The use of numerical models for the fire analysis of reinforced concrete and composite structures

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Numerical methods for the analysis of reinforced concrete and composite structures under fire conditions are presented. They are based on the finite element method using beam elements with subdivision of the cross-section in a rectangular mesh. The structure submitted to increasing temperatures is analysed step-by-step using the Newton–Raphson procedure. A comparison between theoretical and experimental results is made for a reinforced concrete and a composite beam. In both cases there is a good agreement between theoretical and experimental results.

In this article we intend to show that, though the same type of model can be used for both reinforced concrete and composite structures, the difficulty of analysis increases substantially when going from reinforced concrete to composite structures.

Key Words: numerical models, finite elements, computer aided analysis, structural engineering, thermal problems, heat transfer, fire analysis, reinforced concrete, composite structures

1. INTRODUCTION

The behaviour of structures under fire conditions has been studied quite intensively during the last decade and considerable progress has now been made in this field.

The standard fire resistance test according to ISO 834 has been used quite intensively for the evaluation of the fire endurance of structural elements. In many countries, it is still the only legal way to classify structural elements regarding their fire resistance.

Nevertheless, in its present form the test procedure has several shortcomings, concerning for example the preparation of the experiment and the delays involved, the cost of the test, the size of the element to be tested, the heating and restraint characteristics.

Therefore the need for anaytical predictions of thermal and structural responses has grown more and more intensively. Important progress has been made in the development of simple analytical methods, particularly for steel constructions. Unfortunately this type of method is not applicable to all concrete and composite elements. Furthermore it has several limitations even for steel elements when some parameters have to be taken into account.

To improve the prediction of fire resistance it is necessary to develop powerful numerical tools, i.e. computer models able to simulate the real behaviour of the structure in a fire environment. Programs were developed at the University of Liège to study reinforced concrete beams.[3] These models have been modified and improved in order to treat composite elements and to take into account second order effects.[5,6]

In this article we intend to show that, though the same type of model can be used for both reinforced concrete and composite structures, the difficulty of analysis increases substantially when going from reinforced concrete to composite structures.

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2. NUMERICAL PROCEDURE FOR THE STRUCTURAL ANALYSIS UNDER FIRE CONDITIONS

Basic knowledge for the numerical analysis

In the case of steel structures it is usually accepted that they can be analysed using simple methods of calculation. This is due to the fact that the temperature of steel does not vary very much from one point to another in the same element.

This is no longer true when a considerable amount of concrete is present, which is the case for composite and reinforced concrete structures. Furthermore, the study of the mechanical behaviour of concrete at high temperatures is complicated. Therefore it is necessary to use more refined models for these types of structure: a step-by-step analysis is performed taking into account material and geometrical non-linearities.

First of all the structure has to be examined at ambient temperature under static loads representing the loads existing before fire.

To analyse the structure during the development of the fire two distinct problems must be solved:

- a thermal problem consisting in the evaluation of the temperature distribution in the element;
- a mechanical problem consisting in the evaluation of the structural behaviour due to the temperature increase.

To solve these problems it is necessary to use data on thermal and mechanical properties of steel and concrete at high temperatures.

Temperature distribution in the structure

Several questions have to be examined:

- The modelling of the environment created by the fire. The ISO standard temperature–time curve is used, but other types of equations can of course be introduced in the program.

- The density of heat flow transmitted to the element. This problem is quite involved, but it has been shown[3] that constant values can be adopted for the coefficient of convection and resultant emissivity.
- The distribution of temperature inside the element. Numerical methods taking into account the variation of the thermal properties with temperature and the evaporation of moisture in the concrete have to be used.

In this approach a system of finite difference equations is obtained by expressing the heat balance between adjacent rectangular elements (Fig. 1).

Numerical solution for the analysis of the structure under increasing temperatures

The numerical model used to treat reinforced concrete beams is explained in detail in ref. 3. This approach has now been modified and improved in order to simulate the behaviour of composite columns with second order effects. The solution strategy used presently for the thermo-mechanical analysis of the structure is presented hereafter.

The finite element method is used; the structure is divided in beam elements with two nodes and three degrees of freedom at each node (Fig. 2).

