International Association of Hydrogeologists
Comité Belge des Hydrogéologues (CBH) ASBL

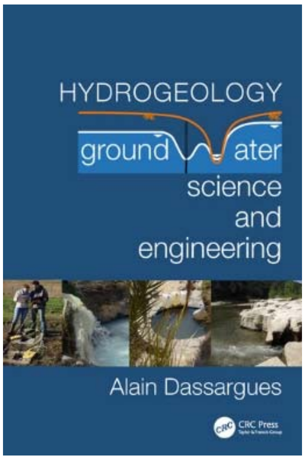
Modélisation écoulement

A. Dassargues

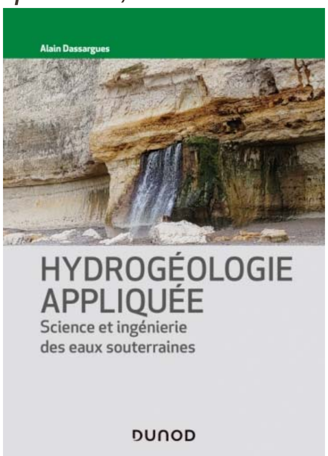
Rappel théorique sur l'écoulement des eaux souterraines et sa modélisation (conceptualisation, construction, calibration)

Modélisation des écoulements souterrains

… sur base d'extraits des chapitres 4, 12 et 13 de:



https://www.routledge.com/Hydrogeology-Groundwater-Science-and-Engineering/Dassargues/p/book/9781498744003



https://www.dunod.com/sciences-techniques/hydrogeologie-appliquee-science-et-ingenierie-eaux-souterraines

… et des slides du cours 'Groundwater modelling' (ULiège)

Modélisation des écoulements souterrains

- Terminologie et méthodologie générale
- Rappel des paramètres et équations d'écoulement
- Conditions aux frontières
- Principe de la méthode des différences finies
- Intégration temporelle
- Références

Un modèle ?

- *a model is a tool for simulating reality in a simplified form* (Wang and Anderson 1982)
- *a mathematical description of the physical reality can already be considered as a mathematical model*
- *a mathematical model can be solved or computed analytically or numerically*
- *'Any type of modeling includes subjective decisions and simplifying assumptions because the true complexity of a natural system is never fully represented and data about properties and variables include uncertainties'* (Fienen 2013)

4

Terminologie

Black-box model: a set of mathematical equations is developed by empirical or statistical fitting of parameters to reproduce historical records of the main variable ('data driven' model) *(Anderson et al. 2015)*

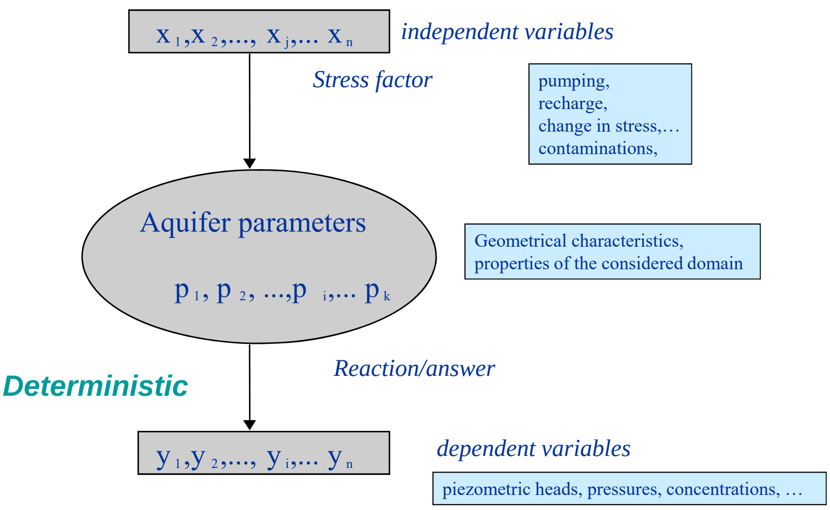
➡ *… all is relative, any model is always a simplification of the reality,*

- *'black box'*
- *'grey box'*
- *physically based but not spatially distributed*
- *spatially distributed but not physically based*
- *spatially distributed and physically consistent*

Various possibilities for catchment scale models

5

Terminologie



Stochastic/probabilistic *using Monte Carlo multiple simulations, the same schema can be used with multiple equally likely sets of parameters, independent variables, and dependent variables.* *(Konikow and Mercer, 1988, Dassargues 2018)*

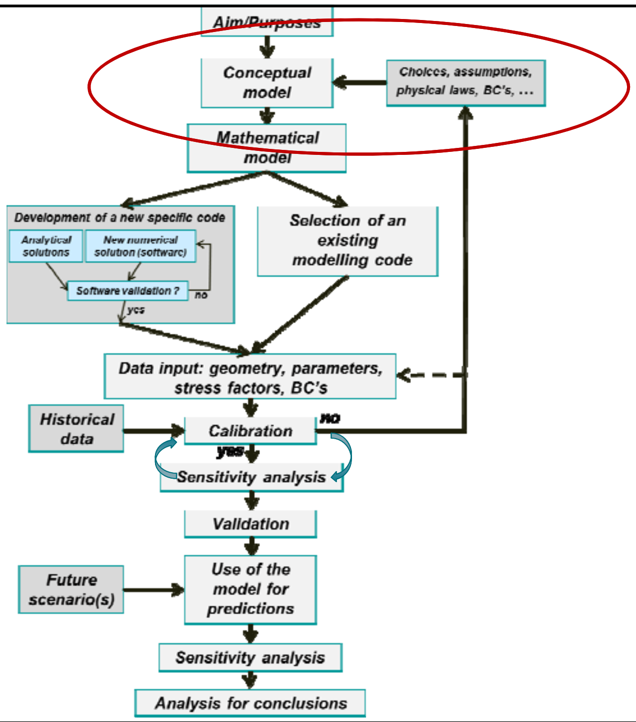
6

Méthodologie générale

Different steps of a groundwater numerical model :

- *clear definition of the final aim*
- *conceptual model*
- *mathematical model*
- *numerical model, development or choice of an existing code*
- *data input*
- *calibration and then validation*
- *sensitivity analysis*
- *application (use) of the model*
- *results analysis with regards to the initial question*
- *redaction of a report*

7

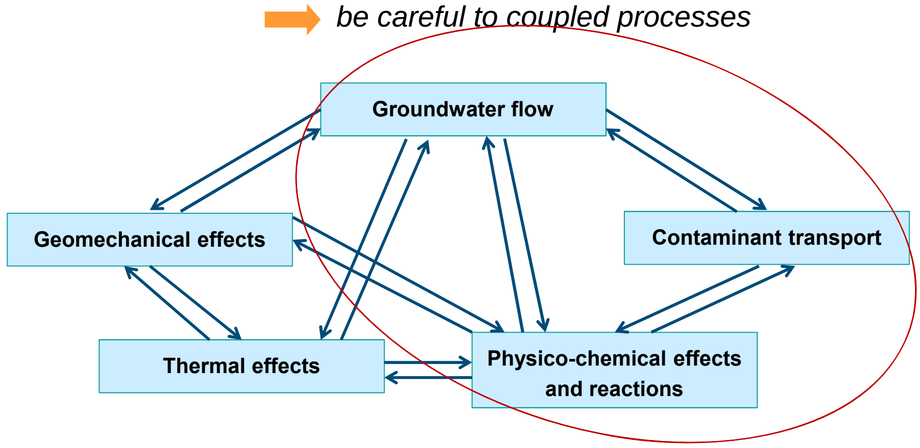


8

Modèle conceptuel

(Anderson et al. 2015)

