UNSUPERVISED LEARNING BASED MODEL ORDER REDUCTION FOR HYPERELASTOPLASTICITY

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Why Model Order Reduction?

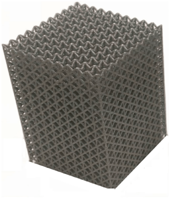

Figure: Metal lattice in the form of periodic mesostructural unit cells

- Real time surgical simulations requires quick computational results.
- Expensive simulations for problems dealing with micro-scale phenomena

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How model order reduction helps?

- Parameterised non-linear mechanical problem:

$$\underset{\sim}{f}_{int}(\underset{\sim}{u}(\mu), \mu) - \underset{\sim}{f}_{ext} = \underset{\sim}{0}$$

  $\underset{\sim}{u}$ is the unknown that has to be computed for any value of parameter μ.

- **Ansatz:** Use precomputed solutions to speed up the online simulation.

Solve non-linear problems efficiently

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How model order reduction helps?

Projection based model order reduction

- Use Galerkin framework, solve for a reduced system:

$$\phi^T \underset{\sim}{f}_{int}(\phi \underset{\sim}{\alpha}) - \phi^T \underset{\sim}{f}_{ext} = \underset{\sim}{0}$$

- Interpolation: $\underset{\sim}{u} = \phi \underset{\sim}{\alpha}$
- Reduced stiffness matrix: $\boldsymbol{K_r} = \phi^T \mathbf{K} \phi$
- Reduced force vector: $\underset{\sim}{f}^r_{int} = \phi^T \underset{\sim}{f}_{int}$
- Solve using Newton Raphson: $\Delta \underset{\sim}{\alpha} = -(\phi^T \mathbf{K} \phi)^{-1} \phi^T \underset{\sim}{R}$
  $\underset{\sim}{R} = \underset{\sim}{f}_{int}(\phi\alpha) + \underset{\sim}{f}_{ext}$

Reduced number of unknowns:$\underset{\sim}{\alpha} << \underset{\sim}{u}$

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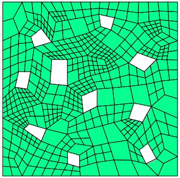
How model order reduction helps?

- ✓ Reduced number of unknowns
- × Number of Gauss points

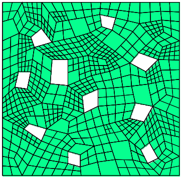
Hyper-reduction strategy to reduce the number of Gauss points

- Utilize the modes to obtain an adaptive non confirming mesh with less number of elements

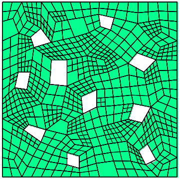
Hyperreduction: Modes and number of elements



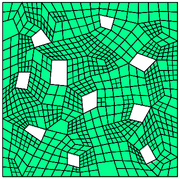
ϕ_1 :530 Elements



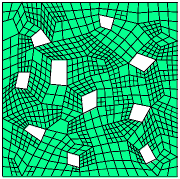
ϕ_2 :668 Elements



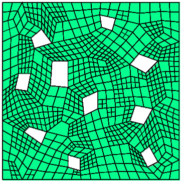
ϕ_3 :728 Elements



ϕ_4 :737 Elements



ϕ_5 :821 Elements



ϕ_6 :872 Elements

Pictorial representation of adaptive non confirming mesh

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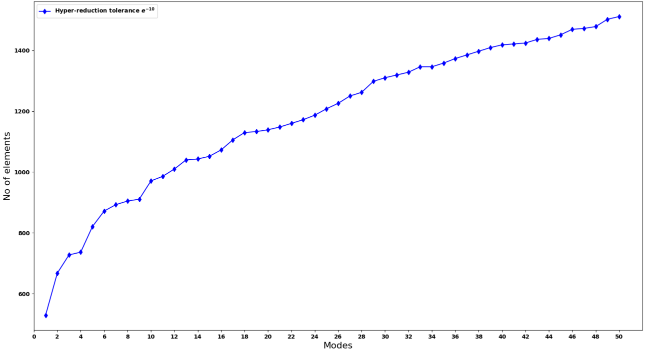
Hyperreduction: Modes and number of elements



- Hyper-reduction:Number of elements (Gauss points) increases with increase in number of modes
- GOAL: Utilize a few modes as possible

