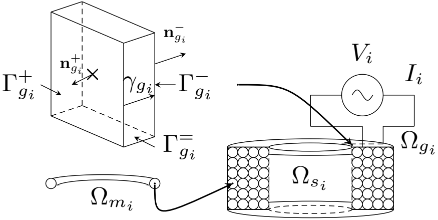
Finite-Element Modeling of Thin Wires Including Skin- and Proximity Effects

Jonathan Velasco

January 7th, 2022

What's a wire?


$\mathbf{n}^-_{g_i}$
$\mathbf{n}^+_{g_i}$
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γ_{g_i}
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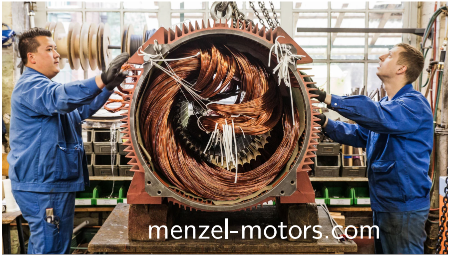
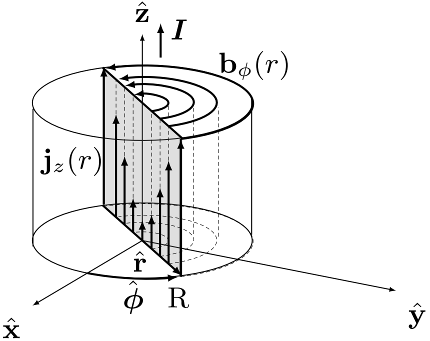
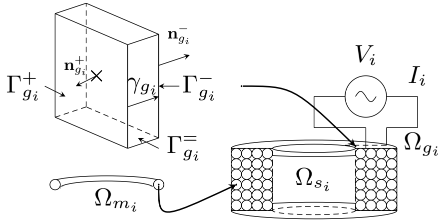
$\Gamma^=_{g_i}$
V_i
I_i
Ω_{g_i}
Ω_{s_i}
Ω_{m_i}

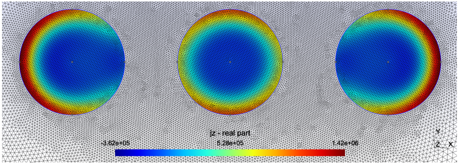

menzel-motors.com

The challenge in modeling conductors: Skin and Proximity Effect



The challenge in modeling conductors: meshing, high aspect ratio and problem size





Introduction

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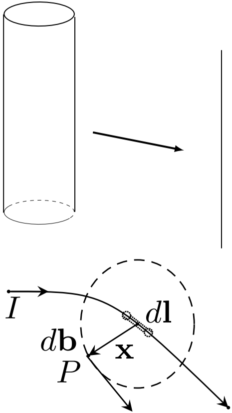
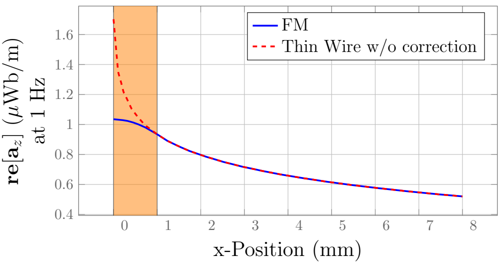
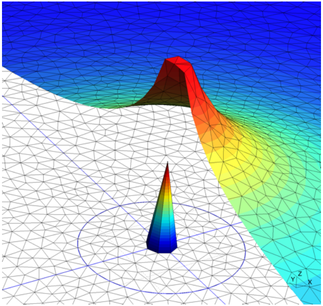
- at a broad range of frequencies
- without the need to discretize the wires explicitly to resolve eddy currents
- with the possibility, then, to work with a much coarser mesh
- without significant loss of accuracy



Modeling Conductors in Frequency Domain: The Problem

- Real wires would advantageously be represented in the model by idealised 1D wires : mesh edges (3D) or nodes (2D)
- BUT FE solutions with 1D conductors yield a non-physical peak of the a-field
- This peak is mesh-dependent and it grows indefinitely in amplitude as the mesh is refined





$$\left(\nu\,\mathbf{curl}\,\mathbf{a}, \mathbf{curl}\,(\alpha_n\hat{\mathbf{z}})\right)_\Omega - A_c\left(I_i, \alpha_n\hat{\mathbf{z}}\right)_{\Omega_{LR}} = 0, \quad \forall \alpha_n \in F_a(\Omega), \qquad \mathbf{a} = \sum_n a_n \alpha_n \hat{\mathbf{z}}.$$

Modeling Conductors in Frequency Domain: Our Solution

- We call *sleeve* the one element thick layer of elements around the 1D conductor carrying current I_i in the ORIGINAL mesh
- The same FE problem is also solved on the sleeve with Dirichlet BC
- The mesh dependent non-physical peak is subtracted from the FE solution
- The obtained truncated field is valid (up to discretization error) outside the sleeve, and it accounts for all currents except I_i inside the sleeve
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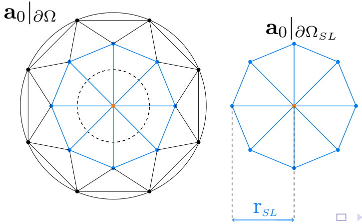

Introduction to the Sleeve Concept

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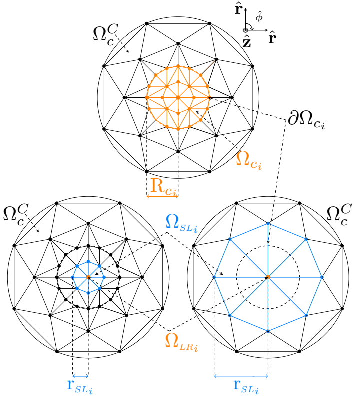

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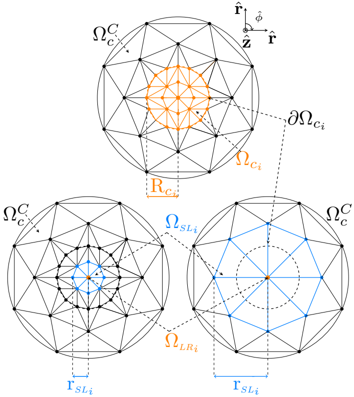

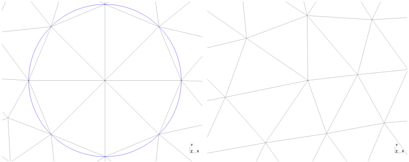

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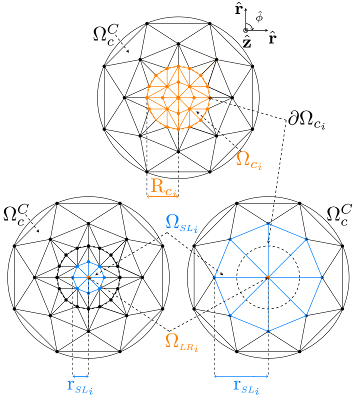
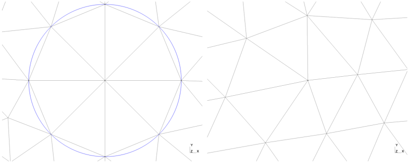
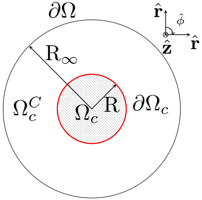


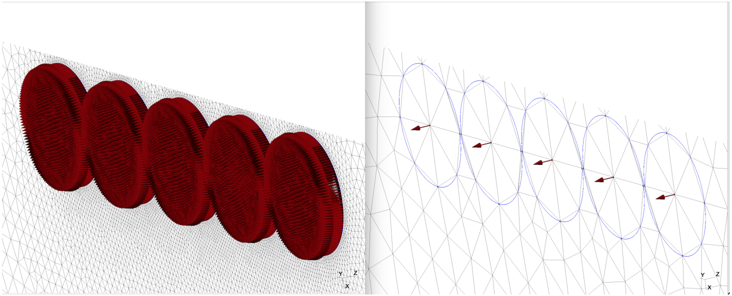




Reference and Proposed Method

Let's define the two main models that will be discussed throughout the description of the method:

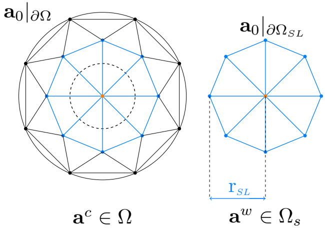
- Full Model: Also known as reference model. The name is related to their all-inclusive nature and fine discretization requirement.
- Semi-analytical Model: The proposed method is a hybrid technique that arises from joining the analytical solution of a single wire in isolation into the FE model.