… the way in which reality is translated in a model
be careful to coupled processes



(Rosbjerg and Madsen 2005, Dassargues 2018)

9

Modèle conceptuel

fundamental step where the main assumptions of the modelling are chosen

- *scale level*
- *steady state or transient analysis*
- *model dimensionality: 1D, 2D vertical, 2D horizontal, quasi-3D, 3D*
- *boundary geometry and location, boundary conditions*
- *geological media (porous / fissured / double porosity / …)*
- *homogeneity/heterogeneity, isotropy/anisotropy, properties changes in function of time*
- *initial conditions within the domain*
- *…*

… 'poorly justified assumptions can potentially discredit an entire groundwater model' (Peeters 2017)

10

Modèle conceptuel

– **Steady state**

- *it does not exist in the reality*
- *ΔRes = 0 and $Q_{in} = Q_{out}$*
- *when piezometric heads and fluxes can be considered as relatively stable*
- *when transient data are lacking (first guess, …)*
- *with data allowing to deduce a 'mean behaviour' of the system : R_{mean}, Q_{mean}, H_{mean}…*
- *for starting with a problem, before going to transient conditions*
- *adopted for simplification, considering extreme conditions and being on the 'security side'*

can be difficult to converge when data are not realistic or when non linearities are not considered

– **Transient simulation**

- *requires generally more data*
- *takes more CPU time*
- *sometimes needed in function of the context*

11

Modèle conceptuel

Extension and dimensionality

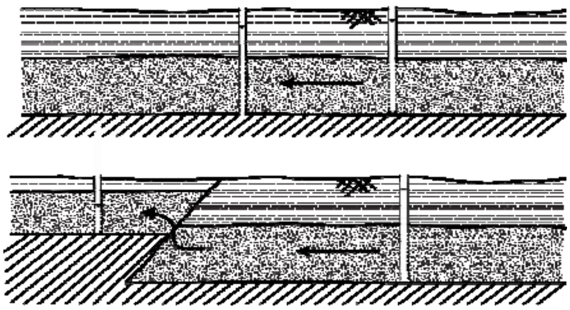
- *1D, … (often in the partially saturated zone)*
- *2D horizontal models:*

groundwater flow considered as mostly horizontal

Is it realistic ?

OK

here a 3D approach is needed



12

Modèle conceptuel

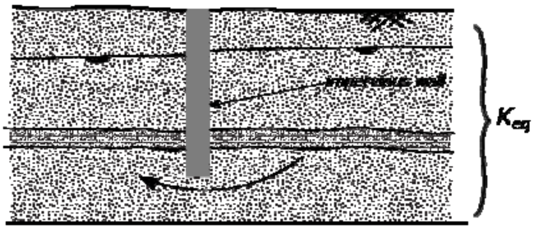
Extension and dimensionality

- *2D horizontal models:*

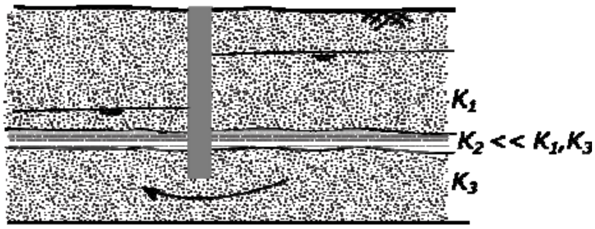
  groundwater flow considered as mostly horizontal

Is it realistic ?

OK



here a 3D approach is needed

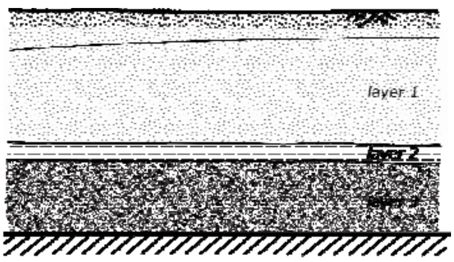


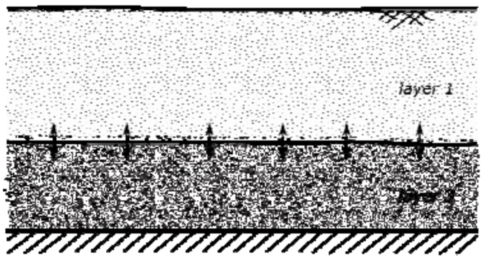
13

Modèle conceptuel

Extension and dimensionality

- *pseudo-3D or quasi-3D*
  - *multi-layers system with 2D gw flow in each of them*
  - *strictly vertical flow in aquitards calculated by applying the Darcy's law*



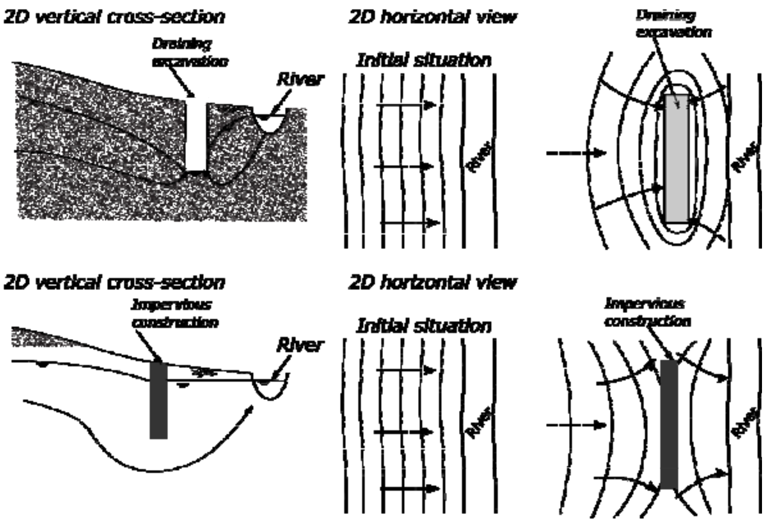


14

Modèle conceptuel

Extension and dimensionality

- *2D vertical models: OK but gw flow not in the considered plane is neglected*



15

Modèle conceptuel

Initial conditions*: initial values of the main variable (generally piezometric head h) in each node of the mesh*

Boundary conditions*: see later*

16

Modèle conceptual (remarque)

(Hill 2006, Gómez-Hernández 2006, Wildemeersch 2012,)