The midplane axial displacement is linear whereas the lateral displacement is a cubic function of x. The Navier–Bernouilli hypothesis is respected, and the shear energy is not taken into account. The equilibrium conditions, based on the principle of virtual displacements, can be written:

$$\mathbf{F}_e = (K)\cdot\mathbf{u} \qquad (1)$$

$\mathbf{F}_e$ vector of nodal forces applied to the structure
(K) structure stiffness matrix; depends upon geometrical and material properties of the elements
$\mathbf{u}$ vector of nodal displacements

In this approach, the cross-section is divided into sub-slices forming a rectangular mesh which is the same as in the thermal analysis (cf. Fig. 3). Therefore the temperatures, strains and stresses can vary from one subslice to

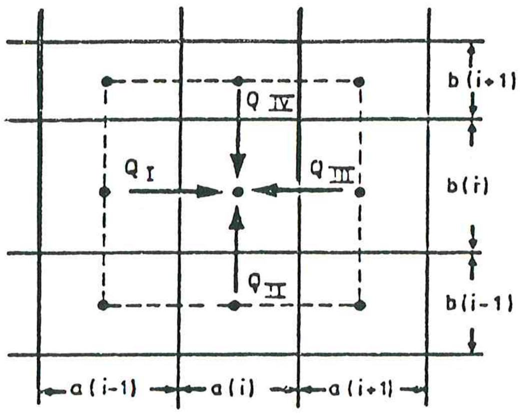
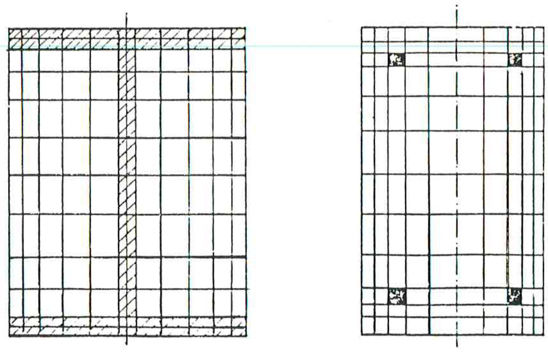


Figure 1. Heat balance between adjacent elements

Figure 2. Beam element



a) composite element   b) reinforced concrete element

Figure 3. Discretisation of the cross-section

another. Thus the integrals on the cross-section appearing in the stiffness matrix, the internal bending moments and axial forces are computed in a numerical way.

The integration along the axis of one element is made by the Gauss method, with two points of integration. Calculating the bending moment M at two points i and j in each element, the shear force T can be estimated using:

$$T = \frac{\Delta M}{\Delta x} = \frac{M_j - M_i}{x_j - x_i} \qquad (2)$$

Stress-strain relations in the materials are non-linear and moreover they vary with temperature. Since it is also desired to take large displacements into account, an iterative approach is essential and the stiffness matrix has to be actualized at each step of the loading before fire occurs, and at each time increment during the development of the fire.

In the problem to be solved the materials are subjected to initial strains due to temperature changes (ϵ_θ) and to creep effects (ϵ_{cr}); in the examples treated here creep effects are not yet taken into account. Thus the stresses will be caused by the difference between the total strains (ϵ_T) derived from the nodal displacements and the initial strains:

$$\sigma = \sigma(\epsilon_\sigma) = \sigma(\epsilon_T - \epsilon_\theta - \epsilon_{cr}) \qquad (3)$$

When the internal nodal forces $\mathbf{F}_i$ are calculated by integrating the internal stresses (3) and compared with the applied nodal loads $\mathbf{F}_e$, it can be observed that equilibrium is not reached.

Thus, at every stage, the difference between the internal forces and the applied loads is determined at all nodes of the structure. These unbalanced residual forces are then redistributed throughout the structure to restore equilibrium. This, combined with the actualisation of the stiffness matrix, gives rise to a Newton–Raphson process. Successive iterations take the form:

$$(\Delta\mathbf{F}_e)_i^{(r)} = (K)_i^{(r)}\cdot(\Delta\mathbf{u})_i^{(r)} \qquad (4)$$

$(K)_i^{(r)}$ structure stiffness matrix updated at the beginning of the rth iteration in the ith increment taking into account the changes in material and geometrical properties
$(\Delta\mathbf{F}_e)_i^{(r)}$ unbalanced residual nodal forces

Comparison between numerical and experimental results for reinforced concrete and composite elements

To demonstrate the accuracy of the numerical results which can be obtained from the described procedure, a