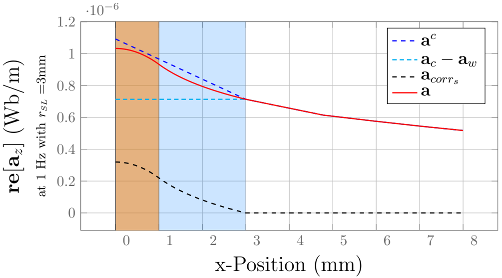


Semi-analytical Method
General Principle in Single Conductor

The general principle of the correction is a one-liner:

$$\mathbf{a} = \mathbf{a}^c - \mathbf{a}^w + \mathbf{a}_{corr}$$





Semi-analytical Method
Analytical Solution of Single Wire in Isolation

$$r^2\partial_r^2\mathrm{a}_z + r\partial_r\mathrm{a}_z + k^2r^2\mathrm{a}_z = -\partial_\phi^2\mathrm{a}_z$$

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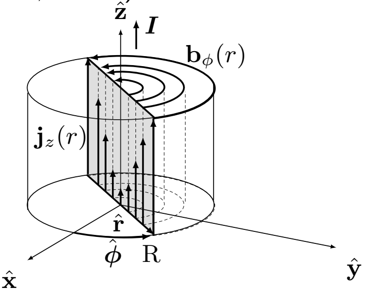
Skin Effect Excitation
($I \neq 0, B = 0$)



$$a_{corr_s}(r) = \frac{\mu_0 I}{2\pi}\left[\frac{\mu_r}{k\mathrm{R}}\frac{J_0(kr) - J_0(k\mathrm{R})}{J_1(k\mathrm{R})} + \log\left(\frac{r_{SL}}{\mathrm{R}}\right)\right], \quad r \leq \mathrm{R}$$

$$a_{corr_s}(r) = \frac{\mu_0 I}{2\pi}\log\left(\frac{r_{SL}}{r}\right), \quad r > \mathrm{R}$$

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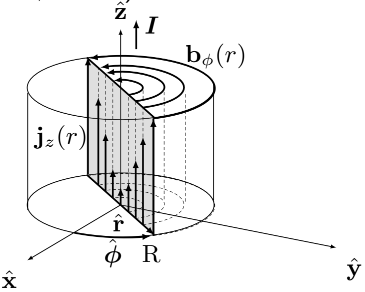

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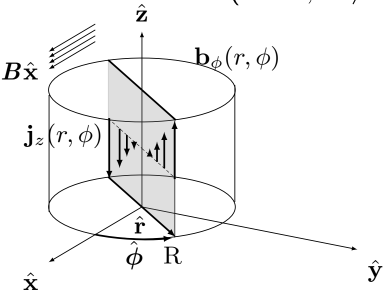
Proximity Effect Excitation
($I = 0, B \neq 0$)



$$a_{corr_p} = -B\left(\frac{2J_1(kr)}{kJ_0(kR)} - r\right)\cos\phi, \quad r \leq \mathrm{R}$$

$$a_{corr_p} = -B\frac{R}{r}\left(\frac{2J_1(kR)}{kRJ_0(kR)} - 1\right)\cos\phi, \quad r > \mathrm{R}$$

Semi-analytical Method including Skin- and Proximity Effects

Let us express the magnetic vector potential as:

$$\mathbf{a} = \mathbf{a}^c - \mathbf{a}^w + \mathbf{a}_{corr}$$

where $\mathbf{a}_{corr} = \mathbf{a}_{corr_s} + \mathbf{a}_{corr_p}$ reconstructs the solution in Ω_{SL}
The $\mathbf{a}v$ magnetodynamics formulation

$$\left(\nu \, \mathbf{curl}\, \mathbf{a}^c, \mathbf{curl}\,(\alpha_n^c \hat{\mathbf{z}})\right)_{\Omega} - A_c\left(I_i, \alpha_n^c \hat{\mathbf{z}}\right)_{\Omega_{LR}} = 0, \quad \forall \alpha_n^c \in F_a(\Omega).$$

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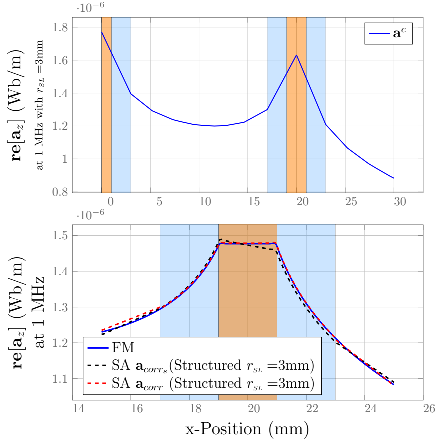

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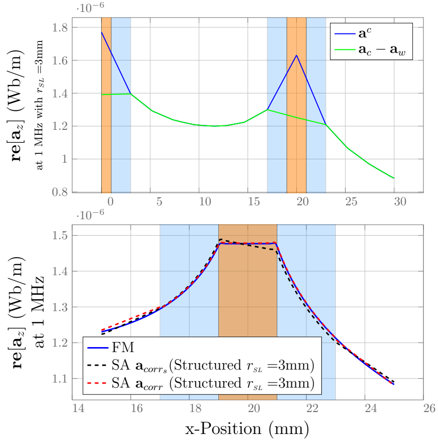

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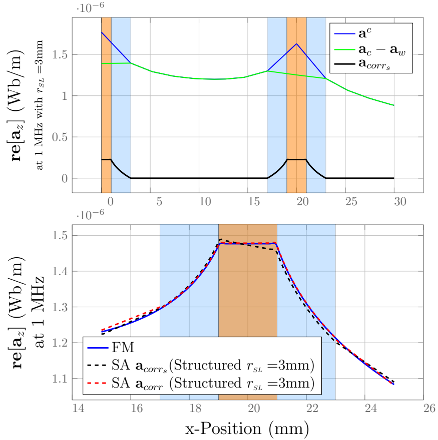

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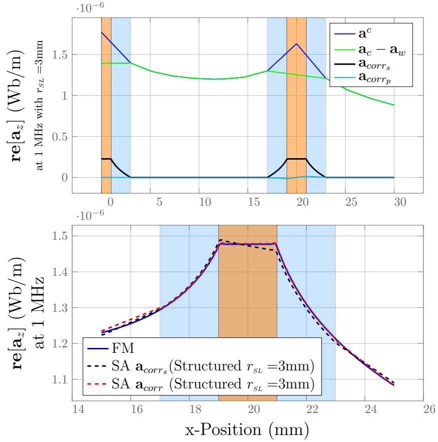

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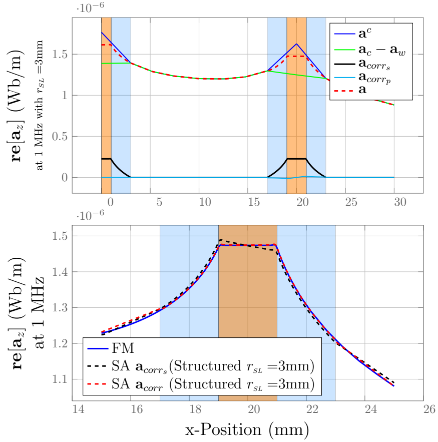


Application Examples

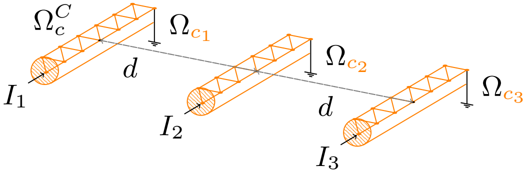
- Single turn wires (3 parallel)
- Multi turn wires (5,10,15 turns)
- Coil radius $\mathrm{R} = 1$mm , an electrical conductivity $\sigma = 5.96e7\,\mathrm{S/m}$, and a relative permeability $\mu_r = 1$ are used in all test cases.