Parsimony or complexity: merits and pitfalls

- *any process-based model becomes complex and remains uncertain*
- *complexity could be considered through the use of stochastic approaches conditioned on the available data*

(Beven and Freer 2001, Gómez-Hernández 2006, Beven and Binley 1992, Hoeting et al. 1999, Neuman 2003, Rojas et al. 2008 and 2010a)

- *complexity could be introduced in a stepwise fashion, from simple to complex*
- *preserve refutability and transparency*

each chosen hypothesis can be tested

modelled processes remain understandable

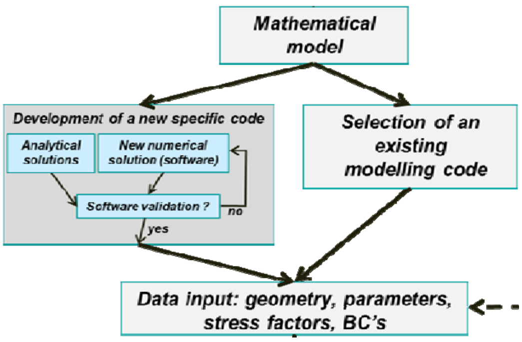
(Ward 2005, Schwartz et al. 2017, Kurtz et al. 2017)

- *to determine if a simple model provides reliable results, its results should be compared to results from a more complex one*

(de Marsily et al. 2005)

17

Choix du software



- *if a new code is developed: it must be validated for the same kind of processes*
- *choose your code in function of your conceptual model*
- *many existing codes for different purposes*

Do not use a hammer to drive a screw
or do not use a screwdriver to drive a nail !

18

Caractéristiques principales d'une modélisation

- *study area represented by a mesh of elements or cells to which nodal points (or nodes) are associated*
- *in those subdomains (cells, elements, volumes) the medium is assumed homogeneous*
- *the continuous variable by a discrete variable (the solution will be found at discrete points of the spatio-temporal domain)*
- *a finer spatial discretization means a better approximation of the solution*
- *partial differential equations are replaced by a system of algebraic equations*
- *the state variables are the unknown*
- *a solution obtained for each specified set of parameter values*
- *…*

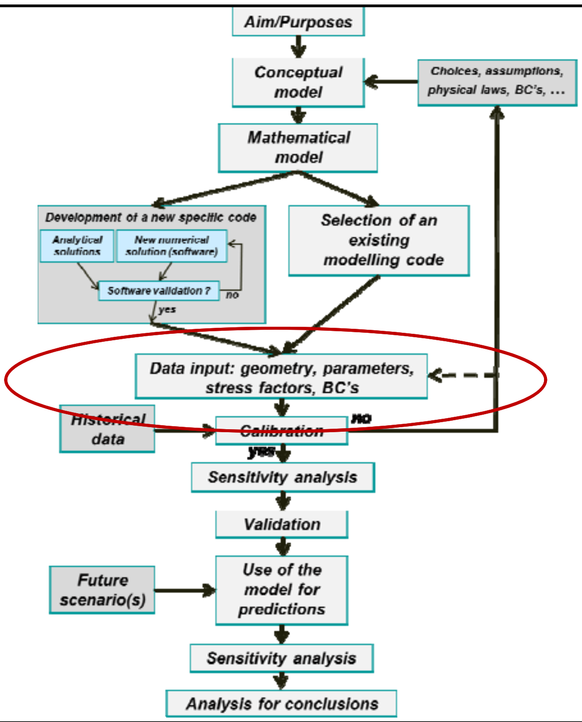
19

Caractéristiques principales d'une modélisation

- *iterative procedures more efficient than direct matrix inversion methods*
- *solution = values at discrete locations in the simulated domain generated from the spatial discretization*
- *if transient problem, the time scale is also discretized in time steps*
- *solution at the n discrete nodes and for all time steps, then interpolations at any location in space and time*

(Wang and Anderson 1982)

20



Données

4 categories :

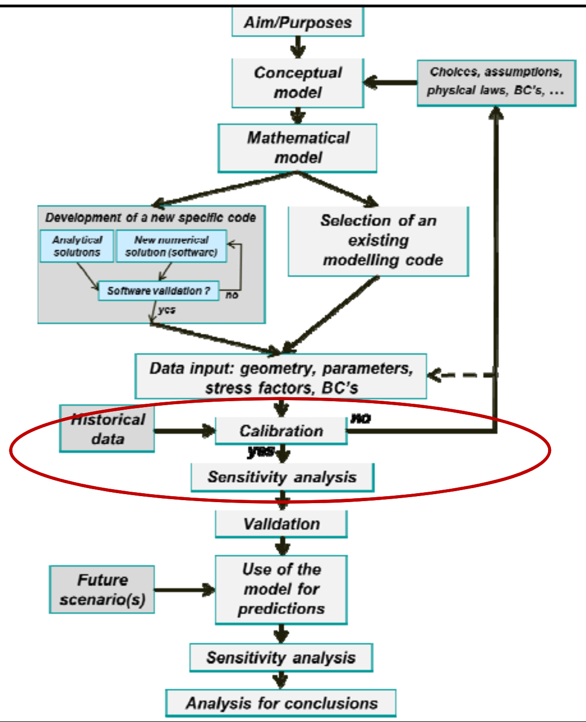
- *1D, 2D or 3D **geometry** of the modelled zone (**geology**, topography, hydrology, concerned problem, scale, …)*
- *values for the **properties** (parameters) playing a role in the modeled processes (i.e. for gw flow: K and S_s or T and S, for solute transport n_e, a_L, a_T, R, …)*
- ***stress factors** applied on the modelled domain (i.e. for gw flow: recharge, pumping, injections, for solute transport mass injection or removal)*
- ***historical measured data** concerning the main problem variable (i.e. for gw flow: piezometric heads, for solute transport: concentrations) or its first derivative (i.e. for gw flow: flow rates or fluxes, for solute transport advective or dispersive mass fluxes) … distributed data in the domain that will be used for calibration (or inverse modeling) procedure*

22

Implémentation du modèle

Conceptual model ➡ *translated in a usable form for modelling:*

➡
- *Spatial discretisation* ➡ *grids with cells*
- *Time discretisation* ➡ *time steps*
- *Boundary Conditions (BC's)*
- *Sink /source terms*
- *Initial values for the main variable*
- *Initial values for possible useful other state variables*



Calibration

Change (adaptation) of the parameters values and distribution … for a better simulation of the reality

➡ *… this reality is considered as represented by historical data sets*

How to quantify objectively the good fit ?

accounting for the discrepancies between observed and computed values of the main variables and/or one or more of their derived variables

Different steps :

- *Objective function formulation (be careful: any objective function is subjective !)*
- *Sensitivity analysis*
- *Change in parameters values (inverse problem);*
- *Validation using another data set (most often another time period, for transient modelling)*

25

Critère pour évaluer la calibration

➡ *weighted least square*

$$\varphi\ (\boldsymbol{b}) = \sum_{i=1}^{n} w_i \left[y_i^{obs} - y_i^{sim}(\boldsymbol{b})\right]^2$$

weighting factors for different kind of data

for the relevant process to answer the initial question !