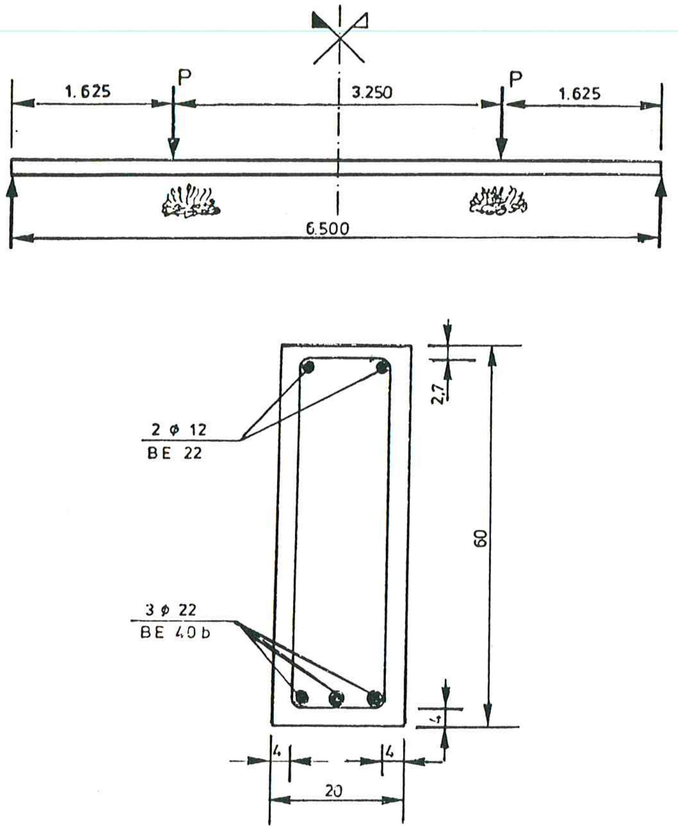


Figure 4. Loading and heating systems of the reinforced concrete beam

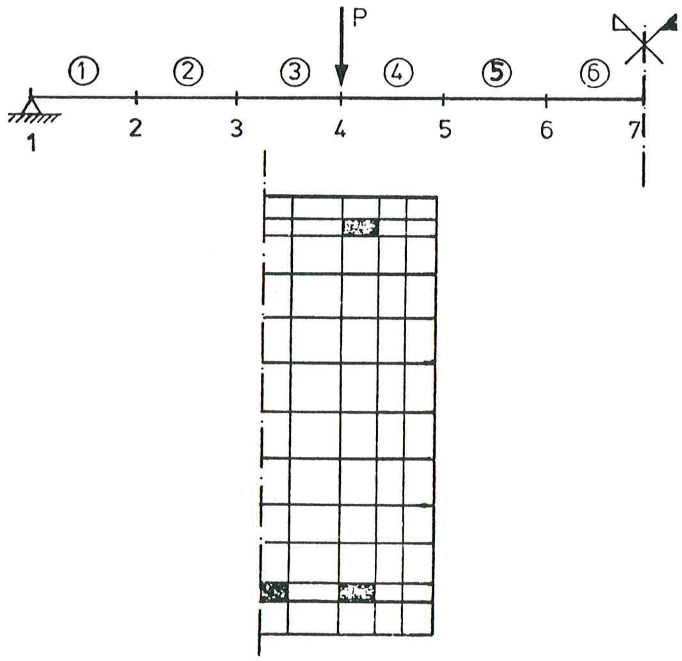


Figure 5. Element and section division

reinforced concrete rectangular beam and a composite T beam have been analysed and the numerical results compared with test results.

Reinforced concrete beam. A reinforced concrete beam of rectangular cross-section had been analysed a few years ago and the numerical results compared with test results obtained at the University of Ghent.[3]

The loading and heating systems are presented in Fig. 4. The beam is loaded and heated symmetrically. The thermal program is applied according to ISO R 834. The dimensions of the cross-section and the reinforcement arrangement are indicated in Fig. 4. Because of the symmetry only one-half of the cross-section has to be considered for the division in subslices and only one-half of the length of the beam is subdivided in six finite elements (Fig. 5).

Figure 6 shows the temperature increase in the central bar. There is a good agreement between theoretical and experimental results.

In a reinforced concrete beam submitted to a fire test there is a steep thermal gradient on the cross-section producing large deflections even at the beginning of the test when the stiffness properties of materials remain unchanged. Figure 7 shows that the numerical procedure can simulate this phenomenon: the agreement is quite good even when the reinforcing bars begin to yield.