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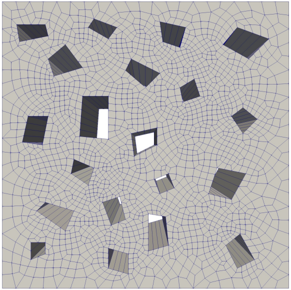
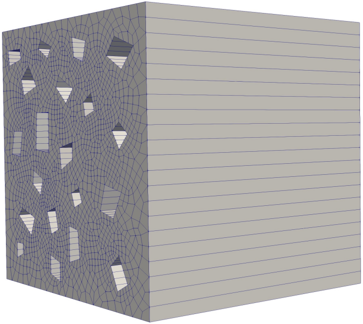
RVE model with multiple voids





Large deformation hyperelastoplastic material model.
Enforced periodic boundary condition using Lagrange multipliers.

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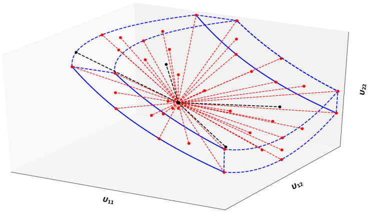
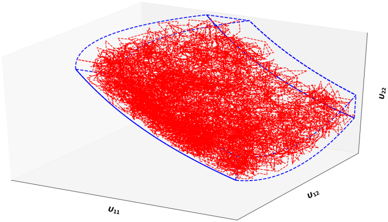
Offline stage: Generation of snapshots

- Input parameters: The coefficients of stretch tensor ($\mathbf{U_M}$).

$$\mathbf{F_M} = \mathbf{R_M U_M}$$



Monotonic loading training parameters for full training set.



Random loading training path for one simulation.

Red lines are training parameters and black lines are test case parameters

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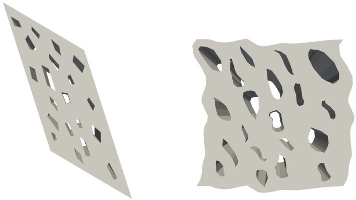
Additive split of the snapshot solutions

$$\underset{\sim}{u} = \boldsymbol{\phi}\underset{\sim}{\alpha} + \boldsymbol{\psi}\underset{\sim}{\beta}$$

Additively split the snapshot solution into fluctuating and homogenised deformation



Homogeneous deformation   Fluctuating deformation

Perform SVD only to the fluctuating deformation

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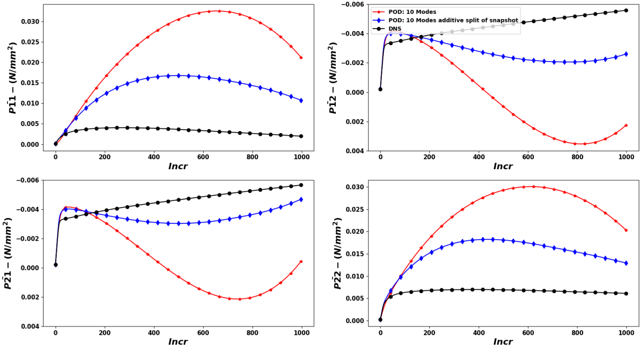
Drawback of POD



POD solution with additive split of snapshots will be used as a reference to compare the results obtained using novel approach.

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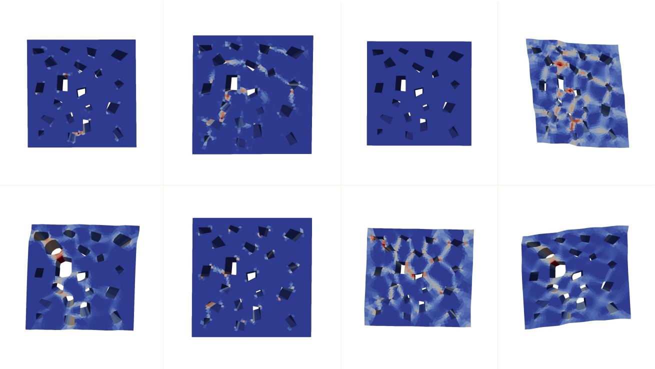
Why unsupervised learning method?

Different snapshot solutions



The localization patterns are different for each case.