(Beven and Binley 1992, Refsgaard and Henriksen 2004, Rojas et al. 2010b and 2010c, Wildemeersch 2012)

If the aim is to simulate the baseflow evolution in a watershed:

$$\varphi_{NS}\ (\boldsymbol{b}) = 1 - \frac{\sum_{t=1}^{nt}\left[q_t^{obs} - q_t^{sim}(\boldsymbol{b})\right]^2}{\sum_{t=1}^{nt}\left[q_t^{obs} - \mu^{obs}\right]^2} \quad \in\ ]-\infty, 1]$$

(Nash and Sutcliffe 1970, Wildemeersch 2012)

26

Modélisation inverse

could be helpful to gain a full understanding of the physical behavior of the simulated system

- *manual trial-and-error procedure*
- *automatically non-linear regression methods = inverse modelling*

more efficient to produce useful statistics

- *main issues : the non-uniqueness of the solution*
- *introduce prior information on the parameter values to avoid as far as possible an ill-posed inversion*

(Carrera et al. 2005, Hill and Tiedeman 2007, Carrera and Neuman 1986b)

27

Analyse de sensibilité = outil utile à la calibration

the amount the simulated value would change given a change in the parameter value

the amount the simulated value would change given a 1% change in the parameter value

- *simple sensitivities*
- *dimensionless scaled sensitivities (dss)*
- *composite scaled sensitivities (css)*

(Hill 1992, Anderman et al. 1996, Hill et al. 1998, Hill and Tiedeman 2007)

the importance of observations as a whole to a single parameter

(example in Goderniaux et al. 2015)

- *calculated using inverse modeling codes as PEST and UCODE*

(Doherty 2005, Skahill and Doherty 2006, Poeter et al. 2005)

the degree of correlation between couple of parameters and/or stress factors

- *+ parameter correlation coefficients*

28

Méthodologie générale

- *Définitions, terminologie, buts*
- *Méthodologie*
- *Modèle conceptuel*
- *Choix du software*
- *Données et implémentation*
- *Calibration + anlayse de sensibilité*
- ***Evaluation/rapport***

Evaluation & rapport

➡ *very important to analyse and evaluate the reliability of model results and adopted conceptual choices <u>with regards to the question to be answered</u> …*

Rapport

➡ *modelling study realised step by step … these steps must be described in the final report to establish clearly the reliability of the results despite the simplifying assumptions of the conceptual model*

➡ *the reader must be able to understand the justification of the conceptual choices and the rigour of the followed approach*

30

Modélisation des écoulements souterrains

- Terminologie et méthodologie générale
- Rappel des paramètres et équations d'écoulement
- Conditions aux frontières
- Principe de la méthode des différences finies
- Intégration temporelle
- Références

Concept d'Elément de Volume Représentatif

➡ *REV = volume of geological medium considered as representative for quantification of the parameters*

➡
- *large enough to be relevant for the studied problem*
- *small enough for avoiding too smoothed values*

⚠ ***… the REV concept implicitly considers the medium as continuous (and porous)***

- *the REV depends on the kind of problem being studied and the study objectives*
- *the REV is used for groundwater flow and solute transport … but also in all other fields*

32

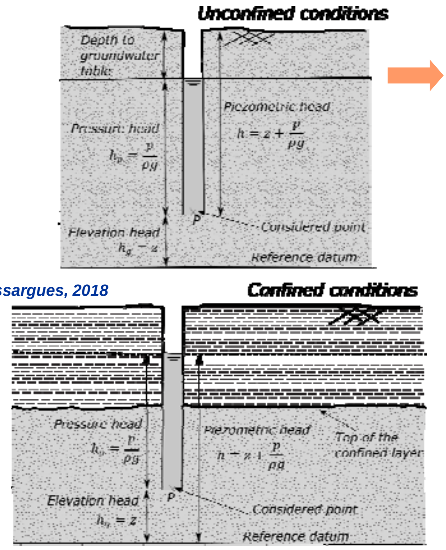
EVR et échelles

Scale

- ***Micro*** ↔ ***Fluid dynamic and pore studies***
  - ↓ *Homogenisation*
- ***Macro*** ↔ ***Lab tests (from dm to a few m)***
  - ↓ *Homogenisation*
- ***Mega*** ↔ ***Detailed numerical models – Physically consistent simulations of the reservoir***
  - ↓ *Homogenisation*
- ***Giga*** ↔ ***Balance models, black-box models, transfer functions, etc.***

Hauteur piézométrique

… the potential energy is expressed usually in 'water head' or hydraulic head or piezometric head:



Dassargues, 2018

$$h = \frac{\Phi_{tot}}{g} = z + \frac{p}{\rho . g}$$

Porosités

… *two components in the total porosity:*

- *effective porosity* n_e
- *retention capacity' or 'specific retention')* S_r

$$n = S_y + S_r = n_e + S_r \quad \text{with} \quad n_e = V_m/V_t \qquad S_r = V_{im}/V_t$$

- *for groundwater flow, the 'effective' porosity is drainage porosity …. also called = "specific yield"* S_y

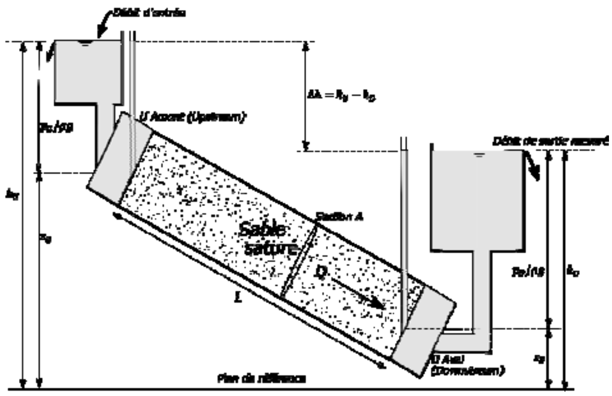
⚠

- *the 'mobile water porosity' to be considered for groundwater flow is typically higher than the 'mobile water porosity' acting in solute transport processes* *(Payne, Quinnan and Potter 2008, Hadley and Newell, 2014, Dassargues 2018)*
  - *'effective transport porosity' < drainage porosity*

35

Loi de Darcy, conductivité hydraulique

$$Q = K.A.\frac{\Delta h}{L}$$



- *… specific flux or flow rate (specific discharge):*

$$q = \frac{Q}{A}$$

in m³/(m².s) … in m/s

36

Loi de Darcy, conductivité hydraulique

This specific discharge is often called 'Darcy's velocity' … it is only a flow rate Q divided by a surface A

this surface is not the groundwater flow section

- *the actual groundwater flow section is :* $A.n_e$
- *… to obtain a mean (averaged/equivalent on the REV) groundwater velocity :*

$$v_e = \frac{q}{n_e} = \frac{K}{n_e}.\frac{\Delta h}{L}$$

m/s

' advection velocity' (effective velocity)