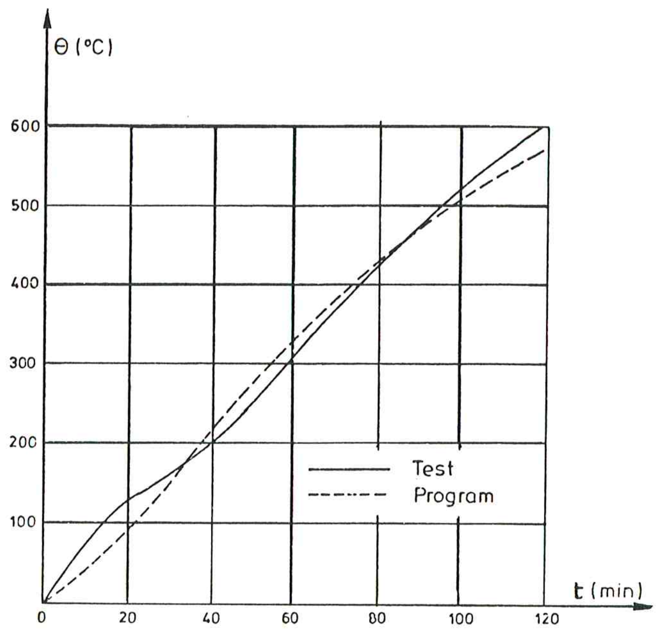


Figure 6. Temperature curve on the central reinforcing bar

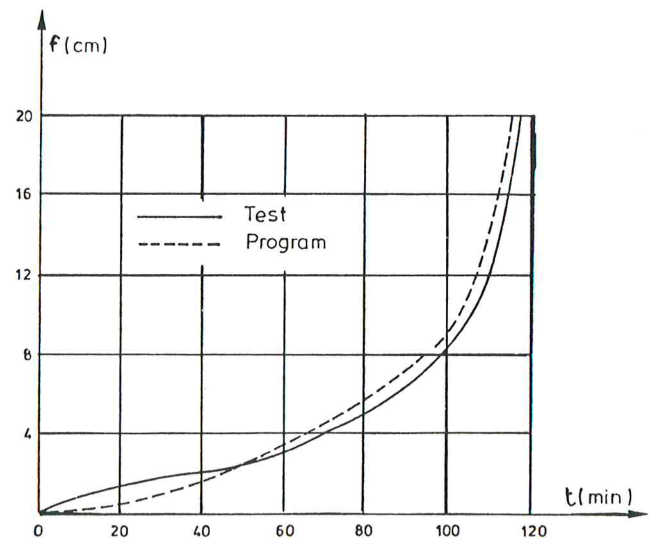


Figure 7. Deflection curve

Composite beam. A composite T beam has been analysed with the improved version of the program and the theoretical results compared with test results obtained at the Technical University of Braunschweig.[6]

The loading and heating systems are presented in Fig. 8. The beam is loaded and heated symmetrically. The thermal program is applied according to ISO R834. The dimensions of the cross-section and the reinforcement arrangement are indicated in Fig. 8. Because of the symmetry only one-half of the cross-section has to be considered for the division in subslices and only one-half of the length of the beam is subdivided in eight finite elements (Fig. 9).

Figure 10 shows the temperature increase on the reinforcing bars. There is a good agreement between theoretical and experimental results, though other comparisons show