All calculations have been performed using Gmsh and GetDP using linear direct solver MUMPS

Hardware Overview:

Model Name:	MacBook Pro
Model Identifier:	MacBookPro11,4
Processor Name:	Quad-Core Intel Core i7
Processor Speed:	2.2 GHz
Number of Processors:	1
Total Number of Cores:	4
L2 Cache (per Core):	256 KB
L3 Cache:	6 MB
Hyper-Threading Technology:	Enabled
Memory:	16 GB

Application Example: 3-Parallel Wires



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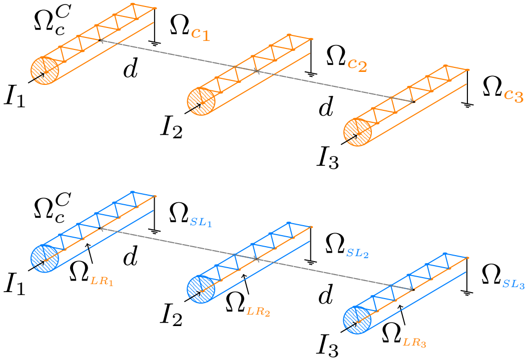

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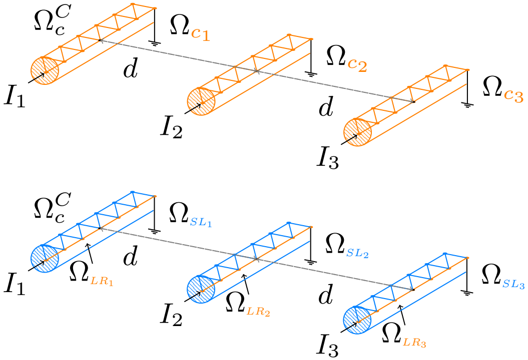

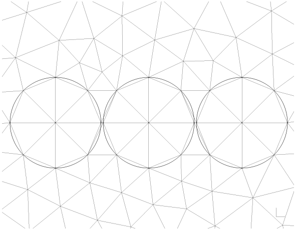

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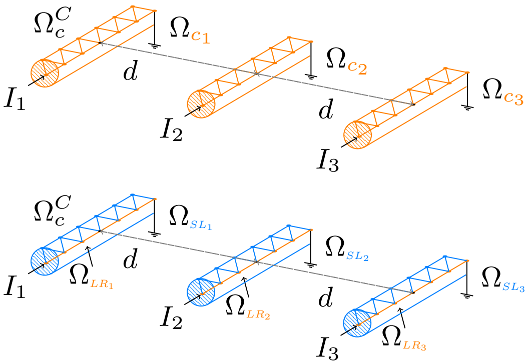
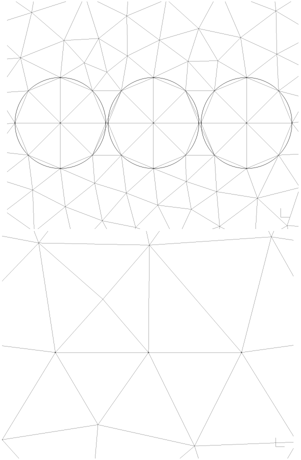
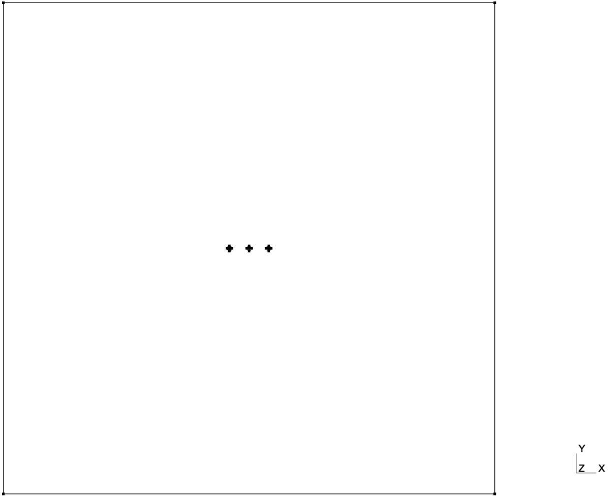
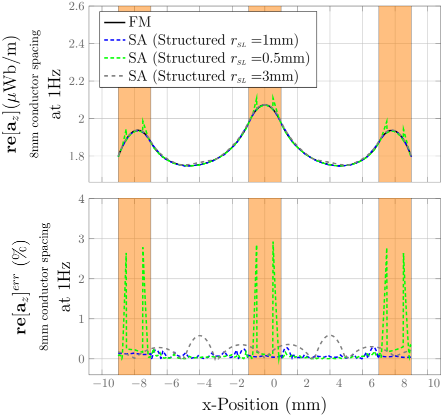




Application Example: 3-Parallel Wires
Geometrical representation with a conductor spacing of 8mm



Application Example: 3-Parallel Wires
Real MVP at 1Hz with conductor spacing of 8mm



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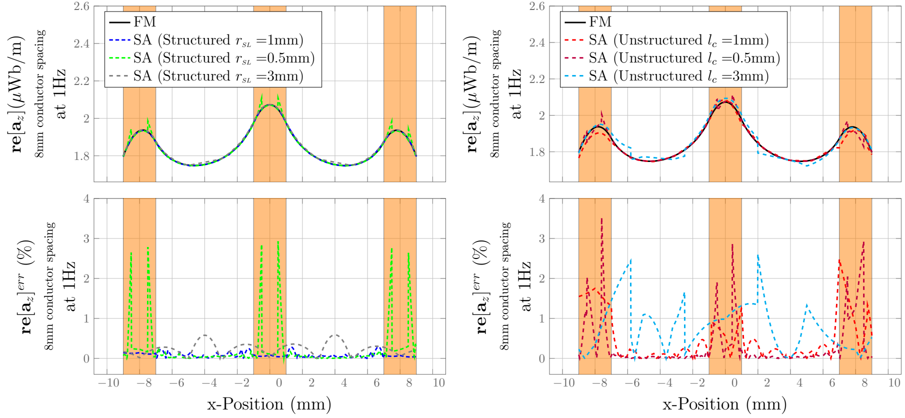
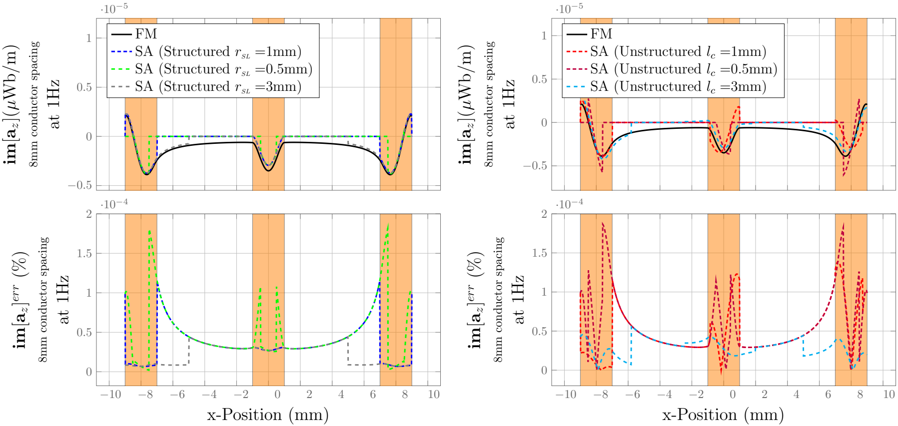


Application Example: 3-Parallel Wires
Imag MVP at 1Hz with conductor spacing of 8mm



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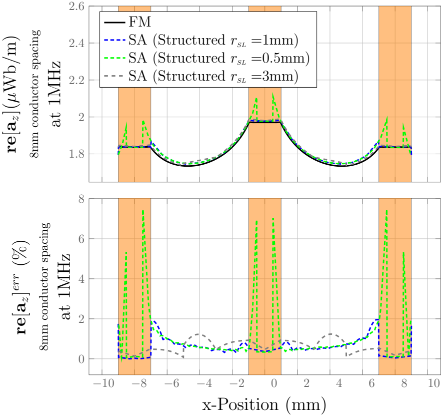