37

Généralisation de la loi de Darcy en 3D

hydraulic conductivity and intrinsic permeability described by tensors:

$$\boldsymbol{K} = \begin{bmatrix} K_{xx} & K_{xy} & K_{xz} \\ K_{yx} & K_{yy} & K_{yz} \\ K_{zx} & K_{zy} & K_{zz} \end{bmatrix}$$

and $\boldsymbol{K} = \boldsymbol{k}\rho g/\mu$

elevation of the considered point (with regards to the reference datum)

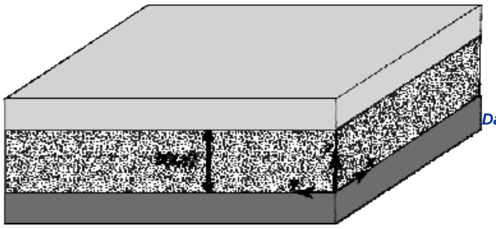
$$\boldsymbol{q} = -\boldsymbol{K}\cdot\nabla h = -\frac{\boldsymbol{k}\rho g}{\mu}\cdot\nabla h = -\frac{\boldsymbol{k}}{\mu}\cdot(\nabla p + \rho g \nabla z)$$

$$\boldsymbol{grad}\, h = \nabla h = \left(\frac{\partial h}{\partial x}, \frac{\partial h}{\partial y}, \frac{\partial h}{\partial z}\right)$$

38

Transmissivité

… for a confined aquifer



Dassargues, 2018

transmissivity (m^2/s) in a point

$$T(x,y) = \int_0^{b(x,y))} K(x,y,z)dz = K_{avg}(x,y)b(x,y)$$

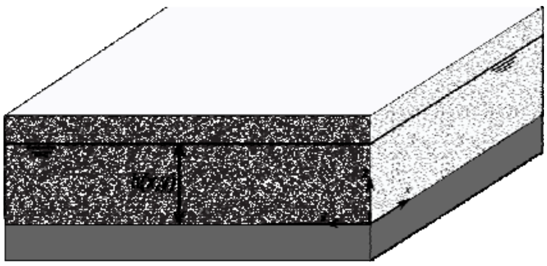
thickness of the confined aquifer at the concerned point

mean value of the hydraulic conductivity on the vertical of the concerned point

39

Transmissivité

… for an unconfined aquifer



Dassargues, 2018

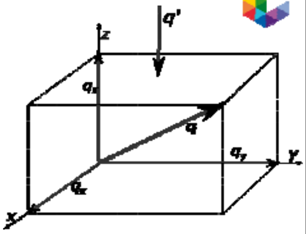
the saturated thickness of the unconfined aquifer at the point (of horizontal coordinates x and y)

$$T(x,y) = \int_0^{h(x,y)} K(x,y).dz$$

depends on the piezometric head!

40

Equation d'écoulement en régime permanent



$$\nabla \cdot (\rho\, \boldsymbol{K} \cdot \nabla h) + \rho q' = 0$$

terms are kg/(m³s)

$$\frac{\partial}{\partial x_i}\left(\rho K_{ij}\frac{\partial h}{\partial x_j}\right) + \rho q'_i = 0$$

in indicial notation

$$\frac{\partial}{\partial x}\left(K_{xx}\frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_{yy}\frac{\partial h}{\partial y}\right) + \frac{\partial}{\partial z}\left(K_{zz}\frac{\partial h}{\partial z}\right) + q' = 0$$

if density is assumed constant and the principal anisotropy directions of the K tensor are known and aligned with the selected coordinate system – terms are in s^{-1}

$$\frac{\partial}{\partial x}\left(K_{xx}\frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial z}\left(K_{zz}\frac{\partial h}{\partial z}\right) + q' = 0$$

if 2D vertical flow, terms are in s^{-1}

$$\frac{\partial}{\partial x}\left(T_{xx}\frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(T_{yy}\frac{\partial h}{\partial y}\right) + q'' = 0$$

if 2D horizontal flow, terms are in m/s

41

… en régime transitoire

… in transient flow,
storage variation in function of the time:

$$\frac{\partial (n\,\rho)}{\partial t}$$

specific storage coefficient (m^{-1})

$$S_s = \rho\, g\, \alpha$$

$$\frac{\partial (n\,\rho)}{\partial t} = \rho^2 g\left(\alpha + \cancel{n\beta_s} + \cancel{n\beta_w}\right)\frac{\partial h}{\partial t} = \rho\, S_s \frac{\partial h}{\partial t}$$

Volume (REV) compressibility of the porous medium (Pa^{-1})

Solid grain compressibility (Pa^{-1})

Water compressibility (Pa^{-1})

42

Coefficient d’emmagasinement spécifique

… often, the influence of the water compressibility and the solid grain compressibility can be neglected with regards to the volume compressibility of the porous medium (as a whole)

$$\Rightarrow S_s = \rho\, g\, \alpha$$

… this link between the volume compressibility and the specific storage coefficient is showing clearly the direct coupling between saturated transient groundwater flow and geomechanical behaviour in compressible porous media

43

Coefficient d’emmagasinement

Storage coefficient = water volume (m³) stored or drained per aquifer surface unit (m²) for a unit variation of piezometric head (m)

$\Rightarrow$ *… vertical integration*

$$S(x,y) = \int_0^{e(x,y)} S_s(x,y).dz$$ **confined aquifer**

$$S = S_s.e$$

$$\Rightarrow S = n_e + \int_{z_1}^{h} S_s.dz$$ **unconfined aquifer**

the most important part of the storage is due to saturation/drainage of the porous medium

44

Equation d'écoulement transitoire en 2D
(écoulement horizontal)

confined aquifer

$$\nabla \cdot (\boldsymbol{T} \cdot \nabla h) + q'' = S\frac{\partial h}{\partial t}$$

terms are in m/s

$$\frac{\partial}{\partial x_i}\left(T_{ij}\frac{\partial h}{\partial x_j}\right) + q''_i = S\frac{\partial h}{\partial t}$$

in indicial notation

$$\frac{\partial}{\partial x}\left(T_{xx}\frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(T_{yy}\frac{\partial h}{\partial y}\right) + q'' = S\frac{\partial h}{\partial t}$$

principal anisotropy directions aligned with the selected coordinate system

unconfined aquifer

$$\nabla \cdot (\boldsymbol{T}(h) \cdot \nabla h) + q'' = n_e\frac{\partial h}{\partial t} = S_y\frac{\partial h}{\partial t}$$

terms are in m/s

$$\frac{\partial}{\partial x_i}\left(T_{ij}\frac{\partial h}{\partial x_j}\right) + q''_i = n_e\frac{\partial h}{\partial t} = S_y\frac{\partial h}{\partial t}$$

in indicial notation

$$\frac{\partial}{\partial x}\left(T_{xx}\frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(T_{yy}\frac{\partial h}{\partial y}\right) + q'' = S\frac{\partial h}{\partial t}$$

principal anisotropy directions aligned with the selected coordinate system

45

Conditions aux frontières (écoulement)