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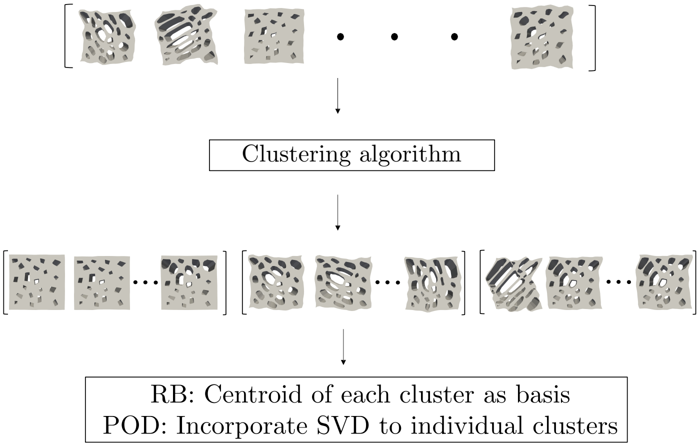
The proposed clustering strategy



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Clustering: The methodology

INPUT

Snapshot solutions

- Fluctuating displacements
- Cumulative plastic strains
- Plastic deformation gradient

Which feature?

Scaling the snapshots?

CLUSTERING ALGORITHM

- Centroid based
- Connectivity based

Which algorithm?

Number of clusters?

Which similarity measure?

OUTPUT

Snapshots grouped based on deformation pattern

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Clustering: The methodology

Clustering approaches

Centroid based clustering

- × Partition the snapshots based on similarity
- × Prespecify the number of clusters

Connectivity based clustering

- × Establish connectivity with nearby snapshots
- × Specify the measure for search radius

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Clustering: The methodology

Evaluation metric and preprocessing

Similarity measure

- × Euclidien distance
- × Absolute projection

Normalization

- × Scale Features from 0 to 1
- × Scale snapshots to unit norm

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Snapshot solutions

Three snapshot matrices

- Cumulative plastic strain values at multiple increments ($\widetilde{\epsilon_p}$)
- Displacement values at multiple increments ($\underset{\sim}{u}$)
- Components of plastic deformation gradient tensor ($\mathbf{F}_P$)

$$\begin{bmatrix} | & | & | & & | & & | \\ \widetilde{\epsilon}_{p1}^{incr} & \widetilde{\epsilon}_{p1}^{incr} & \widetilde{\epsilon}_{p1}^{incr} & \dots & \widetilde{\epsilon}_{p2}^{incr} & \dots & \widetilde{\epsilon}_{pn}^{incr} \\ | & | & | & | & | & | & | \end{bmatrix}$$

$$\begin{bmatrix} | & | & | & | & | & | & | \\ u_1^{incr} & u_1^{incr} & u_1^{incr} & \dots & u_2^{incr} & \dots & u_n^{incr} \\ | & | & | & | & | & | & | \end{bmatrix}$$

$$\begin{bmatrix} | & | & | & | & | & | & | \\ FP_{ii_1}^{incr} & FP_{ii_1}^{incr} & FP_{ii_1}^{incr} & \dots & FP_{ii_2}^{incr} & \dots & FP_{ii_n}^{incr} \\ | & | & | & | & | & | & | \end{bmatrix}$$

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Clustering: Steps involved at the offline stage

Snapshots:
Solutions of multiple
load increments from DNS

$\downarrow$

Group snapshots into clusters based on
$\underset{\sim}{u}_{fluc}$
$\widetilde{\epsilon_p}$
$\widetilde{\epsilon_p}$ and $\mathbf{F_P}$
$\widetilde{\epsilon_p}$, $\mathbf{F_P}$ and $\underset{\sim}{u}_{fluc}$

$\downarrow$

Additively split snapshots
into homogeneous and
fluctuating part

$\downarrow$

SVD to fluctuating
snapshot solutions
in individual clusters

$\downarrow$

Choose the appropriate
modes from each cluster
and orthonormalize

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Selecting the optimal modes

$$
\begin{bmatrix} \Sigma_{c1}^{1} \\ \Sigma_{c1}^{2} \\ \Sigma_{c1}^{3} \\ \Sigma_{c1}^{4} \end{bmatrix}
\quad
\begin{bmatrix} \Sigma_{c2}^{1} \\ \Sigma_{c2}^{2} \\ \Sigma_{c2}^{3} \\ \Sigma_{c2}^{4} \end{bmatrix}
\quad
\begin{bmatrix} \Sigma_{c3}^{1} \\ \Sigma_{c3}^{2} \\ \Sigma_{c3}^{3} \\ \Sigma_{c3}^{4} \end{bmatrix}
$$

$\Sigma = \text{Singular values}$

$c = \text{Cluster}$

- Top singular vectors of each cluster are the first 3 modes
- For 4th mode, Normalize singular values for all clusters and compute:

$$\Sigma_{ci}^{1} - \Sigma_{ci}^{2} \qquad i = 1, 2, 3$$

- Select the one with minimum difference.
- Continue until decided number of modes are reached.