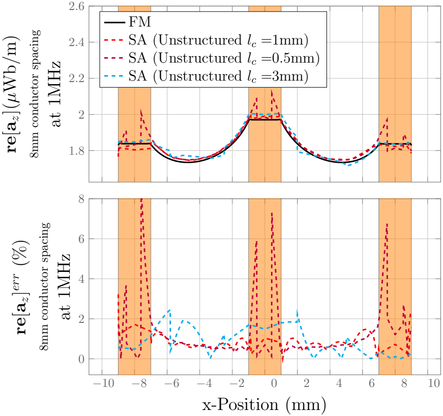

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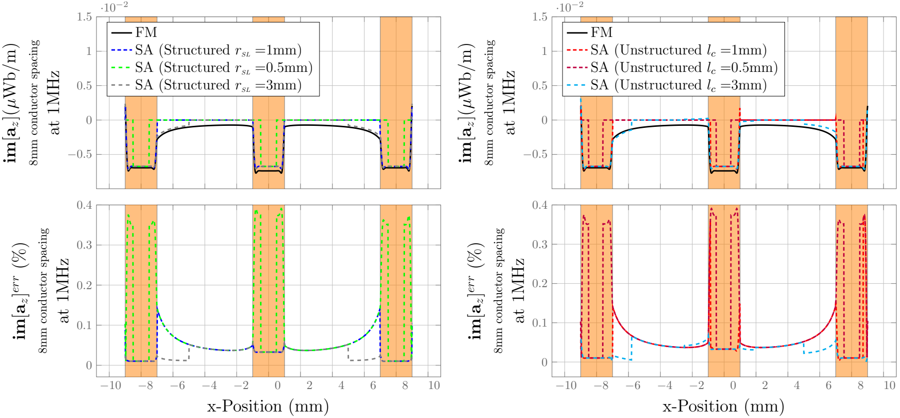
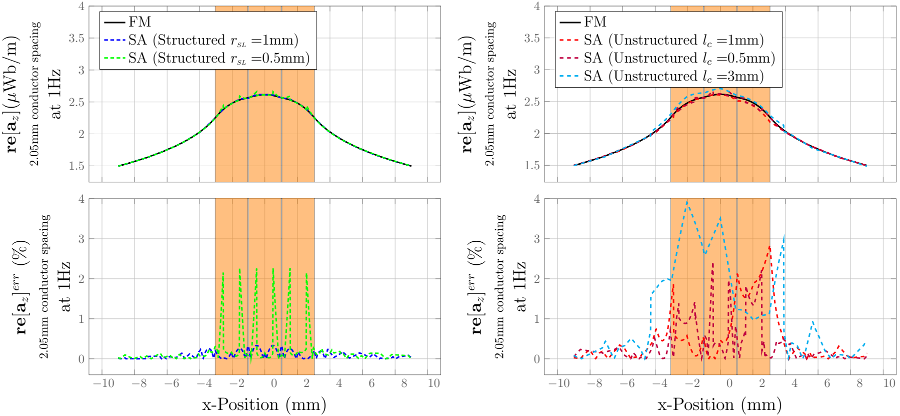


Application Example: 3-Parallel Wires
Real MVP at 1Hz with conductor spacing of 2.05mm



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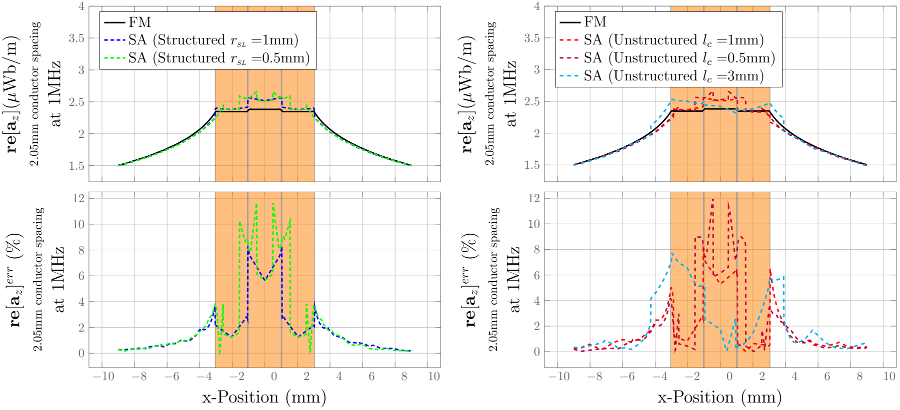

3-Parallel Wires Summary

- A Semi-analytical method to model thin wires was presented

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- A Semi-analytical method to model thin wires was presented
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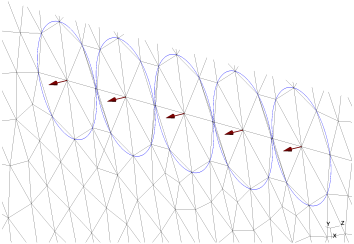
- Then analytical expressions are used to reconstruct the solution within the sleeve to agree with the solution of a fully discretized model
- The application model (3 wires in parallel) is meant to showcase the level off accuracy at different distances and frequencies. This helps to visualize the accuracy of the reconstruction of the solution when both: skin and proximity effects are in action.

Efficient Modeling of Multi-turn Coils
Global Quantities in Thin Wires

- The associated voltage is implicit in the SA formulation and an additional equation is needed



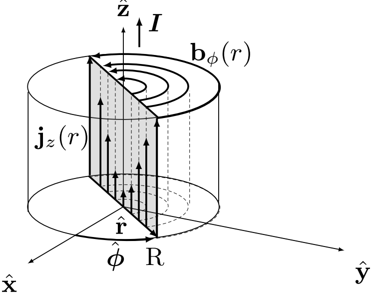
$$\mathbf{grad}\, v = -\left(\frac{\mathbf{j}}{\sigma} - \imath\omega \mathbf{a}_{corr_s}\right) + \imath\omega \mathbf{a}_{corr_p} - \imath\omega(\mathbf{a}^c - \mathbf{a}^w).$$

$$V_i = \left(\frac{k}{2\pi \mathrm{R}\sigma}\frac{J_0(kR)}{J_1(k\mathrm{R})} + \frac{\mu_0}{2\pi}\imath\omega \log(\frac{R_\infty}{R})\right)_{\Omega_{LR_i}} I_i - \imath\omega\left(\mathbf{a}^c - \mathbf{a}^w\right)_{\Omega_{LR_i}}$$

Efficient Modeling of Multi-turn Coils
Power Losses

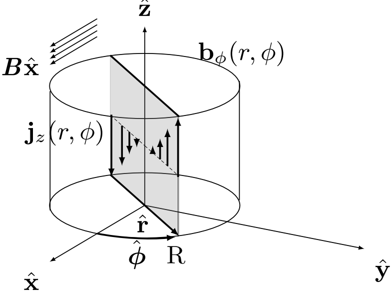
$$P \approx \int_{\Omega_c} \sigma^{-1} \left(|\mathbf{j}_{skin}(r)|^2 + |\mathbf{j}_{prox}(r)|^2 \right) dV = P_{skin} + P_{prox}$$

Skin Effect Excitation
$(I \neq 0, B = 0)$



$$S_{skin} = R_{dc} \frac{kR}{2} \frac{J_0(kR)}{J_1(kR)} |I_i|^2 = P_{skin} + \imath Q_{skin}.$$

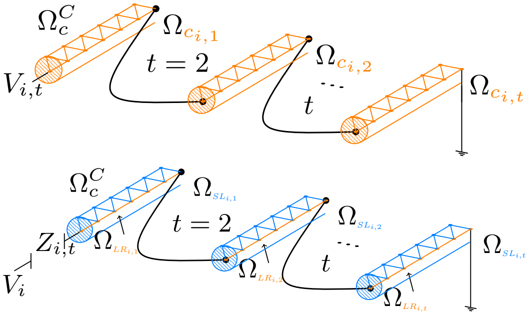
Proximity Effect Excitation
$(I = 0, B \neq 0)$