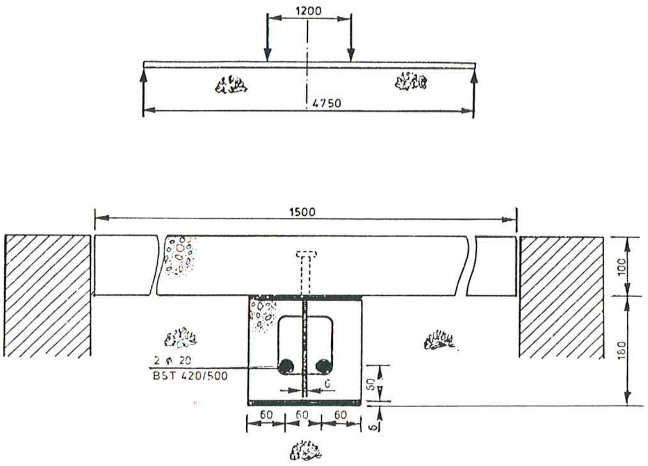


Figure 8. Loading and heating systems of the composite beam

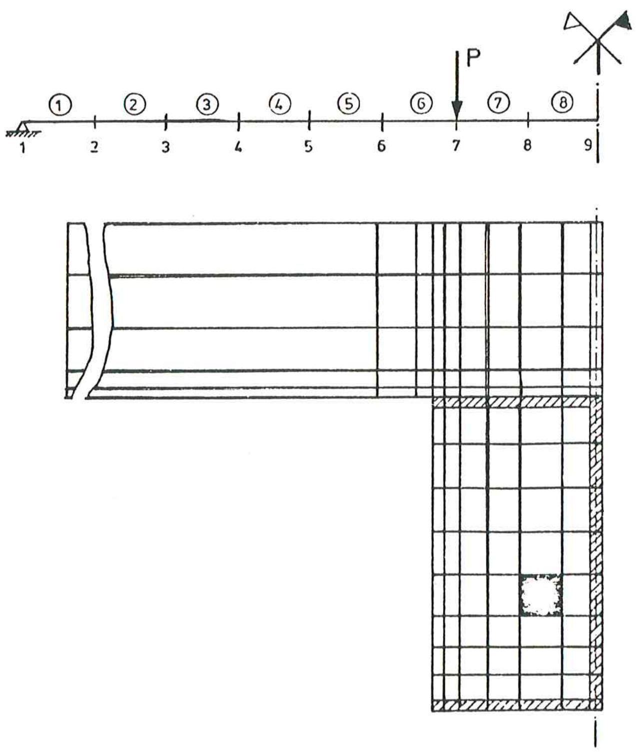


Figure 9. Element and section division

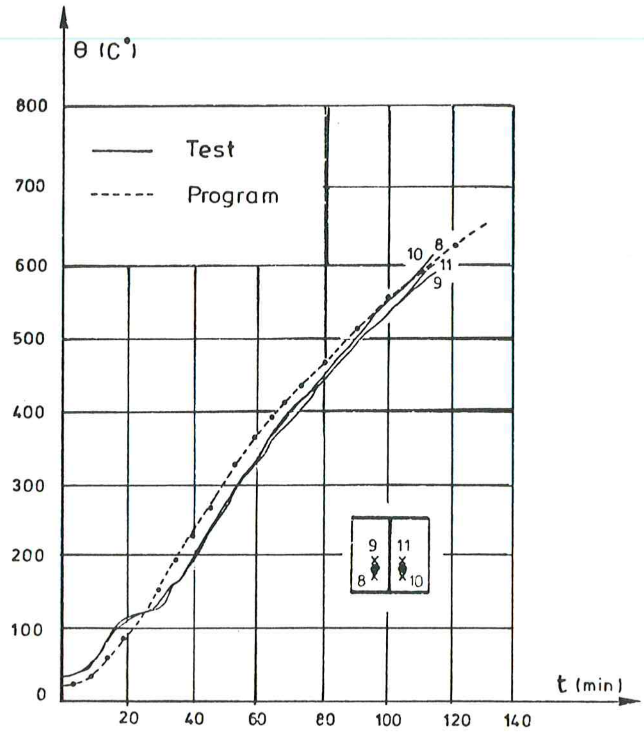


Figure 10. Temperature curves on the reinforcing bars

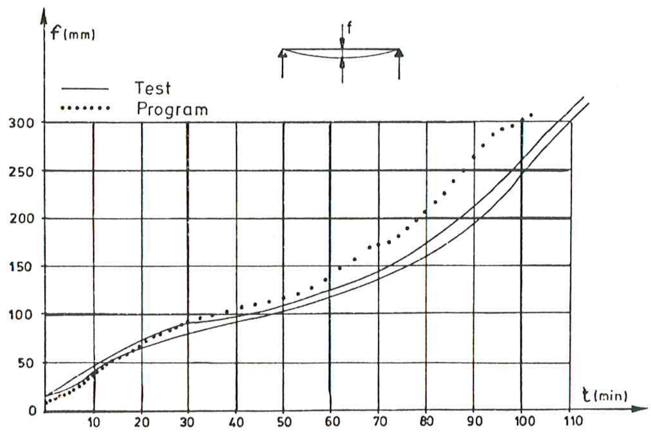


Figure 11. Deflection curve

that the resultant emissivity factor of steel seems to have been chosen a little low.

Figure 11 shows the evolution with temperature of the beam deflection. The agreement between theoretical and experimental results is good. Like a reinforced concrete beam this simply supported composite beam shows a large deflection rate during the first minutes of the test because of the thermal gradient. At the end of the test deflections grow very quickly due to the fact that steel and concrete are losing their stiffness under high temperatures.

3. COMPARISON BETWEEN REINFORCED CONCRETE AND COMPOSITE STRUCTURES REGARDING THE NUMERICAL TREATMENT OF THE PROBLEM

In this paper it is intended to describe not only modifications that have been introduced in the program, but also and mainly the further difficulties arising when going from the analysis of reinforced concrete structures to that of composite structures.

Stability of the numerical scheme for the temperature distribution

To calculate the temperature distribution in the cross-section under non-steady-state conditions the well-known Fourier equation has to be solved:

$$\mathrm{div}(\lambda \ \mathrm{grad}\ \theta) + Q = c\rho \frac{\partial \theta}{\partial t} \qquad (5)$$

where: λ, thermal conductivity; c, specific heat; ρ, specific weight; θ, temperature; t, time; Q, internal heat sources.

In this approach a finite difference method based on the heat balance between adjacent nodes of the discretised domain has been used; in this way the equations can be solved directly.

This formulation has several advantages: the properties of materials can be variable with space and time, the evaporation of moisture in concrete can be taken into account, the boundary conditions can be formulated easily, an irregular size mesh can be used for the discretisation.

The method applied here is called explicit, because the values of θ at a given time are obtained explicitly from the values obtained at the end of the previous time step.[7] This method is advantageous since for each time step it is necessary to solve only n equations to one unknown (n being the number of nodes of the mesh), whereas an implicit method would lead to a system of n equations to n unknowns.

The disadvantage of an explicit method lies in the fact that it is not unconditionally stable and that it leads to a criterion relating the time step to the mesh width.In our case, this criterion can be written as follows for the mesh i:

$$\Delta t < \frac{c_i \rho_i}{4\lambda_i} (M_i)^2 \qquad (6)$$

where M_i is the minimum dimension of mesh i and c_i, ρ_i, λ_i are values of c, ρ, λ in mesh i.

