off-diagonal terms) are omitted to keep the approximation non separable (see Fleury [2]).

$$\tilde{g}^{(k)}(x) \approx g(x^{(k)}) + \sum_{i=1}^{n} \frac{\partial g}{\partial x_i}\left(x_i - x_i^{(k)}\right) + \tfrac{1}{2}\sum_{i=1}^{n} a_i \left(x_i - x_i^{(k)}\right)^2 \quad (8)$$

If the second order sensitivity $\partial^2 g / \partial x_i^2$ is available (which is seldom) or if an estimation $D_{ii}^{(k)}$ of the second order sensitivity (as here) is calculated, one evaluates the second order terms a_i as:

$$a_i = \max(D_{ii}^{(k)}, e) \qquad 0 < e << 1 \quad (9)$$

The *Generalized Method of Moving Asymptotes* (GMMA) has been proposed by Smaoui et al [1] as an extension of the classical Method of Moving Asymptotes by Svanberg [6]. A generalized expression of the MMA scheme can be written in the following form:

$$\tilde{g}^{(k)}(x) \approx g(x^{(k)}) + \sum_{i=1}^{n} \frac{a_i}{x_i - b_i} - \frac{a_i}{x_i^{(k)} - b_i} \quad (10)$$

In order to match the first derivatives and the diagonal second order derivatives (or an estimation of them) of the response function at the current point, one gets the value of the parameters a_i and b_i:

$$a_i = -(x_i^{(k)} - b_i)^2 \frac{\partial g}{\partial x_i} \quad \textit{and} \quad b_i = x_i^{(k)} + 2\frac{\partial g}{\partial x_i} / \max(D_{ii}^{(k)}, e) \quad 0 < e << 1 \quad (11)$$

The advantage of the quasi-Cauchy updating procedure is that the off-diagonal terms, instead of being omitted, are naturally taken into account because of the update procedure. This is consistent with the separable assumption of the approximation. For the GMMA approximation scheme, the combination of the quasi-Cauchy update procedure with the selection rule of the asymptotes (11) provides an automatic selection rule of the moving asymptotes based on the first derivatives only.

Initial value of the second order terms. Initial estimation of diagonal second order terms is very important for two reasons. At first the initial guess of the second order terms have an influence upon the sequence of diagonal Hessian estimations, and later, upon the quality of the approximations, and the number of iterations to come to a stationarity point. Secondly, which is maybe the most important effect, the initial curvature strongly governs the quality of the second iteration point (which is predicted by the solution of the initial sub-problem). If the quality of the approximations of the first sub-problem is bad, its solution can be greatly infeasible or the optimization procedure can even break because no solution can be found. This question has no general and mathematical answer, but one can take benefit of his knowledge of the physical nature of the problem. Moreover a rational justification can be built on the basis of some kind of trust region analysis. One assumes that the response function admits a separable quadratic expansion, which is viewed here, as a linear expansion plus a quadratic trust region term:

$$g(x) \approx g(x^0) + \sum_{i} \frac{\partial g}{\partial x_i}\left(x_i - x_i^0\right) + \tfrac{1}{2}\sum_{i=1}^{n} a_i \left(x_i - x_i^0\right)^2 \quad (12)$$

The trust region terms a_i are adjusted so that the unconstrained one-dimensional minimum $\hat{x}_i$ of the approximated function lies within an admissible distance from current point, i.e. $|\hat{x}_i - x_i^0| < \alpha\, x_i^0$ or $|\hat{x}_i - x_i^0| < \alpha\, (x_i^{\max} - x_i^{\min})$ with α<1. Thus, the initial estimations of the diagonal second order terms can be chosen equal to these trust region terms:

$$D_{ii}^{(1)} = \frac{1}{\alpha} \frac{|\partial g / \partial x_i|}{|x_i|} \quad or \quad D_{ii}^{(1)} = \frac{1}{\alpha} \frac{|\partial g / \partial x_i|}{|x_i^{\max} - x_i^{\min}|} \tag{13}$$

Typically, our numerical experiments showed that α=0.33 is a good choice and the first value of the second order terms in (13) is implemented. One can remark than choosing α=0.5 in the first guess of (13) leads to adopt the curvature introduced by a reciprocal variable expansion scheme if all the derivatives were negative, while with the second guess of (13) one finds the initial curvature introduced by Svanberg in MMA method [6]. Here a bit more conservative choice has been preferred.