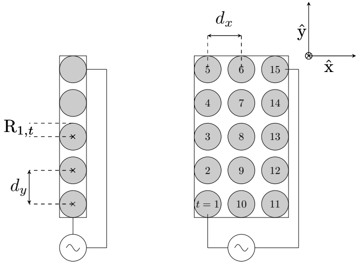


$$P_{prox} = |B|^2 \left(\frac{16\omega^2 \sigma R^2}{D} \right) \frac{\pi R l (k^* J_1(kR) J_0(k^*R) - k J_0(kR) J_1(k^*R))}{k^2 - (k^*)^2}$$

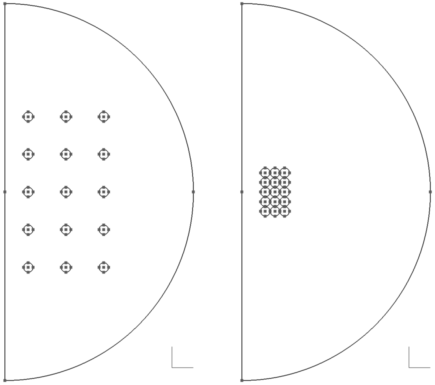
Application Example: Multi-turn Coils

- Constant mesh of 3 elements per radius before $\mathrm{R} = \delta$
- Re-meshing at each frequency past $\mathrm{R} = \delta$ by $l_c = \delta/3$

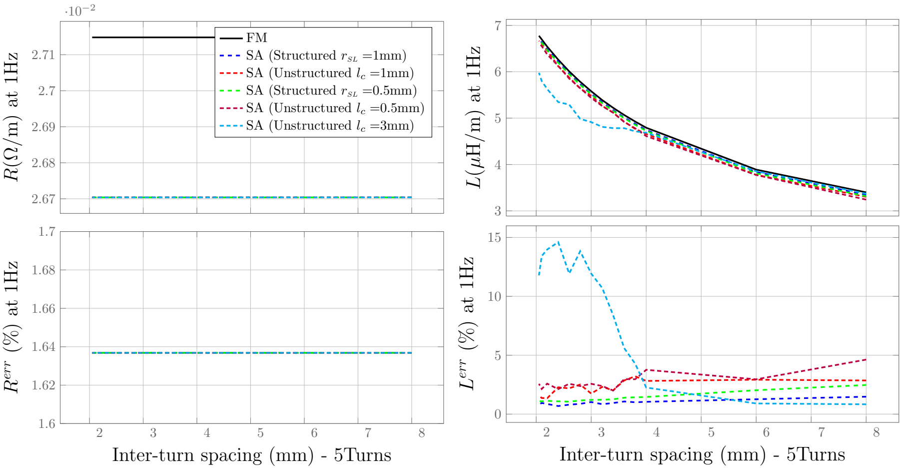




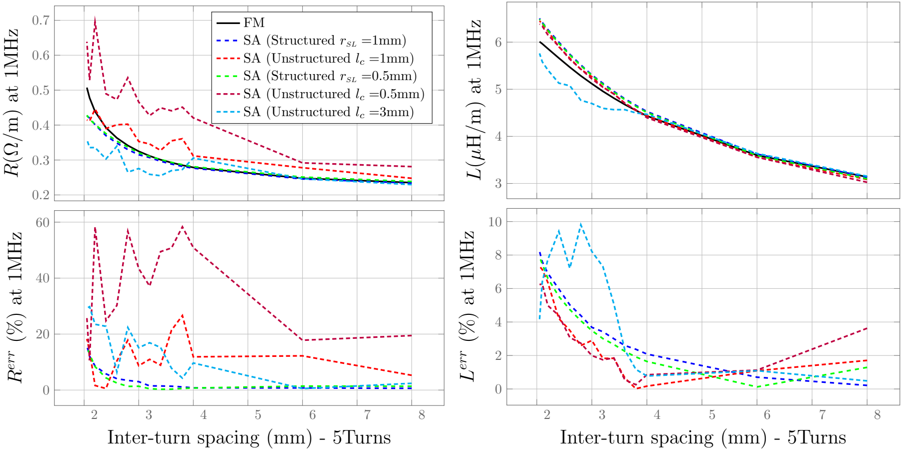
Application Example: Multi-turn Coils
Geometrical representation of a 15-turn coil with an inter-turn distance of: 8mm (left) and 2.05mm (right)



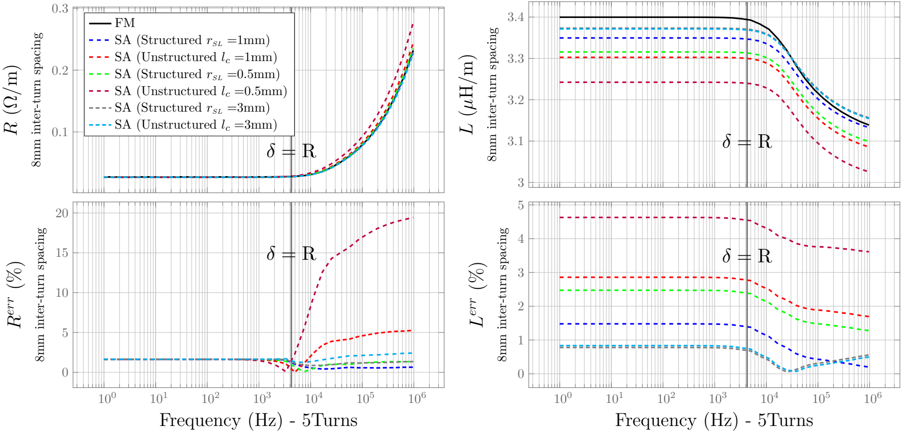
Numerical Test:
Impedance vs. Distance at 1Hz - 5-turn coil



Numerical Test:
Impedance vs. Distance at 1MHz - 5-turn coil



Numerical Tests: Impact of Frequency when conductors are located far from each other - 5-turn coil



5 Turns Inter-turn distance 8mm	FM	SA (Structured) r_{SL} = 1mm				SA (Unstructured) l_c =1mm			
Frequency (Hz)	DoF	DoF	DoF Rel. Diff. (%)	Resistance Rel. Err. (%)	Inductance Rel. Err. (%)	DoF	DoF Rel. Diff. (%)	Resistance Rel. Err. (%)	Inductance Rel. Err. (%)
1.00	1708	665	-61.07	1.64	1.48	425	-75.12	1.64	2.86
4987.89	1755	665	-62.11	1.01	1.38	425	-75.78	0.02	2.76
85222.69	7337	665	-90.94	0.57	0.44	425	-94.21	4.50	1.90
970739.74	48907	665	-98.64	0.64	0.20	425	-99.13	5.26	1.69

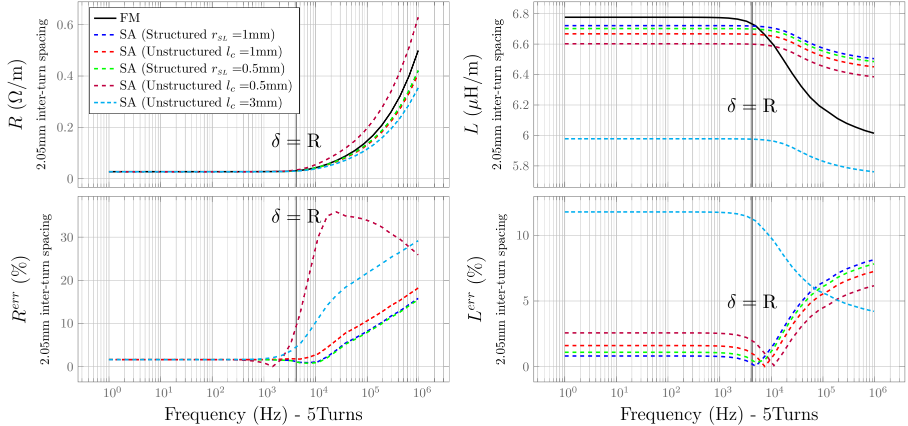
Numerical Tests: Impact of Frequency when conductors are located far from each other - 5-turn coil