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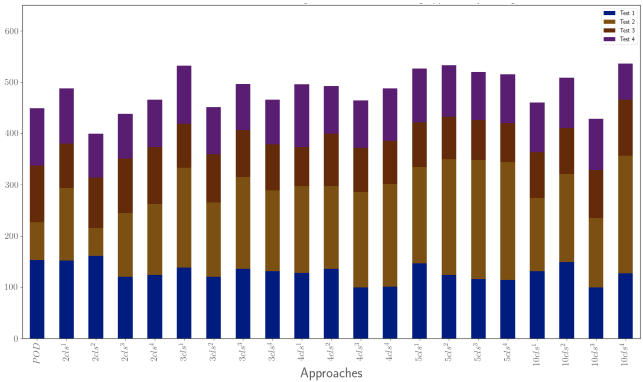
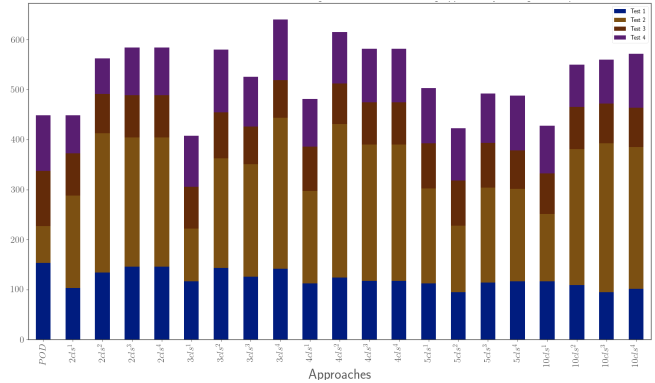
Result of POD based on centroid clustering with Euclidean distance measure by scaling feature between 0 and 1



Sum of error in stress values for all test cases with 10 modes

Clusters^1: Clustering based on $\underset{\sim}{u}_{fluc}$; *Clusters*^{2}: Clustering based on $\widetilde{\epsilon_p}$;
Clusters^3: Clustering based on $\widetilde{\epsilon_p}$ and $\mathbf{F_P}$; *Clusters*^{4}: Clustering based on $\widetilde{\epsilon_p}$, $\mathbf{F_P}$ and $\underset{\sim}{u}_{fluc}$

Result of POD based on centroid clustering with Euclidean distance measure by scaling snapshots to unit norm



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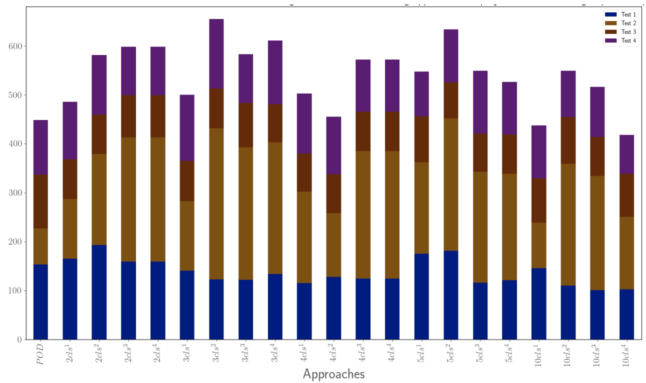
Result of POD based on centroid clustering with projection measure by scaling snapshots to unit norm



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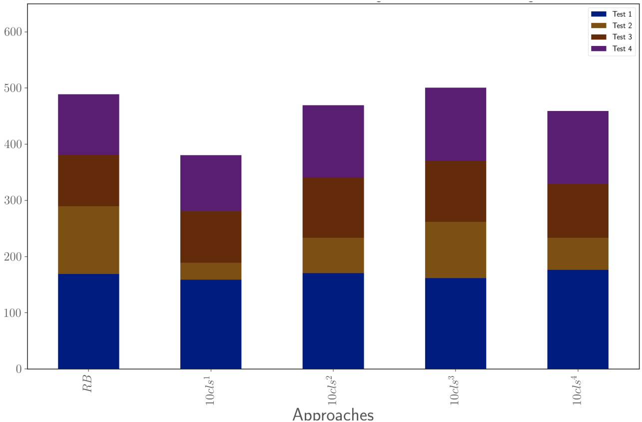
Result of RB based on centroid clustering with Euclidean distance measure by scaling feature between 0 and 1