Restart procedure of quasi-Cauchy updating of $\boldsymbol{D}^{1/2}$. When the response function is non convex along the step, that is $\mathbf{s}^T \mathbf{y} < 0$, it is not possible anymore to find an update for $\boldsymbol{D}^{1/2}$. The question is now to give a set of curvatures to the approximation at the current point, and a new set of second order terms for the later updating. Several possibilities have been evaluated. As the response function is non-convex along the step, one could choose to degenerate the approximation into a linear approximation, and thus to adopt a set of very small values for the new second order terms. The numerical experiments showed that this generally leads to constraint violation, because the approximation becomes very little conservative. Another possibility is to keep the current values of the second order terms and not to do the update in the hope that the next step will show a convex behavior. Numerical experiments showed that this alternative is also not so good either because the updated terms become non consistent. The best results are obtained when restarting of the quasi-Cauchy updating procedure with initial guess proposed in (13).

Second Order Correction (SOC) procedure. The second order correction procedure consists in improving the quality of the approximation by fitting the approximation with the function value at the previous iteration point. Fitting is realized in scaling uniformly the second order terms. Details of the procedure can be found in Ref. [3].
For *separable quadratic approximation*s, the correction factor κ is given by the closed-form solution of:

$$g(\boldsymbol{x}^{(k-1)}) = g(\boldsymbol{x}^{(k)}) + \sum_{i=1}^{n} \frac{\partial g}{\partial x_i} \left(x_i^{(k-1)} - x_i^{(k)}\right) + k\, \tfrac{1}{2} \sum_{i=1}^{n} a_i \left(x_i^{(k-1)} - x_i^{(k)}\right)^2 \tag{14}$$

For *GMMA approximations,* the procedure is a bit more complicated. Scaling second order derivatives is equivalent to move the asymptotes away from the approximation point: $b_i = x_i^{(k)} + k^{-1}\,(b_i^{(k)} - x_i^{(k)})$ where $b_i^{(k)}$ has been calculated to match the reference second order derivatives at the current iteration point. The correction factor is calculated by solving numerically the equation:

$$\tilde{g}^{(k)}(\boldsymbol{x}^{(k-1)}, k) = g(\boldsymbol{x}^{(k-1)}) \tag{15}$$

Too large modifications of the curvature of the approximation would destabilize the convergence process. According to Luenberger [7], it is good to bound the correction factor within the interval: $0.2 \le k \le 5$.

6. Numerical applications

The performances of the second order terms estimation procedure are now evaluated. The updating procedures have been implemented as FORTRAN subroutines. The proposed updates allow calculating the estimated second order terms, and then the mobile asymptotes if necessary, which are used as input data for CONLIN V2.0 optimizer from Fleury [8]. This optimizer supports both kind of second order approximations, quadratic separable scheme and GMMA. The solutions of the convex sub-problems are realized with a primal-dual strategy, i.e. each solution of each sub-problem is itself broken into a sequence of quadratic separable sub-problems, which can be solved efficiently with a dual method [9].
The main characteristics of the quasi-Cauchy updating procedures can be illustrated on the basis of two classic benchmarks, both from Svanberg [6]. The first one is the cantilever beam problem. It consists in minimizing the weight of a beam made of 5 pieces of hollow square cross section and constant thickness x_i. The constraint is a bound upon the tip displacement. The closed form expression of the problem is:

$$\min_{x_i > 0} \quad 0.0624\,(x_1 + x_2 + x_3 + x_4) \quad s.t. \quad 61/x_1^3 + 37/x_2^3 + 19/x_3^3 + 7/x_4^3 + 1/x_5^3 \tag{16}$$

The second benchmark is the two-bar truss problem. This small optimization problem is interesting because it mixes a sizing variables (x_1 is the cross section of the bars) and a shape variable (x_2 is the half distance between the two supports).