5 Turns Inter-turn distance 8mm	FM	SA (Structured) $r_{SL}=1$mm					SA (Unstructured) $l_c=1$mm			
Frequency (Hz)	DoF	DoF	DoF Rel. Diff. (%)	Resistance Rel. Err. (%)	Inductance Rel. Err. (%)	DoF	DoF Rel. Diff. (%)	Resistance Rel. Err. (%)	Inductance Rel. Err. (%)
1.00	1708	665	-61.07	1.64	1.48	425	-75.12	1.64	2.86
4987.89	1755	665	-62.11	1.01	1.38	425	-75.78	0.02	2.76
85222.69	7337	665	-90.94	0.57	0.44	425	-94.21	4.50	1.90
970739.74	48907	665	-98.64	0.64	0.20	425	-99.13	5.26	1.69

5 Turns Inter-turn distance 8mm	FM	SA (Structured) $r_{SL}=0.5$mm					SA (Unstructured) $l_c=0.5$mm			
Frequency (Hz)	DoF	DoF	DoF Rel. Diff. (%)	Resistance Rel. Err. (%)	Inductance Rel. Err. (%)	DoF	DoF Rel. Diff. (%)	Resistance Rel. Err. (%)	Inductance Rel. Err. (%)
1.00	1708	876	-48.71	1.64	2.47	609	-64.34	1.64	4.63
4987.89	1755	876	-50.09	0.68	2.37	609	-65.30	2.36	4.53
85222.69	7337	876	-88.06	1.15	1.49	609	-91.70	16.66	3.76
970739.74	48907	876	-98.21	1.35	1.28	609	-98.75	19.44	3.61

5 Turns Inter-turn distance 8mm	FM	SA (Structured) $r_{SL}=3$mm					SA (Unstructured) $l_c=3$mm			
Frequency (Hz)	DoF	DoF	DoF Rel. Diff. (%)	Resistance Rel. Err. (%)	Inductance Rel. Err. (%)	DoF	DoF Rel. Diff. (%)	Resistance Rel. Err. (%)	Inductance Rel. Err. (%)
1.00	1708	287	-83.20	1.64	0.78	207	87.88	1.64	0.83
4987.89	1755	287	83.65	1.13	0.67	207	88.21	1.31	0.73
85222.69	7337	287	-96.09	1.17	0.30	207	-97.18	2.09	0.24
970739.74	48907	287	-99.41	1.34	0.56	207	-99.58	2.42	0.50

Numerical Tests: Impact of Frequency when conductors are in close proximity - 5-turn coil

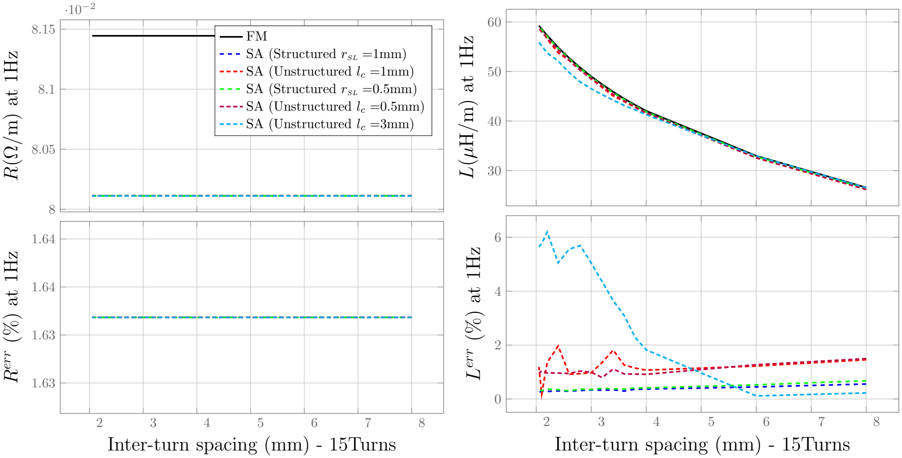


5 Turns Inter-turn distance 2.05mm	FM	SA (Structured) r_{SL} = 1mm				SA (Unstructured) l_c =1mm			
Frequency (Hz)	DoF	DoF	DoF Rel. Diff. (%)	Resistance Rel. Err. (%)	Inductance Rel. Err. (%)	DoF	DoF Rel. Diff. (%)	Resistance Rel. Err. (%)	Inductance Rel. Err. (%)
1.00	924	411	-55.52	1.64	0.81	308	-66.67	1.64	1.61
4987.89	1105	411	-62.81	1.01	0.03	308	-72.13	1.77	0.83
85222.69	5768	411	-92.87	7.74	6.26	308	-94.66	10.14	5.39
970739.74	46614	411	-99.12	15.83	8.14	308	-99.34	18.22	7.25

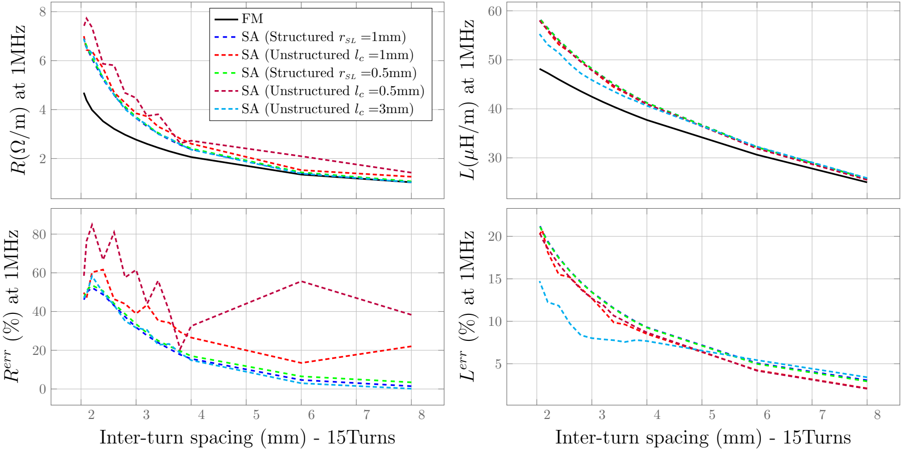
Numerical Tests: Impact of Frequency when conductors are in close proximity - 5-turn coil

5 Turns Inter-turn distance 2.05mm	FM	SA (Structured) $r_{SL} = 1\text{mm}$				SA (Unstructured) $l_c = 1\text{mm}$			
Frequency (Hz)	DoF	DoF	DoF Rel. Diff. (%)	Resistance Rel. Err. (%)	Inductance Rel. Err. (%)	DoF	DoF Rel. Diff. (%)	Resistance Rel. Err. (%)	Inductance Rel. Err. (%)
1.00	924	411	-55.52	1.64	0.81	308	-66.67	1.64	1.61
4987.89	1105	411	-62.81	1.01	0.03	308	-72.13	1.77	0.83
85222.69	5768	411	-92.87	7.74	6.26	308	-94.66	10.14	5.39
970739.74	46614	411	-99.12	15.83	8.14	308	-99.34	18.22	7.25
5 Turns Inter-turn distance 2.05mm	FM	SA (Structured) $r_{SL} = 0.5\text{mm}$				SA (Unstructured) $l_c = 0.5\text{mm}$			
Frequency (Hz)	DoF	DoF	DoF Rel. Diff. (%)	Resistance Rel. Err. (%)	Inductance Rel. Err. (%)	DoF	DoF Rel. Diff. (%)	Resistance Rel. Err. (%)	Inductance Rel. Err. (%)
1.00	924	547	-40.80	1.64	1.10	422	-54.33	1.64	2.57
4987.89	1105	547	-50.50	0.92	0.32	422	-61.81	12.23	1.81
85222.69	5768	547	-90.52	7.46	5.95	422	-92.68	34.19	4.34
970739.74	46614	547	-98.83	15.55	7.82	422	-99.09	25.92	6.16
5 Turns Inter-turn distance 2.05mm	FM	SA (Structured) $r_{SL} = 3\text{mm}$				SA (Unstructured) $l_c = 3\text{mm}$			
Frequency (Hz)	DoF	DoF	DoF Rel. Diff. (%)	Resistance Rel. Err. (%)	Inductance Rel. Err. (%)	DoF	DoF Rel. Diff. (%)	Resistance Rel. Err. (%)	Inductance Rel. Err. (%)
1.00	924	N/A	N/A	N/A	N/A	228	75.32	1.64	11.78
4987.89	1105	N/A	N/A	N/A	N/A	228	-79.37	5.24	11.09
85222.69	5768	N/A	N/A	N/A	N/A	228	-96.05	21.15	5.74
970739.74	46614	N/A	N/A	N/A	N/A	228	-99.51	29.18	4.21