- *Dirichlet conditions: prescribed piezometric head*
- *Neumann conditions: prescribed flux*
- *Cauchy or mixed conditions: flux depending on piezometric head*

46

Hauteurs piézométriques imposées

Prescribed piezometric head

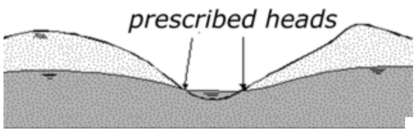
(Dirichlet condition)

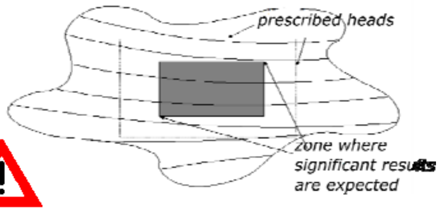
Prescribed piezometric head on the concerned boundary:

$$h(x,y,z,t) = f'(x,y,z,t)$$

f' *can vary in space and time*
(one value per node and per time step)

➡ *a flux will be computed per concerned node*





47

Flux imposés

Prescribed flux (Neumann condition)

The first derivative of the piezometric head is prescribed on the concerned boundary:

$$\nabla h \cdot \boldsymbol{n} = \frac{\partial h}{\partial n}(x,y,z,t) = f''(x,y,z,t)$$

f'' *piezometric gradient normal to the concerned boundary,*
its value can vary in space and time
(one value per concerned node and per time step)
Applying the Darcy's law, it is a way of prescribing the water flux through the boundary:

$$K\frac{\partial h}{\partial n}(x,y,z,t) = q''(x,y,z,t)$$

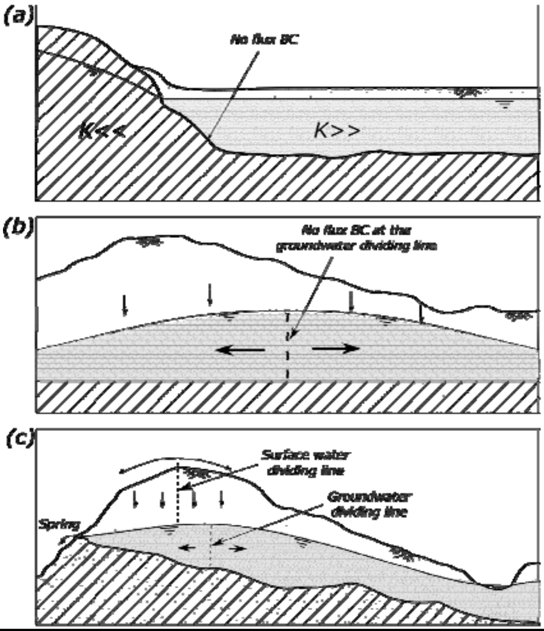
q'' *: precribed flux through the boundary (m/s)*

➡ *particular case:* $f''=0$

48

Flux imposés

Prescribed flux (Neumann condition)

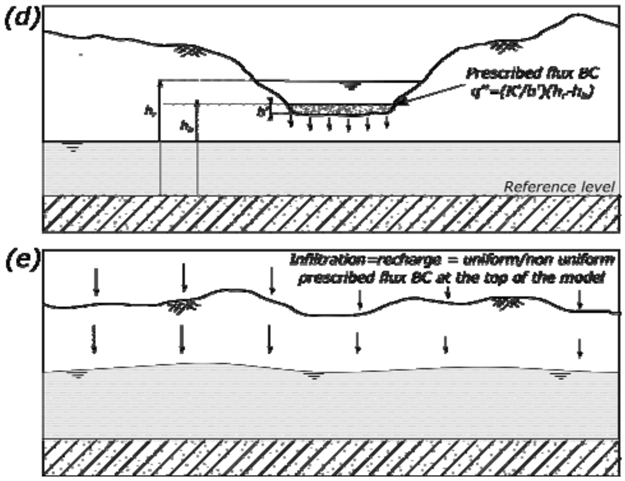


(Dassargues, 2018)

49

Flux imposés

Prescribed flux (Neumann condition)



(Dassargues, 2018)

50

Flux dépendant de la hauteur piézométrique

Flux depending on the piezometric head
(mixed condition or Cauchy condition)

A combination (linear relation) of the piezometric head and its first derivative is prescribed on the boundary:

$$a.\frac{\partial h}{\partial n}(x,y,z,t)+b.h(x,y,z,t)=f'''(x,y,z,t)$$

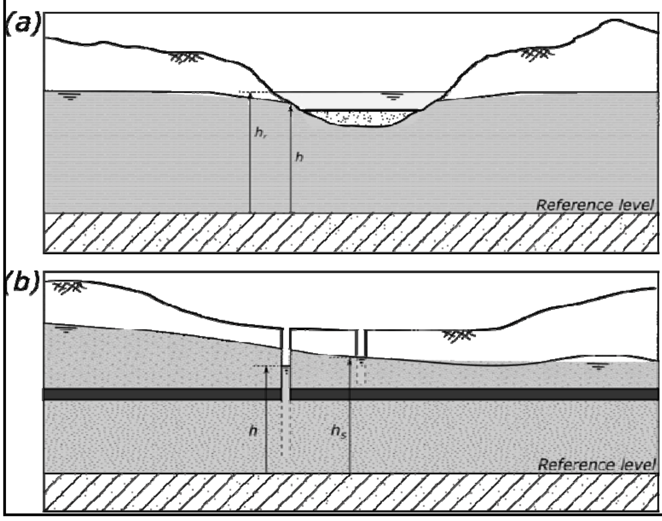
f''' *can vary in space and in time*
(one value per concerned node and per time step)

- *interactions between surface water bodies and groundwater*
- *interactions between different aquifers*

51

Flux dépendant de la hauteur piézométrique

(mixed condition or Cauchy condition)



conductance

$$q'' = -K\frac{\partial h}{\partial n} = \frac{K'}{b'}(h_r - h)$$

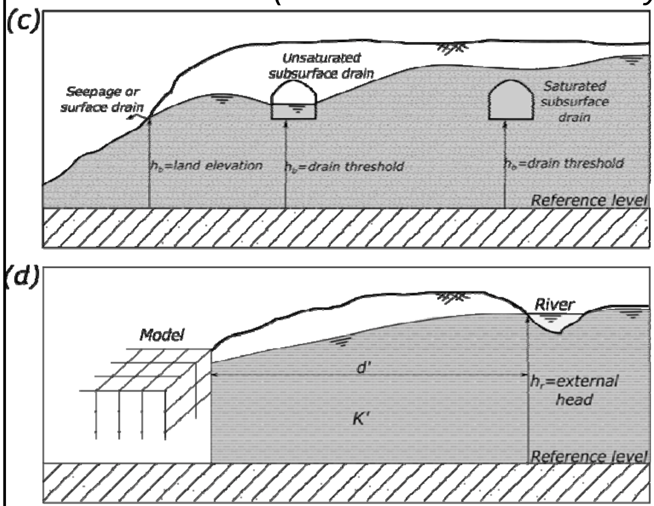
conductance

$$q'' = -K\frac{\partial h}{\partial n} = \frac{K'}{b'}(h_s - h)$$

52

Flux dépendant de la hauteur piézométrique

(mixed condition or Cauchy condition)