This criterion is much more severe for steel than for concrete, and this is due to the high thermal conductivity of steel:

$\Delta t < 1.74$ s for a steel mesh of 1 cm at 20°C

$\Delta t < 27$ s for a concrete mesh of 1 cm at 20°C

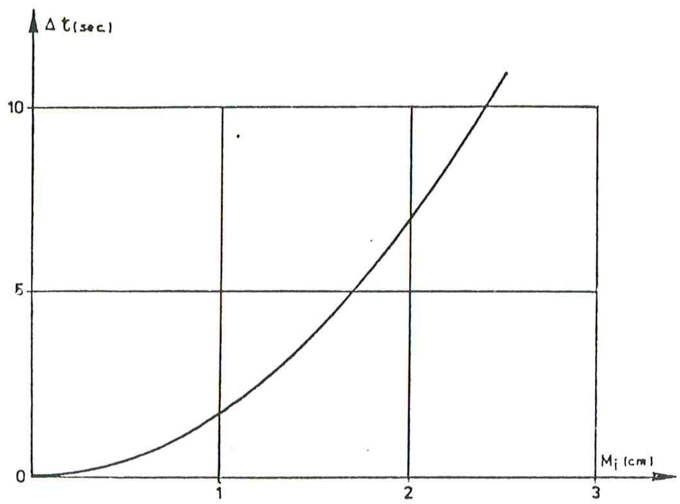


Figure 12. Relation between the maximum time step and the mesh width for steel at 20°C

Table 1. Thermal properties of steel and concrete

ρ	specific weight (kg/m³)
c	specific heat (J/kg K)
λ	thermal conductivity (W/m K)
Steel	$\rho = 7850$
	$c = 470 + 20 \frac{\theta}{100} + 5 \left(\frac{\theta}{100}\right)^2$
	$\lambda = 54 - 3.33 \frac{\theta}{100}$
Concrete	$\rho = 2350$
	$c = 900 + 80 \frac{\theta}{100} - 4 \left(\frac{\theta}{100}\right)^2$
	$\lambda = 2 - 0.24 \frac{\theta}{100} + 0.012 \left(\frac{\theta}{100}\right)^2$

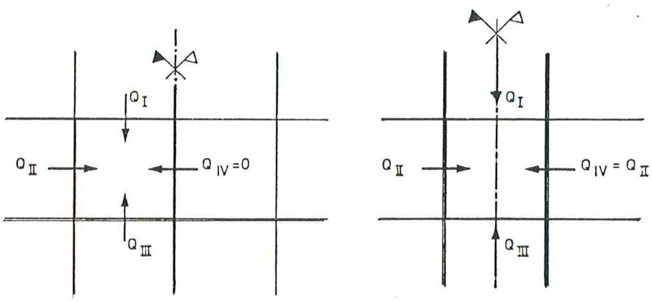


Figure 13. Position of the axis of symmetry in the mesh subdivision

The values and the variation with temperature of the thermal properties of steel and concrete adopted in the program are given in Table 1.

It can be noticed that, for a cross-section discretised in a rectangular mesh, if the size of each mesh is divided by 2, the number of nodes is approximately multiplied by 4 and, at the same time, the time step is divided by 4. Thus, the computation time will approximately be multiplied by 16.

Position of the axis of symmetry in the mesh subdivision

It is possible to analyse only one-half or one-quarter of the cross-section when there exists one or two axes of symmetry (cf. Figs. 3, 5 and 9). Figure 13a explains the very simple way to take symmetry into account in the heat balance. In this situation, and this is frequently the case, the axis of symmetry is put on the boundary between two adjacent meshes and there is no heat flow going through the axis.

In a composite section including a hot-rolled profile (see Fig. 9), the gradient of temperature is not very high through the thickness of the web and of the flanges. Therefore, in the discretisation, one vertical row only can be used for the web and one horizontal row for each flange.

Usually it is necessary to analyse only one-half of the section due to symmetry. If the same scheme as in Fig. 13a is used, the web thickness will be divided by 2 and the time step has to be divided by 4. It can be noticed that, in this case, it is less expensive to consider the whole cross-section than only one-half in the first case. The number of nodes is multiplied by 2, but in the second case the time step is divided by 4; thus the computation is twice less expensive for the whole section.

In order to use only one-half of the cross-section and to avoid the reduction by 4 of the time step, it is desirable to put the axis of symmetry in the middle of a mesh, as shown in Fig. 13b. The formulation of the heat balance is not more complicated than it was for the previous scheme.

Variation of the stability criterion for steel in function of temperature

Though it is possible to consider only one-half or one-quarter of the section, the criterion of stability may lead to particularly short time steps and the calculation of temperatures may yet take an important part of the time of simulation.

However, the computation time can also be reduced by observing that the thermal properties of materials are temperature dependent. In the program the specific heat and the thermal conductivity vary according to the laws given in Table 1.

Figure 15 shows the variation of the temperature dependent term of the stability criterion. It can be noticed that this parameter, and therefore the time step, is multiplied by 2.35 when the temperature increases from 0°C to 550°C. In order to take this into account the time step is evaluated after every calculation of structural effects, i.e. after every time increment (cf. Fig. 14).