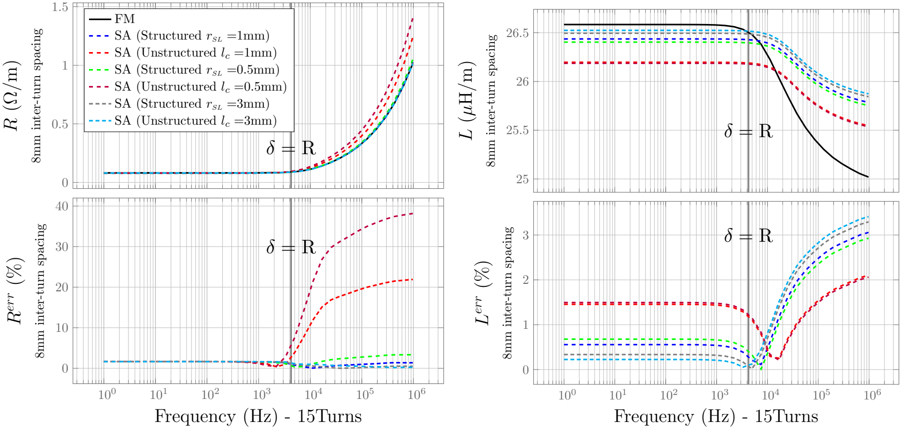
Numerical Test:
Impedance vs. Distance at 1Hz - 15-turn coil



Numerical Test:
Impedance vs. Distance at 1MHz - 15-turn coil



Numerical Tests: Impact of Frequency when conductors are located far from each other - 15-turn coil



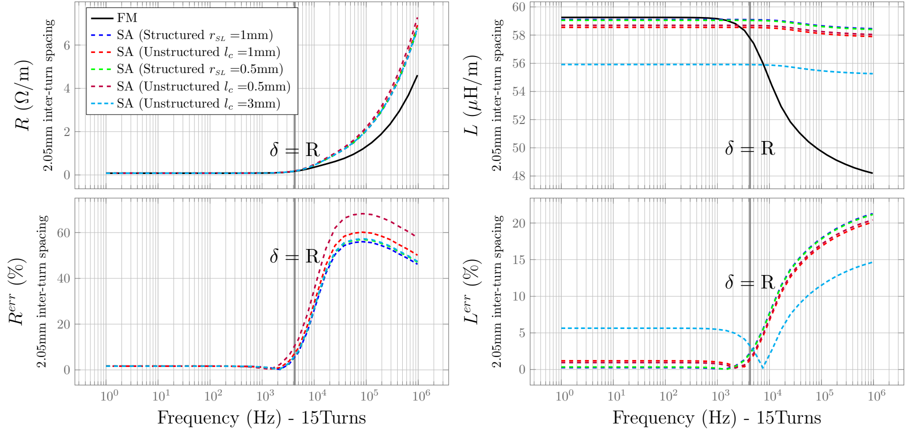
15 Turns Inter-turn distance 8mm	FM	SA (Structured) r_{SL} = 1mm				SA (Unstructured) l_c =1mm			
Frequency (Hz)	DoF	DoF	DoF Rel. Diff. (%)	Resistance Rel. Err. (%)	Inductance Rel. Err. (%)	DoF	DoF Rel. Diff. (%)	Resistance Rel. Err. (%)	Inductance Rel. Err. (%)
1.00	8147	1750	-78.52	1.64	0.56	1018	-87.50	1.64	1.45
4987.89	10878	1750	-83.91	0.78	0.21	1018	-90.64	3.87	1.11
85222.69	113323	1750	-98.46	0.92	2.41	1018	-99.10	19.31	1.47
970739.74	1.02E+06	1750	-99.83	1.36	3.06	1018	-99.90	21.90	2.10

Numerical Tests: Impact of Frequency when conductors are located far from each other - 15-turn coil

15 Turns Inter-turn distance 8mm	FM	SA (Structured) $r_{SL} = 1$mm				SA (Unstructured) $l_c = 1$mm			
Frequency (Hz)	DoF	DoF	DoF Rel. Diff. (%)	Resistance Rel. Err. (%)	Inductance Rel. Err. (%)	DoF	DoF Rel. Diff. (%)	Resistance Rel. Err. (%)	Inductance Rel. Err. (%)
1.00	8147	1750	-78.52	1.64	0.56	1018	-87.50	1.64	1.45
4987.89	10878	1750	-83.91	0.78	0.21	1018	-90.64	3.87	1.11
85222.69	113323	1750	-98.46	0.92	2.41	1018	-99.10	19.31	1.47
970739.74	1.02E+06	1750	-99.83	1.36	3.06	1018	-99.90	21.90	2.10

15 Turns Inter-turn distance 8mm	FM	SA (Structured) $r_{SL} = 0.5$mm				SA (Unstructured) $l_c = 0.5$mm			
Frequency (Hz)	DoF	DoF	DoF Rel. Diff. (%)	Resistance Rel. Err. (%)	Inductance Rel. Err. (%)	DoF	DoF Rel. Diff. (%)	Resistance Rel. Err. (%)	Inductance Rel. Err. (%)
1.00	8147	2785	-65.82	1.64	0.68	2860	-64.90	1.64	1.49
4987.89	10878	2785	-74.40	0.33	0.33	2860	-73.71	7.55	1.15
85222.69	113323	2785	-97.54	2.70	2.28	2860	-97.48	33.87	1.43
970739.74	1.02E+06	2785	-99.83	3.34	2.93	2860	-99.72	38.16	2.06

15 Turns Inter-turn distance 8mm	FM	SA (Structured) $r_{SL} = 3$mm				SA (Unstructured) $l_c = 3$mm			
Frequency (Hz)	DoF	DoF	DoF Rel. Diff. (%)	Resistance Rel. Err. (%)	Inductance Rel. Err. (%)	DoF	DoF Rel. Diff. (%)	Resistance Rel. Err. (%)	Inductance Rel. Err. (%)
1.00	8147	407	-95.00	1.64	0.33	274	-96.64	1.64	0.22
4987.89	10878	407	-96.26	0.98	0.01	274	-97.48	1.14	0.13
85222.69	113323	407	-99.64	0.14	2.64	274	-99.76	0.51	2.76
970739.74	1.02E+06	407	-99.96	0.49	3.29	274	-99.97	0.24	3.41



15 Turns Inter-turn distance 2.05mm	FM	SA (Structured) r_{SL} = 1mm				SA (Unstructured) l_c =1mm			
Frequency (Hz)	DoF	DoF	DoF Rel. Diff. (%)	Resistance Rel. Err. (%)	Inductance Rel. Err. (%)	DoF	DoF Rel. Diff. (%)	Resistance Rel. Err. (%)	Inductance Rel. Err. (%)
1.00	1691	579	-65.76	1.64	0.26	408	-75.87	1.64	1.19
4987.89	2082	579	-72.19	7.69	2.94	408	-80.40	9.61	1.97
85222.69	15885	579	-96.36	56.02	17.56	408	-97.43	60.16	16.45
970739.74	147897	579	-99.61	46.17	21.28	408	-99.72	50.11	20.13

Numerical Tests: Impact of Frequency when conductors are in close proximity - 15-turn coil