The statement of the problem is to find the truss with minimum weight that satisfies restrictions on the allowable stress in the members. The closed form of the problem is the following:

$$\begin{aligned} &\min_{x_1,x_2} \quad x_1\sqrt{1+x_2^2} \\ &s.t. \quad 0.124\sqrt{1+x_2^2}\left(8/x_1+1/(x_1x_2)\right)\le 1 \\ &\qquad 0.124\sqrt{1+x_2^2}\left(8/x_1-1/(x_1x_2)\right)\le 1 \\ &0.2\le x_1\le 4.0 \quad and \quad 0.1\le x_2\le 1.6 \end{aligned} \tag{17}$$

The qualities of the approximation schemes using quasi-Cauchy estimated diagonal second order terms are compared to using exact second derivatives and to using diagonal quasi-Newton techniques as implemented in Ref. [3]. Table 1 and 2 give the convergence histories for the two-bar truss problem when using quadratic approximations (table 1) and when using GMMA approximations. The table reports the weight of the truss and the constraint violation, which is defined as the biggest ratio between the constraint value to the constraint bound. Five values of the second order terms are used: the exact second order sensitivities (exact), the quasi-Newton diagonal estimations, and the 3 quasi-Cauchy estimations, i.e. Oren-Luenberger scaling (OL), quasi-Cauchy updating of D (QC-D), quasi-Cauchy updating of $D^{1/2}$ (QC-D^1/2). The stopping criterion is satisfied when the optimal value of the objective function is known with 4 digits and when the constraint violation is less than 10^{-4}.

Table 1: Two-bar truss with quadratic separable approximation

It.	Exact	QN	QC -OL	QC –D	QC -D^1/2
1	1.6771 (0.9242)	1.6771 (0.9242)	1.6771 (0.9242)	1.6771 (0.9242)	1.6771 (0.9242)
2	1.4981 (1.0120)	1.4723 (1.0266)	1.4762 (1.0839)	1.4723 (1.0266)	1.4726 (1.0264)
3	1.5095 (0.9996)	1.5072 (1.0011)	1.4691(1.0271)	1.5063 (1.0024)	1.4988 (1.0138)
4	1.5087 (0.9999)	1.5087 (0.9999)	1.5088 (0.9999)	1.5090 (0.9984)	1.5146 (0.9974)
5	1.5087 (1.0000)	1.5087 (1.0000)	1.5086 (1.0000)	1.5086 (1.0000)	1.5073 (1.0025)
6			1.5087 (1.0000)	1.5086 (1.0000)	1.5085 (1.0001)
7	Weight (violation)				1.5086 (1.0000)

Table 2: Two-bar truss with GMMA approximation

It.	Exact	QN	QC-OL	QC-D	QC-D^1/2	QC-D SOC	QC-D^1/2 SOC
1	1.6771 (0.9242)	1.6771 (0.9242)	1.6771 (0.9242)	1.6771 (0.9242)	1.6771 (0.9242)	1.6771 (0.9242)	1.6771 (0.9242)
2	1.3932 (1.6180)	1.3986 (1.6117)	1.4405 (1.5649)	1.3986 (1.6117)	1.3932 (1.6179)	1.3986 (1.6117)	1.3986 (1.6117)
3	1.5405 (1.0466)	1.1738 (1.3653)	1.1305 (1.4149)	1.6666 (1.1452)	1.5112 (1.2096)	1.4160 (1.1693)	1.4160 (1.1693)
4	1.5081 (1.0008)	1.4685 (1.0399)	1.2823 (1.2002)	1.5827 (1.0526)	1.5018 (1.0503)	1.4891 (1.0151)	1.4889 (1.0155)
5	1.5086 (0.9999)	1.5163 (0.9968)	1.5521 (0.9923)	1.5067 (1.0233)	1.5106 (1.0077)	1.5075 (1.0007)	1.5074 (1.0008)
6	1.5086 (1.0000)	1.5093 (0.9996)	1.5105 (1.0007)	1.5030 (1.0056)	1.5136 (0.9995)	1.5086 (1.0000)	1.5086 (1.0000)
7		1.5086 (1.0000)	No solution	1.5084 (1.0002)	1.5120 (0.9993)	1.5086 (1.0000)	1.5086 (1.0000)
8			found to	1.5086 (1.0000)	1.5088 (0.9999)		
9			sub-problem		1.5086 (1.0000)		