In the case of a composite structure, however, the advantage is reduced, due to the fact that the central part

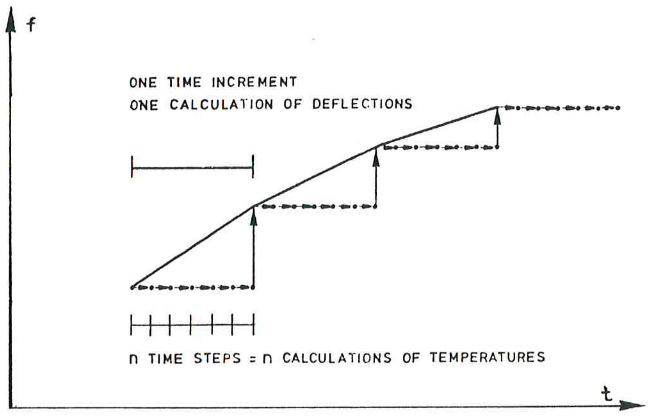


Figure 14. Discretisation of time for the calculation of temperatures and deflections

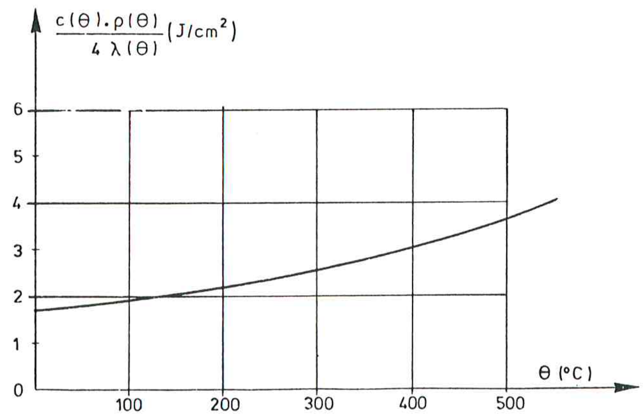


Figure 15. Variation of the stability criterion for steel in function of temperature

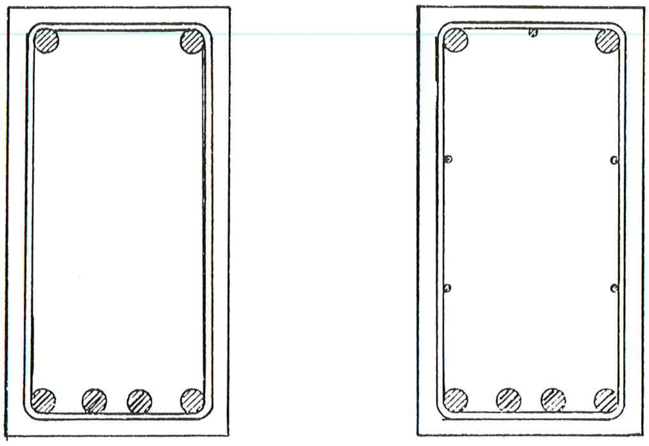


Figure 16. Reinforced concrete rectangular beam

of the web, which is frequently the smallest row, is also the coldest part of the section and its temperature increases slowly.

Discretisation of time for the calculation of temperatures and structural effects

Referring to Fig. 12, it can be seen that, as long as steel is present in the cross-section, the criterion of stability will lead to a time step of the order of magnitude of the second. For the simulation of a fire test it is not necessary to calculate the deflections, the strains, the stresses . . . of the structure every second. By numerical experimentation it has been estimated that the increment of time suitable for the calculation of structural effects in composite structures is of the order of magnitude of the minute.

In the program, the time step related to the stability criterion in every mesh of the structure is calculated automatically. For a certain number of time steps only the temperatures are calculated and this process goes on until the increment of time suitable for the calculation of structural effects is reached, which has been introduced as a data. At that particular moment the structural effects (deflections, strains, stresses) are calculated (Fig. 14).

Discretisation of the cross-section for reinforced concrete and composite sections

In a reinforced concrete section additional steel bars have to be used in order to prevent several disorders in the construction, for example when the section is rather high (Figs. 16a and b). Shear forces are usually carried by vertical stirrups, but it is also possible to use a rectangular mesh of steel bars, in which vertical stirrups and longitudinal additional bars are included.

As the shear energy is not taken into account in the program, the stirrups do not have to be considered in the discretisation. However, they must be able to carry the shear forces, which are determined by equilibrium considerations, and this has to be checked.

Therefore, in a reinforced concrete element, the cross-section usually includes concrete, main reinforcing bars and additional longitudinal bars. The following considerations related to the discretisation of the cross-section have been proved by numerical experimentation.