15 Turns Inter-turn distance 2.05mm	FM	SA (Structured) $r_{SL} = 1\text{mm}$				SA (Unstructured) $l_c = 1\text{mm}$			
Frequency (Hz)	DoF	DoF	DoF Rel. Diff. (%)	Resistance Rel. Err. (%)	Inductance Rel. Err. (%)	DoF	DoF Rel. Diff. (%)	Resistance Rel. Err. (%)	Inductance Rel. Err. (%)
1.00	1691	579	-65.76	1.64	0.26	408	-75.87	1.64	1.19
4987.89	2082	579	-72.19	7.69	2.94	408	-80.40	9.61	1.97
85222.69	15885	579	-96.36	56.02	17.56	408	-97.43	60.16	16.45
970739.74	147897	579	-99.61	46.17	21.28	408	-99.72	50.11	20.13

15 Turns Inter-turn distance 2.05mm	FM	SA (Structured) $r_{SL} = 0.5\text{mm}$				SA (Unstructured) $l_c = 0.5\text{mm}$			
Frequency (Hz)	DoF	DoF	DoF Rel. Diff. (%)	Resistance Rel. Err. (%)	Inductance Rel. Err. (%)	DoF	DoF Rel. Diff. (%)	Resistance Rel. Err. (%)	Inductance Rel. Err. (%)
1.00	1691	1003	-40.69	1.64	0.35	668	-60.50	1.64	0.95
4987.89	2082	1003	-51.83	8.13	2.84	668	-67.92	13.39	2.22
85222.69	15885	1003	-93.69	56.98	17.45	668	-95.79	68.25	16.74
970739.74	147897	1003	-99.32	47.09	21.16	668	-99.55	57.83	20.42

15 Turns Inter-turn distance 2.05mm	FM	SA (Structured) $r_{SL} = 3\text{mm}$				SA (Unstructured) $l_c = 3\text{mm}$			
Frequency (Hz)	DoF	DoF	DoF Rel. Diff. (%)	Resistance Rel. Err. (%)	Inductance Rel. Err. (%)	DoF	DoF Rel. Diff. (%)	Resistance Rel. Err. (%)	Inductance Rel. Err. (%)
1.00	1691	N/A	N/A	N/A	N/A	289	-82.91	1.64	5.64
4987.89	2082	N/A	N/A	N/A	N/A	289	-86.12	8.29	2.62
85222.69	15885	N/A	N/A	N/A	N/A	289	-98.18	57.32	11.17
970739.74	147897	N/A	N/A	N/A	N/A	289	-99.80	47.41	14.65

Multi-turn Coils
On the Resistance Error at Higher Frequencies

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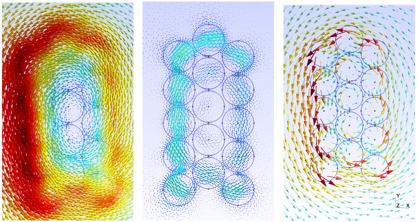
On the Resistance Error at Higher Frequencies

- The error on the resistance is attributed to the over estimation of the excitation field.
- The SA approach integrates over the sleeve and uses the average b-field in the calculation of the resistance due to proximity effects
- At lower frequencies the distribution of the field is fairly uniform and the values are calculated relatively accurately



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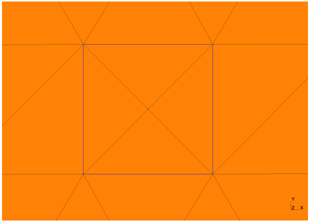
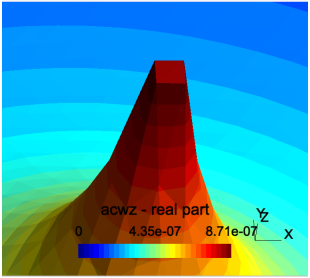
- When the magnetic flux density cannot be assumed to be constant over the wire section, more advanced analytic solution should be used, taking e.g. both the mean and a constant gradient contribution.
- The treatment of the infinity boundary is needed to accurately calculate the inductance value of the conductors

Conductor Cross Section

- The sleeve does not depend on the conductor's shape
- The truncation of the field is performed in the same manner
- Additional challenges arise in the case of unstructured sleeves that might require additional work on the truncation method (e.g., non even-numbered and equilateral polygons, twist or angles, etc)
- The analytical correction MUST be derived for each individual conductor shape.





Dieter Gerling. "Approximate analytical calculation of the skin effect in rectangular wires". In: 2009 International Conference on Electrical Machines and Systems. IEEE. 2009, pp. 1–6.

Extensions, Challenges and Perspectives
Conductors Arrangement

- The connection order between turns is important whenever all turns are not connected in series, e.g. two half-windings in parallel, Litz wire, multiple secondaries, etc.
- The proposed methodology can handle these situations without any modifications.
- Dealing with capacitive effects is an open question.

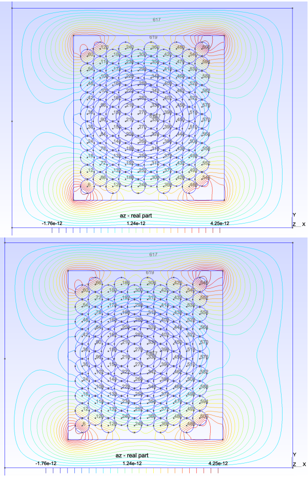


Figure: 100kHz

Magnetic Cores - 50Hz with 500 Turns and AWG5 wire

- The SA method works seamlessly with the magnetic core out of the box
- The resistance and inductance values have a great degree of accuracy

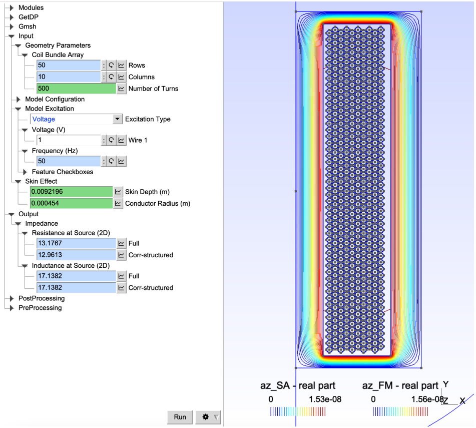
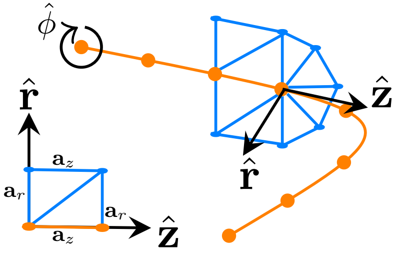


Figure: 50Hz

Application of the Semi-Analytical method in 3D cases

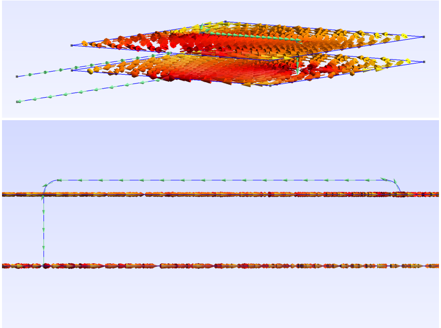
- Just as in the 2D case the sleeve is simply a layer of elements surrounding the Line Region
- In the image we are using prisms for simplicity on the representation of the sleeve
- In the 2D case, the the truncated field needed a correction in the z-direction, however, in 3D, the "z" direction would refer to the tangential component and the r-component the radial component.
- The correction factor contributes to the tangential component of the MVP



- Gauging needs further investigation. Note that the nodes in the sleeve remove the degrees of freedom needed for tree/co-tree or nodal gauging schemes.

Extensions, Challenges and Perspectives
Application of the Semi-Analytical method in 3D cases

- Further extensions should be investigated in the case of Line Region - Shell couplings
- Further development on the full-wave formulation, possibly extending the work of Edelvik, to account for distributed capacitance within the wire.
- Potential applications transmission line type of problems, busbars, PCBs, etc., where not only inductive effects but also capacitive effects are of interest



Fredrik Edelvik. "A new technique for accurate and stable modeling of ar- bitrarily oriented thin wires in the FDTD method". In: IEEE transactions on electromagnetic compatibility 45.2 (2003), pp. 416–423.

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- The results in the case of unstructured sleeves presents larger variance, however the results remain in the same order of magnitude which makes it a powerful tool for quick prototyping.

Thank You!
Questions?