Table 3: Beam problem with GMMA approximation

It	Exact	QN	QC-D	QC-D^1/2	QC-D SOC	QC-D^1/2 SOC
1	1.5599 (1.0000)	1.5599 (1.0000)	1.5599 (1.0000)	1.5599 (1.0000)	1.5599 (1.0000)	1.5599 (1.0000)
2	1.3784 (0.9666)	1.3680 (1.0091)	1.3680 (1.0091)	1.3680 (1.0091)	1.3680 (1.0091)	1.3680 (1.0091)
3	1.3421 (0.9962)	1.3059 (1.7616)	1.3214 (1.1022)	1.3149 (1.1807)	1.3361(1.0154)	1.3338 (1.0273)
4	1.3399 (1.0000)	1.0273 (2.7830)	1.2913 (1.1820)	1.2524 (1.4751)	1.3390 (1.0039)	1.3362 (1.0143)
5		1.1446 (2.1536)	1.3278 (1.0541)	1.2377 (1.8898)	1.3401 (0.9998)	1.3395 (1.0019)
6		1.3154 (1.1153)	1.3432 (0.9985)	1.1685 (2.3955)	1.3399 (1.0000)	1.3400 (0.9999)
7		1.3428 (1.0004)	1.3407 (0.9991)	1.0846 (34.128)		1.3399 (1.0000)
8		1.3409 (0.9989)	1.4000 (0.9999)	0.5300 (68010.)		
9		1.3398 (1.0001)	1.3399 (1.0000)	No solution		
0		1.3399 (1.0000)		found		

At table 3, one reports the results when solving the beam problem with the same set of different values of the second order terms. For both examples, no move-limit strategy has been used. Only the quality of the approximation is measured.
The first conclusion of our experiments that are illustrated at tables 1, 2, and 3 is that QC updates produce good quality diagonal second order terms approximations. However, the approximation procedures based on both QC updating schemes (of D and $D^{1/2}$) generally lead to accumulation points in a bit more important number of iterations than the diagonal quasi-Newton estimation and the second order exact sensitivities. If one expects that exact second order sensitivities are better than quasi-Cauchy estimations, one may be surprised that Quasi-Newton estimation based on implementation given in Ref. [3] is generally better than quasi-Cauchy updates, because making finite differences of the first derivatives has not the same nice elaborated mathematical background as the quasi-Cauchy techniques.
Despite what seems to be suggested by Zhu et al. in [5], updating scheme based on D is not less good than the updating scheme based on $D^{1/2}$ in structural optimization. The first updating scheme is easier to implement and it is generally quite efficient. Even if in the two-bar truss example, the Oren-Luenberger scaling seems to be quite efficient, this performance could not been repeated in other problems. Quite often it leads to sub-problems that can not be solved. This too simple scheme is not robust enough.
Finally QC updating schemes (especially update of D) can be efficiently combined with SOC (second order correction) procedure. This SOC procedure does not destabilize the good convergence properties of QC updating estimation. The conclusions are not always so clear with diagonal QN estimations.

7. Conclusion and on-going work

The theory and the implementation of the recent quasi-Cauchy updates proposed by Zhu et al. [5] have been presented. The QC update schemes, which provide a priori diagonal estimations of the second order terms, are interesting for building structural approximations. To this end an adaptation work has been realized. Preliminary numerical results showed that convergence speed of QC updates can be slow, but they generally yield good quality diagonal Hessian estimates for structural approximations. However, up to now, we have noticed that their performances remain a bit inferior to diagonal quasi-Newton estimations. This leads to think that adaptation work to structural optimization must be pursued, because quasi-Cauchy updating provides a better mathematical foundation to the procedure than making quasi-Newton finite differences.
On going and future works are devoted to use quasi-Cauchy updates with other approximation schemes like the recent Globally Convergent Method of Moving Asymptotes (GCMMA) [10]. In addition, numerical applications with more realistic problems from composite optimization and topology optimization problems will be regarded.

8. Acknowledgements

The work presented in this paper was partly supported by the BRITE-EURAM project COMPOPT (contract BRPR-CT96-0332 of the European Community).

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