- Concerning the thermal calculations, i.e. the calculation of the temperature distribution, there is almost no influence of the additional longitudinal bars. Furthermore, the influence of the main reinforcing bars is also rather small, since the total percentage of steel is not very important. Therefore, for usual thermal computations it is acceptable to consider concrete meshes only in the discretisation.

This is very important for the time step related to the stability criterion described at the beginning of Section 3. For the same mesh dimension the time step can be multiplied by 15 when no steel is considered.

- Concerning the calculation of structural effects, of course the main reinforcing bars have to be considered, but the influence of the additional steel bars is small, and it is not necessary to take them into account.

This is due to the fact that their area is small compared to that of the main bars, that they are usually situated not too far from the boundary of the element and that their temperature increases quite rapidly.

In the case of composite beams such as that shown in Fig. 17, rectangular steel meshes are also used near the upper and lower side of the concrete slab, in order to prevent shrinkage and to distribute the point loads. Additional longitudinal bars are thus introduced in the section. Due to the important width of the slab the total section of these additional bars is not negligible. Their influence is particularly significant at the end of a fire test, when the upper part of the slab has remained cold and is still an efficient zone of the section.

This phenomenon has been observed in the example of Fig. 8. First of all the additional bars were not taken into account in the computation, and after 68 min of simulation the equilibrium was no longer possible. After introducing the additional longitudinal bars the calculation could go on for 102 min (cf. Fig. 11).

Figure 18 shows calculated deflection versus time curves for the composite T beam with and without additional longitudinal bars. As can be seen the differences between the two curves are not significant, while the fire endurance varies substantially from one case to another. This indicates that it is a problem of equilibrium rather than a problem of stiffness.

Discretising individually every single longitudinal bar will lead to two different kinds of inconvenience: the number of rectangular meshes will increase highly and the time step will decrease very severely (small steel mesh in a cold zone. see earlier in this Section).

In order to avoid the second disadvantage the additional bars can be considered only in the calculation of structural effects and can be replaced by concrete meshes in the calculation of temperatures. The error in the temperature distribution is very slight and thus the time step related to the stability criterion will not be influenced.

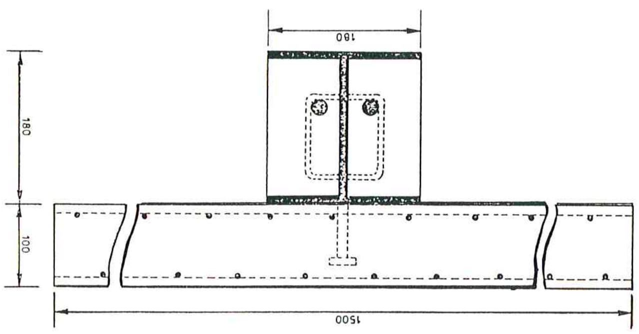


Figure 17. Composite T beam with additional reinforcing steel bars

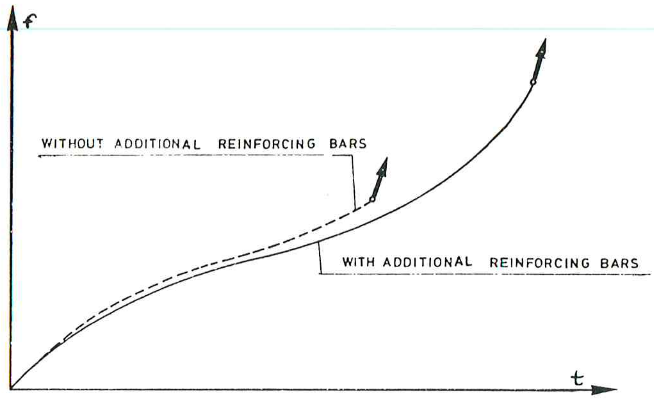


Figure 18. Calculated deflection versus time for a composite T beam

The best way to reduce the number of rectangular meshes could be to represent each layer of additional bars by one or very few concentrated steel meshes, the section of which should be equal to that of all the bars they represent, and placed at the same level as the layer. The stability criterion is then considerably improved, since the equivalent mesh has a larger size. This has almost no influence on the temperature distribution, and the total number of meshes remains approximately the same.

CONCLUSIONS

In this article numerical models for the analysis of reinforced concrete and composite structures under fire conditions have been presented. In both cases the same type of model has been used, i.e. the finite element method with discretisation of the cross-section in a rectangular mesh coupled with an incremental iterative approach with actualisation of the stiffness matrix. The accuracy of the numerical results obtained from the described procedure has been proved by comparing them with test results.

Concerning the numerical treatment of the problem, however, further difficulties arise when going from reinforced concrete to composite structures. They are mainly due to the fact that in composite structures the percentage of steel becomes significant and this leads to a serious increase of the computation time for the thermal analysis. However, this computation time can be reduced if some attention is paid in order to avoid steel meshes of very small size. It must also be pointed out that the analysis of the failure mode is more complicated for composite structures.

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