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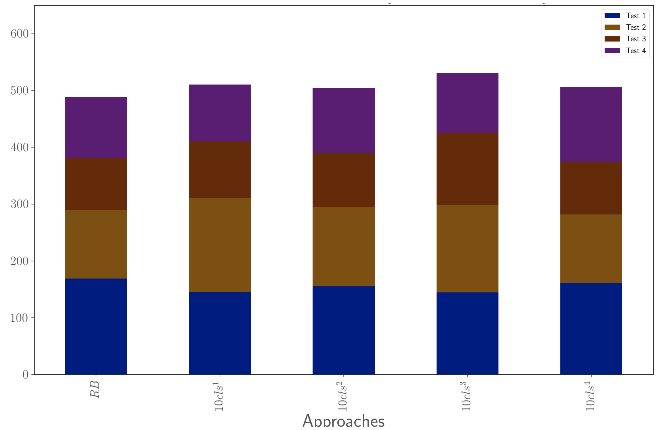
Result of RB based on centroid clustering with Euclidean distance measure by scaling snapshots to unit norm



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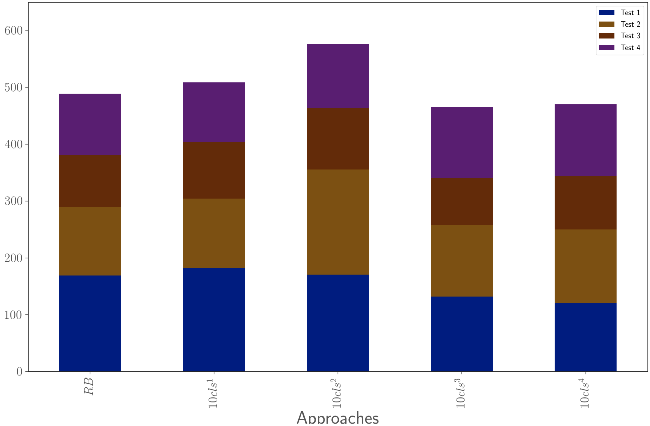
Result of RB based on centroid clustering with projection measure by scaling snapshots to unit norm



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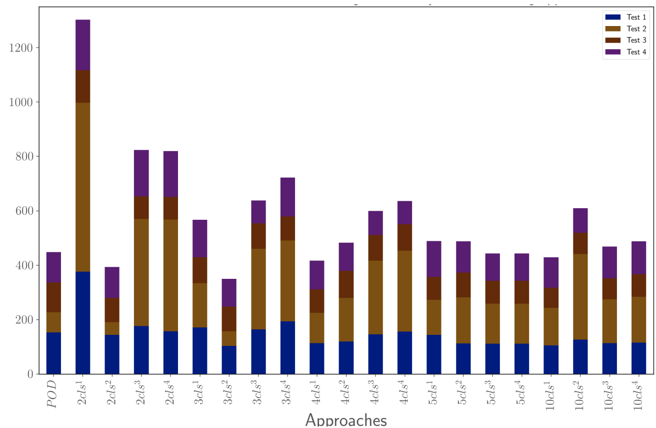
Result of POD based on connectivity clustering with distance measure by scaling feature between 0 and 1



Sum of error in stress values for all test cases with 10 modes

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Inferences from test cases

- Centroid clustering approach with distance measure improves online prediction with two clusters of snapshots grouped based on $\widetilde{\epsilon_p}$.
- Connectivity clustering approach with distance measure improves online prediction with three clusters of snapshots grouped based on $\widetilde{\epsilon_p}$.
- Clustering based on $\mathbf{F_P}$ and $\underset{\sim}{u}_{fluc}$ does not facilitate to improve online prediction for monotonic loading.
- Measure of similarity based on Euclidean distances works better than the measure using absolute projection.

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Future work

- Investigate more on clustering approaches to have a significant improvement in online prediction with less number of modes.
- Incorporate clustering approach for random load path.
- Utilize the results of clustering approach to obtain an adaptive mesh for hyper-reduction strategy.

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