conductance

$$q'' = -K\frac{\partial h}{\partial n} = \frac{K'}{b'}(h_b - h)$$

conductance

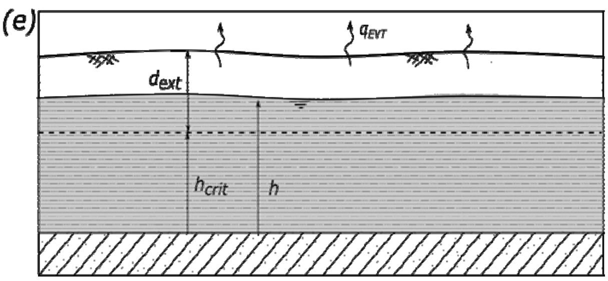
$$q'' = -K\frac{\partial h}{\partial n} = \frac{K'}{d'}(h_r - h)$$

prescribing an 'external head' (i.e. not on the true boundary but outside the modelled zone) so that a groundwater flux across the boundary is computed from the difference between this 'external head' and the piezometric head on the model boundary using a given conductance

53

Flux dépendant de la hauteur piézométrique

(mixed condition or Cauchy condition)



in arid and semi-arid zones

conductance

$$q_{EvT} = \frac{R_{EvT}}{d_{ext}}\left(h(x,y,z,t) - h_{crit}(x,y,z,t)\right)$$

(Anderson et al. 2015)

represent an evapotranspiration flux leaving the model but dependent on the 'depth to water' (i.e. the land surface elevation minus piezometric head). An extinction depth d_{ext} corresponding to a critical head h_{crit} can be defined so that EvT occurs only if the water table is higher

54

Résolution par différences finies

Simple case…

Definition of a partial derivative of a function $h(x)$ *of the variable* x *:*

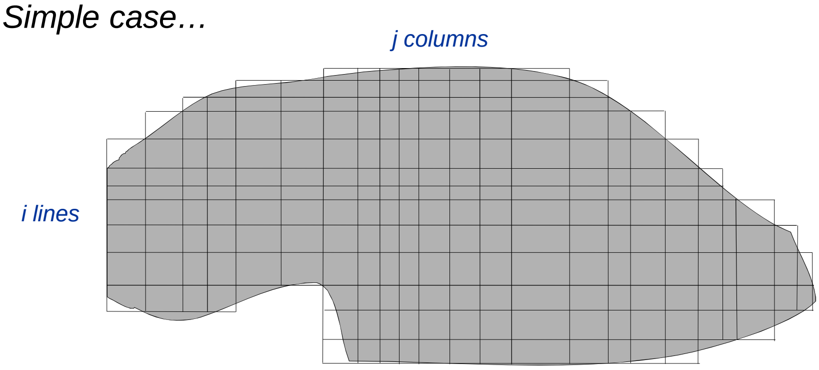
$$\frac{\partial h}{\partial x} = \lim_{\Delta x \to 0}\left(\frac{h(x+\Delta x)-h(x)}{\Delta x}\right)$$

- *spatial discretisation with a grid*
- *the nodes are the central points of rectangular cells (‘Block Centered Finite Difference method’ = BCFD)*
- *the cells are homogeneous ... the continuous variation of the variable is replaced by a discrete variable defined at the central points of the cells*
- *the approximation of the differential equation is better as the cells are small*

55

Résolution par différences finies

Simple case…



- *the nodes are numbered sequentially, index* i, j *and piezometric head values* h_{ij} *are attributed*

56

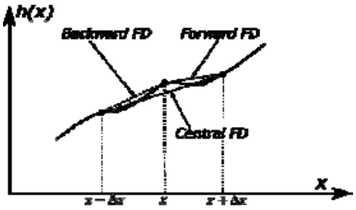
Résolution par différences finies

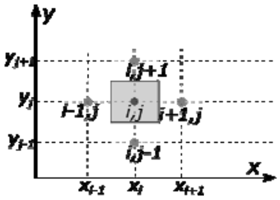
1D spatial approximation of the gradient by a finite difference:

Forward FD $\frac{\partial h}{\partial x} \approx \frac{h(x+\Delta x) - h(x)}{\Delta x}$

Central FD $\frac{\partial h}{\partial x} \approx \frac{h(x+\Delta x) - h(x-\Delta x)}{2\Delta x}$

$$\frac{\partial^2 h}{\partial x^2} \approx \frac{h_{i+1j} - 2h_{ij} + h_{i-1j}}{(\Delta x)^2}$$



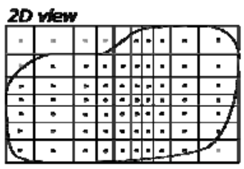


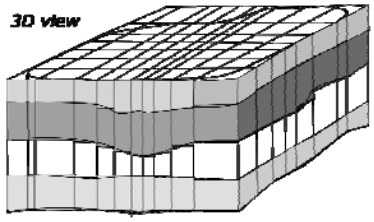
In 2D, with a 2nd order accurate FD:

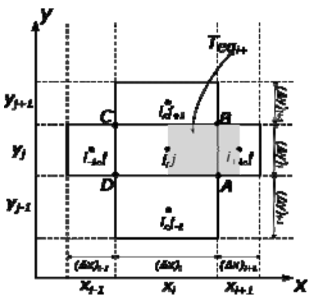
$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} \approx \frac{\left(h_{i+1j} + h_{i-1j} + h_{ij+1} + h_{ij-1} - 4h_{ij}\right)}{(\Delta m)^2} = 0$$

57

Résolution par différences finies: cellules ou blocs centrés







58

$$T_{eq_{i+}} = \frac{2T_{i+1j}T_{ij}}{T_{ij} + T_{i+1j}}$$

Schéma d'intégration temporel

$$\frac{T}{(\Delta m)^2}\left(h_{i+1j} + h_{i-1j} + h_{ij+1} + h_{ij-1} - 4h_{ij}\right) + Q_{ij} = S\frac{h_{ij}(t+\Delta t) - h_{ij}(t)}{\Delta t}$$

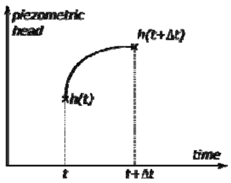
$$\frac{T}{(\Delta m)^2}(1-\theta)\left(h_{i+1j}(t) + h_{i-1j}(t) + h_{ij+1}(t) + h_{ij-1}(t) - 4h_{ij}(t)\right)$$
$$+\frac{T}{(\Delta m)^2}\theta\left(h_{i+1j}(t+\Delta t) + h_{i-1j}(t+\Delta t) + h_{ij+1}(t+\Delta t) + h_{ij-1}(t+\Delta t) - 4h_{ij}(t+\Delta t)\right)$$

$\theta = 0$ ➡ *Full explicit time integration*

$\theta = 1$ ➡ *Full implicit time integration*

$\theta = 1/2$ ➡ *Crank-Nicholson implicit*

$\theta = 2/3$ ➡ *Galerkin implicit*



➡ *stability criterion only for explicit schemes* $\theta < 1/2$

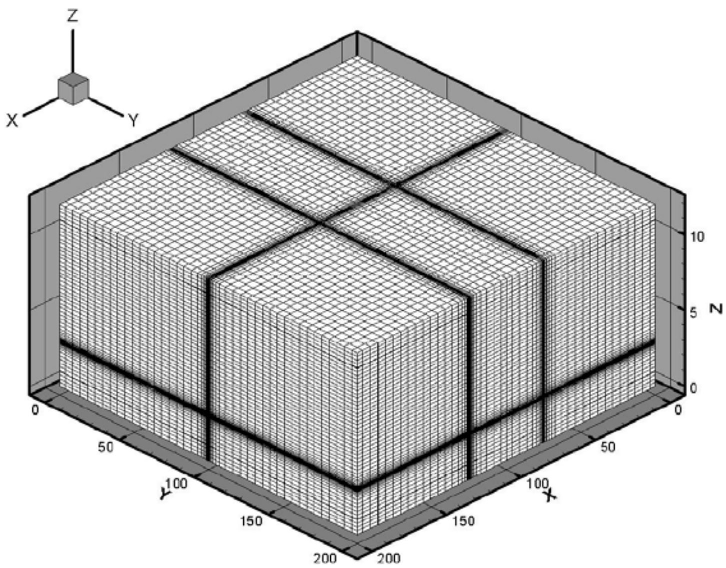
59

Recommandations pratiques

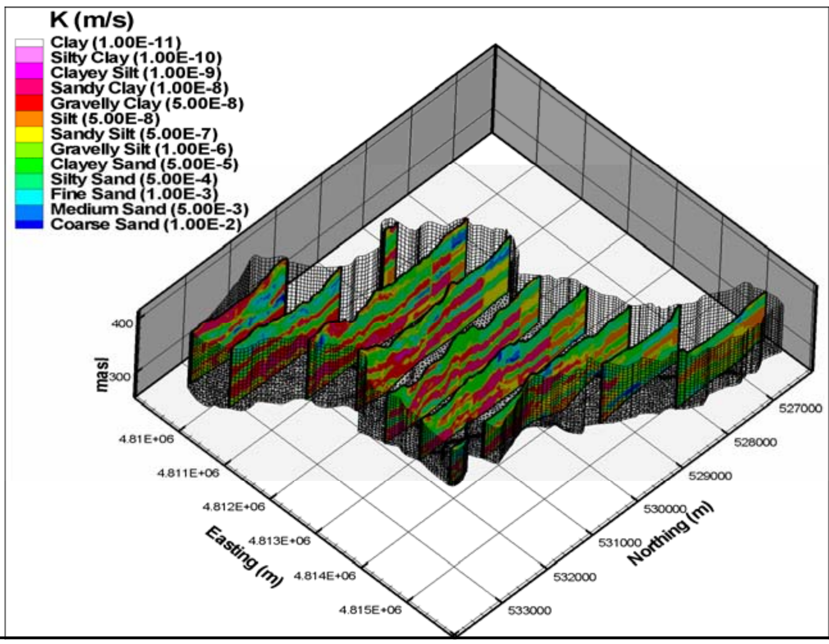
- *an initial field of values for the main unknown variable (piezometric head) needed for initiating the iterative solving*
- *accuracy increases with the number of cells but portability (i.e. computing efficiency) decreases*
- *use smaller cells where a steep gradient of the main variable is expected*
- *spatial discretization: nodes located at pumping wells and observation piezometers*
- *avoid distances between nodes greater than 1.5 the previous one*
- *avoid ratios greater than 1/10 for the cell dimensions (bad numerical conditions for solving the system of equations)*
- *boundaries with a prescribed head should correspond to nodes (central points of the cells, if BCFD)*
- *boundaries with a prescribed flux should correspond to sides of the cells (where the flux condition is calculated) if BCFD*
- *…*

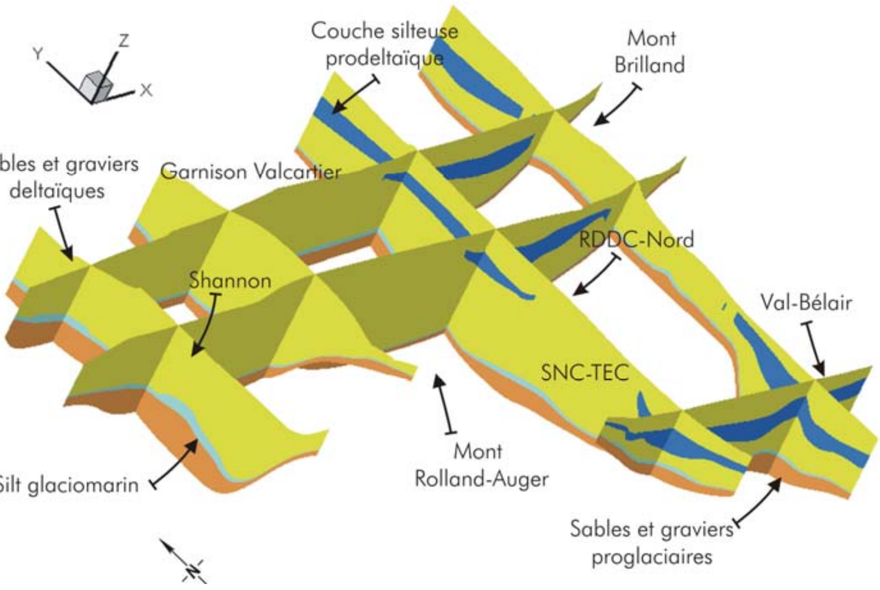
60

Maillage = discrétisation spatiale

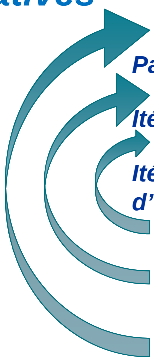


Hydro-stratigraphie: nécessaire avant modélisation





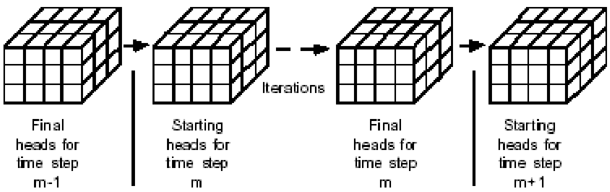
Boucles itératives



Pas de temps

Itérations sur la valeur de T(h)

Itérations sur la résolution du système d'équations



64